Long-Range Surface Water Supply Planning

Recent projections indicate that eastern Massachusetts must expand its surface water supply system to deliver an adequate supply during the coming decades. Understanding the relationships among system storage, reliability and yield is essential to the solution of this long-range planning problem.

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A REPORT developed by the Metropolitan District Commission (MDC) and the Massachusetts Water Resource Authority (MWRA) entitled, "Water Supply Study and Environmental Impact Report," describes an enormous project which, if and when completed, will identify the preferred alternative(s) for meeting the projected water supply needs of the areas serviced by the MDC and MWRA until the year 2020.[1] In 1969, the demand for water from the MDC/MWRA water supply system, which services greater Boston and its neighboring communities, first exceeded 300 million gallons per day (mgd) — a value considered to represent the "safe yield" from the Quabbin and Wachusett Reservoirs, the principal sources of supply.

After a drought that occurred during the mid-1960s that served as a warning to the area water authorities, several studies were undertaken. In 1969, the U.S. Army Corps of Engineers estimated a potential shortage of up to 140 mgd in the MDC/MWRA supply area by 1990.[2] In 1977, the now defunct New England River Basin Commission projected a shortage of 77 mgd by 1990.[3] In 1978, the Massachusetts Executive Office of Environmental Affairs estimated a shortfall of 70 mgd by 1990 for communities serviced by the MDC/MWRA.[4] A current study by the MDC/MWRA, initiated in 1981, projects a shortfall of approximately 120 mgd by the year 2020.[1]

A principal ingredient of the MDC/MWRA study was to determine the "safe yield" that the water supply system can deliver until the year 2020. The choice of water sources to augment the existing water supply system depends primarily on the knowledge of the "safe yield" of both the existing system and the sources under consideration, in addition to the forecasted demand for water. Figure 1 compares the system "safe yield" to the demand for water over the past two centuries and illustrates the increasing shortfall of water that may be anticipated in the future, if the existing water supply system

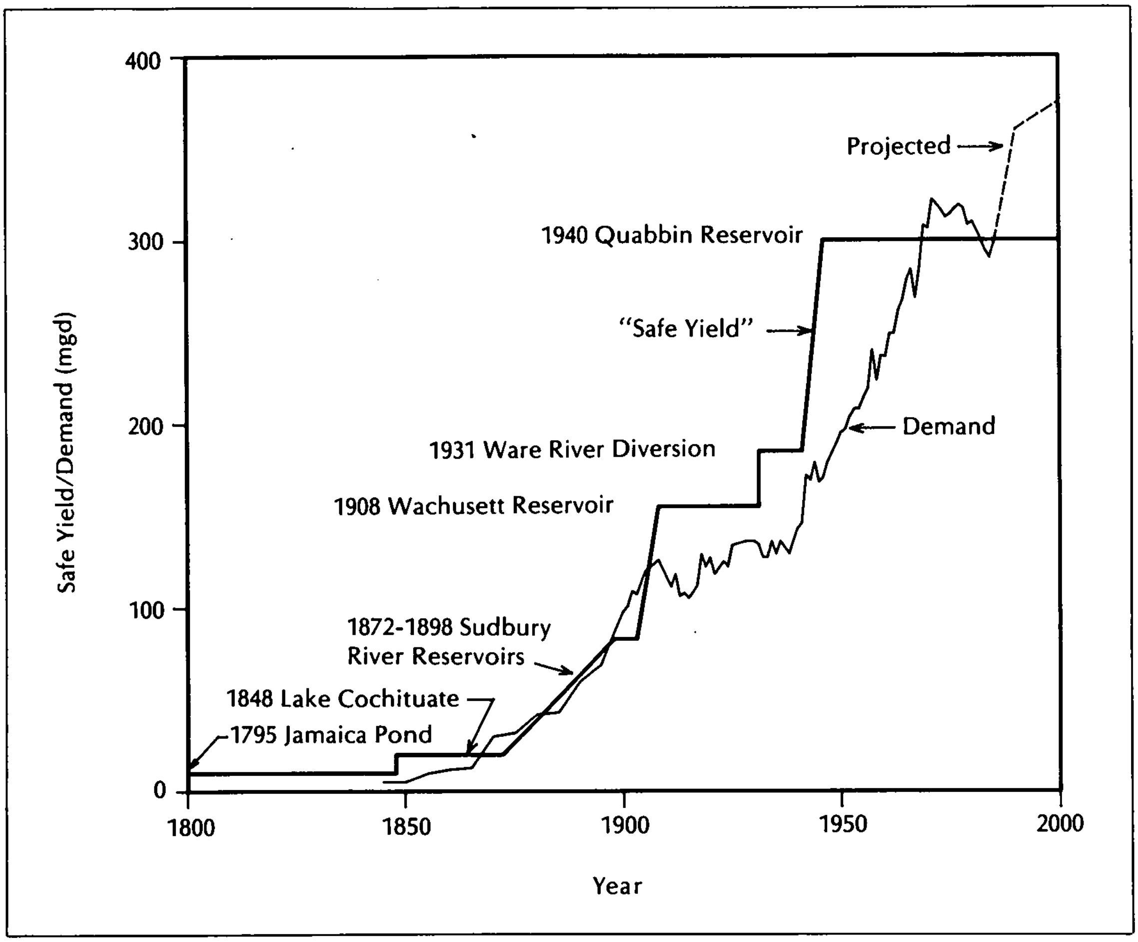

FIGURE 1. A chronologic history of Boston's water supply system.

is not augmented by additional sources of water. It is evident from Figure 1 that the long-range planning problem of determining the optimal plan for augmenting the existing water supply system depends significantly on the value of the existing system yield that can be considered "safe."

Classical procedures for determining the "safe yield" of a water supply system consist of simulating the entire water supply system and routing the historical streamflows through the system to determine the maximum yield that could be sustained without emptying the reservoir(s). The classical "safe yield" estimate for the MDC/MWRA system is simply a *single estimate* of the yield that could be sustained by the system during the worst drought on record, in this case the drought during the 1960s.

Estimating the "safe yield" of the MDC/MWRA system using classical simulation procedures known in practice as Rippl's mass curve,[5] or its automated equivalent sequent peak algorithm,[6] is not new. Historically, whenever an addition to the supply system was contemplated, classical "safe yield" procedures were employed to determine the storage-yield relationship. The estimated "safe yields" of the historic water supply systems are shown in their chronologic sequence in Figure 1. Brutsch presents a comprehensive history of Boston's water supply system from 1630 to today.[7]

Interestingly, over 50 years ago when the Quabbin Reservoir was in its planning phase, the system "safe yield" was estimated to be 300 mgd, an amount that was forecasted to satisfy demands for 50 years. Those calculations that were based on 50 years fewer streamflow measurements than available today were remarkably accurate. Furthermore, the original "safe yield" calculations were made without knowledge of the severe drought that occurred in the 1960s. "Safe yield" determinations were probably viewed less critically in the past, because the demand was seen as an ever increasing amount and each new reservoir added to the system was only another incremental step in the supply curve. Perceived changes in system reliability simply moved the planning horizon forward or backward a few years. In the 1980s, environmental, economic and political factors have imposed severe constraints that make it much more difficult for engineers to implement the conservative designs that herald their profession.

