JOURNAL OF THE
BOSTON SOCIETY OF CIVIL ENGINEERS SECTION
ASCE

Volume 61 JULY 1974 Number 3

HEAT DISPOSAL IN THE WATER ENVIRONMENT

by
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(The ninth John R. Freeman Lecture, presented before the Boston Society of Civil Engineers on April 3, 1974.)

Preface

The author was honored to give the 9th annual John R. Freeman Memorial Lecture before a joint meeting of the Boston Society of Civil Engineers and the Massachusetts Section of the American Society of Civil Engineering in Boston on April 3, 1974. The privilege of a personal remark is requested in view of the unexpected death, only two days later, of the author's friend, teacher and colleague, Professor Arthur T. Ippen.

Professor Ippen attended the lecture and had himself delivered the 6th John R. Freeman Memorial Lecture in 1971. It is entirely fitting that this paper be prefaced by a brief tribute to a man who epitomized the ambition of John R. Freeman to improve the profession of hydraulic engineering. Both men had immeasurable influence on the profession by their encouragement and support of young engineers. In the author's case this happy association as a student and colleague of Arthur T. Ippen covered a period of almost thirty years.

A tribute to one's teacher also requires an acknowledgment of those who represent the future of our profession. Learning is a continuing process, and a teacher should learn as much from his students as they in turn learn from him. This paper is a reflection not only of past influences but of the many things the author has learned from his students. In particular, the contributions of Keith D. Stolzenbach, Patrick J. Ryan and Gerhard H. Jirka are gratefully acknowledged.

Abstract:

The need for continuing development of techniques for predicting temperature distributions due to waste heat discharges into lakes, rivers, estuaries and the oceans is discussed. Emphasis is on the interactive role of basic laboratory experiments, analytical and numerical techniques and field

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observations. Diffusion of buoyant jets is discussed, including heated surface jets and multiple jets issuing from a submerged multiport diffuser. In the near-field analysis of surface jets the important problems are related to the lateral spreading caused by buoyancy. Comparison of theoretical predictions with laboratory and field observations are given. The mechanics of multiport diffusers for heated discharges in shallow receiving waters are discussed in contrast to sewage diffusers. The important problem is the degree to which stratification can be maintained in order to minimize local reintrainment and reduction of dilution capacity. Criteria for stable and unstable flow regimes are provided. A mathematical model for temperature distribution, with or without waste heat addition, in unsteady flows under time-varying meteorological conditions is given.

Introduction

Heat is by far the largest waste product associated with the generation of electricity from a thermal energy source. Modern fossil fuel stations have an efficiency of about 40 percent and the present generation of commercial nuclear stations about 32 percent. Thus in a nuclear power station, for every kilowatt of electrical power produced, the equivalent of two kilowatts is rejected to a heat sink (the surrounding air or water) through the steam condenser system.

Accounting for differences in plant and stack losses between fossil and nuclear units, the following heat rejection rates to condenser cooling water may be used to quantify the heat disposal problem for a unit producing 1000 megawatts (MW_e) of electrical power:

Fossil:	4.2×10^9 BTU/hr (4.4×10^{12} joule/hr)
Nuclear:	6.6×10^9 BTU/hr (7.0×10^{12} joule/hr)

Thus a nuclear unit rejects approximately 1½ times as much waste heat as a corresponding fossil unit. A typical condenser water flow rate for a 1000 MW unit is about 1500 cfs (42.5 m^3/s) and the temperature increase across the condenser is 20°F (11°C) for a nuclear unit.

If a continual supply of new water is available at the condenser water intake from an adjacent body of water, the process is called *once-through* cooling. If the cooling water is recirculated and the heat transferred directly to the atmosphere by an auxiliary mechanism, such as evaporative cooling, the process is called *closed-cycle* cooling.

Under the 1972 amendments to the Federal Water Pollution Control Act, the EPA is directed to establish guidelines for effluent limitations including waste heat, identifying the best *practicable* control technology available. These requirements are to be met by 1977. In addition, EPA must identify the best *available* technology to be met by 1983. Recent EPA guidelines imply that only closed-cycle, evaporative cooling processes meet

the above conditions. However, with respect to thermal discharges, the 1972 Amendments state that if it can be demonstrated that an EPA limitation is more stringent than that necessary to protect the propagation of fish and wildlife, then EPA may permit less stringent control on a case by case basis. The additional economic costs of closed-cycle cooling are usually significant with respect to once-through cooling. Therefore it is important to continue the development of techniques for predicting temperature distributions and the environmental costs of adding heat to natural bodies of water, particularly for power plant sites in the coastal zone and along the Great Lakes.

The objective of this paper is to illustrate some recent developments in the area of temperature prediction in the water environment. These are drawn from recent experience in the Ralph M. Parsons Laboratory at M.I.T. The paper is not intended to be a complete state-of-the-art review, the objective is to emphasize the interactive role of basic laboratory experiments, analytical and numerical techniques and field observations.

The problem of temperature prediction for waste heat disposal is distinguished by the simultaneous occurrence of two factors which add a degree of complexity beyond that for other predictive models for effluent dispersion and water quality in lakes, rivers or coastal waters or air quality in the atmosphere. These two factors are a) the buoyancy of the discharge, and b) the considerable volume and momentum of the effluent. The buoyancy of the discharge which is associated with the temperature change as the cooling water passes through the power plant condenser requires the simultaneous determination of fluid motion (velocity distribution) and heat distribution within the receiving water body. The volume and momentum of the discharge will affect the ambient flow field. Hence, it is not possible to consider the heated discharge as a passive tracer introduced into the ambient flow. This is particularly the case in a zone close to the discharge area (near-field zone) where advection and free turbulence created by the shearing action of the discharge with respect to the ambient water causes jet diffusion of the heated water. Outside this immediate near-field exists a considerably larger far-field zone in which the heat is distributed by buoyancy driven currents and through diffusion and advection by ambient currents.

Predictive models for hydrothermal analysis can be broadly classified into two groups: complete models and zone models. In the *complete models* the governing equations are solved in their more general form over the whole region of interest. These models promise a significant advance through the use of high speed computers with large memories. Yet several problems must be recognized: 1) The state of the art requires many simplifying assumptions regarding turbulent fluid flow and heat fluxes. 2) The flow and temperature field induced by a thermal discharge exhibits distinctly different hydrodynamic zones. Consequently, the simplifying

assumptions utilized in the formulation of the complete model are not uniformly valid throughout the region of interest. This may cause considerable error and thus restrict the utility of such computer models. 3) Boundary conditions at the edge of the solution domain, notably open fluid boundaries, are difficult to specify.

In the *zone models* the whole region of interest is divided into several zones with distinct hydrodynamic properties (such as near-field and far-field). For each zone it is then possible to simplify the governing equations by dropping unimportant terms (through a formal scaling process). This gives a considerable advantage in the mathematical treatment and improved accuracy in the solution. Despite this advantage, problems remain since some of the assumptions which yield simplified governing equations may not be appropriate in the actual application. Furthermore, there may be a lack of criteria on how to establish a correct division of the whole region into zones.

