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A CASE STUDY OF SHEAR WALL-FRAME INTERACTION

by Stephen Chriss*

Abstract

A shear wall-frame system pertaining to an actual building has been investigated for deflection and rotation caused by wind loading. Computational means employed were a desk calculator, a wire model and a computer facility. The analysis method used, consisted of computing stiffness matrices for substitution into an equilibrium equation, which was subsequently solved for system deflections. The contribution of frames in reducing system deflections is significant.

Introduction

Composite shear wall-frame action due to lateral loading is characterized by considerably lower lateral deflections and shear wall rotations than would be obtained if the shear wall was acting alone. Although the principle of the above statement is generally understood and its validity accepted, data in the published literature indicating manifestations of interaction are scarce.

The present study was therefore undertaken in order to determine response characteristics for the system investigated, which possessed rather unusually high beam-to-column and shear wall-to-column stiffness ratios. A simple method of analysis is presented, aiming at determining deflections of the combined shear wall-frame system. Resultant rotations, as well as deflections and rotations for the shear wall acting alone are also presented for comparison. The investigation is of combined analytical-experimental nature and pertains to the Physics Tower of the University of Toronto.

The study was instigated by the necessity of determining whether the rotation of the top of the central core at the 16th floor level under moderate wind load was within the specified limits. Low rotation limits were required due to the operational characteristics of two astronomical telescopes mounted on the top of the core.

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Building Characteristics

The building shown completed in Fig. 1 and in plan in Fig. 2 structurally consists of four supporting perimetric frames, a group of three interior columns and a central core, all interconnected through plane-rigid floor plates.

Perimetric frames and central core terminating at the fifteenth and sixteenth floor levels respectively, are supported at the basement level by a criss-cross wall type foundation, assumed to provide a perfectly flexurally rigid base for the purpose of the present analysis. The central core consists of reinforced concrete sections of 8,12 and 16 in. in thickness, and reinforced as concrete columns at points of high stress such as corners and doorways. Concrete strength changes from 4 to 3 ksi at the 9th floor level.

Perimetric frames consist of 25-in. x 24-in. reinforced concrete columns and 6-ft 8-in. deep composite beams. Column concrete strength changes from 4 to 3 ksi at the 7th floor level. The composite beams consist of 3-ft 3-in. deep, 1-ft wide 3 ksi reinforced concrete spandrel beams supporting the floor plate, and 3-ft 5-in. deep, 8-in. concrete block panel; the latter is bonded to the spandrel beams with cement mortar and topped by cast in concrete 6 x 4 B 8.5 steel beams interconnecting adjacent columns as shown in Fig. 3. Steel beams were introduced as tension-compression members in order to mobilize into flexural action the full depth of the composite beam, thereby increasing the stiffness of perimetric frames. Stiff frames would in turn result in limited rotation at the top of the core as necessitated by the operational characteristics of the telescopes.

Floor plates consist of 15-in. deep stems and 3-in. thick deck concrete waffle type slabs.

The story height between the first basement level (at which frames and core meet the rigid foundation) and the first floor is 17.5 ft., that between first and second is 18.7 ft., and all remaining story heights are 12.9 ft.

Analysis of System

Method of Analysis

The system resisting lateral motion shown in Fig. 4 consists of a planar array of a frame element and a shear wall element, interconnected at floor levels through links. The above elements represent respectively (1) the combination of two opposite physical perimetric frames whose planes are parallel to the direction of wind, and (2) the physical central core.

Frames and central core are assumed fixed at the first basement floor level. The interconnecting links representing the concrete floor plates are assumed hinged at both ends and inextensible, i.e. not altering their length under the influence of interaction forces between the shear wall and frame elements.