The current state-of-the-art of "safe yield" analysis uses stochastic streamflow models to generate alternative, yet likely, streamflow sequences and to route those streamflows through the water supply system using the sequent peak algorithm. The result is a storage-reliability-yield relationship instead of the classical storage-yield relationship produced by simply routing the historical streamflows through the water supply system. Routing synthetic streamflow sequences through the water supply system permits the evaluation of how vulnerable the system is to droughts with different character than, for example, the single severe drought experienced during the mid-1960s.

The application of stochastic streamflow models to water supply problems was introduced by Fiering in the U.S. in 1963.[8] A review of stochastic streamflow models is provided by Matalas.[9] Stochastic streamflow models are no longer limited to research applications; their use in practice is increasing. Recent applications of stochastic streamflow models to determining the reliability of existing reservoir systems include, but are not limited to, studies sponsored by the following agencies: State of California, Bonneville Power Administration, the Bureau of Reclamation, Pacific Gas and Electric Company, and the Brazilian hydroelectric system.[10,11,12,13,14] The MDC/MWRA, along with many other agencies across the nation, have been hesitant to employ stochastic streamflow models that are often complex, difficult and expensive to implement, and really only provide one additional dimension to the problem: a comprehensive description of system reliability. Yet, if adequate management of this large and complex water supply system until the year 2020 is a priority, the system must be examined with regard to its vulnerability to droughts of different character, because it is unlikely that another drought will mimic the drought experienced during the mid-1960s.

This study provides a description of the development of the system storage-yield relationship using the historical streamflows as input to a monthly simulation model of the water supply system, and provides an examination of the system storage-reliability-yield relationship using a stochastic streamflow model. Given our limited knowledge of the character of future droughts the system could experience, the likely range of values of the "safe yield" of the MDC/MWRA water supply system are estimated using a stochastic streamflow model. In short, this study addresses the question of how safe the system "safe yield" really is and provides a measure of how vulnerable the MDC/MWRA system is to future droughts — information that is essential to the long-range planning of surface water supplies in Massachusetts.

Monthly Simulation Model

Classification of Reservoir Systems and Design Procedures for Water Supply Problems. Two general classes of water supply systems exist: short-term and long-term reservoir systems. Short-term reservoir systems operate on an annual or seasonal basis and are characterized by systems with reservoirs that refill at the end of each season or year. Long-term reservoirs do not typically refill at the end of each year; such systems are particularly prone to water supply failures (empty reservoirs) during periods of drought that extend over several years. The existing

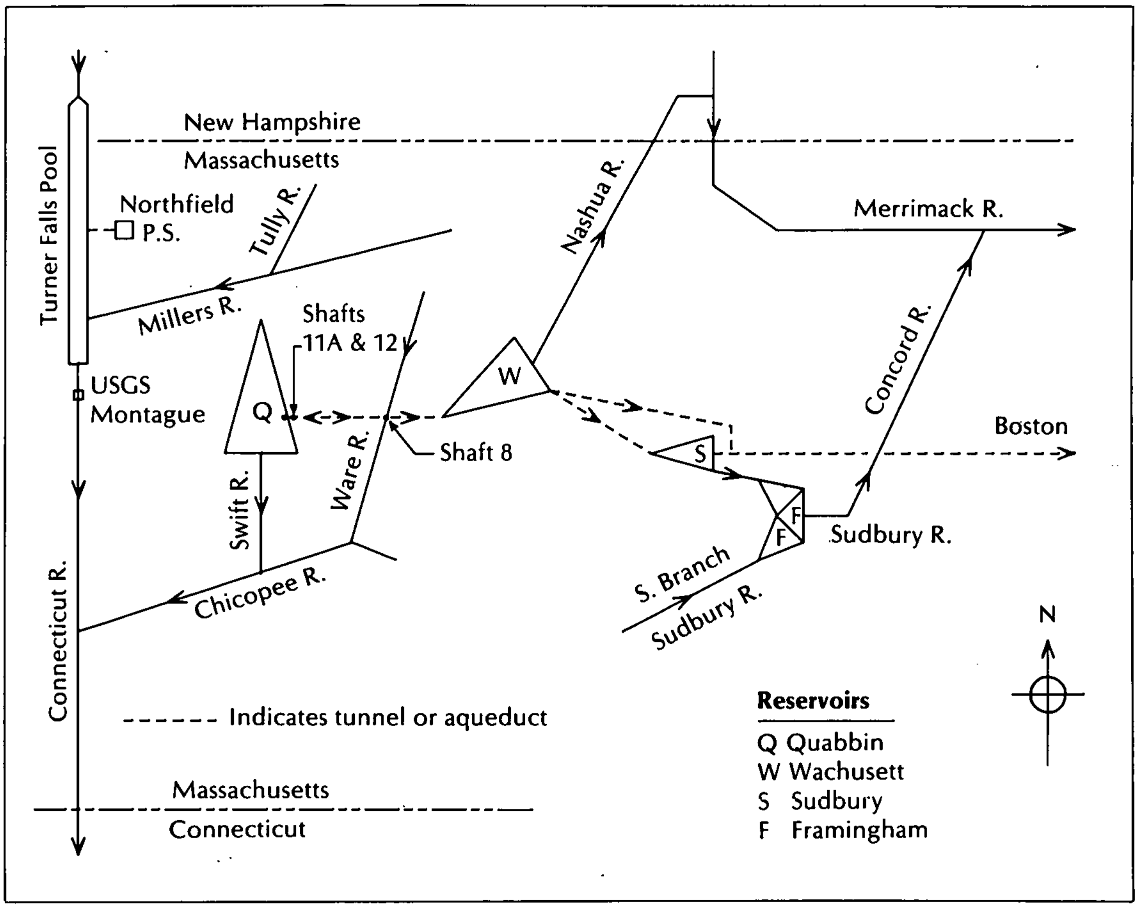

FIGURE 2. Major components of MDC/MWRA water supply system.

MDC/MWRA system consists of long-term reservoirs that have a total active storage capacity of approximately 265 billion gallons or 2.4 years of carry-over storage for a continuous demand of 300 mgd. The MDC/ MWRA reservoirs are rarely full and the traditional "safe yield" analysis that routes the historical streamflows through the system depends almost entirely on the character of the most critical period on record — in this case, the drought of the 1960s.

