In the Footsteps of Giants: The History of the Founders of Earth Pressure Theory From the 17th Century to the Late 19th Century

Studying the work of early engineers on earth pressure and soil behavior can provide a broader and deeper understanding of how to apply earth pressure theory.

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H.Q. Golder once wrote: "Misconceptions of engineering theories arise when we forget the assumptions and postulations on which a theory is based."[1] This statement is particularly appropriate to the ambiguous and controversial "Earth Pressure Theory" that is often carelessly interpreted and used in present engineering work.

Presenting the history of the founders of earth pressure theory (as applied to conventional retaining walls) will help shed some light on this obscure engineering subject. Reviewing these precursors lives, their educational and professional history, and their contributions to general science and civil engineering will help present-day engineers gain a better understanding of the assumptions behind earth pressure theory and, it is hoped, a better appreciation of how to apply the theory.

Ancient Practices

Earth retaining structures are found in the archeological excavations of several ancient civilizations. In Mesopotamia, the hanging gardens of Babylon were built within retaining walls. Major ancient cities were surrounded by walls that had fills placed against the inside faces of the walls. The Romans built fortifica-

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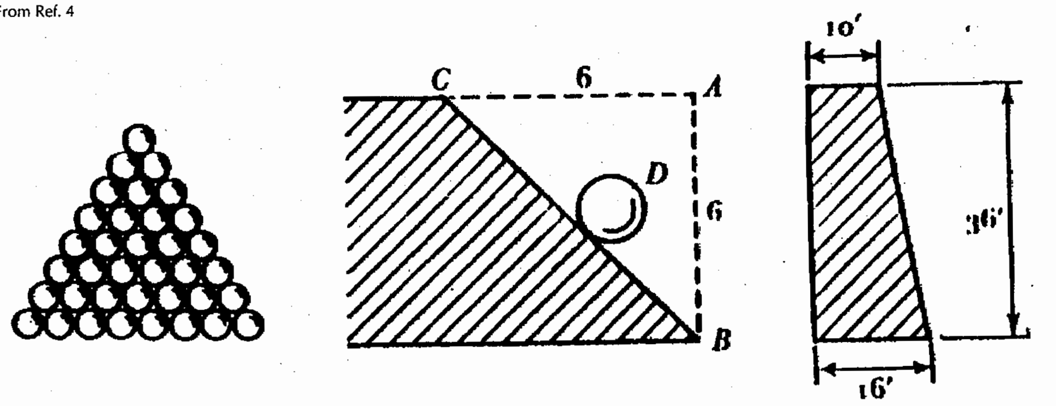

FIGURE 1. Bullet's representation of soil structure and his design parameters for a retaining wall.

tion and bridge walls. Some of these earth retaining structures have withstood the ravages of human and natural actions over time. It is believed that these structures were, most likely, built from knowledge obtained from previously failed structures, rules of thumb or even small-scale prototypes.

Evolutionary Background

It is presumed that the design of retaining walls started with the construction of military fortifications, where the soil mass behind these walls probably caused less distress than did the cannonballs impacting them and the surcharges from military maneuvers. In the 17th century, the French military officer Maréchal Sébastien le Pestre de Vauban (1633-1707) gave empirical rules for building fortification walls from 2 to 25 meters in height.[2] In the late 18th century, the earth pressure issue was brought to the fore due to the construction of canals in Europe. These projects involved large cuttings and embankments and, consequently, earth retaining structures. The development of railways in the 19th century stressed the importance of the earth pressure problem. And, in modern times, the design and construction of various earth retaining structures has skyrocketed.

Early Theories

Pierre Bullet (1639-1716), a member of the French *Académie Royale d'Architecture*, is thought to be the first person to search for an earth pressure theory based on the general principles of mechanics.[3,4] Born in a family of architects, Bullet became Architect of the city of Paris and produced a surveyed map of Paris at the age of 37.[5] He was responsible for the production of construction plans for several buildings, churches and public fountains.

Bullet represented the structure of sand and gravel by a "heap of shot" composed of small frictionless spheres (see Figure 1). Based on arbitrary geometrical assumptions, Bullet's design principles are incorrect. However, his conception that soil consists of spherical particles served as the foundation for the development of subsequent theories and even modern research pertaining to soil mechanics (such as granular soil modeling with glass beads and stochastic packing of granulated materials).

In a 1726 paper to the French *Académie*, Pierre Torteaux de Couplet (?-1744) presented his own earth pressure theory. Couplet believed that Bullet's shot pattern did not represent the most dense packing condition of soil. He believed that equal spheres packed in a tetrahedral shape represented a soil mass in equilibrium.[6] Using that assumption, he determined that the height of such self-supported soil mass is in a 2.8:1 ratio to its base (horizontal:vertical slope). Then, by geometrically decomposing the gravity forces exerted by these spheres on the back of a wall, he calculated the soil thrust against the wall and determined the thickness of a rectangular retaining wall.

EQUATION 1

$$Q = \frac{gh^2}{8}$$

Where:
h = Wall height
g = Earth density

He formulated that the soil lateral pressure, Q, acted at the upper third point of a wall (see Figure 2) and could be expressed as shown in Equation 1. Couplet calculated the base, b, of the wall to be almost equal to half of its height ($b = 0.47h$). Incidentally, this dimensioning is similar to the modern design guidelines for walls.

Another "practical" theory was presented by Bernard Forest de Belidor (1698-1761), a French military and civil engineer. Belidor became an orphan when he was five months old and was adopted by an artillery officer who provided him with early education. When his adoptive father died in 1711, his uncle — then Chief Engineer of the Montreuil region — took Belidor under his private care. When he was 15 years old, Belidor assisted in several sieges while continuing his private studies. The scientific schooling fashioned him into a mathematician and the military operations into an engineer. He applied mathematics to the analysis of cannon-ball trajectories, soil thrust against walls and masonry arches.[7] Named Professor at *L'École d'Artillerie de La Fère,* he soon acquired a European reputation for his geodetic work as well as for his civil and military engineering teachings. He was also member of the academies of science of France, England and Prussia.

From a personal aspect, Belidor's dedication to learning knew few bounds. He was reported to have even paid for soldiers' leave from his own money in order to allow them to study engineering.