In this paper, models for the diffusion of buoyant jets are discussed as one class of zone models which are of particular importance in the prediction of the near-field behavior of heated discharges. The simplifying assumptions pertinent to jet diffusion are discussed and the restrictiveness of these assumptions in the development of actual buoyant discharge models is analyzed. This is done for two types of models: buoyant surface discharges and buoyant submerged discharges. In the final section, a far-field model for temperature distribution in unsteady flows under time-varying meteorological conditions is discussed.

Diffusion of Buoyant Jets

The dominant transport processes in jets are the convection by the mean velocities along the trajectory of the jet and the lateral turbulent diffusion normal to the jet trajectory through the irregular eddy motion within the jet. The convective mechanism is due to the initial discharge momentum and/or the vertical acceleration in the case of submerged buoyant jets.

In general, the governing equations of the boundary layer type are formulated in a local coordinate system following the trajectory of the jet. Exact similarity solutions to these equations can be found if semi-empirical mixing length assumptions are made (Schlichting [15]). For engineering purposes, however, it is more practical to specify similarity profiles *a priori*. By integrating across the jet, the governing partial differential equations are then easily reduced to ordinary ones with the axial distance as the independent variable. This integral technique (method of moments) has been found useful and sufficiently accurate in many applications. Examples include buoyant jets in deep (unconfined) receiving water, either non-stratified or stratified. A further advantage of the integral technique is the possibility of considering a flow of the receiving water by defining a gross

force acting on the jet. As in all problems of turbulent flow, empirical coefficients appear in the analysis and have to be determined from experiments.

Buoyant Surface Jets — Near-Field Analysis

Three-dimensional predictive models of buoyant surface jets which take account of the underlying transport phenomena have been proposed by Stolzenbach and Harleman (17, 18), Prych (12) and Stefan and Vaidyaraman (16). The theoretical premises on which these models are built have been examined in detail by Jirka and Harleman (6).

Figure 1 defines the problem under consideration: Discharge parallel to the free surface of the receiving water which is deep and quiescent. In order to illustrate the various assumptions, only the steady state horizontal momentum equations in the x, y directions are considered in the following:

$$\frac{\partial u^2}{\partial x} + \frac{\partial uv}{\partial y} + \frac{\partial uw}{\partial z} = \frac{g}{\rho_a} \int_z^{-\infty} \frac{\partial \Delta\rho}{\partial x} dz - \frac{1}{\rho_a} \frac{\partial p_d}{\partial x} - \frac{\partial u'^2}{\partial x} - \frac{\partial u'v'}{\partial y} - \frac{\partial u'w'}{\partial z} \quad (1)$$

$$\frac{\partial uv}{\partial x} + \frac{\partial v^2}{\partial y} = \frac{\partial vw}{\partial z} = \frac{g}{\rho_a} \int_z^{-\infty} \frac{\partial \Delta\rho}{\partial y} dz - \frac{1}{\rho_a} \frac{\partial p_d}{\partial y} - \frac{\partial u'v'}{\partial x} - \frac{\partial v'^2}{\partial y} - \frac{\partial v'w'}{\partial z} \quad (2)$$

where x,y,z = cartesian coordinates; u,v,w = time-averaged velocities in x,y,z; u′,v′,w′ = turbulent velocity fluctuations; $\Delta\rho$ = local density difference with respect to ambient density ρ_a; p_d = dynamic pressure. The terms involving $\Delta\rho$ arise from the buoyancy of the flow. An additional effect due to buoyancy is the damping of vertical turbulence, thereby reducing vertical spreading and entrainment. The importance of the buoyant effects is given by the local densimetric Froude number

$$F_L = u_c \left(\frac{\Delta\rho_c}{\rho_a} gh \right)^{-1/2} \quad (3)$$

where u_c, $\Delta\rho_c$ are values of u and $\Delta\rho$ at the jet axis on the water surface and h is a measure of the verticle thickness of the jet.

In the limiting case of the isothermal surface jet, $F_L \rightarrow \infty$, and the buoyancy terms are negligible. In addition, boundary layer arguments lead to the neglect of the lateral momentum equation (2) and the second and third terms to the right of the equal sign in equation (1). Thus, the classical iso-

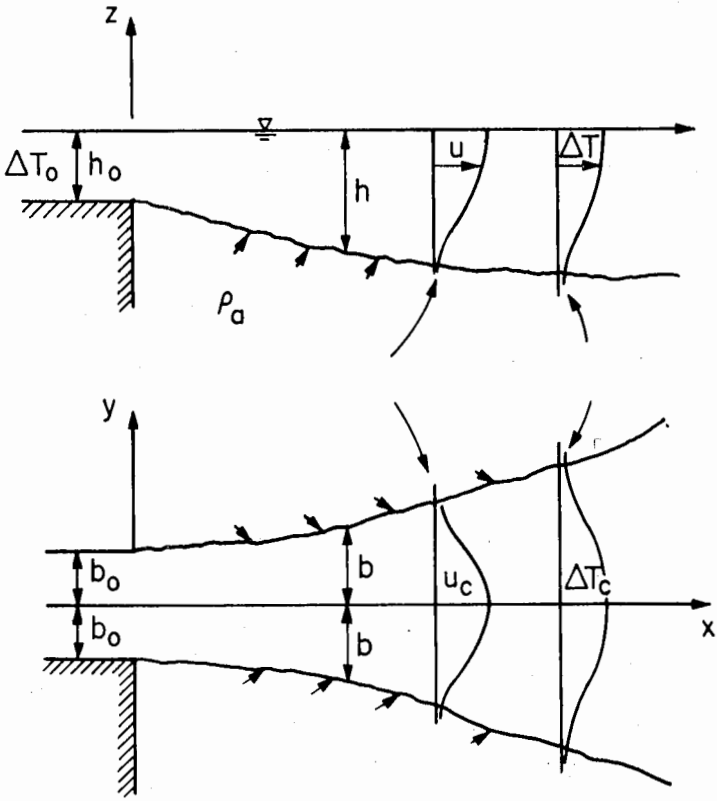

Fig. 1. - Buoyant Surface Jet - Near Field Zone

thermal jet equation expresses the balance between convective transport of axial velocity u and lateral and vertical diffusion.

In the other limiting case of a strongly buoyant surface jet, $F_L \to 1$, and the buoyancy terms are of the same order of magnitude as the convective terms. As the buoyant pressure terms act in both horizontal directions, both the x and y momentum equations must be retained. In contrast to the isothermal case, only vertical diffusion terms are significant for $F_L \to 1$. Thus, the lateral velocity and temperature distributions will not have jet-like sheared profiles.

If profile assumptions on the lateral and vertical distribution of u, v and T (or $\Delta\rho$) are made, the partial differential equations can be reduced to ordinary differential equations (jet integral analysis). However, as the equations should describe the transition in surface buoyant jets from large to small values of F_L, the following difficulties arise: a) Only the vertical jet

profiles are truly of shear type. The lateral profiles change from shear type to a more uniform distribution (shear acting only at the edges). b) The distribution of the lateral velocity, v, is not readily specified in terms of centerline quantities. Some hypothetical assumptions have to be made in this respect.