Linsley *et al.*, in a well-known hydrology text, provide a comprehensive treatment of water supply management.[15] They advocate that a reasonably long record of streamflow is probably an adequate database for the design of short-term reservoir systems, whereas stochastic streamflow models are required for designing long-term reservoir systems. According to Linsley *et al.*:[15]

> The traditional estimate of yield has been based on a critical period, usually the driest period in the historical record. No meaningful estimate of probability can be made for such a critical period. Almost certainly a more severe drought will occur, but the traditional analysis provides no estimate of risk. (p. 444)

They suggest, as have many others, that a realistic stochastic streamflow model be employed to evaluate the reliability of a particular system yield and that the "safe yield" not be referred to as if it were a guaranteed minimum yield.

The Existing MDC/MWRA Water Supply System. Figure 2 depicts the major components of the existing water supply system.

Table 1

Key Reservoir Data

	Reservoir		
	Quabbin	**Wachusett**	**Sudbury**
Drainage Area (square miles)	185.9	107.7	22.0
Surface Area When Full (square miles)	38.6	6.5	2.0
Spillway Storage (mg)*	412,240	64,968	7,254
Active Storage (mg)**	255,040	9,745	3,853

* Spillway storage is total volume of reservoir when water surface is at spillway crest elevation.

** Active storage is spillway storage less minimum pool volume.

The existing MDC/MWRA water supply system draws water from the Quabbin and Wachusett Reservoirs. These reservoirs are connected by a tunnel that passes under the Ware River, and whose flow can be diverted by Shaft 8 to either reservoir.

In practice, water is always diverted from the Ware River to the Quabbin Reservoir where it is then released from Shaft 11A and diverted northerly and westerly around Mt. Zion Island. Water eventually enters the tunnel at Shaft 12 for transfer from the Quabbin to the Wachusett Reservoir. The Ware River water receives significant natural purification due to settling and aeration during its long path through Quabbin Reservoir. Sudbury Reservoir is also available, but because of the inferior quality of its tributary flow, it is only used as an emergency supply. Key data pertaining to these three reservoirs are presented in Table 1.

System Constraints. Legislation restricts Ware River diversions at Shaft 8 to flows over 85 mgd during the period October 15 to June 14. Additional legislation requires minimum downstream releases from Quabbin Reservoir to the Swift River at rates necessary to assure a minimum streamflow of 20 mgd at Bondsville. Similarly, a release of 45 mgd must be made from Quabbin Reservoir when the flow of the Connecticut River at Montague is between 4,650 and 4,900 cubic feet per second (cfs), and that release must be increased to 71 mgd when the streamflow at Montague is less than 4,650 cfs during the period June 1 to November 30. About 14 mgd of water is also drawn from Quabbin Reservoir to feed the Chicopee Aqueduct.

The Wachusett Reservoir receives inflows from the Quabbin-Wachusett tunnel and from its own watershed, which is drained by the Quinapoxet River. The City of Worcester has water rights to 20.7 square miles of the drainage basin and operates three reservoirs within the watershed. The city also has withdrawal rights from the Quabbin-Wachusett tunnel and the Wachusett Reservoir. Other communities also obtain water from Wachusett Reservoir. The demand from these communities depends on the annual availability of water from local sources of supply. For the purposes of modeling, a total of 25 mgd is allocated for use in the general area of Wachusett Reservoir.

Water is transferred to the east from the

Wachusett Reservoir via two conduits: the Marlborough Tunnel that bypasses Sudbury Reservoir, and an aqueduct that supplies three communities before discharging into the Sudbury Reservoir.

Hydrologic Model Input. Simulation of the MDC/MWRA system requires monthly streamflow data for the watershed and river systems that feed the Quabbin and Wachusett Reservoirs in addition to the necessary daily streamflows on the Ware, Swift and Connecticut Rivers in order to assure compliance with the legal constraints. Daily streamflow records for 34 sites were analyzed over the period from 1930 to 1979 (the MDC/MWRA study began in 1981) to obtain the following necessary inflow sequences:

- Monthly inflows to Shaft 8 from the Ware River based on daily inflows in excess of 85 mgd over the period from October 15 to June 14.
- Minimum monthly discharge from Quabbin Reservoir based on analysis of daily flows in the Connecticut River at the Montague gaging station.

Since there are no gaging stations having 50-year streamflow records in the watersheds above the Quabbin, Wachusett and Sudbury Reservoirs, regional hydrologic procedures were employed to transfer streamflow records from nearby hydrologically similar basins.

Since the monthly simulation model is also employed to evaluate the increase in yield that may be obtained from the existing MDC/MWRA system if water is transferred from alternative sources, it was also necessary to estimate the potential transfers of water to the existing system from the Connecticut River via the Northfield Pumped Storage facility, the Millers River, the Tully River and the Merrimack River. In addition, the potential increased monthly flow in the Ware River at Shaft 8 due to the management of 18,000 acres of the watershed is estimated.

Actual transfers of water from these alternative supply sources (as determined by the simulation model) were often significantly less than the potential transfers of water. This discrepancy occurred because during wet years, when high streamflows resulted in high potential transfers, the reservoirs were full (or nearly full), thus negating the value of the potential transfer volumes.

Reservoir Operating Rule. A unique feature of the Quabbin-Wachusett reservoir system is that an external watershed (the Ware River above Shaft 8) can contribute streamflow during the period from October 15 to June 14. These inflows are diverted to the Quabbin Reservoir for water quality reasons, thus prohibiting the use of the tunnel for the transfer of water from Quabbin to Wachusett. When such transfers are made, the Wachusett Reservoir must supply the entire easterly demand on its own. Since the active storage capacity of the Wachusett Reservoir is not extensive, the instinctive rule to keep Wachusett full during the Shaft 8 transfer season leads to wasteful spillage.

An improved operating rule was developed that both minimizes system spillage (downstream releases lost from the system) and system failures (empty reservoir(s)). This algorithm, which essentially seeks to maintain both reservoirs in their normal range, is shown in Table 2. For example, if the Quabbin Reservoir is within its normal range and the Wachusett Reservoir is below elevation 390 (the low end of its normal range), sufficient water is transferred from Quabbin to bring Wachusett up to elevation 390. Such transfers are not allowed when water is scheduled for transfer from Shaft 8 to Quabbin, and these transfers are physically limited by the available difference in head between the two reservoirs.

This operational algorithm generally resulted in spillages when both reservoirs were full. In addition, during failure months, the Quabbin Reservoir tended to empty before Wachusett, as anticipated.

Monthly Simulation Results

Traditional "Safe Yield" Analysis. The traditional approach to determining the "safe yield" of a reservoir system routes the historical streamflows through the system to ascertain

Table 2

Quabbin-Wachusett Operating Rule

Quabbin Reservoir Pool Elevation	Quabbin Reservoir Operational Desire	Wachusett Reservoir Pool Elevation	Wachusett Reservoir Operational Desire
530		395	No Transfers from Quabbin
	Send Water to Wachusett		
528		392.5	
	Normal Range		Normal Range
491		390	
	No Transfer		Receive Water from Quabbin
490		387.5	

the maximum yield that can be sustained without emptying the reservoirs. For the MDC/MWRA system, the 50-year no-failure "safe yield" is 291 mgd. If two, four, five or eight monthly failures were allowed over the 50-year historic period, the yield increases to 292, 294, 295 and 300 mgd, respectively.