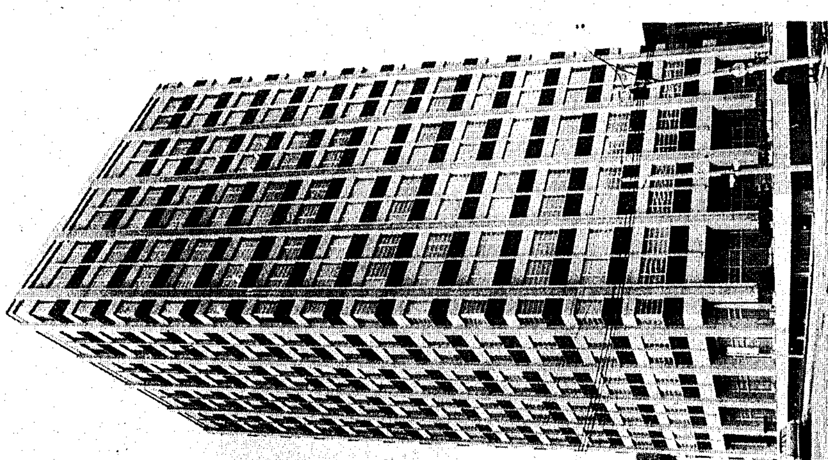

Fig. 1. University of Toronto Physics Tower

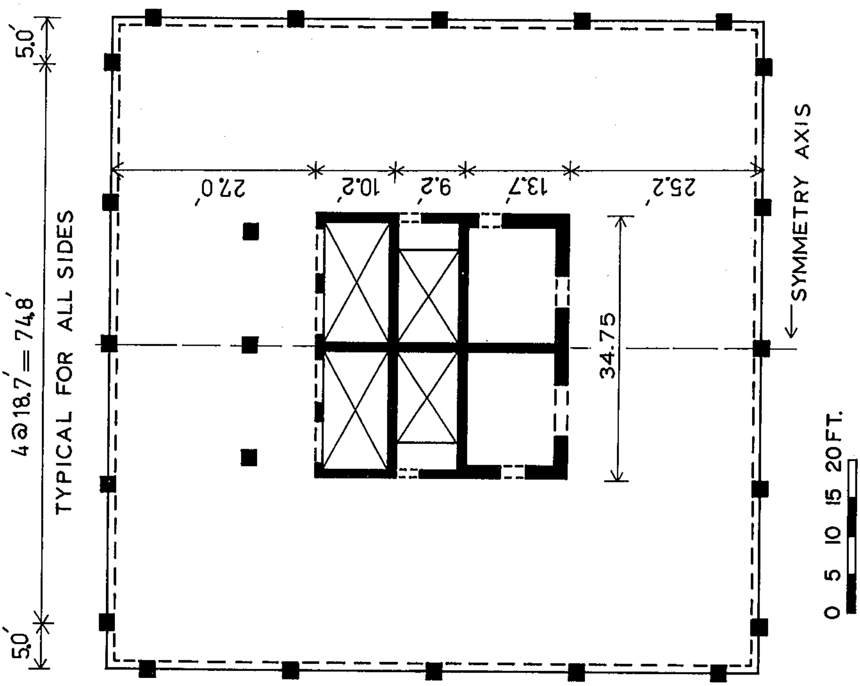

Fig. 2. Floor Plan of Physics Tower

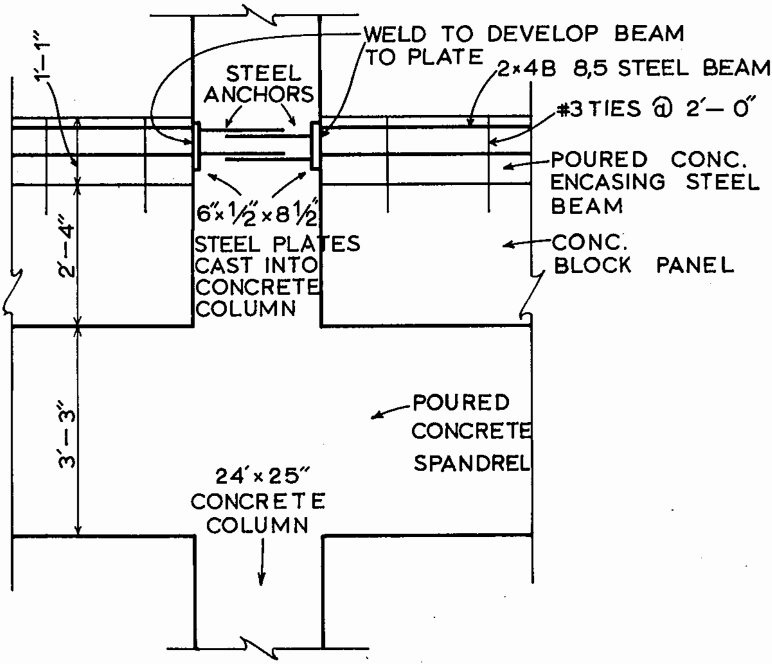

Fig. 3. Typical Column – Composite Beam Intersection

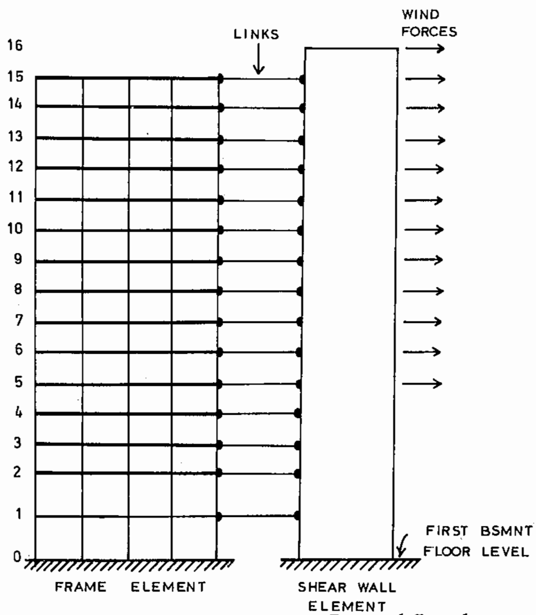

Fig. 4. System Resisting Lateral Loads

System deflections were determined from equilibrium considerations, from the expression:

$$([A^{w}] + [A^{f}])\ \{u\} = \{F\} \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ 1$$

in which $[A^{w}]$ and $[A^{f}]$ are the square stiffness matrices for the shear wall and frame elements respectively, and $\{u\}$ and $\{F\}$ are the column matrices of system horizontal deflections and externally applied forces respectively. The above equation can be obtained by writing the well known stiffness matrix equation[1] for the shear wall and frame elements separately, and adding the two together.

