

# Using Dynamic Measurements for the Capacity Evaluation of Driven Piles

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*Dynamic methods, employing dynamic measurements, are effective tools that require a fundamental understanding beyond the routine automated procedure.*

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**D**ynamic analyses of piles are used to predict pile response based on its behavior during driving. These analyses are enhanced through data provided by dynamic measurements while monitoring the pile during installation and restrikes. The dynamic measurements are utilized for field evaluation of pile capacity and integrity during driving and for post-driving signal matching methods of analysis.

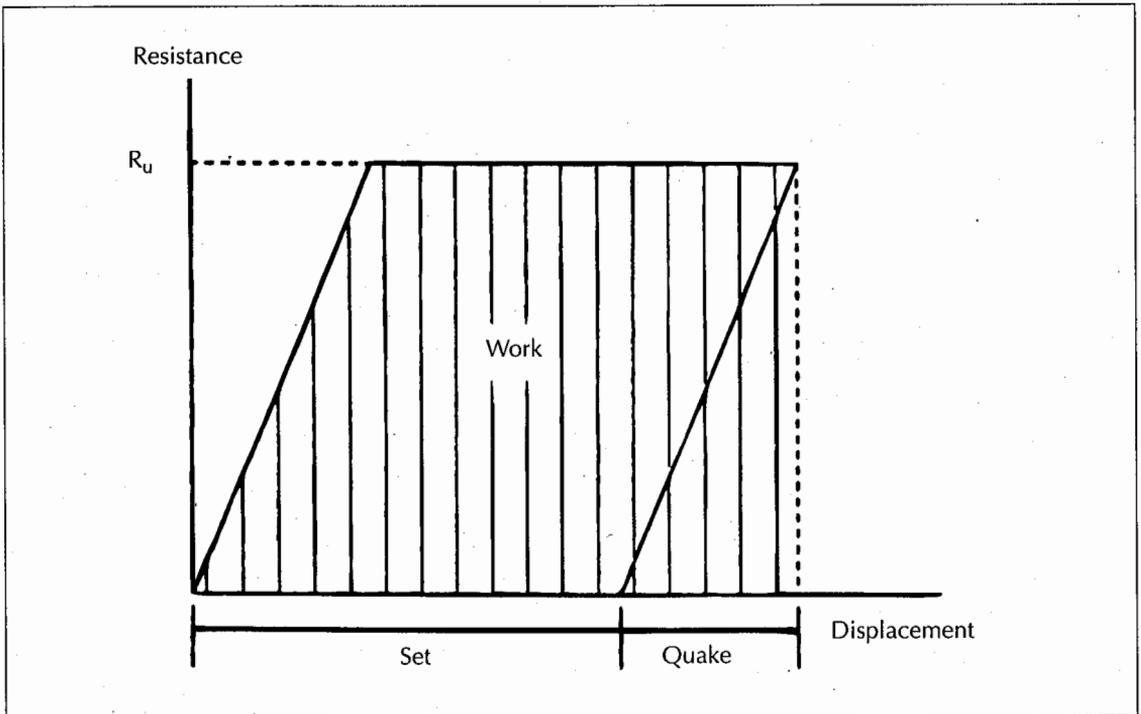
The dynamic measurements and their analyses have great economic and safety benefits during design and construction. The full potential of their application is hampered, however,

by inconsistent performance — depending on the pile and driving conditions, time of driving and method of analysis. Problems include limited accuracy of the primary field method, capacity change in time and the perceived underprediction of the signal matching techniques.

The alternative methods presented here can be used to obtain high accuracy in field predictions during construction, as well as in analyzing the complex behavior of plugged open pipe piles using signal matching techniques. Proper utilization of the dynamic analyses require a fundamental understanding of the theories and their original assumptions; hence, dynamic measurements can not be conducted assuming a "black box" approach.

## Background

Dynamic analyses of piles are methods that predict pile capacity based on behavior during driving. The evaluation of static capacity from pile driving is based on the concept that the driving operation induces failure in the pile-soil system — in other words, a very fast load test is carried out under each blow. There are basically two methods of estimating the ulti-



**FIGURE 1. Resistance versus displacement at the top of the pile.**

mate capacity of piles on the basis of dynamic driving resistance:

- Pile driving formulae (*i.e.*, dynamic equations); and,
- Wave equation (WE) analysis.

*Dynamic Equations.* Dynamic equations can be categorized into three groups:

- Theoretical equations;
- Empirical equations; and,

- Equations that consist of a combination of a theoretical basis with empirical components.

The theoretical equations are formulated on analyses that evaluate the total resistance of the pile, based on the work done by the pile during penetration. These formulations assume elasto-plastic force-displacement relations as shown in Figure 1. The total work is determined by Equation 1.

When the work of the resisting forces,  $W$ , is equated to the energy delivered to the pile ( $W$  equal to  $E_d$ ) the basic familiar elements of the dynamic equations can be extracted as shown in Equation 2.

*The Wave Equation (WE).* Stress propagation in a pile during driving can be described by an equation of motion like Equation 3 – the one-dimensional wave equation.

**EQUATION 1**

$$W = R_u \left( S + \frac{Q}{2} \right)$$

where:

$R_u$  = The yield resistance

$Q$  = The quake denoting the combined elastic deformation of the pile and the soil

$S$  = The set, denoting the plastic deformation (permanent displacement) under each blow.

**EQUATION 2**

$$R_u = \frac{E_d}{\left( S + \frac{Q}{2} \right)}$$

The friction stresses,  $f_s$ , are activated by the pile movement. Under free wave motion (with  $f_s$  equal to zero), Equation 3 becomes the familiar one-dimensional WE. The friction stresses are traditionally represented by a soil model.<sup>1</sup> The static soil resistance-displacement relationship is assumed to be elasto-plastic and is represented by a spring in series with a slider. The dynamic resistance,  $R_d$ , is assumed to be viscous (soil type related) and, therefore, velocity dependent and is represented by a dashpot parallel to the spring. The resisting soil force,  $R_t$ , is a combination of  $R_d$  and  $R_s$  as shown in Figure 2. The different parameters included at the bottom of Figure 2 (for example: J, Q, UR, QR) are the parameters used to specify the assumed behavior of the soil pile interaction for each pile element.

The WE formulation is used in two ways:

- For *pre-driving analysis* — where the entire system is modeled, including the pile, hammer and driving system, as was first suggested by Smith<sup>1</sup> and further improved to include different hammers, splices, and so forth.<sup>3</sup>
- For *post-driving analysis* — which utilizes dynamic measurements obtained near the pile head during driving.

Post-driving signal matching analyses utilize the measured force signal (calculated from strain readings) and the measured velocity signal (integrated from acceleration readings) obtained near the pile top during driving. These analyses model the pile-soil system in a continuous or discrete form as shown at the top of Figure 2, with the element denoted as I-1 representing the point of measurement. The velocity signal is used as a boundary condition at that point while varying the parameters describing the soil resistance in order to solve Equation 3 and match the calculated and measured force signals.