In his book *La science des ingénieurs* (1729), Belidor established, from practical observations, that the angle of repose of soil (loose earth) was 45 degrees (see Figure 3) and that the horizontal soil pressure, Q, which acted at the lower third of the wall, could be expressed as stated in Equation 2.[3,6]

He then proceeded, like Couplet, to determine the thickness of a rectangular retaining wall with a base, b, equal to three-quarters of its height. This dimensioning is, inexplicably, more conservative than his predecessor's.[6]

Military and civil engineering practices in the mid-17th century France were more advanced than those in other countries. Vauban's tables for fortification wall dimensions and Belidor's engi-

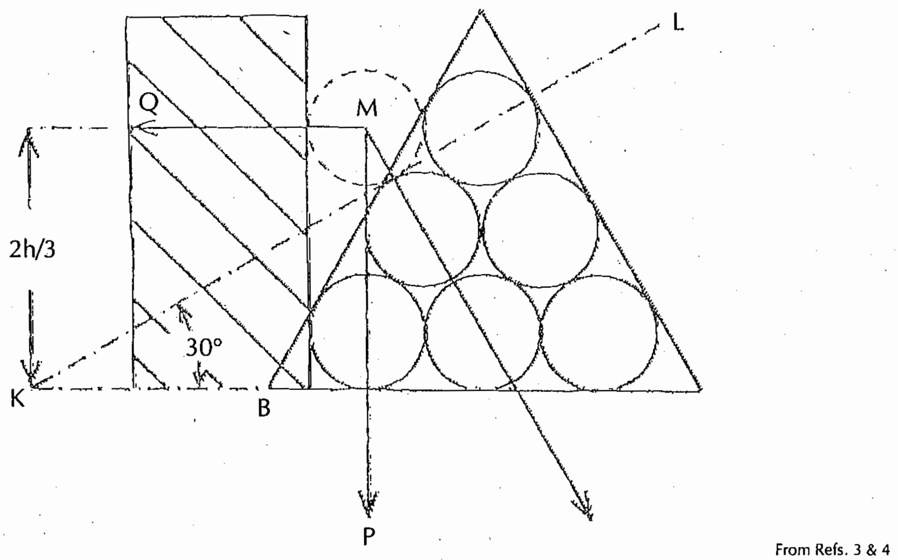

FIGURE 2. Couplet's earth pressure theory and his formulation of soil thrust on a wall.

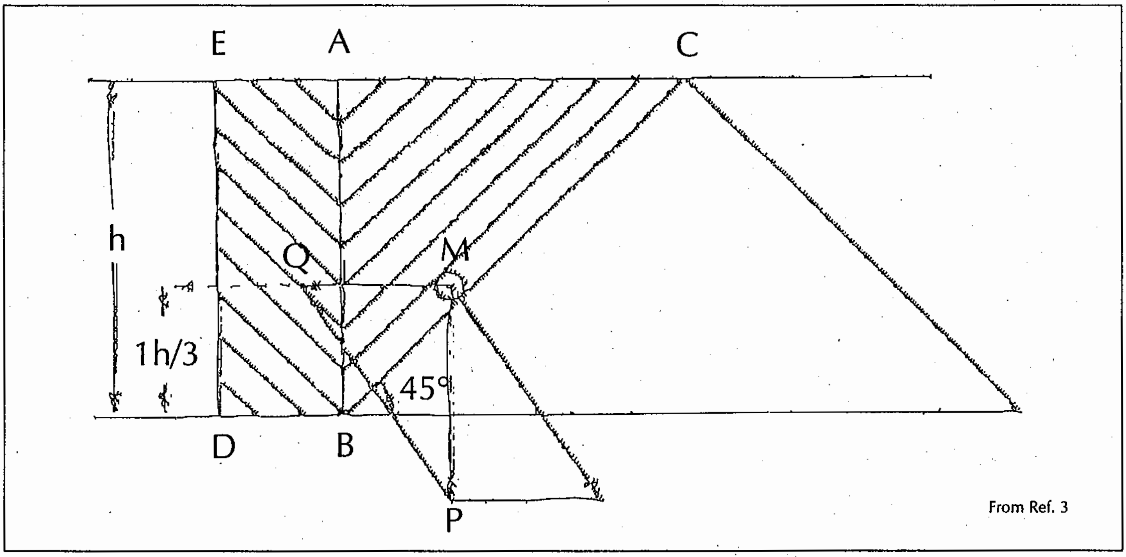

FIGURE 3. Belidor's soil pressure resultant acting at the lower third of a wall.

neering manual were in common use. These design methods, however, were based on rules of thumb and did not take into account actual soil properties. Although these theories were based on questionable experiments and geometrical assumptions, they remained mostly unchallenged until the late 18th century.

Coulomb's Theory

Charles Augustin Coulomb was born at Angoulême in France on June 14, 1736. He entered the military profession as an engineer, serving for 12 years in the Caribbean islands of Martinique, Guadeloupe and Santa Lucia. He was also a theoretician, devoting his attention to very diverse subjects, such as electricity, magnetism, torsion and strength of materials. Coulomb's military engineering field experience, blended with his mathematical education, caused him to solve engineering problems through a "*mélange du calcul et de la physique*" (combination of mathematics and physics).[3,4] This process led to one of his not well known works, which was his "Wedge Theory of Earth Pressure," developed more than 220 years ago.

EQUATION 2

$$Q = \frac{gh^2}{4}$$

Where:
h = Wall height
g = Earth density

On March 10, 1773, Coulomb presented the paper titled "Essay on the Application of the Rules of Maxima and Minima to Some Problems in Statics" to the French Academy of Sciences (to which he was elected in 1786).[8] This essay contained fundamental propositions in the fields of strength of materials, soil mechanics and arch design. Coulomb's theory pertaining to the rupture of masonry piers dealt with construction materials, which he observed to fail in sliding along a plane surface. The conceptual similarity between this theory and the theory of earth failure behind retaining walls is apparent in Coulomb's essay (see Figure 4).

Instead of utilizing the concept of an angle of internal friction, ϕ, of the soil, Coulomb used two soil parameters: a coefficient of friction, n, (where $n = \text{cotg}\phi$) and a cohesion, δ, which he assumed to be constant. Coulomb theorized that, given an indefinitely long supported mass of soil and its unit weight, friction coefficient and cohesion, the horizontal thrust acting normally to the back of the retaining wall could be determined.