Stiffness coefficients for the central core disregarding the effect of shear were determined as follows. One point at a time was deflected while all other points were locked against translation only; the stiffness coefficient A_{kj} is then equal to the reaction required to keep point k locked against translation, while a load applied at point j is producing a unit displacement at j. Reactions were determined from moment distribution. All arithmetical operations were carried out through a desk calculator rather than a slide rule due to the required accuracy in formulating the stiffness matrix. A constant flexural rigidity EI = $\ell.30\text{x}10^{10}$ kip-ft^{2} was used in the calculations. This figure representing an average of the rigidities in the two building directions, was arrived at by conventional methods after due allowance for openings and doorways.

Stiffness coefficients for the perimetric frames were determined by means of a wire model as described further. Stiffness expressions for the central core (shear wall element) and the two perimetric frames combined (frame element) are given by Eqs. 2 and 3 below which indicate the nature of the coefficients rather than the complete coefficient layout.

An outline of the method of computation of rigidities is given in Appendix I.

$$\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ \cdot \\ \cdot \\ P_{14} \\ P_{15} \\ P_{16} \end{Bmatrix} = \begin{bmatrix} +43.855 & -31.842 & +16.911 & \cdot\ \cdot & 0 & 0 & 0 \\ -31.866 & +53.571 & -52.095 & \cdot\ \cdot & 0 & 0 & 0 \\ +16.921 & -52.100 & +82.156 & \cdot\ \cdot & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot\ \cdot & +84.566 & -56.536 & +14.930 \\ 0 & 0 & 0 & \cdot\ \cdot & -56.537 & +57.697 & -20.786 \\ 0 & 0 & 0 & \cdot\ \cdot & +14.934 & 20.787 & +\ 9.010 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ \cdot \\ \cdot \\ u_{14} \\ u_{15} \\ u_{16} \end{Bmatrix} \ldots 2$$

$$
\begin{Bmatrix} P_1 \\ P_2 \\ P_3 \\ \cdot \\ \cdot \\ P_{13} \\ P_{14} \\ P_{15} \end{Bmatrix} = \begin{bmatrix} +0.082 & -0.40 & 0 & \cdot\ \cdot & 0 & 0 & 0 \\ -0.040 & +0.190 & -0.150 & \cdot\ \cdot & 0 & 0 & 0 \\ 0 & -0.150 & +0.300 & \cdot\ \cdot & 0 & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & 0 & \cdot\ \cdot & +0.300 & -0.150 & 0 \\ 0 & 0 & 0 & \cdot\ \cdot & -0.150 & +0.300 & -0.150 \\ 0 & 0 & 0 & \cdot\ \cdot & 0 & -0.150 & +0.150 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ \cdot \\ \cdot \\ u_{13} \\ u_{14} \\ u_{15} \end{Bmatrix} \quad .3
$$

In the above expressions, forces P are in kips and deflections u are in millionths of a foot.

It is noted from Eq. 3 that the frame element corresponds to a discrete spring type system[2], since the influence of deflecting a floor level is only spread to the floors above and below. Such an action is attributable to the fact that the ratio of composite beam to column stiffness is of the order of 10.

The lateral forces acting on the building were assumed to be those due to a 25 mph wind producing a pressure increasing with the floor level according to the recommendations of the National Building Code of Canada. The wind load was considered negligible up to the 50-ft. level (4th floor) due to the shielding effect of neighboring buildings.

Wire Model

Due to the nature of the composite beams, the columns were considered free to deform flexurally in their height corresponding to the depth of the block panel; in addition, the steel beam, its concrete encasement, and its tributary width of concrete block panel were considered to provide an elastic restraint to column lateral deflections. The above considerations, outlined in Appendix I, correspond to an idealized frame section as shown in Fig. 5. Resort, therefore, was made to a wire mechanical model for the determination of stiffness coefficients of the perimetric frames.

A typical wire model section representing the physical prototype section (Fig. 5) is shown in Fig. 6. Flexible arms A correspond to the springs of Fig. 5. Arms A offer elastic restraint to wires C through cantilever action, and were made effective through post-tensioning of a fine steel wire after soldering had been completed. Wire bands B and C correspond to the flexural rigidities of the prototype composite beam and column respectively and the gusset plate G corresponds to the column-spandrel beam poured joint.