Allowance for Drought Management. The traditional definition of "safe yield" is not entirely plausible because, in practice, consumers can generally be persuaded to curtail water use during times of drought. A feasible drought management program thought to be acceptable to consumers was developed for this study. It is typical of drought management programs adopted by communities throughout the U.S. The estimated effect of this drought program if placed in operation over the 50-year (600-month) planning period is summarized in Table 3 where the demand coefficients represent the fraction of normal usage attributed to each of the four drought stages. Implementation of this program increased the no-failure "safe yield" from 291 to 294 mgd. This 3-mgd increase is roughly equivalent to one year's growth in system demand.

Allowance for Drought Severity. Russell *et al.* examined the severity of the 1960s drought in Massachusetts by analyzing the four-year cumulative sums of annual precipitation over the period from 1871 to 1966.[16] Their results indicated that the four-year cumulative sum of precipitation from 1963 to 1966 was the most severe event of the 96-year period. Russell *et al.* pooled the precipitation records from Fitchburg, Worcester and Fall

Table 3

Drought Management Program

Stage	Contingency Plan	Active Drought Management Months	Active Drought Management % of Time	Demand Coefficient Winter	Demand Coefficient Summer
0	No Conservation	549	91.5	1.00	1.00
1	Voluntary Conservation	30	5.0	0.997	0.995
2	Outdoor Use Banned	17	2.83	0.950	0.935
3	Partial Rationing	4	0.67	0.68	0.72

Table 4

Incremental "Safe Yield" of Alternative Sources of Supply

Alternative	Incremental "Safe Yield" (mgd)
Existing System (without Sudbury)	300
Connecticut River Diversion	63
Millers River Diversion	33
Millers & Tully River Diversion	38
Merrimack River Diversion	120
Watershed Management	5
Sudbury (including 3 Framingham reservoirs)	20

River in their analysis.

Given this relatively long historical record of precipitation, it is apparent that the historical period from 1930 to 1979 used in the MDC/MWRA study contains a drought (the 1960s event) that is more severe than could be anticipated in a "typical" 50-year planning period. Therefore, the MDC/MWRA study argued that additional monthly failures should be allowed during the 50-year historical simulation period in order to account for the unusual severity of drought of the 1960s. If eight monthly failures are allowed, the "safe yield" of the system increases to 300 mgd. A more rational determination of the severity of the drought during the mid-1960s on the system yield can be achieved using a stochastic streamflow model.

There are many problems associated with the determination of drought frequency using four-year cumulative sums of precipitation. The sequences of cumulative sums have very high serial correlations, hence the statistics computed by Russell *et al.* are known to be substantially biased as shown by Loucks *et al.*[16,17] Furthermore, the traditional expressions to compute the recurrence interval of the drought event that occurred from 1963 to 1966 cannot be employed, because the sequences of cumulative sums are not independent. A previous study provides alternative procedures for evaluating the recurrence interval of water supply failure (or cumulative precipitation departures) for dependent failure sequences.[18]

The Impact of Alternative Sources of Supply. The MDC/MWRA employed the monthly simulation model to determine the incremental "safe yield" of the existing system augmented by the alternative sources that are described in Table 4. Again, eight monthly failures were allowed over the 50-year historic period. The rationale used for the existing system alone was assumed to also apply to the existing system, plus an alternative.

Analysis of Traditional "Safe Yield." Hellstrom used the approach described above for the MDC/MWRA in order to evaluate the "safe yield" of their water supply system; further details of the simulation model and results may be found in the MDC Safe Yield report.[19] Overall, the monthly simulation model has enabled the MDC/MWRA to evaluate how the existing and proposed water supply systems would have performed over the past fifty years under a continuous demand of 300 mgd. However, the problem of determining how the existing and proposed systems would perform over the 32-year planning horizon from 1988 to 2020 still remains. In the following discussion, the yield of the reservoir system is considered to be a random variable, instead of a guaranteed minimum yield as was done in the discussion above. Such analyses should provide decision makers with the information regarding the trade-offs among system storage, reliability, yield and drought management procedures that is necessary for the proper solution of this important long-range planning problem.

Water Supply System Reliability

Stochastic Streamflow Models. Stochastic streamflow models are no longer simply research tools; their role in water resource engineering is now well established. Again, quoting Linsley *et al.*:[15]

> It has been customary to refer to the "safe yield" of a reservoir as if it were a

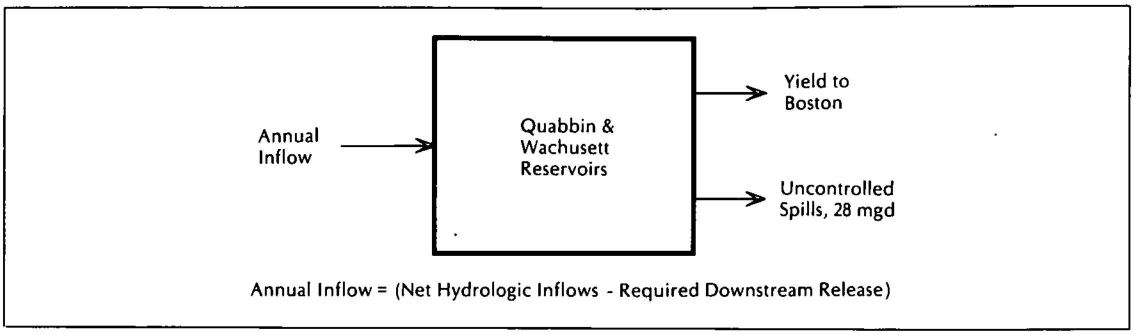

FIGURE 3. A simplified representation of the Quabbin-Wachusett Reservoir system.

> guaranteed minimum yield. Determinations based on analysis of the historic record provide no evidence regarding the reliability of a reservoir. (p. 403)

Instead of the traditional approach of routing the historical streamflows through the water supply system, Linsley *et al.* and others recommend routing at least 1,000 sets of N-year synthetic streamflow sequences through the water supply system to generate a storage-reliability-yield relationship instead of simply a storage-yield relationship. In this analysis, N is defined as the length of the planning horizon. The advantages associated with the use of stochastic streamflow models in water supply problems, as opposed to just the historical streamflow sequence, are well documented by Fiering, Vogel, Vogel and Stedinger, and others.[8,18,20,21] Short-term reservoir systems that refill each year require the use of monthly, weekly or even daily stochastic streamflow models. Such models can be extremely difficult to implement as evidenced by one such state-of-the-art model recently developed by Stedinger *et al.* that is currently being applied to California's Central Valley Project.[22,13] For long-term reservoir systems, such as the MDC/MWRA system, which does not typically refill on an annual basis, it may be adequate to use an annual stochastic streamflow model instead of a more complex monthly model. In fact, several studies have found only modest differences in the performance of reservoir systems that result from the application of different, yet reasonable, stochastic streamflow models. Current developments in the application of stochastic streamflow models emphasize methods for acknowledging and incorporating the uncertainty inherent in all model parameter estimates instead of emphasizing which model is best.[22]