The soil parameters (describing a simple elasto-viscoplastic material) include the side and tip quake, side and tip damping, and the pile shaft and tip ultimate resistances. The tip model is identical to that of the shaft without the upward resistance (with UR, equal to zero, as shown in Figure 2). Additional parameters may be used to describe the soil resistance and

### EQUATION 3

$$E_p \frac{\partial^2 u}{\partial x^2} - \frac{S_p}{A_p} \cdot f_s = \rho_p \frac{\partial^2 u}{\partial t^2}$$

where:

$u(x,t)$  = Longitudinal displacement of infinitesimal pile segment

$A_p$  = Pile area

$S_p$  = Pile circumference

$E_p$  = Modulus of elasticity

$\rho_p$  = Unit density of the pile material

$f_s$  = Friction stresses

rebound ratio for unloading (which differs from loading). Further improvements are possible through better modeling of the physical phenomena — for example, considering the inertia of the displaced soil's mass.<sup>4</sup> Other improvements of the soil model itself are restricted due to:

- The limited available information from dynamic acceleration and strain response at one point of measurement; and,
- The poor ability to simulate soil following the complex loading sequences it undergoes during driving, set-up and static loading.

The signal matching process is depicted in the form of a flow chart as shown in Figure 3. The subscripts *msd* and *cal* denote measured and calculated values, respectively. Iterations are performed by changing the soil model variables for each pile element in contact with the soil until the best match between the force signals (calculated and measured) is obtained. The results of these analyses are assumed to represent the actual distribution of the ultimate static capacity along the pile. This procedure was first suggested by Goble, Likins and Rausche utilizing the computer program CAPWAP.<sup>5</sup> Similar analyses were developed by others utilizing the program code TEPWAP<sup>2,6</sup> and TNOWAVE.<sup>7</sup>

The linear elastic pile model and the obtained elasto-plastic "static" soil resistance dis-

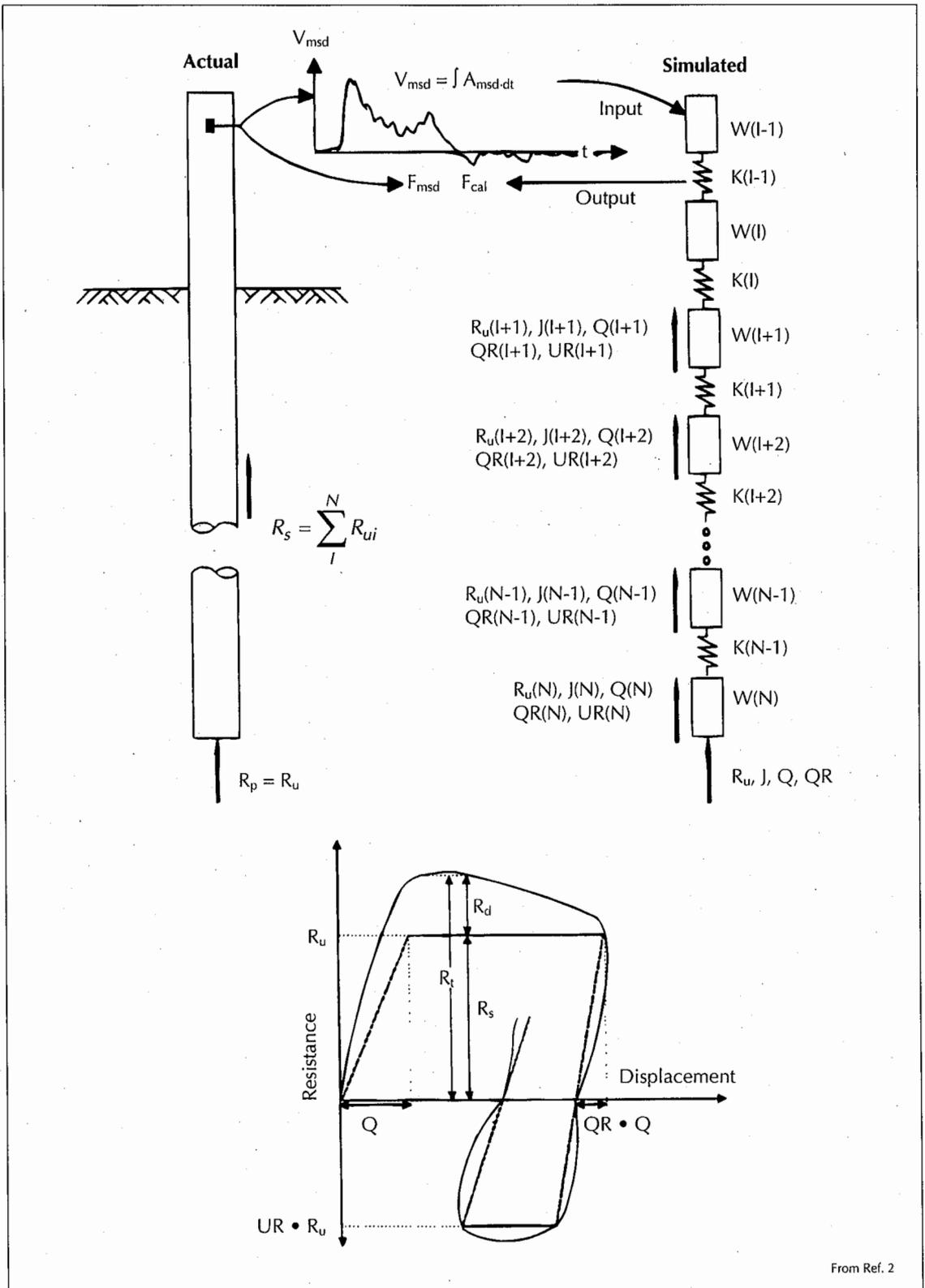
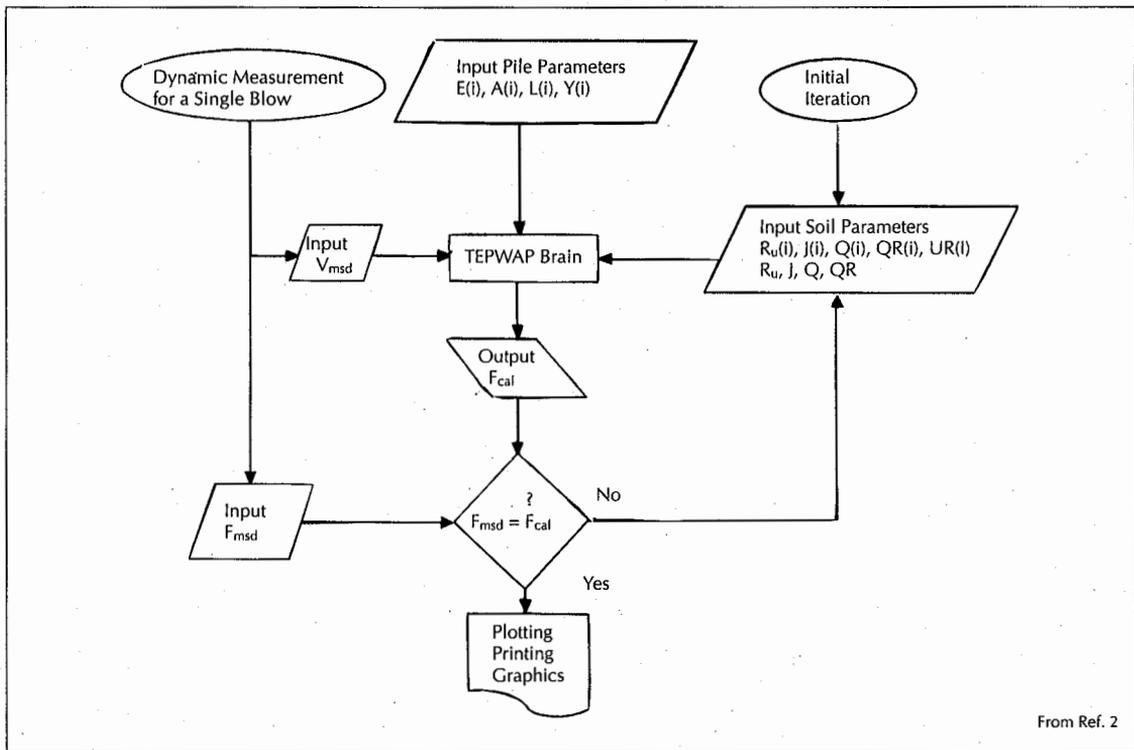