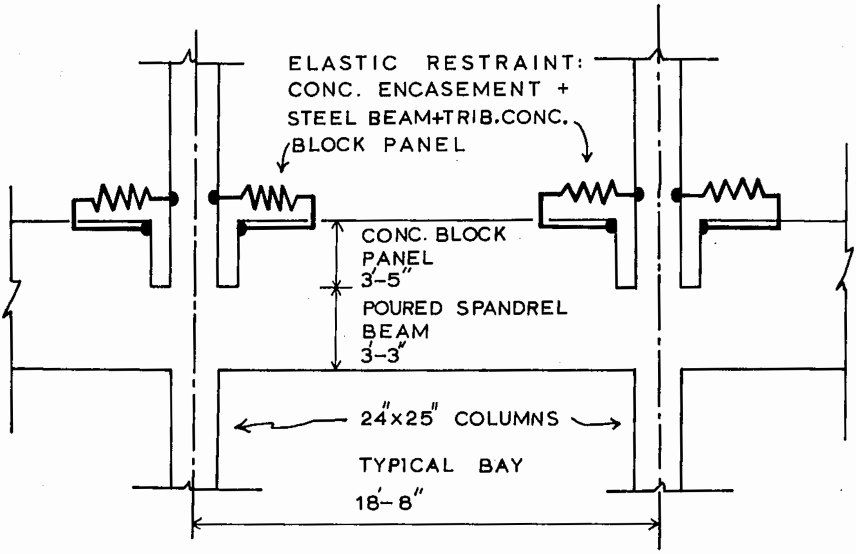

Fig. 5. Idealized Typical Prototype Frame Section

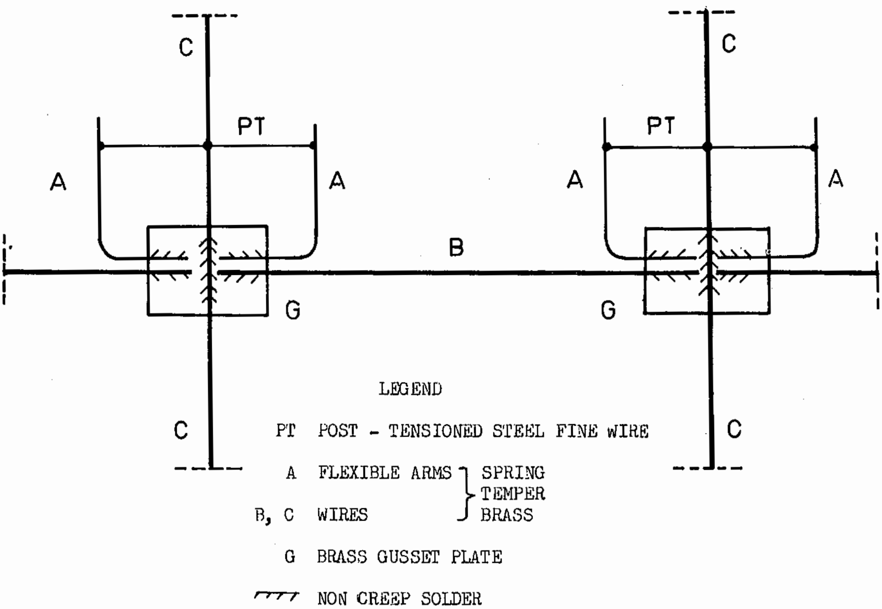

Fig. 6. Wire Model – Typical Section Corresponding to Fig. 5

Model characteristics were prescribed by the scales of length $1/K_\ell$, stiffness $1/K_k$, and spring constant $1/K_s$. The first two scales were independently chosen, while $1/K_s$ is equal to K_ℓ^3/K_k as follows from similitude theory[3,4]; $1/K_s$ is relating the elastic column restraint of the prototype (springs of Fig. 5) to the corresponding element of the model (flexible arms of Fig. 6.).

Numerical values of the scales were chosen on considerations of convenience of construction and loading, practicability of model size, availability of model members and readability of deflections; the values chosen were:

$$\frac{1}{K_\ell} = \frac{L_m}{L_p} = 0.025 \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad 4$$

$$\frac{1}{K_k} = \frac{(EI)_m}{(EI)_p} = 0.555\text{x}10^{-10} \quad \ldots\ldots\ldots\ldots\ldots \quad 5$$

$$\frac{1}{K_s} = \frac{C_m}{C_p} = \frac{K_\ell^3}{K_k} = 3.55\text{x}10^{-6} \quad \ldots\ldots\ldots\ldots \quad 6$$

in which L, (EI) and C represent length, flexural rigidity and spring constant respectively, and the subscripts m and p apply to model and prototype respectively.

The model was constructed of spring temper wires jointed by means of non-creep solder. Since the frames were assumed to act essentially as discrete spring type systems, only five stories were incorporated into the model. This assumption was verified during experimentation by the fact that the stiffness coefficients for levels two stories away were 4 percent or less of the coefficients one story away from the loaded level.

Experimental arrangement and schematic representation are shown in Fig. 7 and 8 respectively. A simple electrical circuit was utilized to indicate contact of the model with the pins P, while the model was under the influence of weights placed on the trays.