Despite these theoretical restrictions it is useful to retain the assumptions of jet-like lateral profiles in order to describe the deviation due to buoyancy from the more non-buoyant behavior, without attempting to describe the limiting case of strongly buoyant behavior. The utility and range of applicability of such an approach have to be demonstrated by comparison with experiments. If this procedure is followed, the distributions for velocity and density are given by

$$u/u_c = f(\eta, \zeta);$$

$$\Delta T/\Delta T_c = \Delta\rho/\Delta\rho_c = g(\eta, \zeta);$$

where $\eta = y/b$ and $\zeta = z/h$ and f and g define bell-shaped jet distributions. After integration in the lateral direction the governing equations acquire the following general form

$$\frac{dQ}{dx} = c_1 \alpha_o u_c h + c_2 \alpha_v u_c b \tag{4}$$

$$\frac{dM}{dx} = \frac{dP}{dx} \tag{5}$$

$$\frac{dH}{dx} = -c_3 k \Delta T_c b \tag{6}$$

$$\frac{db}{dx} = \epsilon + \left(\frac{db}{dx}\right)_B \tag{7}$$

where c_1, c_2 and c_3 are profile-dependent coefficients, k is a coefficient for surface heat loss, ΔT_c is the excess centerline temperature of the surface and ϵ is the rate of lateral spreading for a non-buoyant jet.

The above equations are amenable to numerical solution using appropriate initial conditions at the discharge point. The integral quantities are defined as follows:

Volume flux $$Q = \int_A u \, d\eta \, d\zeta \tag{8}$$

Momentum flux $$M = \int_A \rho_a u^2 \, d\eta \, d\zeta \tag{9}$$

Pressure force $$P = \int_A \left[\int_{-\infty}^{\zeta} g \Delta\rho \, d\zeta \right] d\eta \, d\zeta \tag{10}$$

Temperature flux $$H = \int_A \Delta T \, u \, d\eta \, d\zeta \tag{11}$$

where A is the cross-sectional area of the jet.

The continuity equation (4), uses the entrainment concept proposed by Morton, Taylor and Turner (10) which relates the normal velocities at the jet boundary to the centerline velocity by means of a proportionality coefficient. α_o is the constant coefficient for lateral entrainment, and α_v is the variable coefficient for vertical entrainment which is a function of the local buoyant damping of turbulent entrainment, characterized by F_L, so that $\alpha_v = \alpha_o f(F_L)$ as indicated by the data of Ellison and Turner (2). Equation (5) expresses the balance between longitudinal momentum flux and buoyant pressure force. The heat conservation equation (6) allows for excess heat decay to the atmosphere. The jet spreading equation (7) represents the assumption that the buoyancy of the jet causes spreading of the jet width $\frac{db}{dx}_B$, in addition to the usual non-buoyant turbulent spreading, ϵ. Closure of the equations requires specification of $(db/dx)_B$ through the use of the lateral momentum equation. Different hypotheses are possible:

i) Stolzenbach and Harleman (17, 18) assume the local lateral velocity, v, to be proportional to the local lateral density gradient, $\partial\Delta\rho/\partial y$, and the local longitudinal velocity, u; the proportionality constant being equal to $(db/dx)_B$. This specification allows integration of the lateral momentum equation.

ii) Prych (12) and Stefan and Vaidyaraman (16) solve the lateral momentum equation under the simplifying assumption that $\left(\frac{db}{dx}\right)_B = f(F_L)$.

Buoyant Surface Jets — Experimental Results

Figure 2 shows a comparison of surface jet theoretical predictions and laboratory experimental results for a buoyant jet having an initial densimetric Froude number of 1.8. Good agreement is obtained for the longitudinal decrease of the excess surface temperature $\Delta T_c/\Delta T_o$ along the jet centerline and for the jet thickness. Both theories (refs. 17 and 12) tend to over-predict the lateral width of the jet when the ratio $\frac{b}{h} \sim 100$. The half-width ($b_{1/2}$) shown in Figure 2 is the distance from the centerline to the point at which the surface temperature excess is equal to ½ the centerline temperature rise. The half-depth ($h_{1/2}$) is defined in a similar manner. The over-prediction of the lateral width is probably due to two facts: (1) The retention of lateral jet-like profiles at small F_L;(2) the neglect of lateral shear $\overline{v'w'}$ in the lateral momentum equation which is done in all of the analyses described above.

Figures 3, 4 and 5 show examples of field data for a heated surface discharge at Pilgrim Nuclear Power Station collected by Pagenkopf, et al (11). Comparison between the field data and three-dimensional, steady state mathematical models is complicated by the fact that the densimetric Froude number of the discharge varies over a range from 2 to 11 during the six-hour change from high to low tide. Nevertheless, data obtained during a period of an hour on either side of the time of high tide represents reasonably steady state conditions. Figure 3 shows the centerline temperature decrease, for a jet near high tide having an initial densimetric Froude number of 2.2, as a function of longitudinal distance in comparison with the Stolzenbach-Harleman (17, 18) prediction. A plot of the observed vertical temperature distribution along the centerline of the plume is shown in Figure 4. This indicates that for the low Froude number jet there is very little interaction between the development of the jet plume and the sloping ocean bottom offshore of the discharge channel.

Frequently, thermal discharge criteria formulated by regulatory agencies are specified in terms of a permissible water surface area within a certain isotherm of excess temperature. The analytical models described above can be used to predict such areas as shown in Figure 5. The Stolzenbach-Harleman plume area prediction is shown in comparison with areas determined from successive field measurements taken during the period near high tide. The tendency to over-predict the isotherm areas having small values of $\Delta T_c/\Delta T_o$ is again evident.

Additional analytical and experimental studies on both the near and far-field behavior of buoyant surface jets are underway in the R. M. Parsons Laboratory. These include: The interaction of the jet plume with the bottom boundary of the receiving water, either an abrupt step or a gradual slope; the interaction of the jet plume with an along-shore current in the