A Simplified Annual Water Supply System Simulation Model. To approximate the reliability of the MDC/MWRA water supply system and to determine how safe its "safe yield" really is, the system is simplified so that an annual stochastic simulation model can be applied. A more comprehensive analysis would employ a monthly stochastic streamflow model in combination with the monthly simulation model. It was found, however, that a simple annual model is adequate to approximate the relationship among yield, storage and reliability for the MDC/MWRA system. Figure 3 depicts the simplified annual simulation model where the Quabbin and Wachusett Reservoirs are treated as one reservoir with an annual inflow equal to the total hydrologic input to both reservoirs less the legislated minimum downstream releases. The hydrologic input consists of watershed inflows and the Shaft 8 inflow from the Ware River in addition to the precipitation that falls on each reservoir surface less evaporation losses. All historical inputs and outputs to the annual model are obtained from the monthly system simulation model. This simplified annual simulation model could not be developed without the more complex monthly model that performs the necessary daily and monthly accounting to provide a realistic description of the system. Therefore, the annual model is not

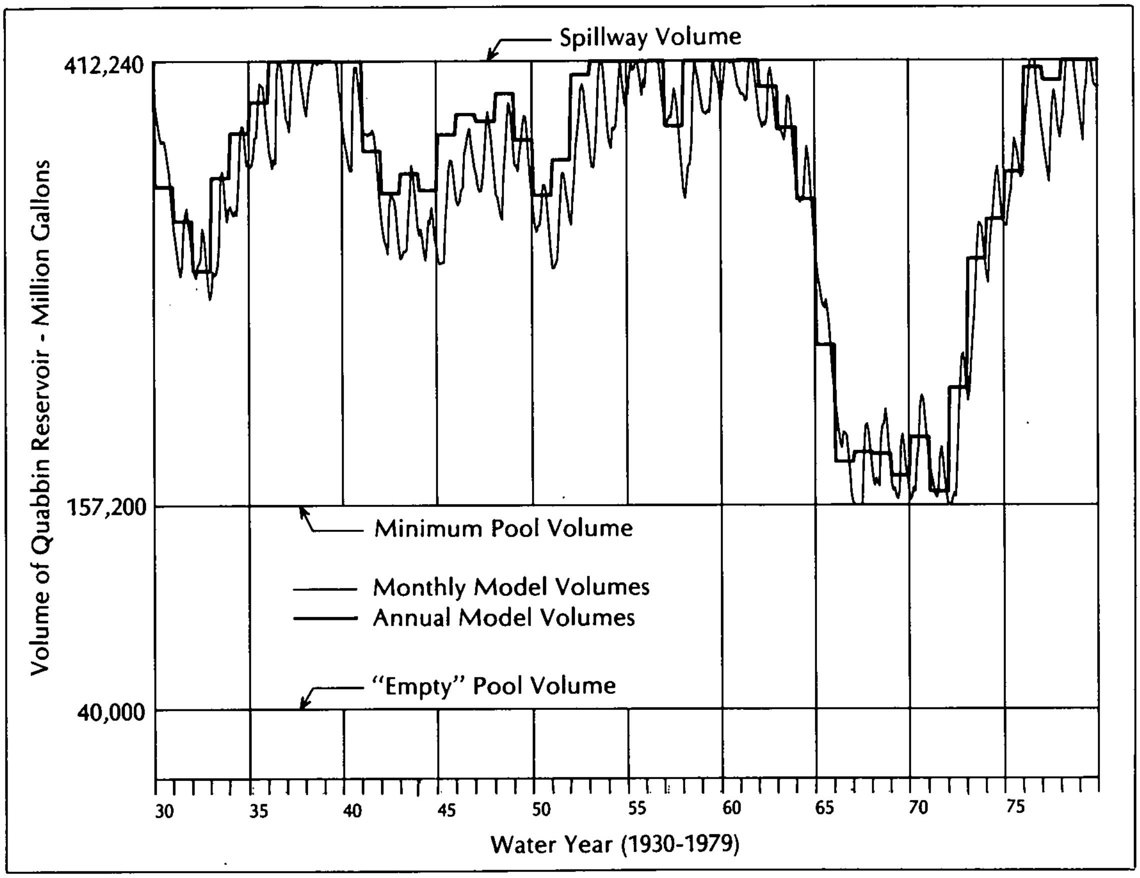

FIGURE 4. A comparison of Quabbin Reservoir storage levels using monthly and annual simulation models.

intended to replace the monthly model; it is used simply as an adjunct to approximate system reliability. Alternative procedures for approximating the reliability of large and complex water supply systems are possible, such as the approaches used by Palmer and Lettenmaier on the Seattle water supply system, the indexed sequential modeling approach employed by Labadie *et al.* for the California Central Valley Project, or the simplified procedures recently introduced by Vogel.[23,12,18]

Figure 4 demonstrates the performance of the annual simulation model compared with the monthly water supply simulation model over the historical period of record. Using both models, it documents what the storage levels of Quabbin Reservoir would have been over the historic period if a continuous yield of 300 mgd were delivered. The monthly model clearly provides more detail than the annual model. However, the annual model captures the overall behavior of the system. Using either model, it is evident that the system is vulnerable to severe droughts such as the one that occurred in the 1960s. If the yield (demand) had been 300 mgd during the historical period, the Quabbin Reservoir would have been drawn down below its spillway from 1961 through 1976. Furthermore, Quabbin Reservoir would have remained operationally nearly empty at its minimum pool elevation for five consecutive years from 1967 to 1971. The profound impact of drought on the MDC/MWRA system, such as shown in Figure 4, provides ample justification for a careful determination of the long-range reliability of the system.

An Annual Stochastic Streamflow Model. To characterize the hydrologic inputs to the MDC/MWRA water supply system, a relatively simple annual stochastic streamflow model was developed that can mimic the sequence of observed annual inflows to the system. Figure 3 defines the annual inflows. The 50 historical annual inflows are well approximated by a normal distribution with a mean, $\hat{\mu}$ = 328 mgd, and standard deviation, $\hat{\sigma}$ = 111.5 mgd. The normal hypothesis was substantiated using probability plots and probability plot correlation coefficient tests that have been recently advocated by Filliben and by Vogel.[24,25] This result is not unexpected: Markovic found that the hypothesis that annual streamflows are normally distributed could not be rejected at the 5 percent significance level for 72 percent of 446 sites in the western portions of the U.S.[26]

The annual inflows appear to be characterized by a dependent process with first-order serial correlation, $\hat{\rho}_1$ = 0.351, corrected for bias.[27] This result is to be expected: using 106 basins in New England, Vicens *et al.* found the mean and standard deviation of estimates of the first-order serial correlation coefficient of annual streamflows, ρ_1, to be 0.22 and 0.14, respectively.[28]

The simplest stochastic streamflow model that will reproduce these estimated statistics, in addition to the normal hypothesis, is a first-order autoregressive (AR(1)) model of the form:

$$Q_{t+1} = \mu + \rho_1(Q_t - \mu) + \sigma\epsilon_t\sqrt{(1 - \rho^2_1)} \qquad (1)$$

where Q_t is the annual streamflow in year t and ϵ_t is a normally distributed random variable with zero mean and unit variance. The model may be implemented by replacing the population values of the parameters in Equation 1 by the sample estimates of the parameter values reported earlier.