The following technique was utilized. With the switches RS (regular) and MS (momentary) closed, enough weights were placed on tray T2 to just break the circuit between the pin and the model (light of bulb B off). Weights were then placed on tray T1 creating a deflection Δ, reestablishing the contact between pin and model. Counter-balancing weights W were then placed on tray T2 to just break again the contact. The stiffness coefficient A_{43} for model would then be equal to W/Δ. Two deflection operations were performed to test

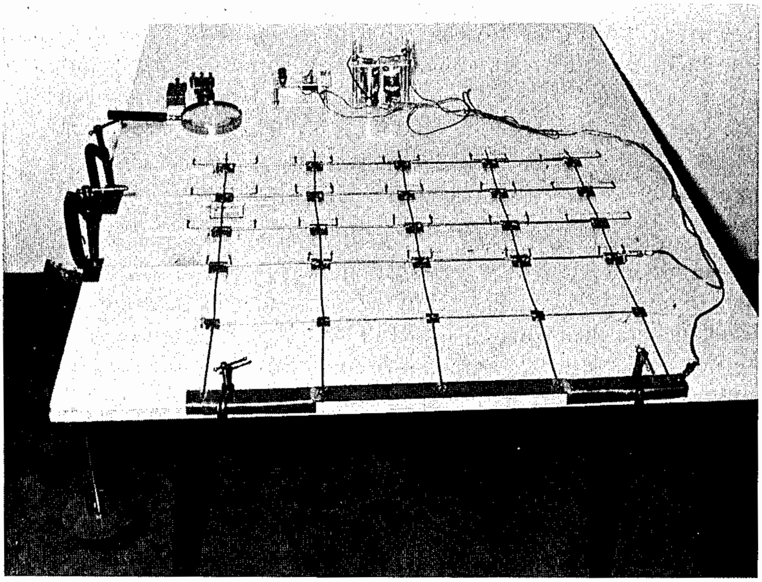

Fig. 7. Wire Model – Experimental Arrangement

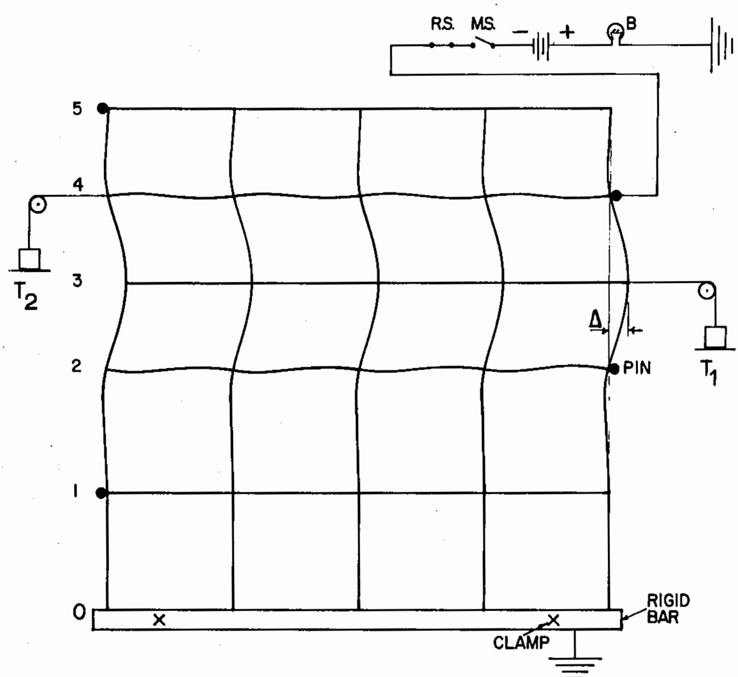

Fig. 8. Wire Model – Schematic Representation

the linearity of the W-Δ relationship, and the average W/Δ value was taken. The difference between the values obtained for each deflection operation was in most cases between zero and 7%. Deflections were measured on a scale graduated in hundredths of an inch and read through a magnifying glass.

System Solution

The expressions given by Eqs. 2 and 3 and the force vector given by the second column of Table 1 were combined into the form of Eq. 1 for the solution of which a digital computer facility was employed. Determined values of deflections u were then back-substituted into Eq. 3 to yield forces acting on the frame element. These forces were in turn subtracted from the externally applied loads, to yield the forces acting on the shear wall element. Force distribution and system deflections are shown in Table 1.

To check the correctness of the solution, the deflection of the shear wall element under the calculated set of forces was determined by cantilever deflection formulas and compared to the computer solution set of deflections: a very close agreement was obtained.