FIGURE 2. Notations used for modeling piles and soil in TEPWAP analysis.



From Ref. 2

FIGURE 3. Flow chart describing the analysis process using TEPWAP.

tributions can then be used to simulate a static load test using the load-transfer method.<sup>8</sup> The obtained load-settlement relations represent the pile response under static loads (excluding time effects).

*Capacity Evaluation in the Field.* The procedure of monitoring pile driving by dynamic measurements is well established. Early large-scale studies (for example, by the Michigan State Highway Commission in 1965,<sup>9</sup> the Texas Highway Department in 1973<sup>10</sup> and the Ohio Department of Transportation in 1975<sup>11</sup> — see also Highway Research Record<sup>12</sup> and Goble *et al.*<sup>5</sup>) led to the development of commercial systems that enable complete and relatively easy acquisition of dynamic measurements and analysis during driving. The pile driving analyzer (PDA),<sup>13</sup> which is the most commonly used device in this country, utilizes a simplified pile capacity evaluation method known as the Case method.<sup>5</sup> The formulation of the method is based on a simplification of the WE and employs the force and velocity measurements taken at the pile top in order to obtain the total resistance. The static resistance is then evalu-

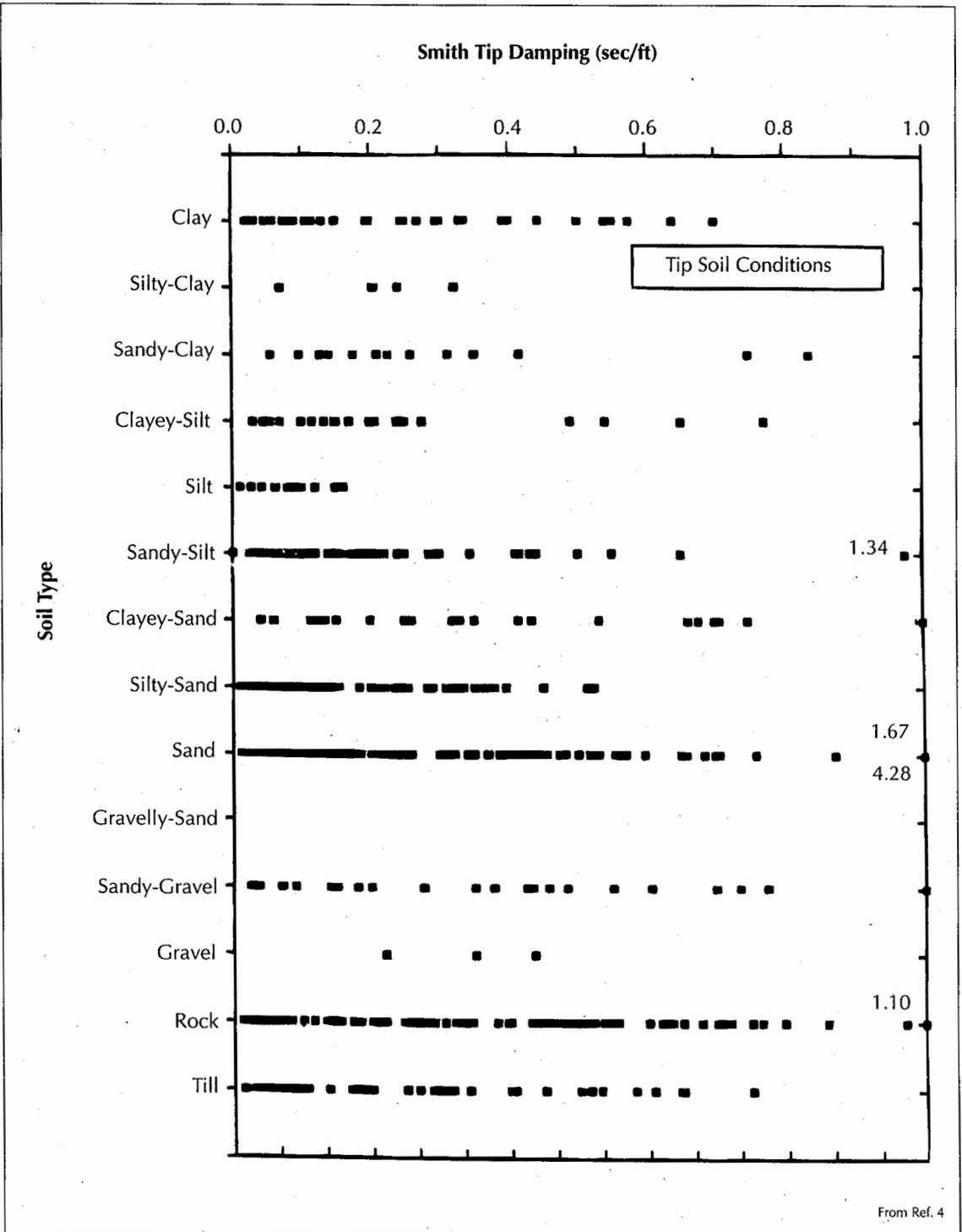
ated based on a dimensionless damping coefficient,  $J_c$  (Case damping), that was correlated to the soil type at the pile tip.<sup>14,15</sup>

### Encountered Difficulties

The dynamic analyses encountered three fundamental difficulties:

- Static versus dynamic resistance;
- Problem formulation; and,
- Time-dependent pile capacity.