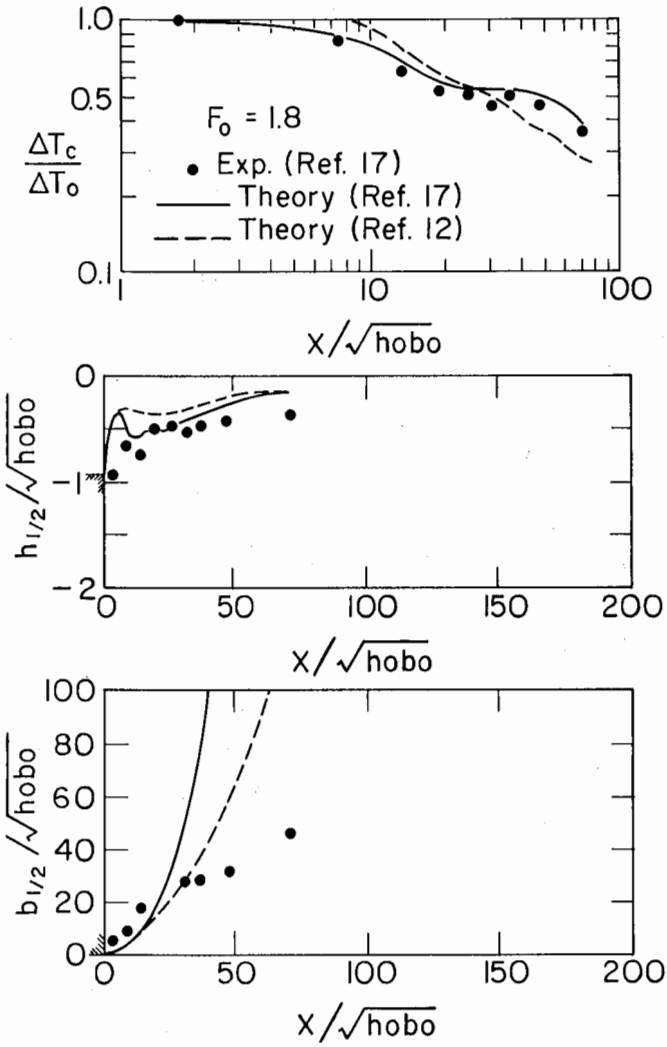

Fig. 2. - Comparison Between Laboratory Experiments and Theoretical Predictions: Buoyant Surface Jet, $F_o = U_o\left(\frac{\Delta\rho_o}{\rho_a} gh_o\right)^{-1/2} = 1.8$; $h_o/b_o = 0.87$ and $k/U_o = 6.2 \times 10^{-5}$

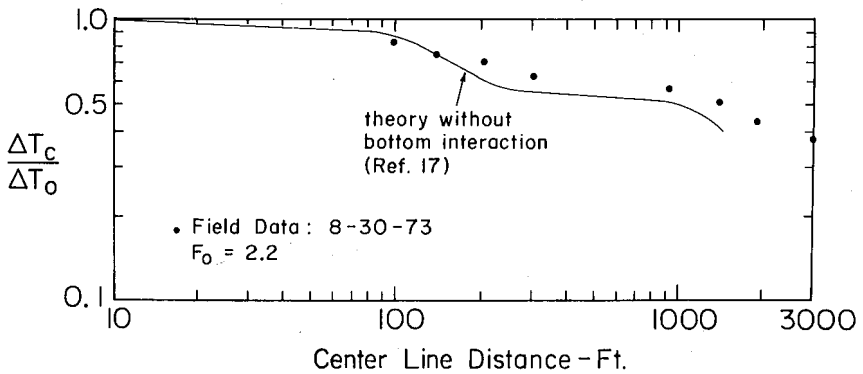

Fig. 3. - Comparison Between Field Data and Theory for Surface Excess Temperature Along Plume Centerline During High Tide at Pilgrim Nuclear Power Station: $F_o = U_o\left(\frac{\Delta\rho_o}{\rho_a} gh_o\right)^{-1/2} = 2.2$;

$h_o/b_o = 0.47$ and $k/U_o = 1 \times 10^{-5}$

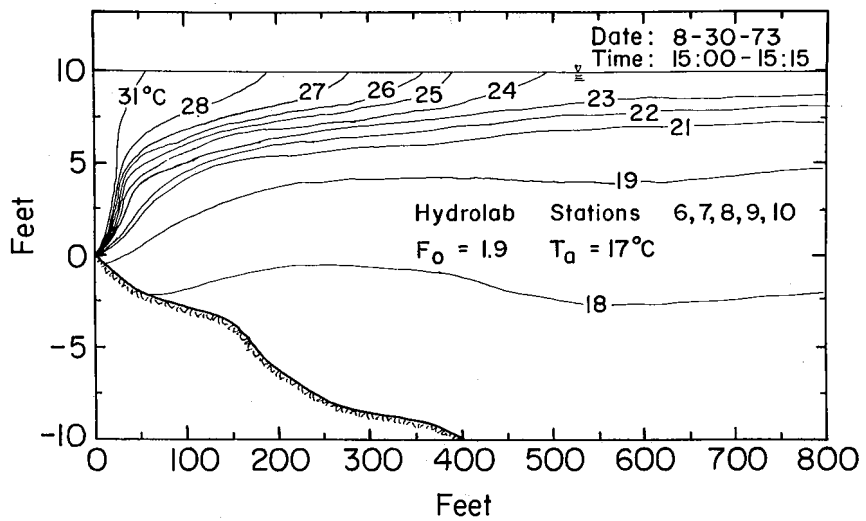

Fig. 4. - Observed Vertical Temperature Distribution Along Plume Centerline During High Tide at Pilgrim Nuclear Power Station

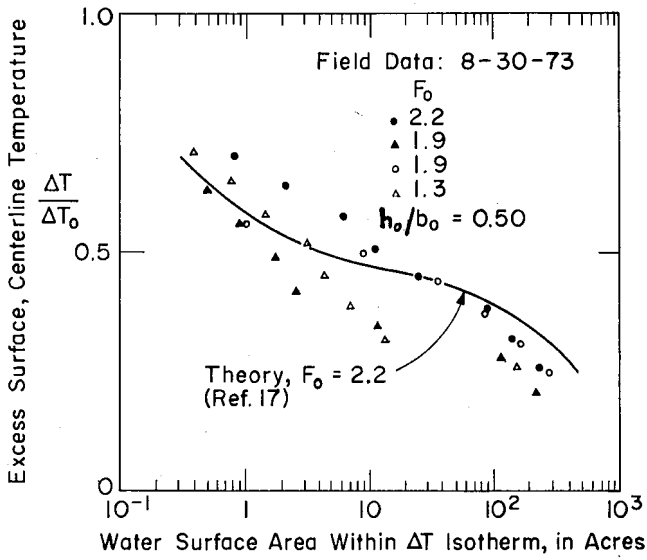

Fig. 5. - Comparison Between Field Data and Theory for Water Surface Areas Within ΔT Isotherms During High Tide at Pilgrim Nuclear Power Station

receiving water and the extension of the temperature prediction to the far-field where ambient currents, diffusion and surface heat loss dominate the process. Some preliminary results have been published by Harleman, Adams and Koester (4) in connection with studies for the Atlantic Generating Station, a floating nuclear power station off the New Jersey coast.

Multiport Diffusers in Shallow Water

Basic analytical and experimental studies on the mechanics of submerged multiport diffusers has been conducted by Jirka and Harleman (6, 7). A multiport diffuser consists of a pipeline, laid on the bottom of the receiving water, with many nozzles of diameter, D, attached at a regular spacing, ℓ. The individual round jets emanating from the nozzles interfere after a short distance and form a two-dimensional jet zone. It has been shown that the jet parameters in this two-dimensional zone are equal to those of an "equivalent slot diffuser" with slot width $B = \pi D^2/4\ell$ and equal discharge velocity, U_0. Using the concept of the "equivalent slot diffuser" reduces the number of dimensionless parameters characterizing a multiport diffuser and thus provides a means to compare different designs.