Table 1. System Forces and Deflections

FLOOR LEVEL	FORCES (KIPS)			SYSTEM HORIZONTAL DEFLECTIONS u (FEET)
	EXTERNALLY APPLIED FORCES F	ON SHEAR WALL ELEMENT FORCES P	ON FRAME ELEMENT FORCES P	
16	4.14	4.14	—	0.001948
15	4.54	−13.44	17.98	0.001824
14	4.36	4.12	0.24	0.001704
13	4.36	3.98	0.38	0.001583
12	4.36	3.85	0.51	0.001459
11	4.36	3.73	0.63	0.001307
10	4.36	3.74	0.62	0.001200
9	4.36	3.87	0.49	0.001063
8	4.36	4.01	0.35	0.000925
7	3.55	3.52	0.03	0.000783
6	3.47	3.95	−0.48	0.000642
5	3.47	4.46	−0.99	0.000503
4	0	1.63	−1.63	0.000373
3	0	2.35	−2.35	0.000253
2	0	11.29	−11.29	0.000148
1	0	2.78	−2.78	0.000038

Rotations at the top of the central core were obtained from the difference of horizontal deflections divided by the story height. For comparison, deflection, and rotation at the top of the central core if it were acting alone, were also computed from cantilever deflection formulas. Both sets of response characteristics are shown in Table 2.

Table 2. Comparison of Response Characteristics

SYSTEM	HORIZONTAL DEFLECTION AT TOP OF CORE (FEET)	ROTATION AT TOP OF CORE (RADIANS)
FRAME PLUS SHEAR WALL ELEMENTS	0.00195	0.92×10^{-5}
SHEAR WALL ELEMEMENT ONLY	0.00680	4.5×10^{-5}

Summary and Conclusions

A combined analytical-experimental approach to the problem of determining deflections of a shear wall-frame system has been outlined. Computational means employed were a desk calculator, a wire model and a computer facility for the determination of stiffness coefficients for the shear wall element, the frame element and system solution respectively. The above computational means could be interchanged or interreplaced.

Although the stiffness coefficients for the frame element are more than two orders of magnitude smaller than those for the shear wall element, the contribution of the former element has a marked effect on decreasing system response characteristics (Table 2).

References

1. Argyris, J.H. and Kelsey S., *"Energy Theorems and Structural Analysis"*, Butterworths, London, Toronto, 1960, p. 22.

2. Could, P.L., "Interaction of Shear Wall-Frame Systems in Multistory Buildings", *Journal of the American Concrete Institute*, Vol. 62, Title 62-4, January 1965, pp. 45-70.

3. Hetenyi, M. *"Handbook of Experimental Stress Analysis"*, John Wiley & Sons, New York, 1950, p. 693.

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Appendix I

Outline of the Method of Computation of Rigidities

1. Central Core (Shear Wall Element)

Calculation of the moments of inertia of the central core of the building, was based on the following assumptions:

1. Plane sections before bending remain plane after bending, i.e. warping is absent.
2. Material over doorways does not contribute to stiffness.

Accordingly, the centroidal axis XX, was located by means of conventional methods for the cross section shown in Fig. A1. Axis YY is an axis of symmetry.

Subsequently the moments of inertia were evaluated for the cross section. Due to the first of the above mentioned assumptions, the calculated moment of inertia is overestimated, since during bending about the YY axis, in reality, sections such as ABC, DEF, G, H (Fig. A1) will not be fully effective due to warpage (Fig. A2). The same is true for bending about the YY axis.

Due to the second of the above mentioned assumptions, the calculated moments of inertia are underestimated, since in reality, the material existing over doorways will contribute to stiffness.

The two above mentioned opposite effects were considered to cancel each other at least partially, thereby resulting in a moment of inertia approaching the true condition.

The moments of inertia for the concrete alone were found as:

$$I_{XX} = 2.200 \times 10^4 \text{ ft}^4$$

$$I_{YY} = 2.346 \times 10^4 \text{ ft}^4$$

Taking into account the effect of vertical reinforcement by consideration of transformed steel areas:

