Designing and Analyzing Prestressed Concrete Members Using a Capacity Diagram

A capacity diagram can provide a useful tool for preliminary design and for determining induced stresses in precast concrete members.

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The use of precast concrete for structural members is a widespread technique in the construction industry. The advantages of using precast concrete, in particular the reductions in construction time, make precast concrete a preferred construction material for many developers.

However, the choices of the precast concrete members available to the structural engineer who specifies their use in a job are limited by the shapes offered by the producers of the members. This limitation of choice places additional weight on the preliminary design, making it an important step in the process of designing prestressed concrete structures.

Background

Prestressed concrete members are considered to act as elastic elements whose behavior is in accordance with the straight line theory. The general equation that depicts the relation of the stresses and that is used to compute the stresses at a given point y from the neutral axis is as follows:

$$f_y = (P/A) + (Pe/I)y + f_{ey} \quad (1)$$

$$f_y - f_{ey} = P((1/A)+(e/I)y) \quad (2)$$

where:

f_y = Concrete stress at point y from the neutral axis

f_{ey} = Stress due to external loads at point

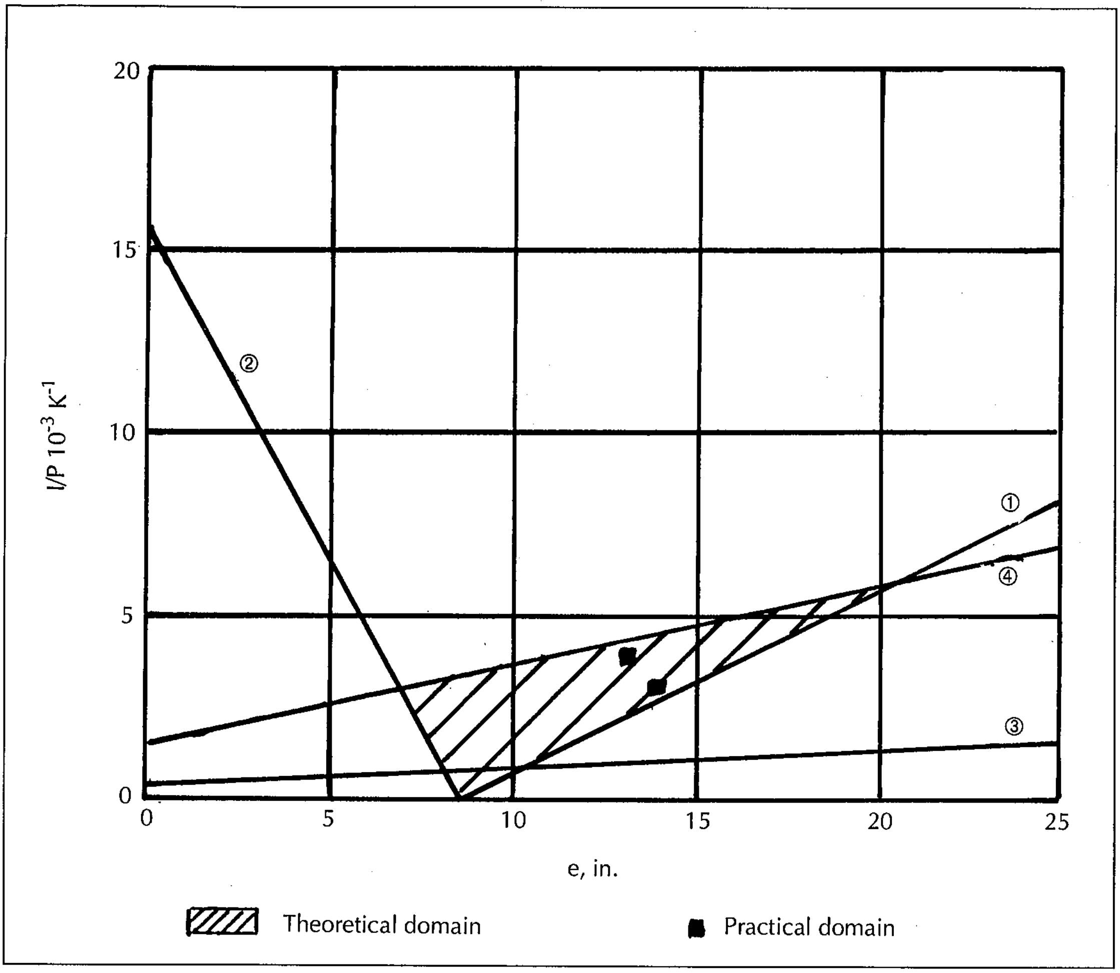

FIGURE 1. A Magnel diagram.

y from the neutral axis

P = Total prestressing force

A = Area of the prestressed concrete members

e = Prestressing force eccentricity

I = Moment of inertia of the prestressed concrete member

y = Distance from the neutral axis to the point considered

Equation 2 represents the general relationship between the prestressing force and its eccentricity for any given load system. This equation can be found in any prestressed concrete manual.