Over the years a considerable body of knowledge has been built up on the design of offshore multiport diffusers for sewage. Only recently have

multiport diffusers been considered for thermal discharges in coastal waters and in the Great Lakes. The design problems for the two types of diffusers are quite different. Sewage diffusers generally require a dilution at the water surface of the order of 100. This has generally limited their application to water depths of 100 feet or more where the large dilution is due to the long trajectory of the buoyant jet rising toward the surface. The vertical thickness of the mixed zone at the surface is a relatively small fraction of the total depth as shown in Figure 6 (a) and (c) for vertical and non-vertical discharges in deep water. In contrast, thermal diffusers generally require a dilution at the water surface of the order of 10 and, particularly in the East coast and Great Lakes, are located in water depths much less than 100 feet. Another important difference is the relative buoyancy of the two types of discharges. In the case of a sewage diffuser, the density difference between the effluent and the receiving water is about an order of magnitude higher than for a heated discharge. These two factors, the greater depth and buoyancy, usually insure that the mixed layer at the surface is stably stratified and the near-field dilution is little affected by diffuser orientation or currents in the receiving water.

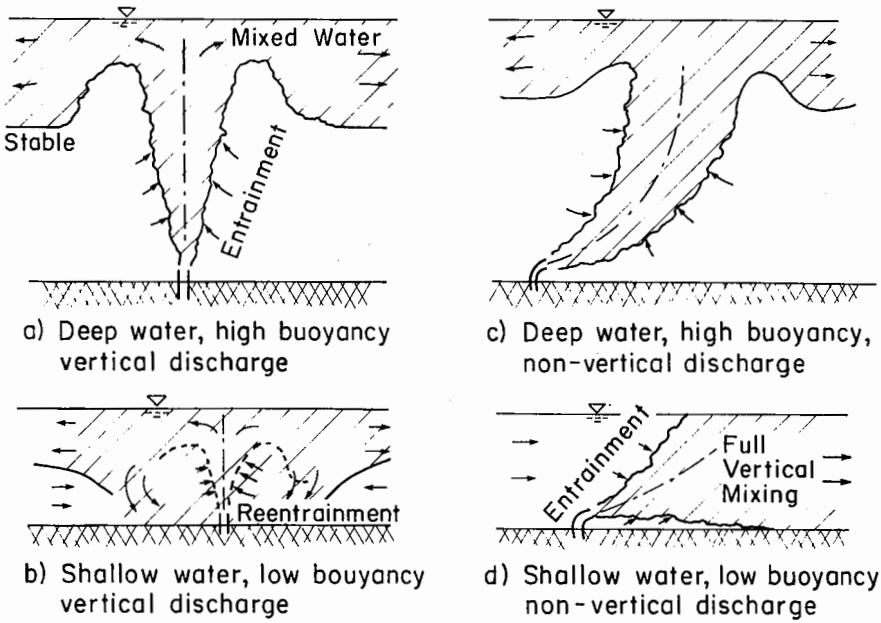

Fig. 6. - Submerged Buoyant Jets: Effects of Relative Buoyancy, Submergence and Angle of Discharge

Stability Analysis and Prediction of Dilution

The analytical and experimental studies of Jirka and Harleman (6) have shown that, because of low buoyancy, thermal diffusers in shallow water almost always exhibit an unstable stratification in the near field, thus leading to reentrainment or full vertical mixing as shown in Figures 6 (b) and (d). Prediction of the dilution of such diffusers is based on the two-dimensional stratified flow regions shown in Figure 7. These regions are (1) a

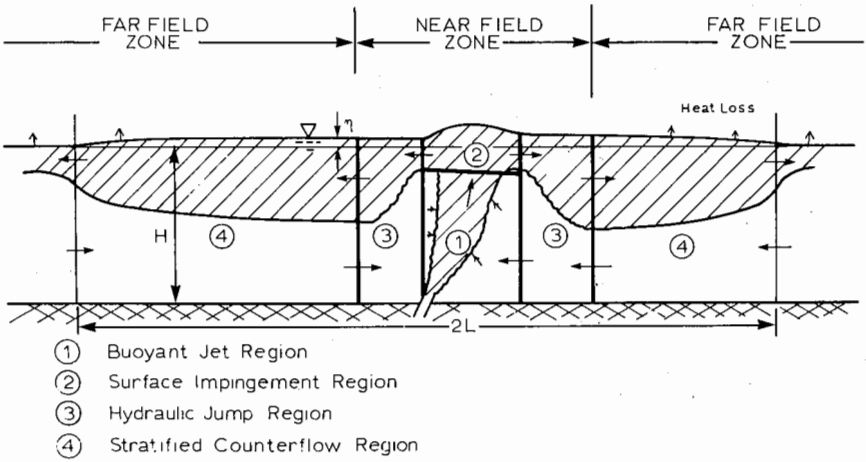

Fig. 7. - Zones of Stratified Flow in Two-Dimensional Analysis of Submerged, Buoyant Slot Jet

buoyant jet region, (2) a surface impingement region, (3) an internal hydraulic jump, and (4) a stratified counterflow region. These regions, each with different hydrodynamic properties allowing simplifying approximations, can be analyzed separately. Successive matching of the individual solutions yields a description of the total flow field. The objectives of the analysis are to determine the limiting condition of a stable flow field, that is, the criterion line between stable and unstable regimes and the dilutions which occur in the two regimes.

Stability is primarily dependent on regions (1), (2) and (3). Inspectional analysis of the problem gives the following governing parameters:

Slot densimetric Froude number: $$F_s = U_o \left(\frac{\Delta\rho_o}{\rho_a} gB \right)^{-1/2} \qquad (12)$$

Relative water depth: H/B

Angle of discharge: θ_o

where $\Delta\rho_o$ = initial density difference and H = water depth.

(i) Buoyant Jet: A hydrostatic pressure distribution is assumed in this region. This is tantamount to the assumption that the pressure disturbance due to the rise in surface elevation as a result of impingement is limited to the impingement region. This assumption is essentially verified by experiments. A buoyant jet analysis, utilizing the entrainment concept, is performed to give predictions of jet dilutions and trajectories.

(ii) Surface Impingement: The surface impingement region provides the transition between the jet flow, with a strong vertical component, and the horizontal spreading motion. The process is complex and is most conveniently analyzed by a control volume approach, using a continuity equation, a horizontal momentum equation and two energy equations (account being taken of the energy loss in the flow transformation). Results of the analysis give the thickness of spreading layer, h_1, and thus the elevation to which effective jet entrainment occurs. Furthermore, the dynamic characteristics of the spreading layer, represented by a densimetric Froude number

$$F_1 = u_1 \left(\frac{\Delta\rho}{\rho_a} gh_1\right)^{-1/2} \tag{13}$$

where u_1 = layer velocity, $\Delta\rho$ = relatively density difference between upper and lower layer, can be calculated. The upper layer thickness is about ⅙ of the total water depth and is only weakly dependent on F_s and H/B. The Froude number F_1 is strongly dependent on F_s and H/B and generally supercritical.