Coulomb's basic findings are summarized as follows:

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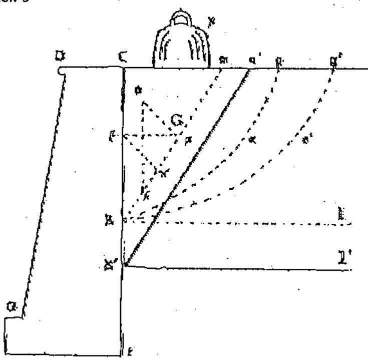
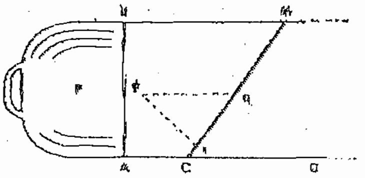


Coulomb's Figure Illustrating His Earth Pressure Theory



Coulomb's Figure for Rupture of Masonry Piers (Rotated)

FIGURE 4. Similarity between soil wedge and masonry pier failures in Coulomb's memoir.

- Considering the statics of a wedge of earth such as *CBa* (see Figure 5 on the next page), he obtained an expression for the horizontal force, *A*, which would keep the wedge in equilibrium when the full shear resistance was mobilized along *Ba*. He differentiated his expression for the force, *A*, equated the result to zero and finally obtained the particular expression for the "active earth pressure" on the back of a vertical wall (with no wall friction and no surcharge load) as shown in Equation 3, which can be reduced to the modern trigonometrical form as shown in Equation 4. Coulomb gave examples showing that the expression gave design parameters for walls in accordance with then-accepted practice.
- He showed that force, *A*, could increase to a value, *A'*, and that equilibrium would be maintained until this value was exceeded when the wedge would move up the plane *Ba*. Coulomb's force, *A'*, is what is now called the *passive earth pressure*.

EQUATION 3

$$A = ah^2 - b\delta h$$

Where:

h = The height of the soil wedge
a, b = Functions of n

From Ref. 6

EQUATION 4

$$A = 0.5\gamma \frac{1-\sin\phi}{1+\sin\phi} h^2 - 2c\frac{1-\sin\phi}{\cos\phi} h$$

Where:

γ = Soil weight unit
c = Soil cohesion

EQUATION 5

$$w = 15\frac{h}{100}$$

Where:

w = Wall thickness

- He realized that the surface of rupture is curved but assumed a straight slip line. He stated that cohesion has no influence on the position of the plane of failure and that for a purely cohesive material the plane is inclined at 45 degrees.
- The case of wall surcharge (see Figure 5) was solved along with the case of wall friction. He realized that wall friction reduces the total pressure and increases the wall stability. He did not, however, consider walls with inclined backs or sloping backfills. Coulomb deduced the design rule as shown in Equation 5. Therefore, according to Coulomb's theory, a wall 1 meter thick would support a horizontal fill 7 meters high. This design rule is not a conservative dimensioning for a wall, in comparison with modern rules, even with a front batter wall.
- He also discussed the precautions in dimensioning a battered wall without addressing practical soil-structure issues (friction of soil on masonry, hydrostatic

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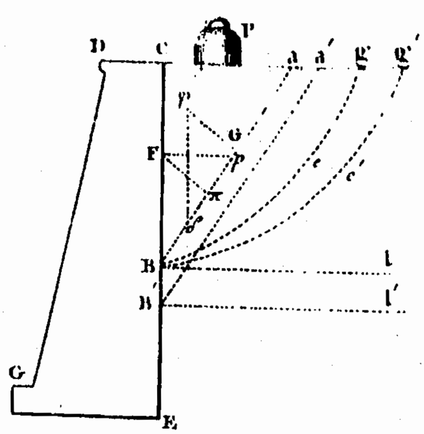
EXEMPLE.

Si l'adhérence δ eſt ſuppoſée nulle, comme dans les terres nouvellement remuées; ſi le frottement eſt égal à la preſſion, comme dans toutes celles qui prennent 45^d de talus naturel, abandonnées à elles-mêmes; ſi n eſt ſuppoſé égal à v, l'on trouvera pour lors $A = \frac{gx}{4}(a - x)$, & $x = \frac{a}{2}$, l'angle $CBa = 36^d\ 34'$, & $A = \frac{ga^2}{16}$, le *momentum* total de la preſſion des terres autour du point G ſera $\frac{b^3}{3.16}$; d'où l'on tireroit, pour un mur de terraſſe ſans talus, dont l'épaiſſeur ſeroit c, en ayant égard à la réaction du frottement qui contribue à augmenter le *momentum* de la réſiſtance du revêtement, de la quantité $\frac{cb^2}{16}$, l'équation

$$\frac{b^3}{3.16} = \frac{ccb}{2} + \frac{cb^2}{16};$$

& par conſéquent $C = \frac{15}{100}\, b$, à peu-près; c'eſt-à-dire qu'un mur de trois pieds de largeur ſeroit, dans cette hypothèſe, en équilibre avec la pouſſée d'une terraſſe de vingt pieds de hauteur.



From Ref. 4

FIGURE 5. An excerpt from Coulomb's memoir "On Earth Pressure and Retaining Walls."

pressure of percolating water, wet soil effect on friction, frost action). He stated that it was safe to construct a wall with a front batter of 1:6 and a thickness at the top equal to 1/7th of the height.

Coulomb died on August 23, 1806. He attained such positions and honors as *Lieutenant-Colonel des Ingénieurs, Chevalier de l'Ordre de Saint Louis,* Member of the *Légion d'Honneur* and of *L'Académie des Sciences.*

Coulomb's geometrical approach to the earth pressure problem prompted other engineers in France and Germany to develop geometrical constructions to determine the earth pressure thrust.

Later Works on Earth Pressure

In 1794, the Prussian engineer Reinhard Woltmann published an essay on earth pressure theory using the same nomenclature as Coulomb's.[9] Woltmann's work, which included experiments conducted on a large-scale wall apparatus, was sponsored by the Royal Scientific Academy of Petersburg. It is likely that no reference was given to Coulomb's work because of the contemporary war with revolutionary France. Woltmann was the first to express the earth pressure thrust in its modern trigonometrical form, omitting the soil cohesion term.