In determining member stresses, it is well known that there are two crucial stages in the life of a prestressed concrete member:

1. *Initial stage* when the prestressing force is transferred from the prestressing steel to the prestressed concrete member.

2. *Final stage* when the prestressed concrete member is acted on by the design service loads and all losses have occurred in the prestressing force.

Equation 2 can be rewritten for the top and bottom fibers of a prestressed concrete member at the initial and final stages within the framework of the given load system. This process is used to obtain the four governing equations of the prestressed concrete members. The best representation of these equations was formulated by Magnel.[1] The diagrams presented in his work still constitute the most popular tool for evaluating the response of

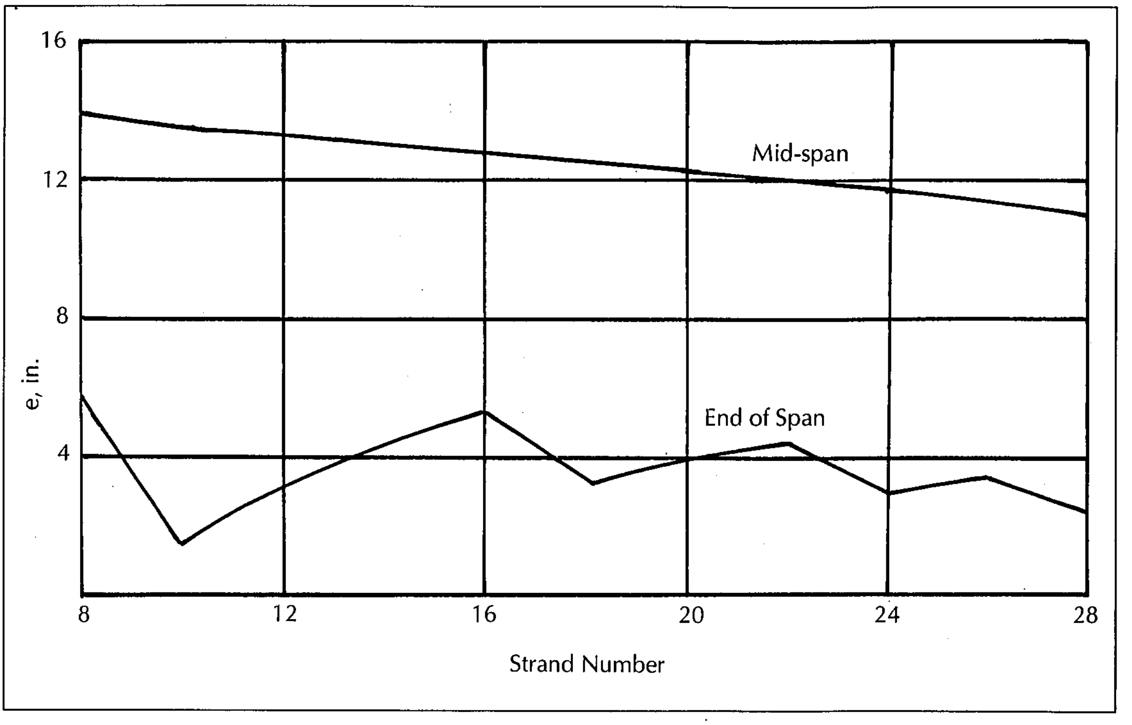

FIGURE 2. An eccentricity diagram for an AASHTO Type II prestressed concrete member.

prestressed concrete members to applied external loads.

Basically, Magnel's diagrams are theoretical representations of the prestressing force/eccentricity relationship. Figure 1 presents such a diagram illustrating the force/eccentricity relationship for a 50-foot AASHTO Type II span member acted on by a 1.2 kips/lf uniform distributed load. It should be noted that the difference between the theoretical and practical domains — of the prestressing force and its eccentricity — that makes Magnel's diagram almost an impractical design tool.