Stochastic Simulation of System Yield and Reliability. The stochastic approach to water supply problems employs a stochastic streamflow model to generate N years of synthetic streamflows, where N is equal to the planning period of interest. Those synthetic streamflows are then routed through the annual water supply system model employing the sequent peak algorithm to determine the yield that could have been delivered without failure over the N-year period. Repeating this procedure 1,000 times leads to a cumulative probability distribution of the no-failure system yield. If the planning period happens to equal the length of the historical record (50 years in this case), then the traditional "safe yield" estimate based on the single historical streamflow sequence would be expected to be approximately equal to the median yield derived from the stochastic analysis. To argue otherwise would require either a longer historical streamflow record or further knowledge regarding the severity of the historical streamflow record.

The stochastic streamflow model in Equation 1 can be employed to derive general storage-reliability-yield curves for the MDC/MWRA system using current procedures advocated by Vogel, and Vogel and Stedinger.[20,29] The results of these simulations are presented in Figures 5, 6 and 7.

Figure 5 displays the storage-yield function using the annual and the monthly historical simulation models compared with the storage-yield function for the median yield derived from the stochastic model. Again as evidenced earlier in Figure 4, the annual historic simulation model approximately mimics the behavior of the system as does the annual stochastic simulation model. Figures 4 and 5 provide evidence that an annual model approximates the overall storage-yield behavior of this system. However, the stochastic model furnishes additional information regarding system reliability that the historical simulation models cannot.

Figure 6 documents the information that may be obtained from a stochastic analysis. In this case, the Quabbin-Wachusett reservoir system reliability is plotted as a function of the system yield and the planning period. In one sense, Figure 6 clarifies the decision problem of determining the system "safe yield;" namely, that the determination of system "safe yield" is contingent on *a priori* decisions regarding the length of the plan-

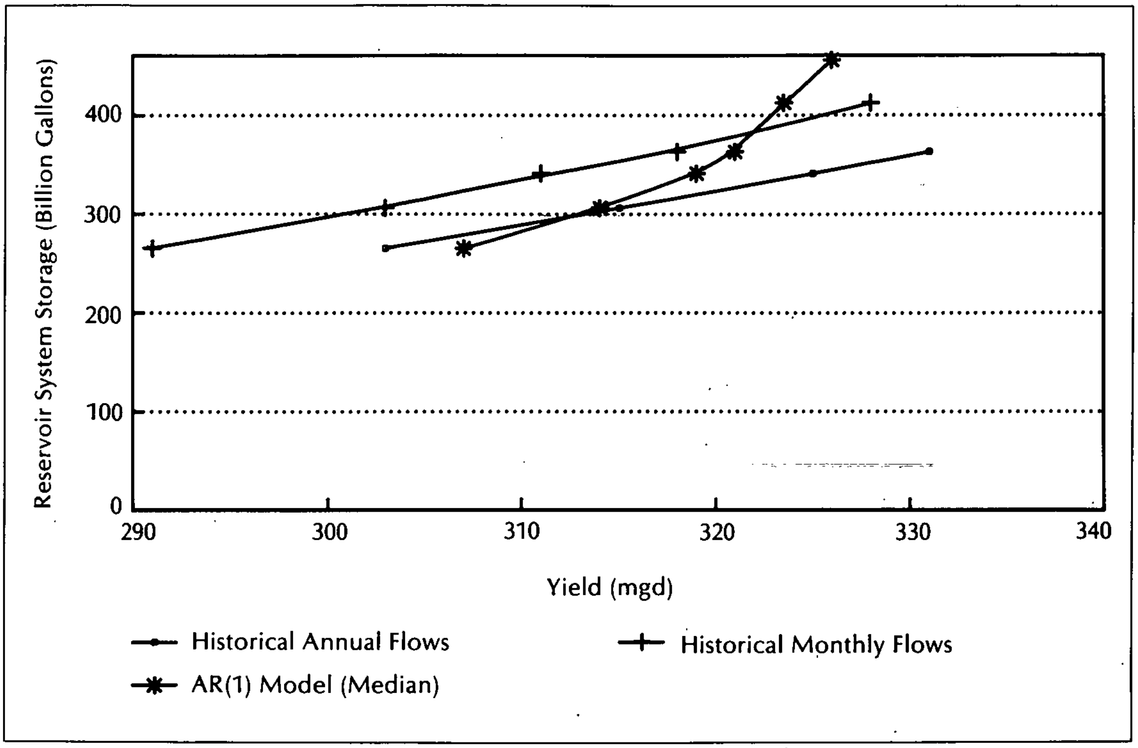

FIGURE 5. The storage-yield function for the Quabbin-Wachusett reservoir system.