Coulomb's findings were not applied until they were used by another French engineer, Baron Riche de Prony (1755-1839). Having completed a brilliant science education, Prony participated, at the age of 28, in the design and construction of two major bridges, and in the rehabilitation works of the port of Dunkerque. In 1791, he was in charge of the cadastral works necessitated by the establishment of the metric system by the French Revolution. He became Professor of mathematical sciences at the famous *L'École Polytechnique* (in 1794) and director of *L'École des Ponts et Chaussées* (in 1798).

In his book *Rechèrches sur la poussée des tèrres* (1802), Prony introduced the use of angles rather than lengths (as used in Coulomb's equation), simplified the analysis of earth pressure and presented a graphical method of designing retaining walls. He proved that the plane of rupture bisects the angle between the vertical and the line of natural slope.

Prony applied the inclined plane theory and the rule of maximum to calculate the earth thrust against a wall and the corresponding angle of rupture.[10] He taught his findings on the dimensioning of earth retaining structures in a class entitled "Public Work Constructions."

In 1808, M. Mayniel, a French military engineer, wrote the most comprehensive book about earth pressure theory, wall experiments and practical design rules.[6] He critically reviewed earlier theories and experiments, from Bullet to Coulomb. He was the first to include the effect of wall friction in the trigonometrical form of the earth pressure equation. He also provided design rules and examples for wall design and construction, including buttressed retaining walls. He observed failed walls and concluded that they mainly slide or overturn at their interface with the bearing soil and that the main cause for later failures was that soil excavation had exposed the wall toe.

In 1840, Jean-Victor Poncelet (1788-1867), a French mathematician and military engineer, published his memoir on the stability of walls and their foundations. Admitted at an early age to school, he completed two grades per year and entered, at the age of 19, *L'École Polytechnique*, and then graduated as Lieutenant Engineer from *L'École de Metz*. He joined Napoleon's army expedition to Moscow. A prisoner of war at Saratov, he recapitulated his mathematical studies for 18 months from memory. During this intellectual solitude, he established the bases of modern "projective geometry." In 1825, as Professor of Applied Mechanics at *L'École de Metz*, he worked on researches and wrote his memoirs on geometry, earth structures and hydraulics. Promoted to General of brigade in 1848, he was also active in politics and was elected member of parliament.

Poncelet gave a graphical and analytical solution for the maximum earth pressure on a wall based on Coulomb's wedge theory. Interestingly, this graphical method for determining retaining wall thickness was adopted in the U.S. Army Corps of Engineers "Paper No. 3 on Practical Engineering" published in 1845 (see Figure 6 on the next page).[11]

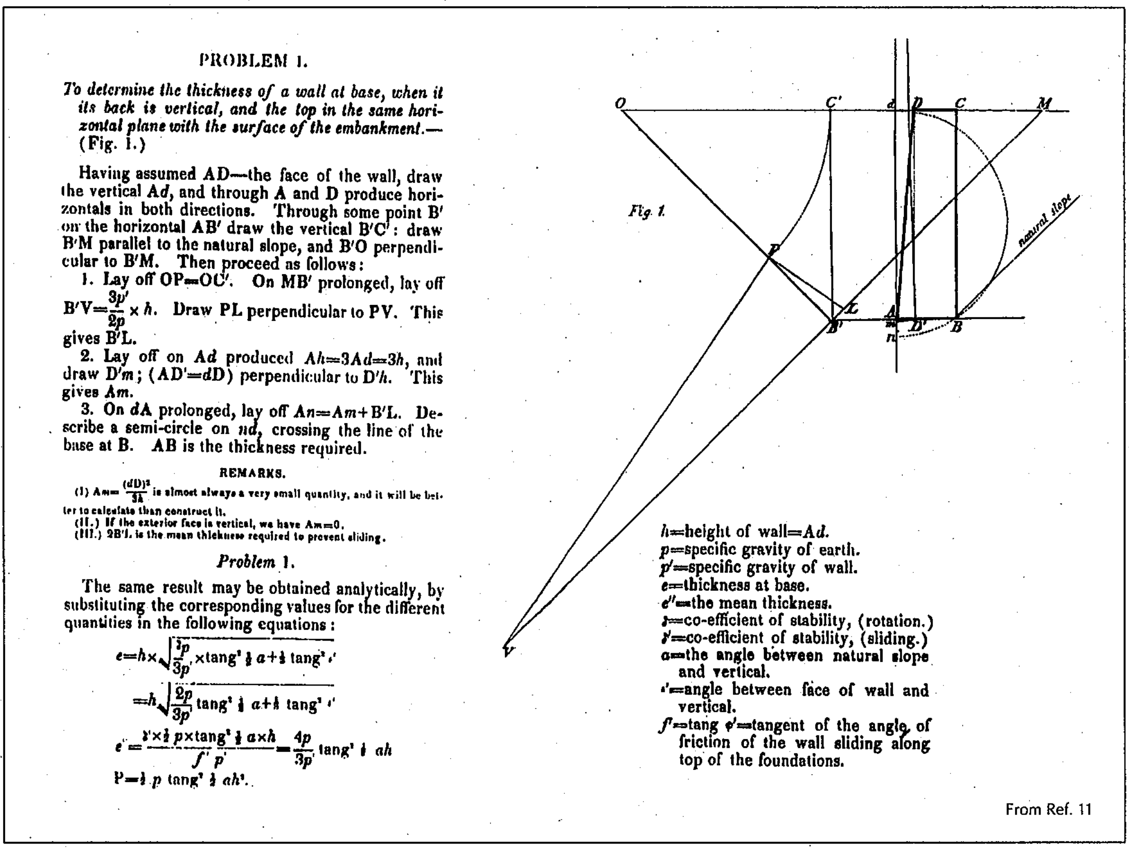
In his 1866 book *Die Graphische Statik*, Carl Culmann (1821-1881), a Bavarian bridge engineer and later professor of engineering sciences at the Swiss Federal Institute of Technology in Zurich, presented a graphical solution for the general case of the wedge theory. This graphical solution is still currently used and published in geotechnical handbooks.[12] Culmann's graphical approach was influenced by his study of geometry, strength of materials and stability of arches.

Finally, in 1871, a general graphical solution of the general wedge case (inclined backwall, wall friction and irregularly surfaced backfill) was given by the German engineer G. Rebhann.[13] This simple graphical solution, based on an interesting mathematical theorem, provides the location of the line of sliding and the magnitude of the wall reaction force (see Figure 7). The line of sliding, *BC*, divides the area *BACD* in half and the wall reaction, *R*, is equal to the area *KCD* times the soil unit weight.