Eccentricity Diagrams

The real relationship between the prestressing force and its eccentricity is the one that can be accommodated by the given shape of the prestressed concrete member. An eccentricity diagram provides one means of depicting that relationship by graphically representing the prestressing force eccentricity as a function of the number of strands following the strand pattern of a given cross-section.[2] The eccentricity diagram is based on the geometry of the member and should be furnished by the manufacturer together with its geometric characteristics which are indispensable for designing prestressed concrete members. Figure 2 provides an example of an eccentricity diagram for an AASHTO Type II member.

In fact, the diagram depicted in Figure 2 represents a domain where a given strand number yields the maximum eccentricity, at mid-span and at the end of the member, that can be accommodated by the cross-section. A smaller value can be used, but its use would reflect a poor utilization of the prestressing technique.

Design Equations

Equation 2 can be rewritten in the following general form:

$$f = Pk \qquad (3)$$

where:

k = Prestiffness coefficient (rigidity coefficient for prestressed concrete members)

This general equation expresses the relationship between the induced stress in the prestressed member, the prestressing force and a stiffness type coefficient describing the geometry of the cross-section.

The stiffness coefficient has different values for the top and bottom of the prestressed concrete member and can be computed using Equations 4 and 5.

$$k_t = (1/A) - (ey_t/I) \quad (4)$$

$$k_b = (1/A) + (ey_b/I) \quad (5)$$

where:

t = Top of the member
b = Bottom of the member

Using Equations 3, 4 and 5, two values of the prestressing force can be computed for the top and bottom of a member for the initial stage. The minimum value computed represents the maximum prestressing force that can be accommodated by the member at release.

Following the same procedure for the final stage, two values of the prestressing force can be computed for the top and bottom of the member. In this case, the maximum value represents the minimum prestressing force that can be required by the given external load system. The domain between the two values obtained for the initial and final stages reflects the range of prestressing forces that can be accommodated by the member within the given geometric and loading constraints.[2]

Capacity Diagrams

As shown above, Equation 3 expresses the stress/prestressing force relationship and can be calculated for any given cross-section. For practical purposes, the relationship between the induced stress and the number of strands that generate the state of stress is more helpful for the designer and manufacturer of the prestressed concrete members. Having this relationship in mind, Equation 3 can be rewritten taking into account the number of strands as follows:

$$f = kP_1N \quad (6)$$

while:

$$P_1 = A_s f'_c jl \quad (7)$$

$$m = f/P_1 \quad (8)$$

where:

f = Unit stress in prestressed concrete member
f'_c = Ultimate strength of the prestressed concrete
P_1 = Prestressing force developed by one single prestressing strand
j = Jacking coefficient (jacking stress to ultimate strength ratio)
l = Loss coefficient (prestressing force to initial prestressing force ratio)
m = Coefficient representing the concrete stress to strand force ratio
N = Number of prestressing strands

Using the notation from Equation 8, Equation 6 then becomes:

$$m = kN \quad (9)$$

The capacity diagram of the prestressed concrete member is the graphical representation of Equation 9. This equation reflects the relationship between the introduced stresses that are due to the design loads, including the stresses due to prestressing force, the number of strands and the stiffness of the member. It should be noted that the stiffness coefficient is a function of the geometric characteristics of the member, including the eccentricity of the prestressing force.

For a prestressed concrete member, the top and bottom fibers of the member are the ones that govern the design of that member, thus permitting the derivation of the following two equations from Equation 9:

$$m_t = k_tN_t \quad (10)$$

$$m_b = k_bN_b \quad (11)$$

Equations 10 and 11 are implicit functions of the prestressing force eccentricity and can be calculated at the midspan and at the end of the prestressed concrete member for both the initial and final stages.