*Static Versus Dynamic Resistance.* Due to very fast loading (the pile penetrates at approximately 150 cm/sec), the reacting forces to the penetration consist of dynamic components in addition to those that would be observed under pseudo-static loading conditions.<sup>16</sup> Hence, the dynamic analyses attempt to tackle the load rate effect, distinguishing between the two responses, thus predicting the pile behavior based on the estimated static components. The use of viscous damping and the assumption that it is dependent on soil type has no physical basis. Figure 4 presents the damping factors



**FIGURE 4. Tip soil conditions versus Smith tip damping based on CAPWAP or TEPWAP results for 581 pile cases.**

used to obtain wave matches for 581 CAPWAP and TEPWAP analysis cases.<sup>4</sup> The data in Figure 4 clearly show that no correlation exists between the soil type at the pile's tip and the

damping parameter. Similar relations to that shown in Figure 4 were obtained for the damping parameters along the pile's shaft.

*Problem Formulation.* A mathematical formulation of a physical problem inevitably requires a set of assumptions to be made. Some of those assumptions can either be wrong or invalid under certain conditions. The following are examples of such difficulties as encountered for the dynamic equations and the WE analyses:

1. The low reliability of the dynamic equations (see, for example, Housel<sup>17</sup>) is due to several reasons:

- Their parameters, such as the efficiency of energy transfer and the quake, are assumed and, therefore, may not reflect the high variability of the field conditions;
- The theoretical analysis of the "rational" pile formula (see, for example, Bowles<sup>18</sup>) relates the energy transfer mechanism to a Newtonian analysis of ram-pile impact. This formulation is theoretically invalid for representing the "elastic" energy transfer mechanism that actually takes place; and,
- No differentiation is made between static and dynamic soil resistances.

A clear distinction is required, therefore, between the underlying valid energy analysis and additional estimations of the different parameters, many of which are either invalid theoretically or practically limited in their accuracy.

2. The one-dimensional WE is a simplified case of the generic three-dimensional wave formulation for linear elastic material. This formulation is obtained under the assumptions that the waves are propagating in a homogenous prismatic shaped slender body. It is also assumed that, under loading, plane parallel cross sections remain plane and parallel and that a uniform distribution of stress exists across each plane. It should be noted that the assumption of uniaxial stress does not imply uniaxial strain. Thus, owing to Poisson's effect, there are lateral expansions and contractions arising from the axial stress

that are associated with lateral inertia.<sup>19</sup> The additional friction term in Equation 3 was included under the assumption that the soil is stationary (has no inertia effects) and the action of the friction forces does not violate any of the previous assumptions.

These underlying assumptions used for the theoretical description of the pile condition during driving are adequate for most practical purposes. These assumptions are violated, however, when analyzing open-ended plugged piles (and in some cases steel H-piles), resulting in an estimated capacity that is substantially lower than the actual capacity.<sup>6,20</sup>

The Case method is a further simplification of the one-dimensional WE for which the velocity of the pile head is calculated from the measured force, assuming that the pile head is free and the soil response is perfectly plastic in nature and time independent.

The Case method encountered two fundamental difficulties:

- The total resistance is dependent on the "impact" and reflection times and different variations of the method produce different results; and,
- The dimensionless damping coefficient was found to have questionable correlations to soil type that render the method impractical without site specific "calibration."<sup>2,4,21,22</sup>

*Time-Dependent Pile Capacity.* The static component obtained from the encountered resistance during driving is often different than the long-term static resistance of that pile. Set-up (freeze) or relaxation will cause the pile capacity determined from a static load-test to differ from the results obtained during the end of driving. A large study (sponsored by the Massachusetts Highway Department) is currently being carried out at the University of Massachusetts-Lowell in order to assess the gain of capacity with time based on static and dynamic methods.

Figure 5 shows an example of the change in the predicted capacity based on dynamic measurements as a function of time after driving. The obtained relations demonstrate the need to

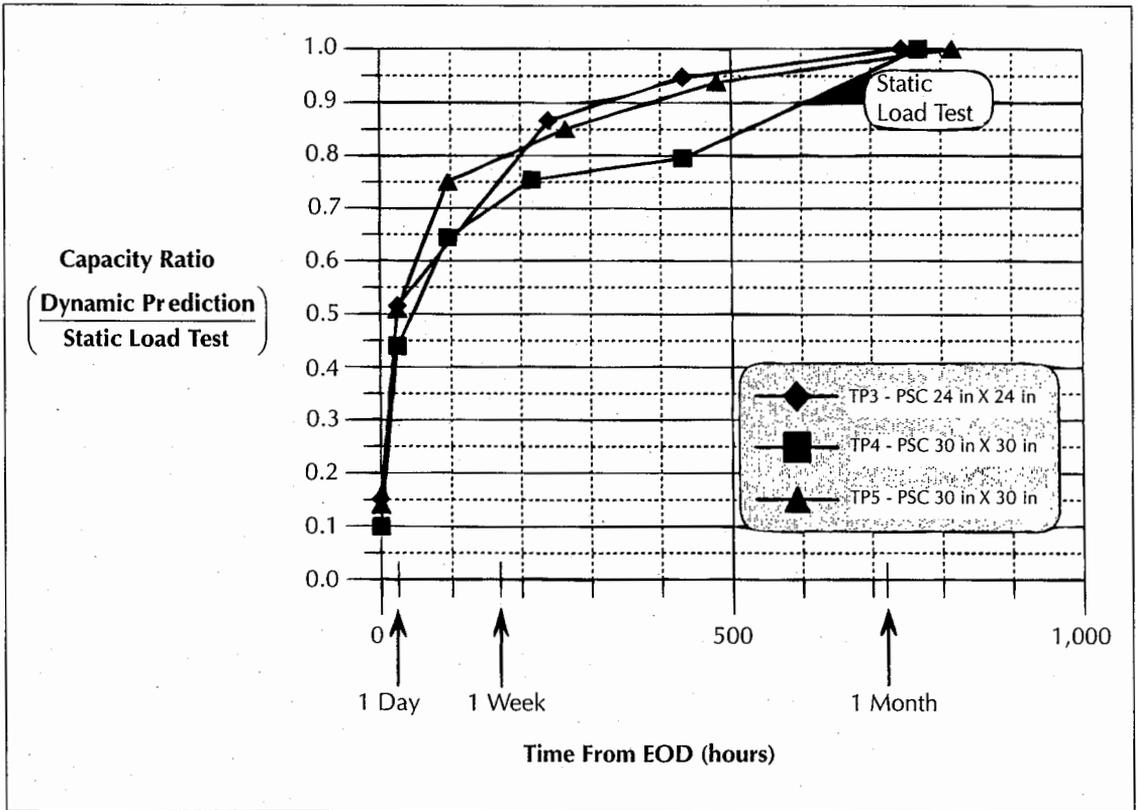


FIGURE 5. The ratio of dynamic predictions at the end of driving and following restrikes compared to static load tests.

adjust the dynamic prediction to the time of driving; otherwise, such analyses could be misinterpreted as poor and inadequate.<sup>23</sup> The dynamic analysis of records at the end of driving and subsequent restrikes resulted in a rate of capacity gain with time that matches the results that were obtained from static load tests.<sup>24</sup> The implementation of time adjustments is not discussed further here and Figure 5 serves only as an illustration for the time dependency issue.