(iii) Internal Hydraulic Jump: Experimental observations indicate that following the surface impingement the thickness of the surface layer suddenly increases by the formation of an internal hydraulic jump.

The governing equations for internal hydraulic jumps have been derived by Yih and Guha (21). A simplified solution for small density differences has been obtained by Jirka and Harleman (6). The equations indicate that for certain upstream conditions no solution is possible, that is, a stable subcritical downstream condition does not exist.

The dividing line between stable and unstable conditions (in which reentrainment occurs) is shown in Figure 8 for a vertical discharge. The parameters defining stability are the relative submergence H/B and the equivalent slot densimetric Froude number F_s given by equation (12).

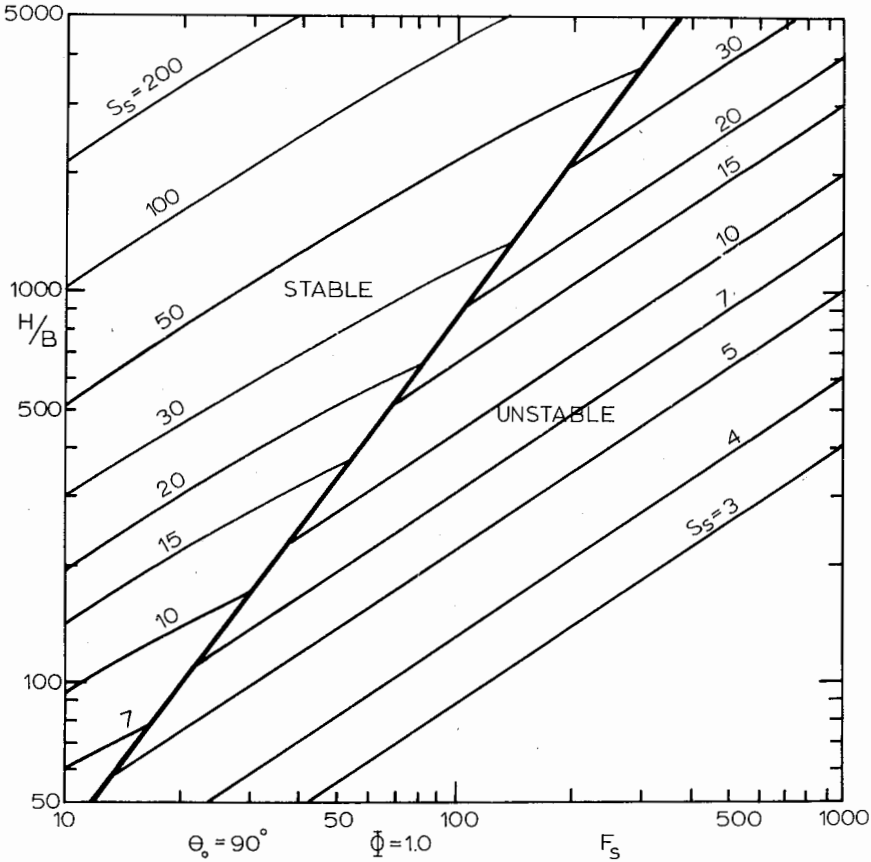

Fig. 8. - Surface Dilution S_s as a Function of Relative Submergence H/B and Slot Densimetric Froude Number F_s: Dividing Line Between Stable and Unstable Regimes

The variable of major importance in the design of submerged multiport diffusers is the near-field surface dilution S_s. For the case of a stable near-field S_s can be directly obtained from the buoyant jet analysis, if account is taken of the thickness of the impingement layer.

Whenever the near-field is unstable and reentrainment into the jet zone occurs, a simple buoyant jet analysis is not valid. In this case the near-field dilution is directly dependent on the stratified counterflow system in the far-field (region 4). This counter-flow presents a balance between buoyancy forces and frictional forces (dependent on geometry, boundary and interfacial friction). Values for S_s, evaluated through analysis of these different

far-field conditions by Jirka and Harleman (6) are shown in Figure 8. Upon crossing the dividing line from the stable to the unstable zone, the surface dilution decreases due to reentrainment. For typical conditions encountered in condenser water discharges, thermal diffusers are almost always in the unstable zone. For a specified dilution S_S, it is generally desirable to design as close to the dividing line as possible in order to promote vertical stratification. This requires reducing the diffuser nozzle velocity U_O; however, practical considerations of internal diffuser hydraulics dictate that U_O should not be less than approximately 2 m/sec.

Three-dimensional aspects of multiport diffusers, including the interrelation between the diffuser length, the water depth and the total waste heat load were also considered by Jirka and Harleman (6). Other important design considerations are the orientation of the diffuser nozzles and the diffuser axis with respect to currents in the receiving water. In the case of reversing ambient currents (such as unsteady wind-driven or tidal currents), it is desirable to design the diffuser to promote stratification and to be effective in either current direction. This objective can be met by the use of alternating direction nozzles (no net horizontal momentum). The studies described above are useful in making preliminary design estimates and for screening alternative discharge schemes for further investigation in a hydrothermal scale model. A number of such model studies for specific power plant sites have been carried out in the R. M. Parsons Laboratory (3, 8, 20).

Analytical and experimental analysis of the four flow regions shown in Figure 7 for a single round buoyant jet discharging vertically upward has been conducted by Lee (9). Ungate (19) has experimentally investigated the effect of Reynolds number, in the laminar-turbulent transition range, on the entrainment of buoyant jets covering a wide range of densimetric Froude numbers. This study is helpful in the choice of length scales for physical models.

Far-field Temperature Distribution in Unsteady Flow

The mathematical model developed by Harleman, Brocard and Najarian (5) is a one-dimensional, transient numerical model in which water temperature is a function of longitudinal distance and time. The model is applicable to rivers, shallow reservoirs and estuaries in which temperature variations in the longitudinal direction are more important than the vertical.

In post-operational studies of once-through cooling processes, it is generally necessary to demonstrate compliance with established thermal discharge criteria by determining the incremental temperature rise due to condenser water discharge. This implies that ambient temperatures are known and can be subtracted from observed temperatures to determine the

incremental rise. Because of buoyancy and unsteady flow reversals (due to tidal motion in estuaries and hydroelectric regulation in rivers and reservoirs) thermal effects of power plants may extend both upstream and downstream of the site. Therefore, it is difficult to determine natural or ambient water temperature by direct measurement. Temperature prediction models are also required to provide input to water quality models where the rate constants governing biochemical reactions are temperature dependent.