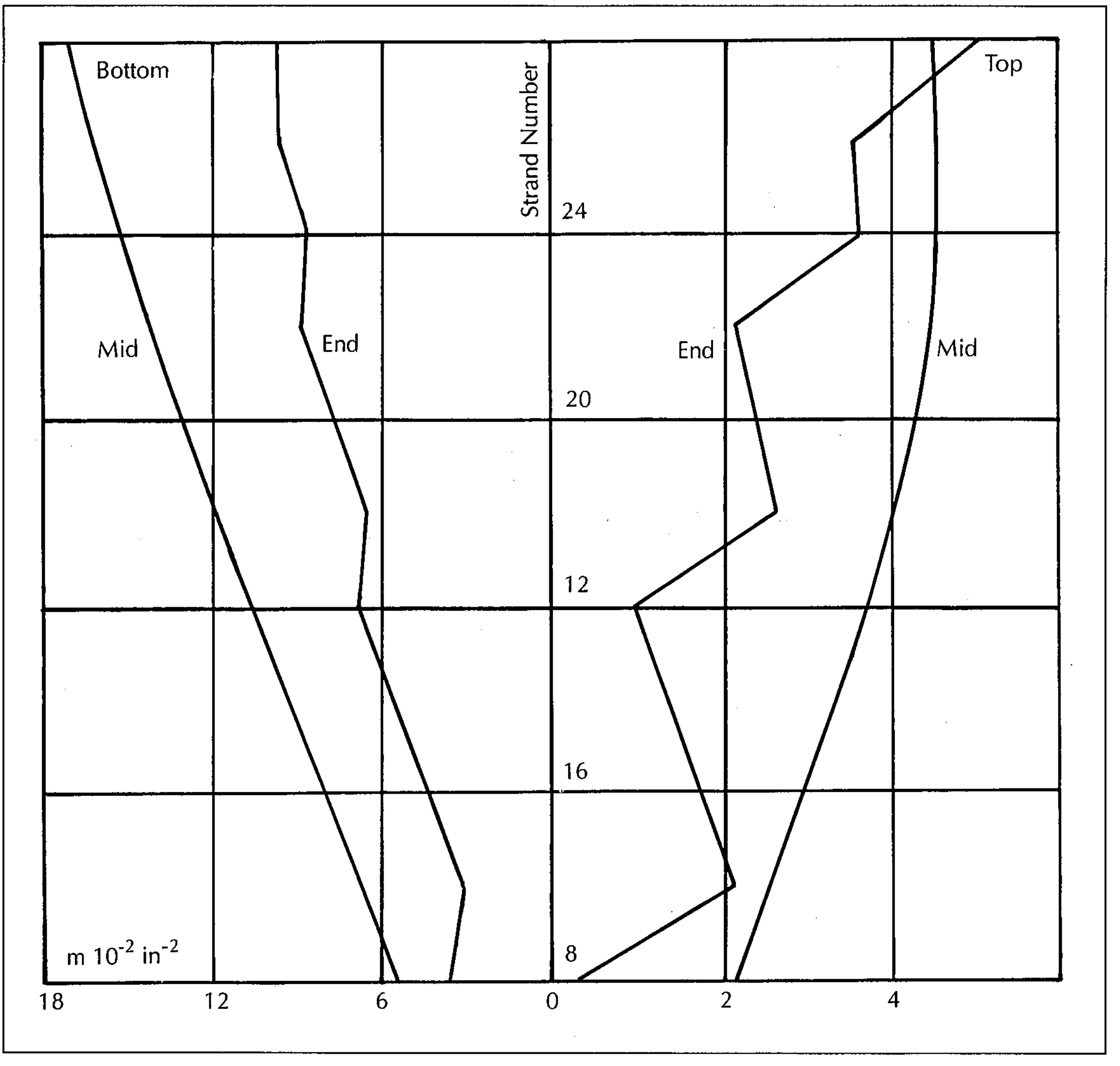

FIGURE 3. A capacity diagram.

Using Equations 10 and 11 together with the eccentricity diagram shown in Figure 2, the capacity diagram for an AASHTO Type II member was plotted for the top and bottom fibers of a prestressed concrete member (see Figure 3). The maximum number of strands that can be accommodated by the prestressed concrete member at release is given by Equation 12 and is the minimum value from the number of strands computed for the top and bottom fibers using Equations 10 and 11.[2]

$$N_i = min\ (N_{ti}, N_{bi}) \quad (12)$$

The maximum number of strands that can be accommodated by the prestressed concrete member at the final stage is expressed by Equation 13. This equation reflects the maximum value of the number of strands computed for the top and bottom fibers using Equations 10 and 11.[1]

$$N_f = max\ (N_{tf}, N_{bf}) \quad (13)$$

The domain of the number of strands for the given constraints is expressed by Equation 14. It is obvious that when $N_i > N_f$, the member cannot accommodate the given load system and that a new member should be tried in the design.

Table 1
Specifications for an Example AASHTO Type II Prestressed Concrete Member

Area = 369 sq. in.
y_b = 15.83 in.
Moment of inertia = 50,979 in.^{4}
y_t = 20.17 in.
Span = 65 ft.