### Alternative Methods

One-dimensional WE analyses might not be adequate for field analysis or appropriate for the analysis of complex piles.

*Simplified Field Analysis.* In order to simplify field analysis and increase its accuracy, the energy approach method could be adopted. An energy balance must exist between the total energy transferred to the pile and the work done by the pile-soil system. This relationship

is true for the dynamic equations as well as for the WE type of analyses. Both analyses assume static resistance to follow elasto-plastic force-displacement relations. While the WE formulation distributes the resistances along the pile as it simulates the propagation of the stresses, the energy balance equation lumps it for the entire system. Although the losses due to the dynamic phenomena can be taken into consideration by viscous damping, they are not represented in the dynamic equations since the particle velocity along the pile cannot be evaluated.

In practice, however, the dynamic component of the resisting forces (even though represented by viscous damping) accounts for other energy losses such as radiation, soil inertia, true damping and more. These energy losses are determined by the combination of pile shape and the surrounding soil type in addition to the penetration velocity. The WE type of solutions (including CAPWAP and TEPWAP) consider the damping at each depth and, therefore, indi-

rectly account for the energy losses in the different pile types and surrounding soils. As a result:

- Very little correlation can be found between the soil type and the damping parameters (true for Smith damping and even more so for the Case method damping);<sup>4</sup>
- While the total capacity of the pile can be found accurately by analyses like CAPWAP (since it matches the energy delivered to the work produced), the distribution of the resistances is not necessarily accurate;<sup>2,25,26</sup> and,
- The parameters used to obtain matches of energy delivered to work produced will be a function of pile type, especially large versus small displacement (for the difference in parameters between small to large piles, see, for example, Liang<sup>27</sup>).

Equation 2 can be used as the basic energy balance equation. However, the parameters of this equation are replaced by measured values obtained during driving and, hence, can be used in conjunction with the PDA.

The energy delivered is taken as  $E_d$  equal to  $E_{max}$ , the maximum energy obtained by the maximum value of Equation 4.

The quake,  $Q$ , is evaluated from the difference between the maximum pile top displacement and the permanent set at the top (see Equation 5).

In Equation 5, the maximum displacement is obtained by the maximum value from Equation 6.

The set can be obtained by the final displacement of the integrated velocity signal for the full measured time (see Equation 7).

However, in practice the displacement is already the second integration of a measured value (acceleration) and the accuracy of the final displacement is questionable. The use of the field blow count is, therefore, recommended for the set.

The maximum resistance under the above assumptions is obtained by Equation 8 – the energy approach equation. First developed by the author,<sup>2</sup> further examination regarding the suitability of its use was carried out as part of

#### EQUATION 4

$$E_{d(t)} = \int_0^t V(t)F(t)dt$$

where:

$V(t)$  = Velocity signals at the pile top

$F(t)$  = Measured force at the pile top

$E_{d(t)}$  = Delivered energy as a function of time

#### EQUATION 5

$$Q = D_{max} - S$$

where:

$D_{max}$  = Maximum pile top displacement

$S$  = Permanent set

#### EQUATION 6

$$D(t) = \int_0^t V(t)dt$$

where:

$D(t)$  = Pile top displacement as function of time

#### EQUATION 7

$$D_{fin} = \int_0^{t_{max}} V(t)dt$$

where:

$t_{max}$  = Time length of the velocity signal

#### EQUATION 8

$$R_u = \frac{E_{max}}{Set + \frac{D_{max} - Set}{2}}$$

studies for the Federal Highway Administration (FHWA) and the Massachusetts Highway Department.<sup>4,22,25</sup>

Equation 8 takes into account only the elastoplastic energy losses of the pile-soil system and can be regarded as yielding the maximum pos-

### EQUATION 9

$$R_s = K_{sp} \cdot R_u$$

where:

$R_s$  = Static pile capacity

$K_{sp}$  = Static pile capacity coefficient

sible resistance. Examination of the predicted static capacity is achieved via a single correlation factor — which is the ratio between the actual capacity to the predicted one. This ratio can also be viewed as a factor that represents all dynamic related energy losses in the soil (see Equation 9).

Table 1 summarizes the performance of the simplified energy approach compared to that

of the wave matching technique for 95 piles related to 206 pile cases (including restrrike analyses). The analyzed cases include a wide range of pile types and sizes driven in a wide range of soil types. The data consist of all available cases for which good quality dynamic measurements and static load test were available. All piles were statically loaded to failure and the ratio between the predicted to the actual static capacity is denoted by  $K_{sw}$  and  $K_{sp}$  for the wave matching techniques and the simplified energy approach, respectively. The energy approach method provides more accurate predictions than the elaborate wave matching analysis, especially for construction purposes for which the end of driving (EOD) predictions proved to be highly accurate. A risk analysis carried out on that data suggests that, using the

### EQUATION 10

$$\frac{\partial^2 u_z}{\partial t^2} = k_1 \cdot \frac{\partial^2 u_z}{\partial z^2} + k_2 \cdot \frac{\partial^2 u_r}{\partial r \partial z} + k_2 \cdot \frac{1}{r} \cdot \frac{\partial u_r}{\partial z} + k_3 \cdot \frac{\partial^2 u_z}{\partial r^2} + k_3 \cdot \frac{1}{r} \cdot \frac{\partial u_z}{\partial r}$$

where:

$$k_1 = \frac{\lambda + 2\mu}{\rho}$$

$$k_2 = \frac{\lambda + \mu}{\rho}$$

$$k_3 = \frac{\mu}{\rho}$$

$z$  = Longitudinal coordinate

$r$  = Radial coordinate

$t$  = Time

$u_z$  = Axial (longitudinal) displacement

$u_r$  = Radial displacement

$\rho$  = Material unit density

$\mu, \lambda$  = Lamé constants

### EQUATION 11

$$\frac{\partial^2 u_r}{\partial t^2} = k_1 \cdot \frac{\partial^2 u_r}{\partial r^2} + k_1 \cdot \frac{1}{r} \cdot \frac{\partial u_r}{\partial r} - k_1 \cdot \frac{1}{r^2} \cdot u_r + k_2 \cdot \frac{\partial^2 u_z}{\partial r \partial z} + k_3 \cdot \frac{\partial^2 u_r}{\partial z^2}$$