0 to 9th floor

$$I_{XX} = (2.200+0.200) \times 10^4 = 2.400 \times 10^4 \text{ ft}^4$$

$$I_{YY} = (2.346+0.200) \times 10^4 = 2.546 \times 10^4 \text{ ft}^4$$

9th to 15th floor

$$I_{XX} = (2.200+0.140) \times 10^4 = 2.340 \times 10^4 \text{ ft}^4$$

$$I_{YY} = (2.346+0.140) \times 10^4 = 2.486 \times 10^4 \text{ ft}^4$$

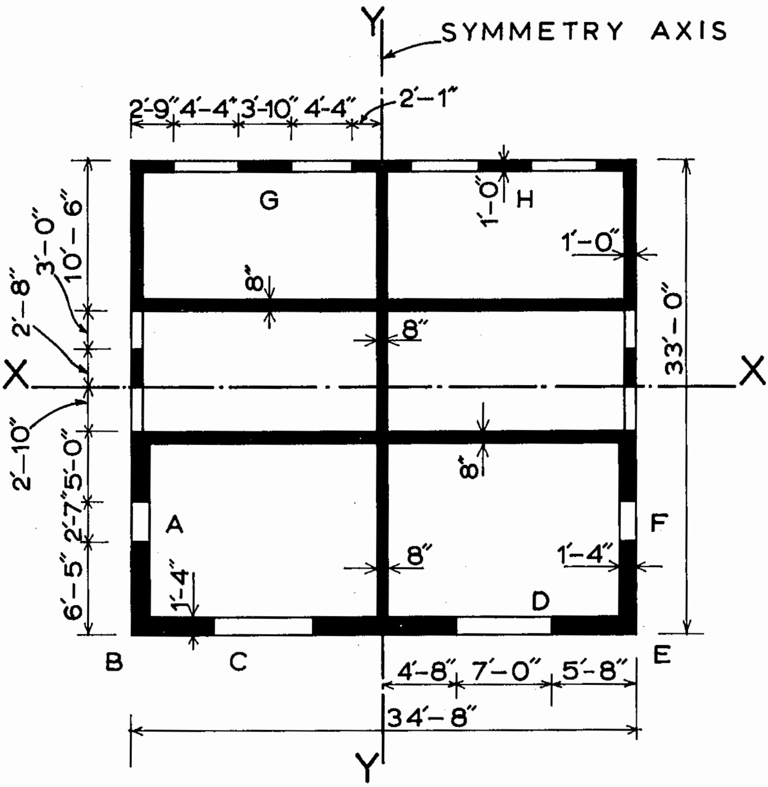


Fig. A1. Central Core Plan Used in Evaluation of Moments of Inertia

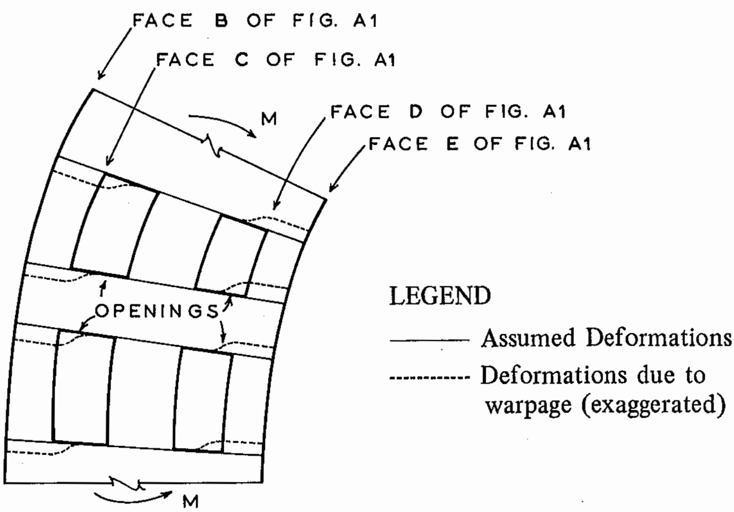


Fig. A2. Central Core Flexural Deformations

The modulus of elasticity for concrete was taken as:

0 to 9th floor (f'_c = 4 ksi) : $E = 5.3 \times 10^5$ kip/ft^2

9th to 15th floor (f'_c = 3 ksi) : $E = 4.8 \times 10^5$ kip/ft^2

Thus flexural rigidities become:

0 to 9th floor: $(EI)_{XX} = 1.24 \times 10^{10}$ kip-ft^2, $(EI)_{YY} = 1.35 \times 10^{10}$ kip-ft^2

9th to 16th floor: $(EI)_{XX} = 1.12 \times 10^{10}$ kip-ft^2, $(EI)_{YY} = 1.20 \times 10^{10}$ kip-ft^2

Although the arithmetic average of the above figures is 1.23×10^{10}, the value used in assembling the stiffness matrix was: $EI = 1.30 \times 10^{10}$ kip-ft^2 since the bottom (stiffer) part of the cantilever, has a stronger influence on deflections than the top.

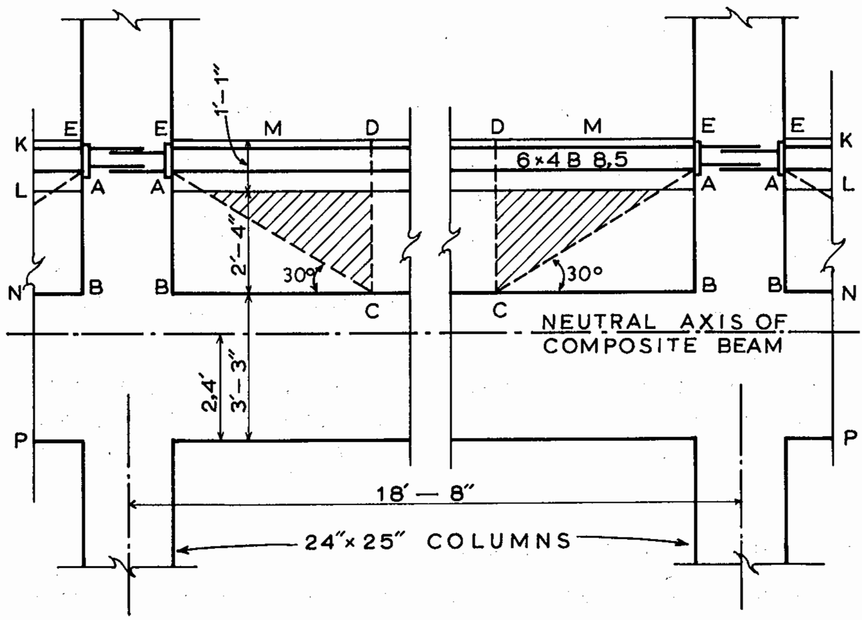
2. Perimetric Frames (Frame Element)