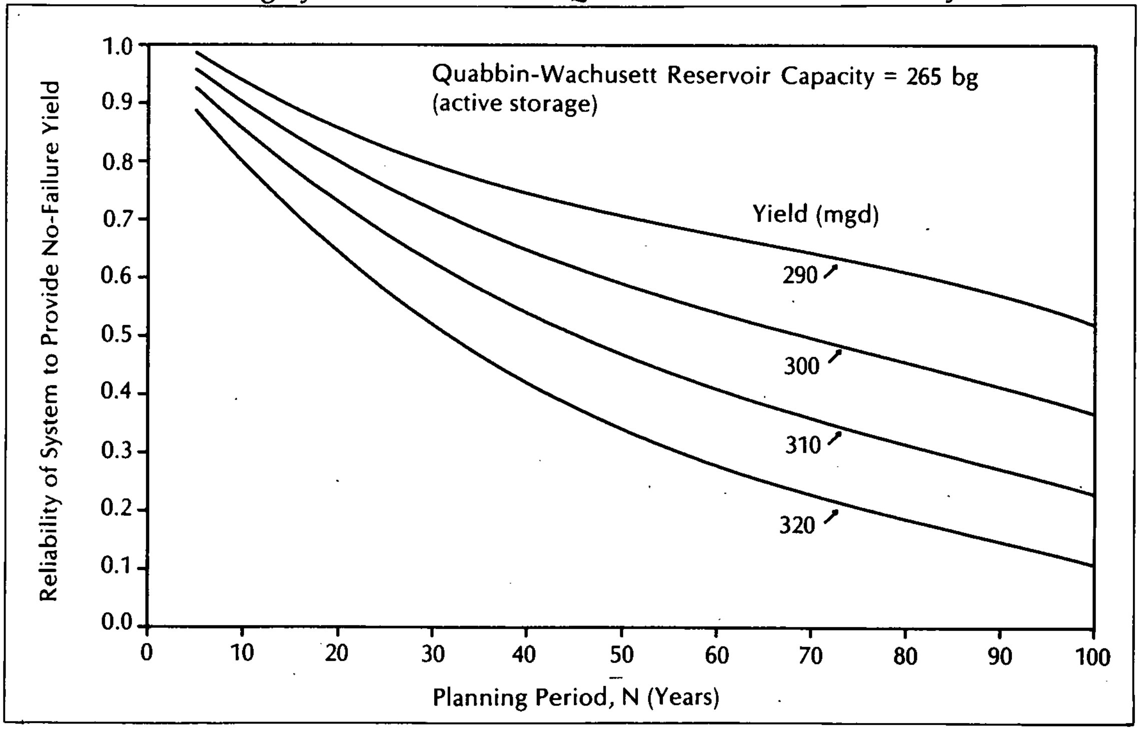

FIGURE 6. The reliability of the Quabbin-Wachusett reservoir system as a function of system yield and planning horizon.

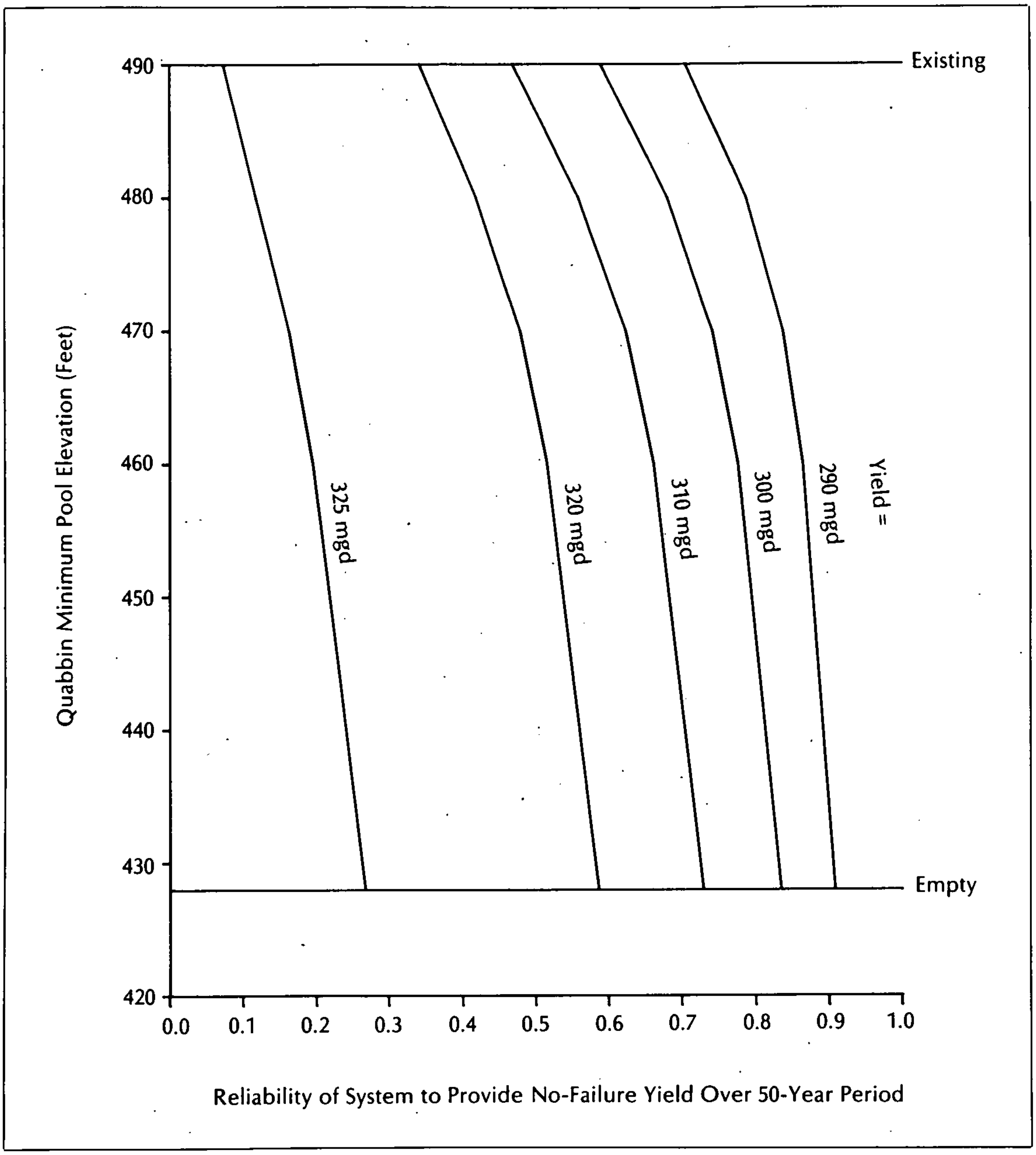

FIGURE 7. The reliability and yield of the Quabbin-Wachusett reservoir system as a function of Quabbin Reservoir minimum pool elevations.

ning period and an acceptable long-range system reliability. History can provide some guidance. For water supply systems in the U.S., 50 years is the planning period that is used most often.[30] Furthermore, since most systems were designed using the traditional approach of routing the historical streamflows through the reservoir system to determine the no-failure yield, the default value of long-range reliability is approximately 0.5 (the cumulative probability of the median yield). Such reasoning produces a system "safe yield" of approximately 305 mgd from Figure 6. However, that yield drops significantly if a long-range reliability much greater than 0.5 is required.

Impact of Quabbin Minimum Pool Elevation on System Yield and Reliability. Once the storage-reliability-yield relationship for the system is determined, the impact of increasing active system storage can be examined by lowering the minimum pool elevation in Quabbin Reservoir. Figure 7 depicts the relatively marginal impacts of lowering the minimum pool elevations in Quabbin Reservoir. For a fixed reliability, yield increases corresponding to drops in the minimum pool elevation are rather minimal. Likewise, for a fixed yield, lowering the minimum pool elevation has only a marginal impact on long-range system reliability. These results are to be expected for any long term reservoir system that accommodates several years of carry-over storage. The maximum yield for this system, obtainable by emptying the Quabbin Reservoir, for a reliability of 0.5, is about 321 mgd as shown in Figure 7.

How Safe Is a "Safe Yield"?