**TABLE 1**  
Summary of Large-Scale Data Analysis

Driving Time	Ksw = Load Test/(CAPWAP or TEPWAP)			Ksp = Load Test/(Energy Approach)		
	Number of Cases	Mean	Standard Deviation	Number of Cases	Mean	Standard Deviation
All Times	206	1.367	0.533	208	0.925	0.293
EOD	95	1.441	0.619	96	1.001	0.316

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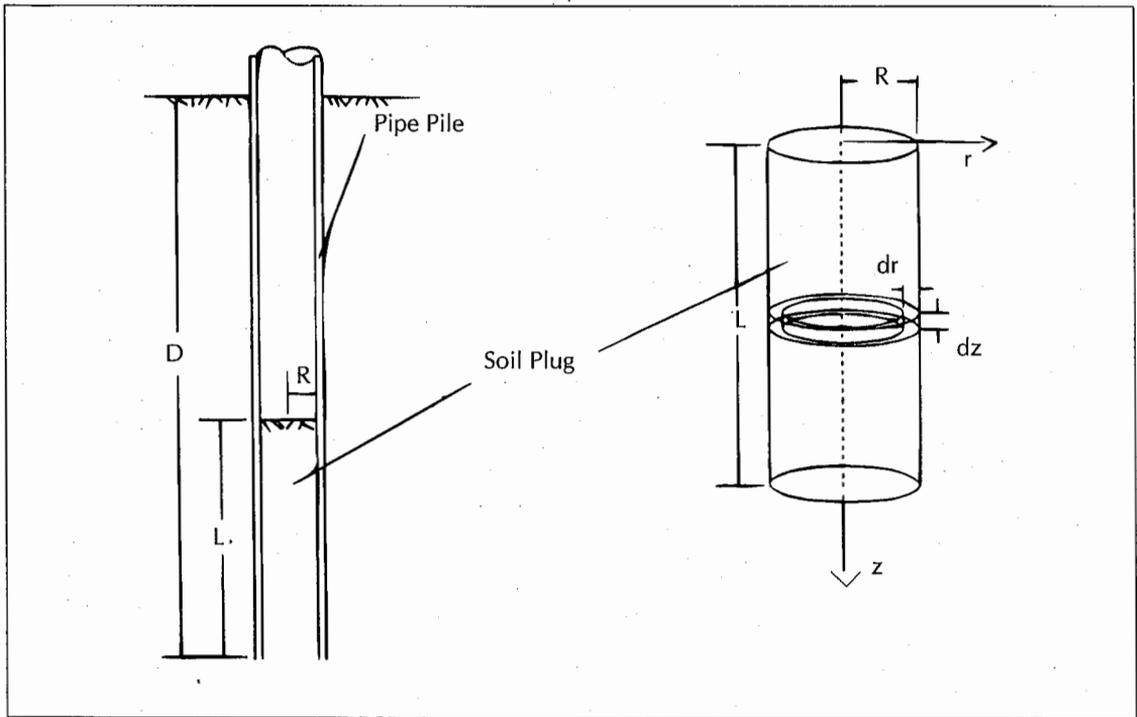


FIGURE 6. A geometric representation of the pile-plug system as a cylinder within a pipe.

energy approach, a factor of safety of 2.0 for the EOD records would be associated with a zero risk. Capacity gain effect was not considered in the analyses summarized in Table 1. As a result, the data of Figure 5, for example, would be included for all the measurements (EOD and restrikes).

*Complex Plugging Formulation.* Open-ended piles are the preferred solution in coastal and offshore construction. The assumptions that lead to the one-dimensional WE are violated in the case of driving open-ended plugged piles. A complete new formulation was developed, founded on the notion that the pile-plug system can be thought of as a cylinder within a pile (see Figure 6). The governing equations of motion for slender elastic cylinders are shown in Equations 10 and 11.<sup>28</sup>

The formulation of boundary conditions, corresponding equations of motion and their numerical solutions are not discussed here. A well documented case study carried out by a consortium of oil companies in Empire, Louisiana, is used to demonstrate the effectiveness of the solution in light of the limitations of the one-dimensional WE.<sup>29-31</sup> The analyses pertain

to test pile No. 4 — a 388-foot long, 14-inch diameter (0.544-inch wall thickness) open pile driven below an 18-inch casing to a depth of 320 feet. Figure 7 presents the measured force and velocity signals at the pile top at the end of driving. The best possible match between the force measured to the force calculated using TEPWAP is shown in Figure 8. This analysis resulted in a static capacity evaluation of 140 kips. An independent CAPWAP analysis on the same blow showed a similar (poor quality) match as the one presented in Figure 8 with the same static capacity prediction. Figure 9 shows the TEPWAP best match when the plug mass was lumped together with that of the pile (based on full plugging conditions).<sup>16</sup>

Though an excellent match was obtained between the calculated and the measured forces, the analysis resulted in a static capacity of 140 kips, identical to that obtained for the wave match in Figure 8.

Figure 10 presents the wave match results of the Plug Wave Analysis Program (PWAP) based on the aforementioned formulation. This match does not differ much from that presented