Rankine's Theory

In the late 19th century, engineers were attempting to determine the distribution of stresses in cohesionless materials through an analytical approach. Earth material supported by a retaining wall structure was of particular interest for a well known Scottish civil engineer and university professor.

William John Macquorn Rankine was born in Edinburgh, Scotland, on July 5, 1820. His fa-



PROBLEM 1.

To determine the thickness of a wall at base, when it its back is vertical, and the top in the same horizontal plane with the surface of the embankment.—(Fig. 1.)

Having assumed AD—the face of the wall, draw the vertical Ad, and through A and D produce horizontals in both directions. Through some point B′ on the horizontal AB′ draw the vertical B′C′: draw B′M parallel to the natural slope, and B′O perpendicular to B′M. Then proceed as follows:

1. Lay off OP=OC′. On MB′ prolonged, lay off $B'V=\frac{3p'}{2p}\times h$. Draw PL perpendicular to PV. This gives B′L.

2. Lay off on Ad produced $Ah=3Ad=3h$, and draw $D'm$; ($AD'=dD$) perpendicular to $D'h$. This gives Am.

3. On dA prolonged, lay off $An=Am+B'L$. Describe a semi-circle on nd, crossing the line of the base at B. AB is the thickness required.

REMARKS.

(I) $Am=\frac{(dD)^2}{3h}$ is almost always a very small quantity, and it will be better to calculate than construct it.

(II.) If the exterior face is vertical, we have $Am=0$.

(III.) 2B′L is the mean thickness required to prevent sliding.

Problem 1.

The same result may be obtained analytically, by substituting the corresponding values for the different quantities in the following equations:

$$e=h\times\sqrt{\frac{\gamma p}{3p'}\times\text{tang}^2\tfrac{1}{2}\alpha+\tfrac{1}{3}\text{tang}^2\epsilon'}$$

$$=h\sqrt{\frac{2p}{3p'}\text{tang}^2\tfrac{1}{2}\alpha+\tfrac{1}{3}\text{tang}^2\epsilon'}$$

$$e'=\frac{\gamma'\times\frac{1}{2}p\times\text{tang}^2\frac{1}{2}\alpha\times h}{f'p'}=\frac{4p}{3p'}\text{tang}^2\tfrac{1}{2}\alpha h$$

$$P=\tfrac{1}{2}p\ \text{tang}^2\tfrac{1}{2}\alpha h^2.$$

h=height of wall=Ad.
p=specific gravity of earth.
p'=specific gravity of wall.
e=thickness at base.
e''=the mean thickness.
γ=co-efficient of stability, (rotation.)
γ'=co-efficient of stability, (sliding.)
α=the angle between natural slope and vertical.
ϵ'=angle between face of wall and vertical.
f'=tang φ'=tangent of the angle of friction of the wall sliding along top of the foundations.

From Ref. 11

FIGURE 6. Poncelet's geometrical solution for the soil wedge theory.

ther was Lieutenant in the Rifle Brigade and Superintendent of the Edinburgh and Leith Railway. Rankine's early instruction in arithmetic and mechanics was mainly obtained from his father. His uncle gave him, at the age of 14, a copy of Newton's *Principia* in the original Latin, which established the foundation of his knowledge in dynamics and physics. He attended Edinburgh University, after which, while engaged on railway projects with his father, he invented at the age of 21 the method of setting out rail curves. He joined the Institution of Civil Engineers, where his first independent paper was read, as an Associate in 1843.

He became, in 1855, Professor of Engineering at Glasgow University (the Engineering Chair at the university was founded in 1840 by Queen Victoria), where he taught applied mechanics and thermodynamics. He became one of the three greatest authorities on those subjects, along with Lord Kelvin and Clausius. In his 30 years of research and writing, Rankine published 158 original papers, several manuals and books, and contributed to the weekly magazine *The Engineer*. After 17 years of dedicated teaching career, he died in Glasgow on December 24, 1872.

Rankine investigated the stress conditions corresponding to a state of plastic limit equilibrium in which every part of a semi-infinite mass of soil is on the verge of failure. His memoir titled "The Stability of Cohesionless Soil," which was presented on June 10, 1856, at the Royal Society of London, had the objective of establishing the mathematical theory of equilibrium of a granular soil.[16]

In his *Manual of Civil Engineering* (first published in 1861), Rankine applied his theory of conjugate pressures to a mass of earth having an indefinitely extended upper plane surface.[17] He considered a section of a prismatic (deformed rectangle) mass of earth defined by the plane of greatest and least soil pressures. He extended this conjugate theory to the determination of pressure of earth against a vertical wall plane assuming a triangular prism of soil (see Figure 8). He also used a graphical method, with analogy to electrical equipotential lines, to determine lines of equal pressures and equal thrust against a wall. He concluded his findings with the following statement:

> "There is a mathematical theory of the combined action of friction and *adhesion* in earth; but for want of precise experimental data, its practical utility is doubtful."

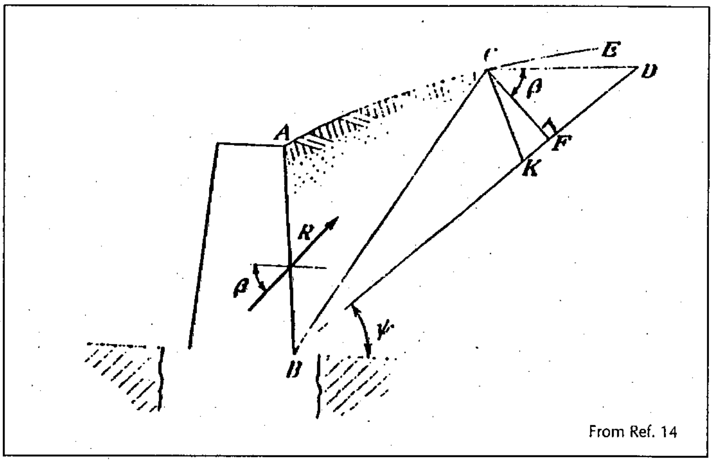


FIGURE 7. Rebhann's construction method determining the soil wedge plane and the wall's reaction force.

Rankine upheld his thesis's validity based on:

> "the concordance of true theory and perfected practice, against the misconceptions of those who believed in separating between the investigators in pure science and the men who directed the arts of construction."