The flow field is unsteady and non-uniform and is determined by simultaneous solution of the continuity and momentum equations. The governing equations for one-dimensional flow in a variable area channel are: the continuity equation

$$\frac{\partial A}{\partial t} = \frac{\partial Q}{\partial x} = q \tag{14}$$

and the longitudinal momentum equation

$$\frac{\partial}{\partial t}(AU) + \frac{\partial}{\partial x}(QU) = -gA\frac{\partial h}{\partial x} - g\frac{Q|Q|}{AC^2R_h} \tag{15}$$

where, x = distance along longitudinal axis; t = time; h = elevation of water surface with respect to horizontal datum; Q = cross-sectional discharge; q = lateral inflow per unit length of channel; U = average cross-sectional velocity in the channel, $= Q/A$; g = acceleration of gravity; A = cross-sectional area of channel; C = Chezy coefficient; and R_h = hydraulic radius of channel.

In equation (15) a term which represents the effect of a longitudinal density gradient and which is significant only within the salinity intrusion region of estuaries has been ignored. Boundary conditions must be specified (either water surface elevation h or discharge Q) at the upstream and downstream sections of the river, reservoir or estuary being modeled. The solution of equations (14) and (15) for $Q = f(x,t)$ and $A = f(x,t)$ can be obtained numerically by schemes as described by Dailey and Harleman (1). The solution requires the specification of initial conditions for h and Q and advances in time in accordance with the values of the time varying boundary conditions.

The quantity ρcT represents the amount of heat per unit volume of water and the one-dimensional conservation of heat equation is

$$\frac{\partial}{\partial t}(A\rho cT) + \frac{\partial}{\partial x}(Q\rho cT) = \frac{\partial}{\partial x}\left[AE_L\frac{\partial}{\partial x}(\rho cT)\right] + \phi_n b + S \tag{16}$$

where, ρc, the product of density and specific heat of water has the dimensions of heat content, BTU (joules) per unit volume per degree; T = water temperature; E_L = longitudinal dispersion coefficient; ϕ_n = net heat influx per unit water surface area, BTU/ft².sec (joules/m².sec); b = width of water surface and S = source term accounting for waste heat discharges and lateral heat input from tributary inflows. For a power station with once-through cooling S is the heat rejection rate in BTU/sec (joules/sec) divided by the effective longitudinal length over which the heat injection occurs.

The steady state form of eq. (16) has been used to determine the longitudinal temperature distribution in rivers under natural and artificial heat inputs. Since ρc can be assumed constant and the dispersive term can be neglected in rivers, the steady form for a reach with no tributary ($\partial Q/\partial x = o$) can be written as

$$\frac{dT}{dx} = \frac{\Phi_n b}{\rho c Q} \tag{17}$$

This first order differential equation can be solved by specifying a temperature boundary condition at the upstream end such that $T = T_o$ at $x = o$. Additional simplifications to facilitate an analytical solution include the assumption that the surface width b is constant and linearization of the net heat flux into the water surface.

$$\Phi_n = -K \quad T - T_E \tag{18}$$

where K is the surface heat transfer coefficient in BTU/ft².day.°F (joules/m².day.°C) and T_E is the equilibrium temperature for which $\Phi_n = o$. The solution of equation (17) is

$$T = T_E + \left(T_o - T_E\right) \exp\left(-\frac{bKx}{\rho c Q}\right) \tag{19}$$

provided both K and T_E are independent of x.

For unsteady flow problems with time varying meterological conditions, the solution of equation (16) must be carried out simultaneously with the continuity and longitudinal momentum equations (14 and 15) by numerical techniques. In the unsteady case the linearization of the surface heat flux ϕ_n by means of equation (18) is of little value since both K and T_E are functions of time. Since meterological data are necessary to determine T_E, the same data can be used to compute ϕ_n in equation (16) directly. Ryan and Harleman (13) and Ryan, et. al. (14) have made a careful study of surface heat transfer mechanisms and have proposed formulae by which ϕ_n can be

calculated as a function of time from standard meterological observations. The net influx of heat per unit surface area is the summation of five terms: the net incident solar (short wave) radiation; the net incident atmospheric (long wave) radiation; the long wave, back radiation from the water surface; the evaporative and conductive heat fluxes.

The transient model has been used to predict ambient temperatures in Conowingo Reservoir (5) on the Susquehanna River. Unsteady flows, including flow reversals, occur due to the transient operation of a pumped-storage plant and the hydroelectric power stations at the upstream and downstream boundaries of the reservoir. The objective of the study was to determine the temperature rise (above ambient) due to the discharge of waste heat from Peachbottom No. 2 Atomic Power Station which is located on the reservoir. The mathematical model computes natural reservoir temperatures under given meterological conditions and boundary conditions. The temperature rise due to Peachbottom is determined by subtracting the calculated ambient temperatures from temperatures measured in the reservoir during operation of the nuclear power plant.

The reservoir length is about 12 miles (19.2 km) and values of $\Delta x = 1500$ ft. (460 m) and $\Delta t = 12$ min. were used in the unsteady flow calculations. For the temperature calculations the same time step was used; however, Δx was reduced to 500 ft. (165 m).

Velocities and temperatures were measured in Conowingo Reservoir during 1972 prior to the operation of Peachbottom No. 2. A comparison of calculated and observed mean velocities at two different cross sections for a three day period in September 1972 is shown in Figure 9. Flow reversals due to the hydroelectric operation are clearly indicated. A comparison of the calculated and observed longitudinal temperature profile at the beginning of September 3, 1972 is shown in Figure 10. Another verification of the ambient temperature prediction model was made during a 10 day period in April 1972. Figure 11 shows the predicted and observed temporal variations in temperature at a section located about 5 miles below the upstream boundary.

Conclusions

A number of analytical and experimental techniques for predicting water temperature distributions due to waste heat discharges have been discussed. Predictive techniques are needed in the preparation of environmental impact statements for pre-operational site studies in order to evaluate the economic and environmental costs of alternative cooling water systems. These techniques are also useful in post-operational studies, inasmuch as field observations can be carried out only under a limited number of ambient conditions. Mathematical models can be used in interpreting field data and for providing additional information for receiving water conditions other than those measured.

The use of mathematical and/or physical models for the planning and design of field monitoring programs has received relatively little attention. It is suggested that a considerable amount of time and expense could be saved by making use of temperature prediction models in planning field surveys.