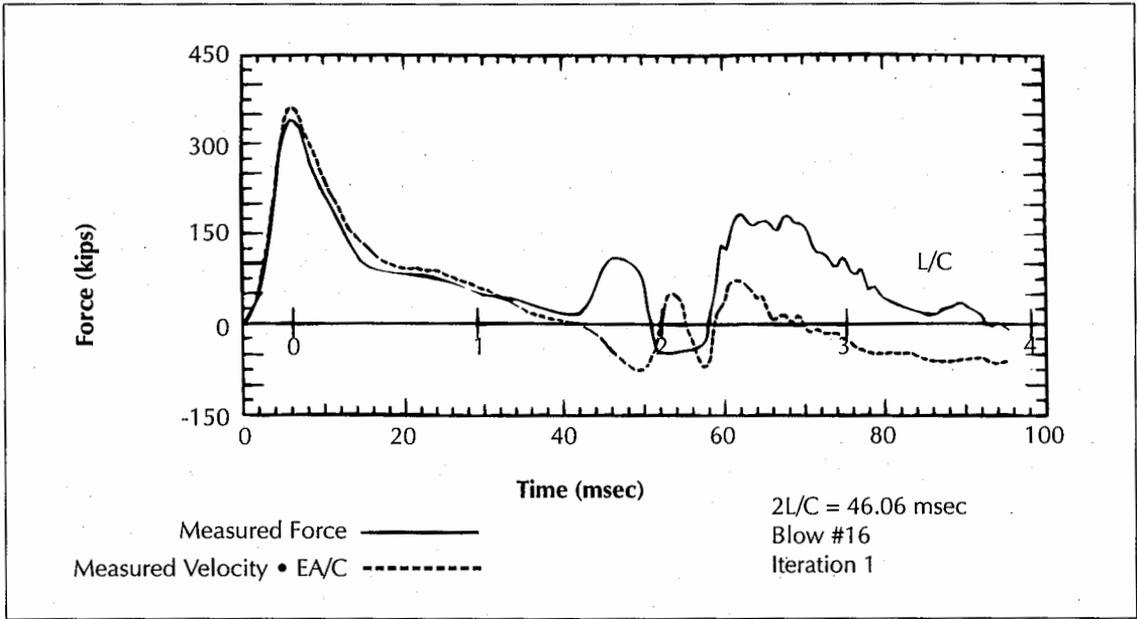


FIGURE 7. Measured force and measured velocity times the pile impedance ( $V \cdot EA/C$ ) at the pile top versus time at the EOD for Empire Site Test Pile No. 4.

in Figure 9. However, it resulted in a capacity prediction of 396 kips and a predicted load settlement relation that matches very nicely the two load tests carried out in the field as indicated in Figure 11.

### Summary & Conclusions

The dynamic analyses based on measurements obtained during pile driving are powerful methods that permit the evaluation of pile re-

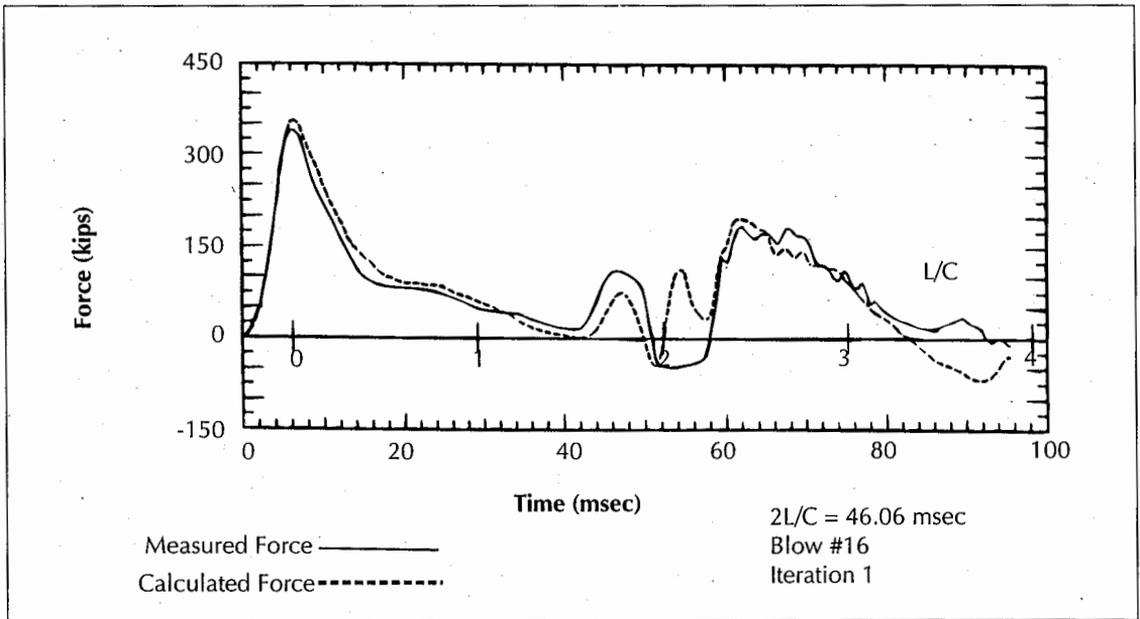
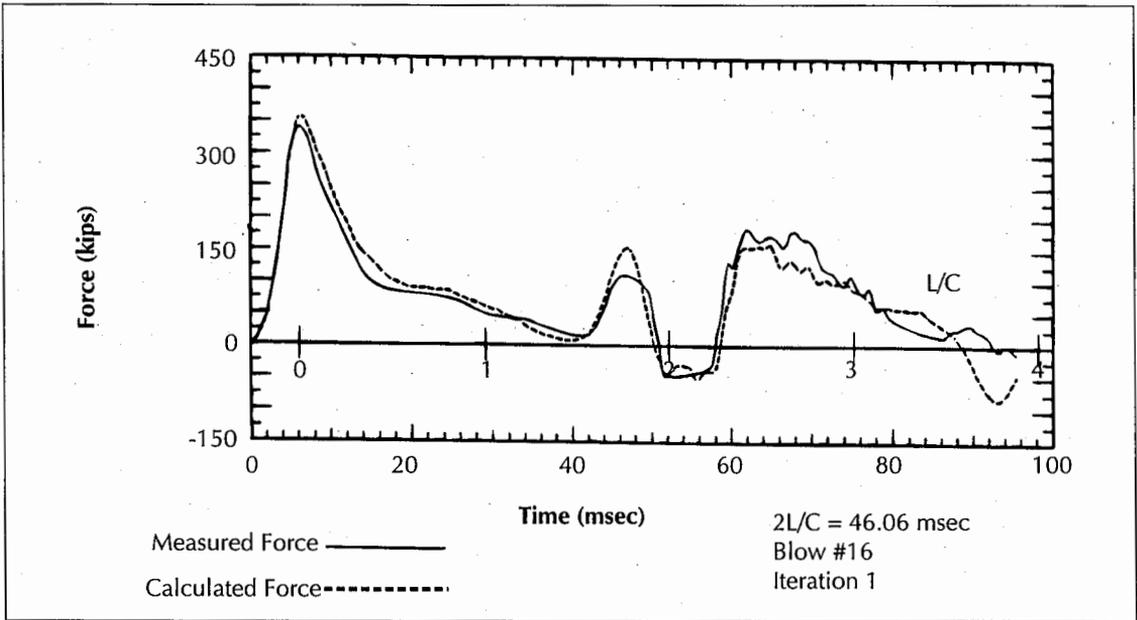