Being a mathematical physicist, he brought the theoretical science into contact with engineering practice (lateral earth pressure, chimney and arch stability, and soil bearing capacity). Rankine believed that previous theories were limited in their hypothesis and applications and wanted to lay the ground of a *rational* soil theory. His study of masonry arches and their equal thrust lines influenced greatly his earth pressure work.

Rankine's theory of designing retaining walls on the basis of the analysis of stresses in cohesionless materials was further discussed and refined by several European engineers: Saint-Venant (1870), Considère (1870), Lévy (1873), Winkler (1871) and Mohr (1872).

In particular, in 1882, M. J. Boussinesq established his theory based on the differential equations of stress distribution in an elastic cohesionless semi-space.[18] Boussinesq, then Professor at the Faculty of Sciences in Lille, France, introduced the friction angle between the fill and the back of wall into the earth pressure theory to account for a more precise wall design (see Figure 9 on page 89).

Experimental Work

Several engineers pursued their work on earth pressure acting against retaining walls by conducting related experiments for the purpose of examining the validity and consequences of the earth pressure theories.

The earliest recorded experimental work on retaining walls was presented by Belidor in his book *La science des ingénieurs* (1729).[19] Belidor mentioned that, as a result of his wall experiments using gun shot as backfill, the prism of rupture of the backfill was on a slope of 1:1 (45 degrees) and the wall reaction would be one-half of the prism's weight, which is consistent with his theory mentioned above.

In 1746, Gadroy published a memoir in France that described experimental tests conducted with a sand backfill in a box 7.5 centimeters square and 28 centimeters long, of which the square end was hinged.[6] He was the first to describe a plane of rupture behind the wall to be steeper than the angle of repose of the backfill.

Emiland-Marie Gauthey (1732-1806), a French engineer known for his construction of the Charolais Canal (which involved several locks and bridges), introduced bridge and wall design as professor at *L'École des Ponts et Chaussées*.[5,19] He published a comprehensive bridge design manual, *Traité de la construction des ponts*, prefaced by his nephew, the famous Navier. In this manual Gauthey gave design

IV. **Pressure of Earth against a vertical Plane.**—In fig. 156, let O X represent a vertical plane in, or in contact with, a mass of earth, whose upper surface Y O Y is either horizontal or inclined at any angle θ, and is cut by the vertical plane in a direction perpendicular to that of steepest declivity. It is required to find the pressure exerted by the earth against that vertical plane *per unit of breadth*, from O down to X, at a depth O X $= x$ beneath the surface, and the direction and position of the resultant of that pressure.

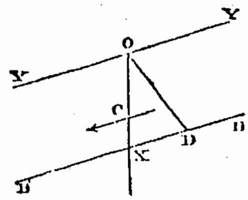


Fig. 156.

The *direction* of that resultant is already known to be parallel to the declivity Y O Y.

Let B B be a plane traversing X, parallel to Y O Y. In that plane take a point D, at such a distance X D from X, that the weight of a prism of earth of the length X D and having an *oblique* base of the area unity in the plane O X, shall represent the intensity of the conjugate pressure per unit of area of a vertical plane at the depth X; that is to say, construct fig. 155 as already described, and make

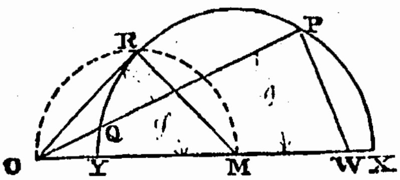


Fig. 155.

O P : O Q in fig. 155 : : O X : X D in fig. 156.

Draw the straight line O D; then will the ordinate, parallel to O Y, drawn from O X to O D at any depth, be the length of an oblique prism, whose weight, per unit of area of its oblique base, will be the intensity of the conjugate pressure at that depth. Let O D X be a triangular prism of earth of the thickness unity; the weight of that prism will be the *amount* of the conjugate pressure sought, and a line parallel to O Y, traversing its centre of gravity, and cutting O X in the *centre of pressure* C, will be the *position* of the resultant of that pressure. The depth O C of that centre of pressure beneath the surface is evidently two-thirds of the total depth O X.

To express this symbolically, make

$$\mathrm{X\,D} = x \cdot \frac{p'}{p} = x \cdot \frac{\cos\theta - \sqrt{(\cos^2\theta - \cos^2\varphi)}}{\cos\theta + \sqrt{(\cos^2\theta - \cos^2\varphi)}}; \;\ldots\ldots(17.)$$

From Ref. 17

FIGURE 8. Rankine's conjugate pressures theory in a mass of earth.

guidelines for bridge abutments, piers and retaining walls.

In 1785, using a box 75 centimeters high and 30 centimeters wide, Gauthey was the first to perform a complete set of earth pressure experiments with sand. In order to determine what part of the backfill was responsible for the pressure against the box wall, he built a hinged sloping bottom and recorded that the pressure against the vertical face was the same for a number of increasing bottom slopes until it reached 67.5 degrees with respect to the horizontal. This experimental result is close to the theoretical failure angle of a dense sandy fill behind a retaining wall.

The earliest large-scale apparatus — a box 1.72 meters long, 1.15 meters high and 1.15 meters wide — was built by the Prussian engineer Reinhard Woltmann in 1791.[19] The front wall was hinged on the top and was prevented from rotating by an adjustable stop at one third of the height. Materials tested included sand, gravel, soil and rye. The measured forces were half of the thrust given by Coulomb's thrust formula, disregarding the vertical component. This reduction in thrust was probably due to friction loss along the sides of the box.

In 1807, Mayniel completed several sets of experiments (a total of 33 tests) on a hinged box

3 meters long, 1.5 meters wide and 1.5 meters high.[6] An iron strut, located at one-third of the height, was hinged to the gate and pushed against weights on a friction block (see Figure 10). From his experiments Mayniel found that Coulomb's theory was the only true and simple earth pressure theory: the experimental resultant thrust acted at one-third of the height and equaled one-quarter to one-third of the weight of the wedge of rupture for loose fill and one-seventh to one-twentieth of the wedge weight for packed fill.