Calculation of rigidities of perimetric frames was based on the following considerations and assumptions:

1. Steel beam, encasing concrete, block panel and poured concrete, spandrel, Fig. A3, act integrally as a composite beam.
2. Concrete column-concrete block interfaces AB (Fig. A3) cannot develop either tensile or compressive forces. Therefore, columns can deform flexurally in the lengths AB, the latter being physical gaps, as shown in Fig. 5.
3. Blocks AEDCA (Fig. A3) consisting of the steel beam, its concrete encasement and its tributary width of block panel act as elastic restraint for the column, when it (column) is laterally deflected. The extensional rigidity EA of the block divided by its length, i.e., its spring constant is that of a strut having length ED and cross-sectional area and modulus of elasticity, those corresponding to the middle position M.

For the evaluation of the flexural ridigity of the composite beam, the neutral axis was first located by considering transformed areas. The moduli of elasticity of poured concrete, concrete block and steel beam were taken in the ratios 1:0.5:10 respectively. Subsequently, the moment of inertia was calculated as $I = 19.3$ ft^4; the corresponding flexural rigidity was then: $EI = 19.3 \times 4.7 \times 10^5 = 9.1 \times 10^6$ kip-ft^2. The spring constant Cp of blocks AEDCA was calculated as:

$$Cp = \frac{\Sigma EA}{L} = \frac{I}{L} (E_1 A_1 + E_2 A_2 + E_3 A_3)$$



LEGEND

KP Composite Beam Consisting of:

1. Steel Beam
2. KL Concrete Encasement 1' - 1" Deep, 8" Wide
3. LN Concrete Block Panel 2' - 4" Deep, 8" Wide
4. NP Poured Concrete Spandrel 3' - 3" Deep, 12" Wide

ADECA – Elastic Restraint to column lateral deflection, consisting of concrete encasement, steel beam, tributary block panel shown shaded.

Fig. A3. Resisting Elements of Frame

where E_1A_1, E_2A_2, E_3A_3 refer to concrete encasement, steel beam, and concrete block respectively. Since $L = 4.8$ ft, $E_1 = 4.7 \times 10^5$ kip/ft^2, $E_2 = 10E_1$, $E_3 = 0.5E_1$, $A_1 = 0.72$ ft^2, $A_2 = 0.017$ ft^2, $A_3 = 0.60$ ft^2, the value of Cp becomes: $Cp = 1.16 \times 10^5$ kip/ft.

The flexural rigidities for the columns were calculated as follows:

0 to 7th floor

For the concrete $I_c = \dfrac{2^4}{12} = 1.33$ ft^4; for the circular reinforcement $I_s = 0.17$ ft^4; total moment of inertia $I_t = 1.33 + 0.17 = 1.50$ ft^4.

For the concrete, $E = 5.2 \times 10^5$ kip/ft^2. Thus $EI = 1.50 \times 5.2 \times 10^5 = 8.06 \times 10^5$ kip-ft^2 per column.

7th to 15th floor

For the concrete $I_c = \dfrac{2^4}{12} = 1.33$ ft^4; for the tied reinforcement $I_s = 0.33$ ft^4; total moment of inertia $I_t = 1.33 + 0.33 = 1.66$ ft^4.

For the concrete $E = 4.7 \times 10^5$ kip/ft^2. Thus $EI = 4.7 \times 1.66 \times 10^5 = 7.8 \times 10^5$ kip-ft^2 per column.

An average $EI = 7.9 \times 10^5$ kip-ft^2 per column was assumed throughout the height of the building.

Appendix II – Notation

A	=	stiffness coefficient;
$[A]$	=	square stiffness matrix;
C	=	spring constant = $\frac{f}{\delta}$;
EI	=	flexural rigidity;
F	=	external forces;
$\{F\}$	=	column matrix of external forces;
K_k	=	scale of flexural rigidities = $\frac{(EI)_p}{(EI)_m}$;
K_ℓ	=	scale of lengths = $\frac{L_p}{L_m}$;
K_s	=	scale of spring constants = $\frac{C_p}{C_m}$;
L	=	length;
P	=	internal forces;
u	=	horizontal deflection;
$\{u\}$	=	column matrix of horizontal deflections;
W	=	weights loading model;

Greek Symbols

δ	=	axial deformation
Δ	=	model deflection

Superscripts

w	denotes shear wall
f	denotes frame

Subscripts

p	denotes prototype
m	denotes model

Key Words

Analysis, displacements, experimental, frame, investigation, shear wall, stiffness coefficients, system, wind load, wire model.