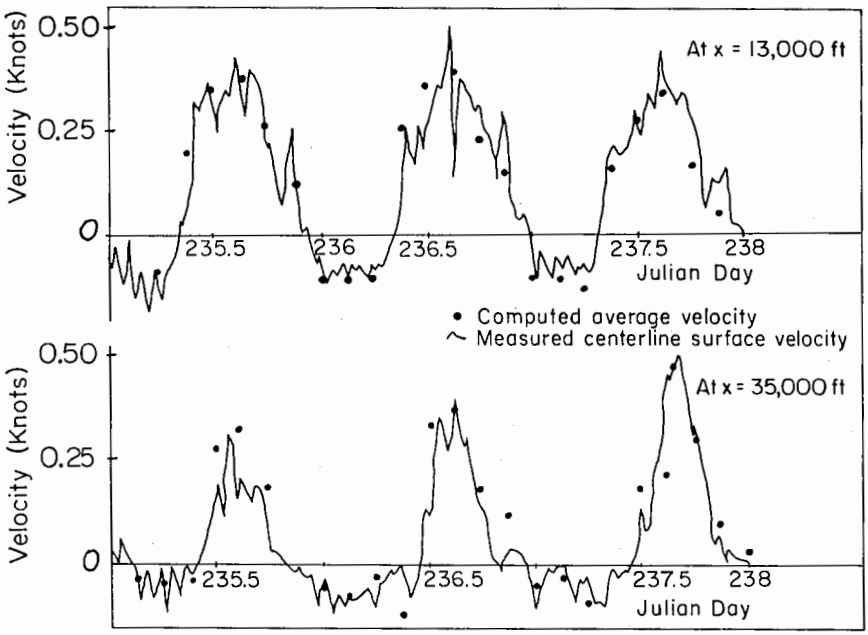

Fig. 9. - Comparison Between Measured and Predicted Velocities in Conowingo Reservoir from September 1-3, 1972: Upper Curve at x = 13,000 ft., Lower Curve at x = 35,000 ft.

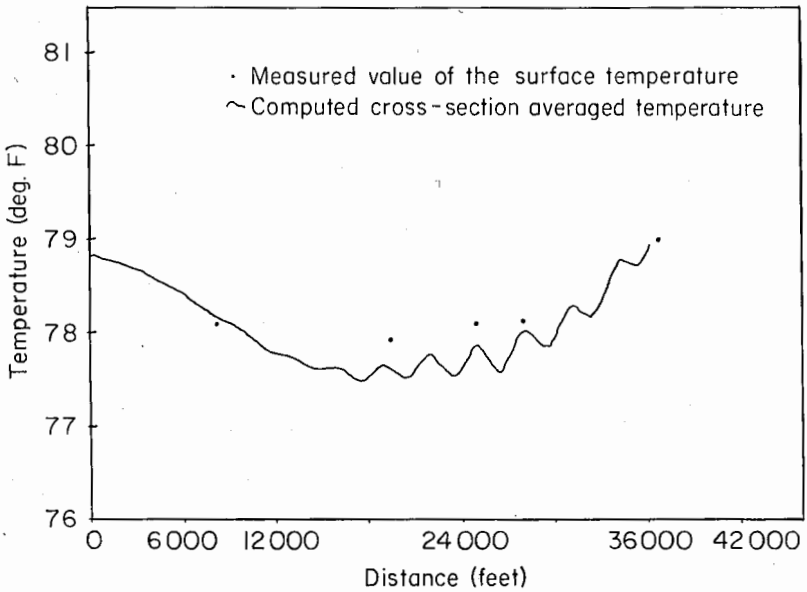

Fig. 10. - Comparison Between Measured and Predicted Longitudinal Temperature Profile in Conowingo Reservoir at 0 hrs. on September 3, 1972

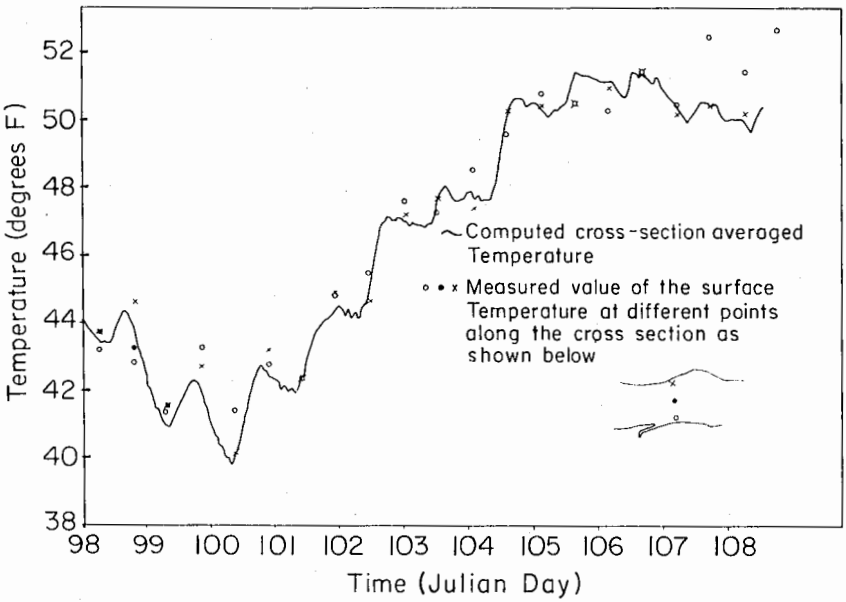

Fig. 11. - Comparison Between Measured and Predicted Temperatures in Conowingo Reservoir from April 8-18, 1972 at x = 25,000 ft.

Appendix I. - References

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Appendix II. - Notation

The following symbols are used in this paper:

A	=	cross sectional area;
b	=	lateral width of jet;
$b_{1/2}$	=	value of b on water surface where $\Delta T = \Delta T_c/2$;
B	=	slot width;
c	=	specific heat of water;
C	=	Chezy coefficient;
D	=	nozzle diameter;
E_L	=	longitudinal dispersion coefficient;
F_1	=	densimetric Froude number of spreading layer;
F_L	=	local densimetric Froude number;
F_s	=	slot jet densimetric Froude number;
g	=	acceleration of gravity;
h	=	vertical thickness of jet;
$h_{1/2}$	=	value of h where $\Delta T = \Delta T_c/2$;
h_1	=	vertical thickness of spreading layer
H	=	temperature flux; or total water depth;
k	=	surface heat loss coefficient;
K	=	surface heat transfer coefficient;
ℓ	=	nozzle spacing
M	=	momentum flux
p_d	=	dynamic pressure
P	=	pressure force

q	=	lateral inflow per unit length of channel;
Q	=	volume flux
R_h	=	hydraulic radius;
S	=	source term for waste heat discharge;
S_s	=	near-field surface dilution;
t	=	time;
T	=	water temperature
T_E	=	equilibrium temperature;
T_o	=	water temperature at $x = 0$;
ΔT	=	local temperature difference with respect to ambient;
ΔT_c	=	value of ΔT at jet axis on water surface;
u,v,w	=	velocity components in x,y,z directions;
u', v', w'	=	turbulent velocity fluctuations in x,y,z directions;
u_i	=	horizontal velocity of spreading layer;
u_c	=	value of u at jet axis on water surface;
U	=	average cross-sectional velocity;
U_o	=	nozzle discharge velocity;
x,y,z	=	rectangular Cartesian coordinates;
α_o	=	lateral entrainment coefficient;
α_v	=	vertical entrainment coefficient;
ϵ	=	rate of lateral spreading for a non-buoyant jet;
ζ	=	z/h;
η	=	y/b;
ϕ_o	=	discharge angle of slot;
ρ_a	=	ambient density;
$\Delta\rho$	=	local density difference with respect to ambient;
$\Delta\rho_c$	=	value of $\Delta\rho$ at jet axis on water surface;
$\Delta\rho_o$	=	initial density difference with respect to ambient; and
φ_n	=	net heat influx per unit water surface area