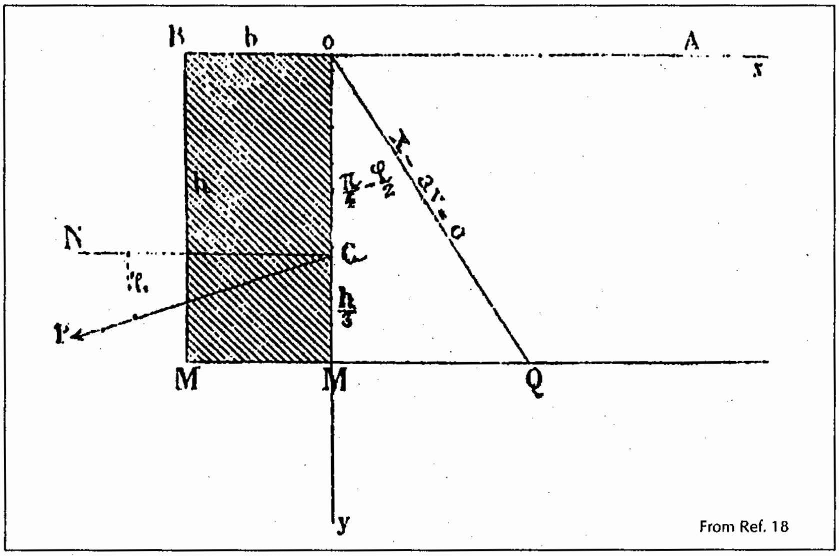


FIGURE 9. Boussinesq's elastic theory and his representation of the soil stress field zones.

Mayniel's work represents one of the earliest identification and analyses of the effect of compaction of fill on its thrust magnitude. His results are somehow scattered but they do show that compacted fill exerts a thrust half of that exerted by loose fill. These results are in general agreement with modern experiments and design rules.

In 1834, full-scale wall tests were conducted in England by Sir John-Fox Burgoyne (1782-1871), an English field marshal.[5,19] Lieutenant of the military engineers at the age of 16, he was distinguished in sieges of fortifications in Egypt and Portugal. He became Chief Engineer in the British expedition that attempted to reacquire New Orleans. He was later in charge of the Public Works Commission in Ireland where he worked on several railroad retaining structure projects.

The full-scale field tests were performed on four rubble masonry walls backfilled with loose earth. The walls were built in different shapes but each was 6 meters high and 6 meters long (see Figure 11). The tests were conducted during and after heavy rains. Wall *A* in Figure 11 was backfilled level and remained stable after 40 days of rain. Wall *B* showed some fissures but it was stable. Wall *C* failed when the fill reached 5.2 meters high by overturning at 1.7 meters from the base. Wall *D* overturned as a unit when the fill was 5.2 meters high. The test results were consistent with the overturning moments computed by the earth wedge theory.

These tests are an early example of "load tests to failure" that are conducted nowadays on earth structures and foundations. They constitute a *parametric* study of various wall designs that affect the stability of retaining walls. Basically, these tests imply a practical design rule — that the base of a front batter wall is equal to the third of its height ($b = 0.3h$).

In 1899, A. Steel built a 4.4-meter high test wall at the University of Nebraska.[19] He measured horizontal and vertical forces on test boards inserted in two openings located 15 and 75 centimeters above the base of a backfilled bin. Materials used were dry and damp earth and mud, with backfills levelled and sloped.

He compared the experimental pressure curves to the wedge theory and found that the total soil thrust is correctly predicted by the theory. However, higher soil pressures were measured at the top part of the wall and lower pressures were detected at its bottom part (see Figure 12).

These early simple experiments led to 20th century research work that is characterized by critical findings on earth pressure magnitude and distribution, passive pressure against bridge abutments and soil dynamic thrust.

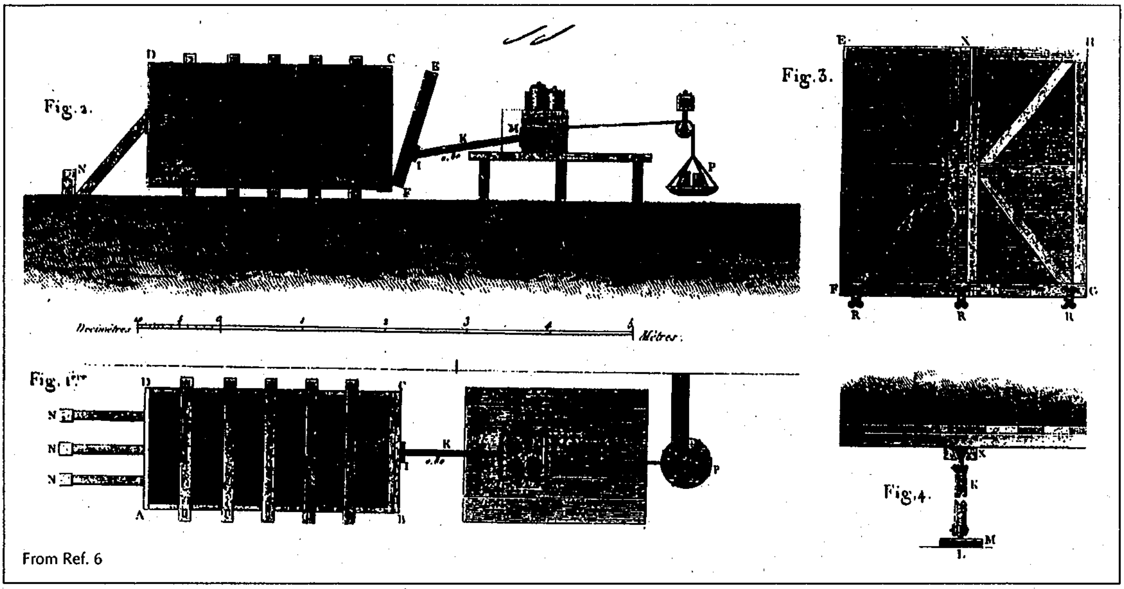


FIGURE 10. Mayniel's large-scale wall apparatus.

Lessons & Thoughts

The significant lessons that can be learned from these engineers and their achievements are:

1. Interestingly, some of these precursors worked on the earth pressure theory driven by their affinities and abilities in mechanics and mathematics. Others were mainly interested in the applied side of the theory on public work projects. However, most of these engineers succeeded by pursuing basic science education.

2. We can observe the influence, since the 17th century, of the military, academia, and public work and transportation entities in the design and construction of civil engineering projects (for example, bridges, canals and retaining structures). More fascinating is the thrust of these entities toward science advancement and research work.