Allowable stresses in concrete (in ksi):
Initial stage top = -0.2
Initial stage bottom = 2.6
Final stage top = 2.6
Final stage bottom = -0.4

Prestressed steel specifications:
f'_c = 270 ksi
A_s = 0.153 sq. in.
c_j = 0.7
l_i = 0.9
l_f = 0.75

$$N = domain\ (N_f \ldots N_i) \qquad (14)$$

The capacity diagram for AASHTO prestressed concrete members can be found in a previous work[3] and for ACI prestressed concrete members can be found in another earlier work.[4] Using the procedure described herein, a capacity diagram for a prestressed concrete member can be plotted (with consideration given to its transformed cross-section).[5]

Design Example

The following step-by-step procedure shows how to utilize the equations listed above to construct a capacity diagram for designing a prestressed concrete member. For example, given an AASHTO Type II prestressed concrete member with the specifications as shown in Table 1, the problem is to design the prestressed concrete member for the given constraints and for a 1.0 k/foot uniform distributed load.

The first step consists of computing the induced stresses due to the weight of the beam:

$w = (369/144) \times 0.15 = 0.383$ k/lf.
$M_w = 0.383 \times 65^2 / 8 = 203$ k/ft.
$f_{tw} = 12 \times 203 \times 20.17 / 50{,}979 = 0.96$ ksi
$f_{bw} = -12 \times 203 \times 15.83 / 50{,}979 = -0.76$ ksi

Then the induced stresses due to the external loads is calculated:

$M_c = 1 \times 65^2 / 8 = 528.13$ k/ft.
$f_{tc} = 12 \times 528.13 \times 20.17 / 50{,}979 = 2.51$ ksi
$f_{bc} = -12 \times 528.13 \times 15.83 / 50{,}979 = -1.97$ ksi

The next step consists of computing the prestressing force per strand for the initial stage and then for the final stage:

$P_{1i} = 0.153 \times 270 \times 0.7 \times 0.9 = 26.03$ kip
$P_{1f} = 0.153 \times 270 \times 0.7 \times 0.75 = 21.69$ kip

The third step consists of determining the number of strands for the initial stage. The calculations for the top fiber follow:

$f = -(f_{ti} - f_{tw})$
$f = -(-0.2 - 0.96) = 1.16$ ksi
$m_{it} = f/P_{1i}$
$m_{it} = 1.16/26.03 = 0.4456$ in.$^{-2}$

From the capacity diagram N_{ti} = 22. The calculations for the bottom fiber follow:

$f = f_{bi} + f_{bw}$
$f = 2.6 + 0.76 = 3.36$ ksi
$m_{ib} = f/P_{1i}$
$m_{ib} = 3.36/26.03 = 0.1291$ in.$^{-2}$

From the capacity diagram N_{bi} = 19. The maximum number of strands for the initial stage are as follows:

$N_i = min\ (N_{ti}, N_{bi})$
$N_i = \min(22,19) = 19$

The fourth step is to determine the number of strands for the final stage. The calculations for the top fiber follow:

$f = -(f_{tf} - f_{tw} - f_{tc})$
$f = -(2.6 - 0.76 - 2.51) = 0.87$ ksi
$m_{ft} = f/P_{1f}$
$m_{ft} = 0.87/21.68 = 0.0402$ in.$^{-2}$

From the capacity diagram N_{bf} = 18. The calculations for the bottom fiber follow:

$f = f_{bf} + f_{bw} + f_{bc}$
$f = -0.4 + 0.76 + 1.97 = 2.33$ ksi
$m_{fb} = f/P_{1f}$
$m_{fb} = 2.33/21.68 = 0.1075$ in.$^{-2}$

From the capacity diagram N_{bf} = 16. The maximum number of strands for the final stage are as follows:

$N_f = max\ (N_{tf}, N_{bf})$
$N_f = max\ (18{,}16) = 18$

The last step required is to compute the domain for the number of strands:

$Domain = (N_f \ldots N_i)$
$Domain = (18 \ldots 19)$

Conclusion

The capacity diagram serves as a tool that can be used for designing and analyzing prestressed concrete members. The diagram is independent of the type and size of the strands, prestressing steel stress, the prestressing loss and span length of the member. Within acceptable approximations, it can be used for bonded as well as debonded strands. It can be plotted for any shape if taking into account the tendon pattern, hence the eccentricity diagram. For the given constraints, the capacity diagram can be used for the evaluation of the member response to the external loads and for understanding how to introduce selected stresses in prestressed concrete members.

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