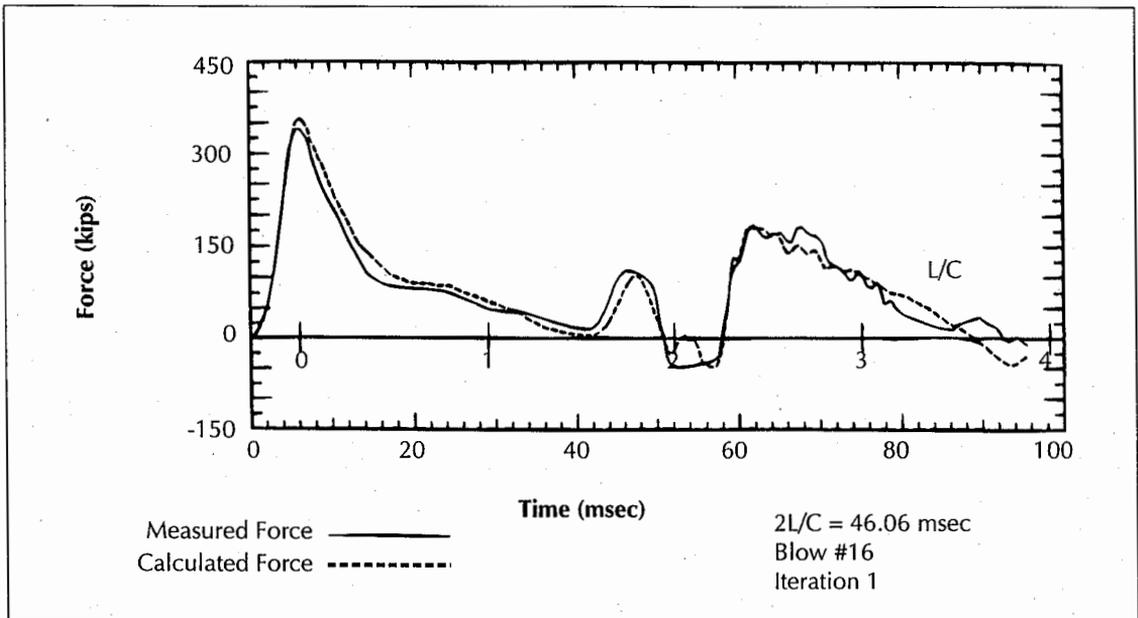
FIGURE 8. Measured force and force calculated by TEPWAP (best match) at the pile top versus time using the data presented in Figure 7.



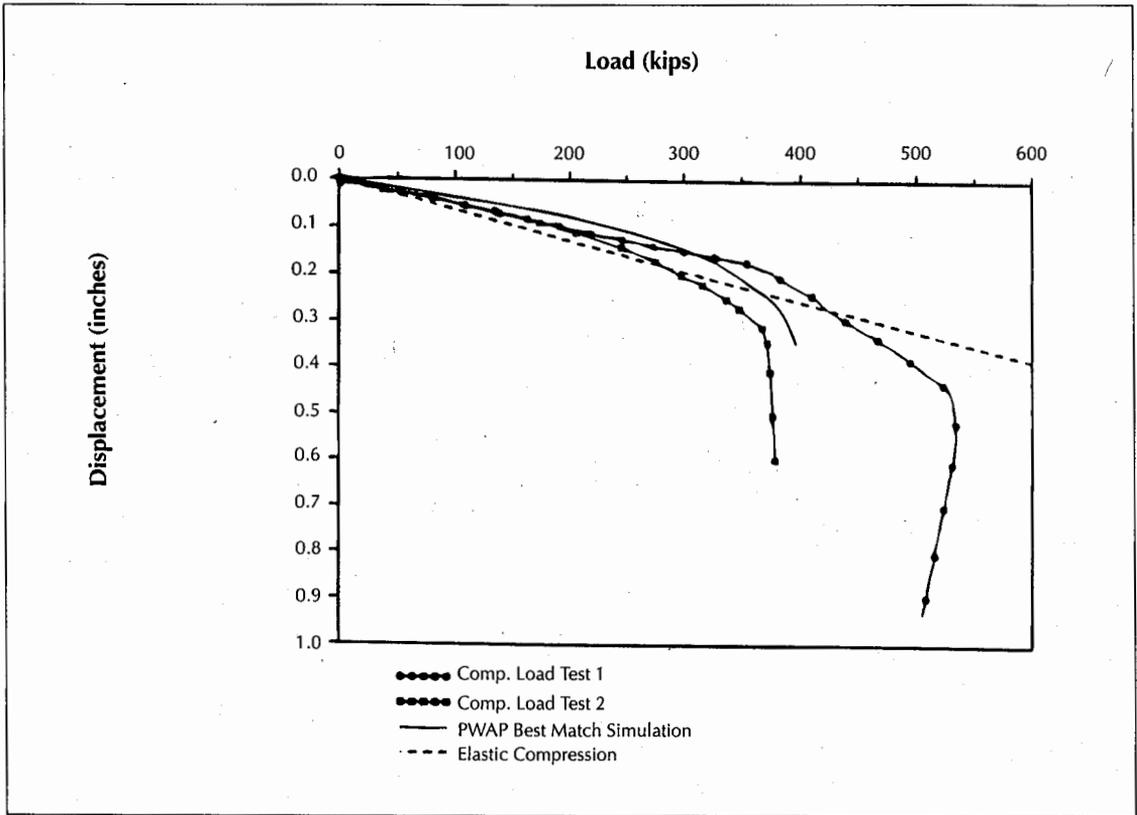
**FIGURE 9. Measured force and force calculated by TEPWAP (lumping the plug's mass together with that of the pile) at the pile top versus time using the data presented in Figure 7.**

sponse under static loads. Accurate analysis during and after driving have great economic and safety benefits for design and construction. Their utilization, however, encountered difficulties resulting from:

- Load rate effect;
- Underlying assumptions of the fundamental formulations; and,
- Time-dependent pile capacity.



**FIGURE 10. Measured force and force calculated by PWAP (best match) at the pile top versus time using the data presented in Figure 7.**



**FIGURE 11. Load test simulation for Empire Test Pile No. 4 as obtained from PWAP analysis compared to the actual static load test.**

In order to overcome some of these difficulties, two alternative methods of analysis were presented. The first was a simplified method called the energy approach, which allows instantaneous field analysis during driving. The evaluation of the method was carried out on 95 piles and 206 pile cases and resulted in its validation as an accurate analysis method (superseding the accuracy obtained by the wave matching techniques). The wave matching techniques were demonstrated to be extremely effective in monitoring the capacity gain with time (although the data presented here are limited in scope).

The second method presented here is a complex analysis based on the rigorous solution of the plug's equations of motion. Since the physical phenomena of a driven plugged pipe pile violate the underlying assumptions of the one-dimensional WE, analyses based on that formulation lead to erroneous results — even when a very close match between the measured and

calculated signals can be obtained. The proposed solution that more accurately models the physical phenomena leads to excellent agreement between the measured and calculated load settlement relations.

The simple and complex analysis methods presented here clearly illustrate the need to question the theories and go back to the original assumptions. The prediction of pile capacity using dynamic measurements requires experience, engineering judgment and a fundamental understanding of the assumptions and methodology. Therefore, in no way should it be considered to be a routine automated procedure.

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