3. The main precursors of this theory (Coulomb and Rankine) had scientific educations and field experience upon which they built their theories and design methods, which are contrasted by their geometri-

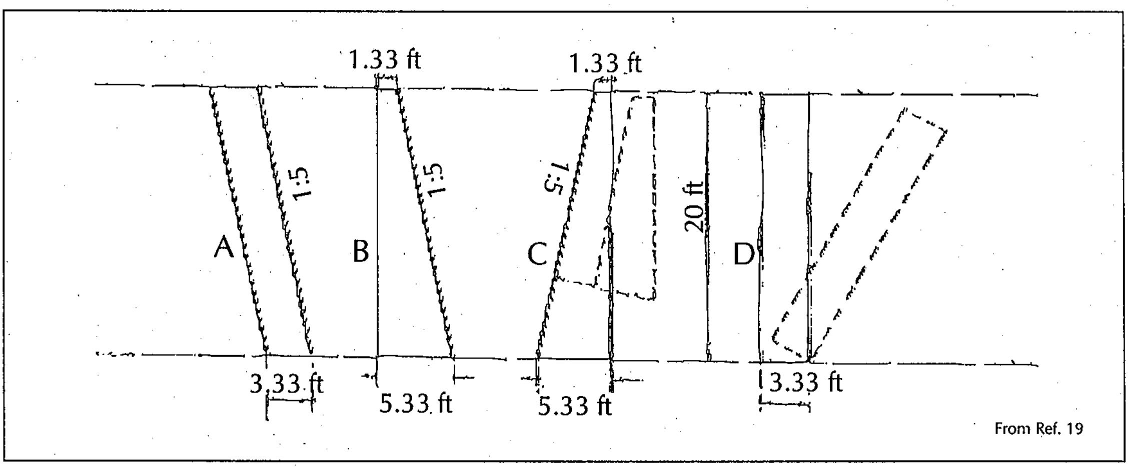


FIGURE 11. Burgoyne's full-scale tests on four walls.

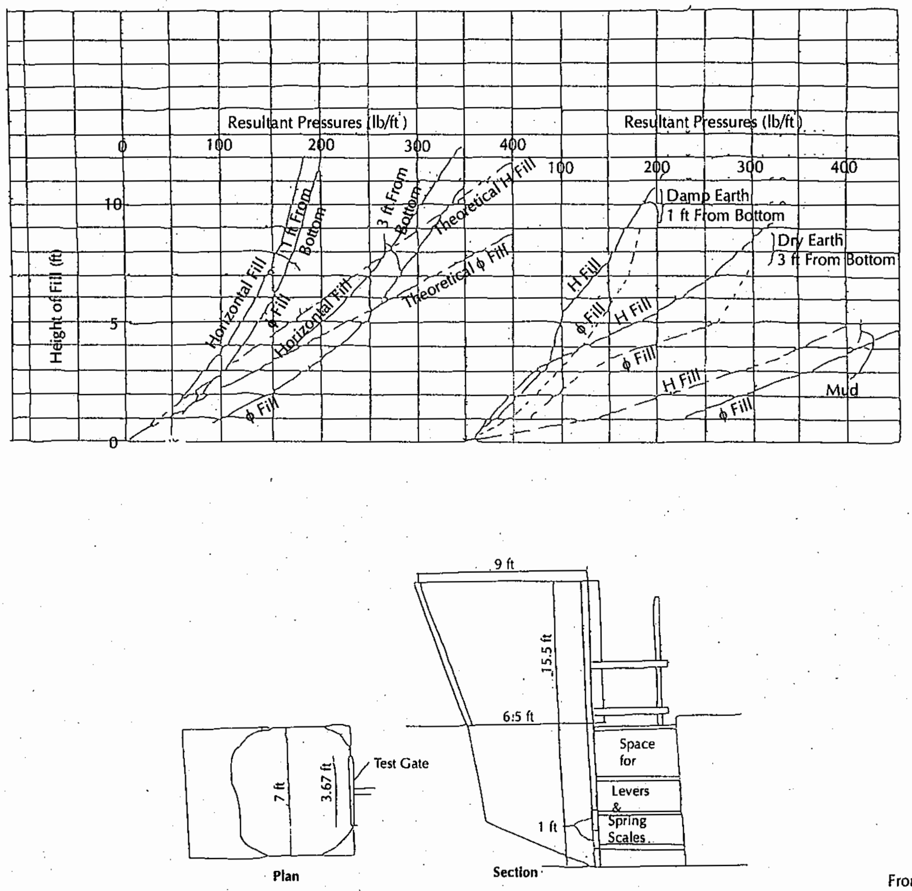


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FIGURE 12. Large-scale experiments by A. Steel.

cal and analytical approaches. Followers were theorists and/or experimentalists (such as Mayniel and Burgoyne) who still did critically review previous practices and propose new improvements.

4. These engineers were engaged in many social and professional activities, such as engineering societies and public services, as well as service for their country. Their life, as a whole, seems an encompassing active existence.

Some of the thoughts that can be discussed about this earth pressure theory and related research work are:

1. Earth pressure theory is probably the oldest with regard to earth structures and is related to the other old and important theory of earth slope stability. Earth pressure is still a current issue for conventional walls as well as for innovative earth retaining structures, for which further research work is needed.

2. Early design methods were based on basic soil data. These methods were developed before the advent of in-situ and lab testing results, and, therefore, did not address the modern issue of factor of safety. While some early designed walls have failed, the fact that many have withstood the ravages of time provides incentive to further research such special wall stability factors as stepped backwall, soil arching, construction sequencing, etc.

3. These precursors conducted their research work by focusing on a particular subject, but they used a multi-disciplinary approach that helped them resolve the related

problems they encountered. Making civil engineering research and development projects a multi-disciplinary effort and environment is, most probably, the basis for a plan of action in current engineering practice.

Door to the 20th Century

The evolution of experimental work on earth pressure, the understanding of soil behavior and the adoption of methods for soil properties measurements have slowly but surely opened the way for the revival of the earth pressure theory initiated by the modern geotechnical Swedish School in the beginning of the 20th century, by Terzaghi's classic and historic work and by Mononobe/Okabe's introduction of the additional dynamic thrust acting on retaining walls.

The legacy of these predecessors should be inspiring since "it is by studying the work of these great persons that we can develop our knowledge and intelligence."[8]

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