Variability of the Theoretical Maximum Yield. The theoretical upper limit on the yield of a water supply system is the average annual inflow to that system. In this case, average annual inflow is the net hydrologic inflow (streamflow input and reservoir precipitation minus reservoir evaporation, seepage and other losses) minus the required downstream release (see Figure 3). Even if a reservoir system with infinite storage capacity could be constructed, the long-term system yield could not exceed the average annual inflow. For the MDC/ MWRA system, 328 mgd is an *estimate* of the average annual inflow to the reservoir system based on a 50-year historical record. The true value of the average annual inflow has an approximate 95 percent confidence interval that ranges from 289 to 367 mgd. Using these straightforward statistical calculations, the rather fuzzy random interval in which the theoretical maximum system yield lies can be determined. Acknowledgement of this profound uncertainty inherent in any streamflow record and resulting model parameter estimates is the current focus of research in stochastic streamflow modeling.[22]

It should be apparent from these calculations and from Figure 7 that increasing system storage will not, by itself, be sufficient to augment the existing supply. Instead, attention should focus on increasing the average annual inflow through major river diversions that are currently proposed. Such diversions, if not ultimately rejected, require years or even decades to earn approval, as is evident from the Connecticut River diversion project plan that was first proposed in 1964 and whose future still remains uncertain.[31]

A determination of the range of maximum system "safe yields" that can be expected is useful in pointing out that a yield greater than 328 mgd is too optimistic. The favored "minimax" decision criterion results in a maximum system yield lower than 328 mgd.

Variability of the System "Safe Yield." Engineers are often reluctant to provide a single value for the system "safe yield" because system "safe yield" is a random variable. Nevertheless, decision makers expect engineers to provide a single value upon which to base necessary decisions. Engineers should be aware of the uncertainty associated with a particular "safe yield" estimate prior to divulging a single value for public scrutiny. By analogy, a surgeon may elect to perform surgery knowing in advance the range of possible outcomes and their associated likelihood. In cases of elective surgery, it is considered poor medical practice not to present the potential outcomes prior to surgery, otherwise family members and/or the patient could witness catastrophic results. Thus, any single value of "safe yield" should always be qualified by a clear account of its statistical significance to avoid potential surprises.

Monte-Carlo Experiment: Variability of System "Safe Yield." A Monte-Carlo experiment is performed to determine the range of "safe yield" estimates that can be anticipated for the MDC/MWRA water supply system. The stochastic streamflow model in Equation 1 was employed to generate 10,000 equally likely 50-year annual inflow sequences. The generated annual inflows have the same mean, standard deviation and first-order serial correlation as do the 50 observed

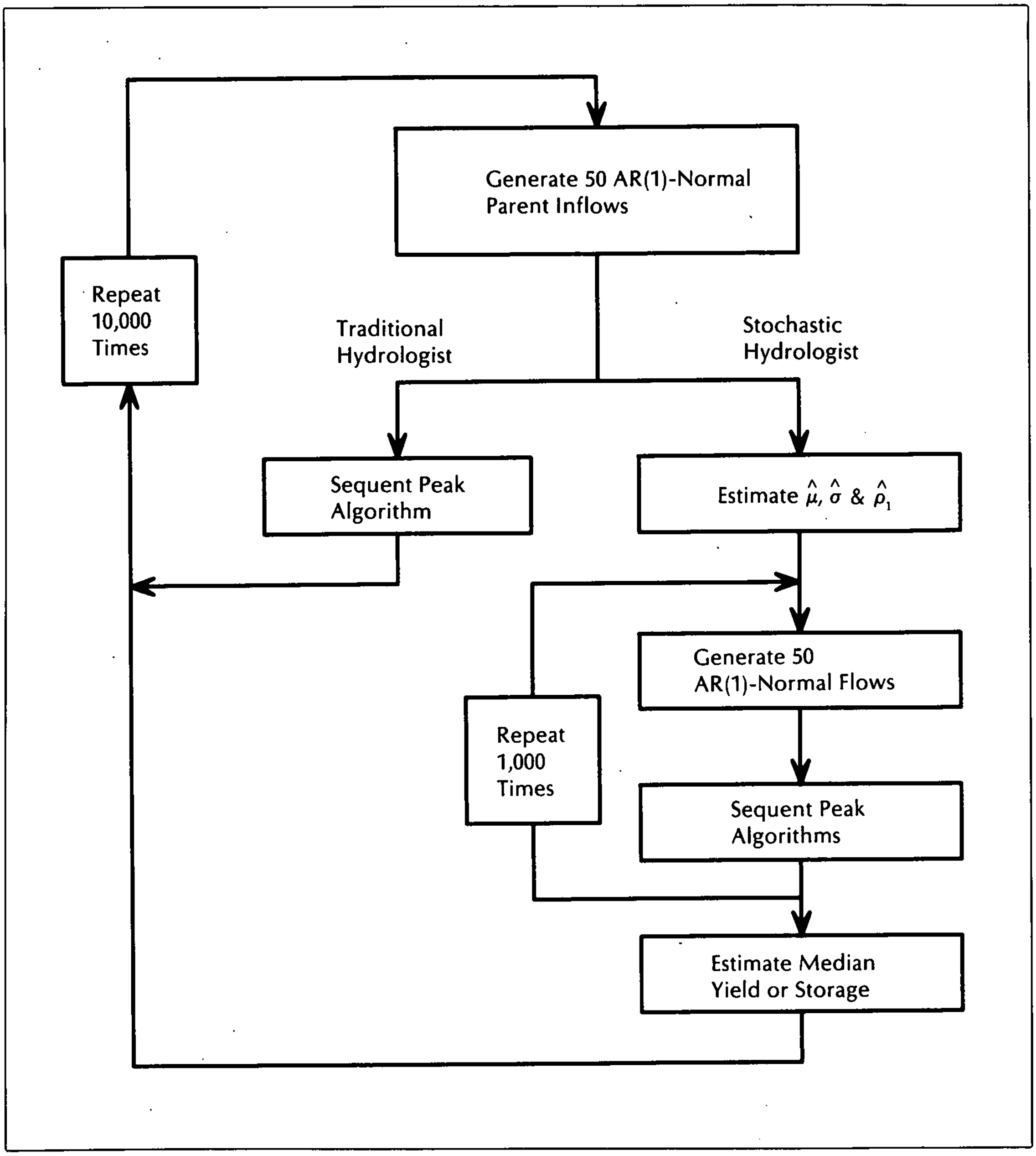

FIGURE 8. Monte-Carlo experiment to determine the likely range of "safe yield" estimates.

annual inflows obtained from the monthly simulation model. The 10,000 generated sequences are considered the parent or true inflows in this experiment. Each 50-year parent trace contains drought sequences that are comparable to, but slightly different from, those contained in the single observed 50-year historical sequence. Each of the 10,000 synthetic traces (parent inflows) was routed through the annual water supply simulation model using the sequent peak algorithm to obtain 10,000 estimates of the traditional "safe yield." This procedure is summarized in Figure 8 where it is contrasted with the alternative approach of estimating the "median yield" by fitting a stochastic streamflow model. The stochastic hydrologist fits the assumed stochastic

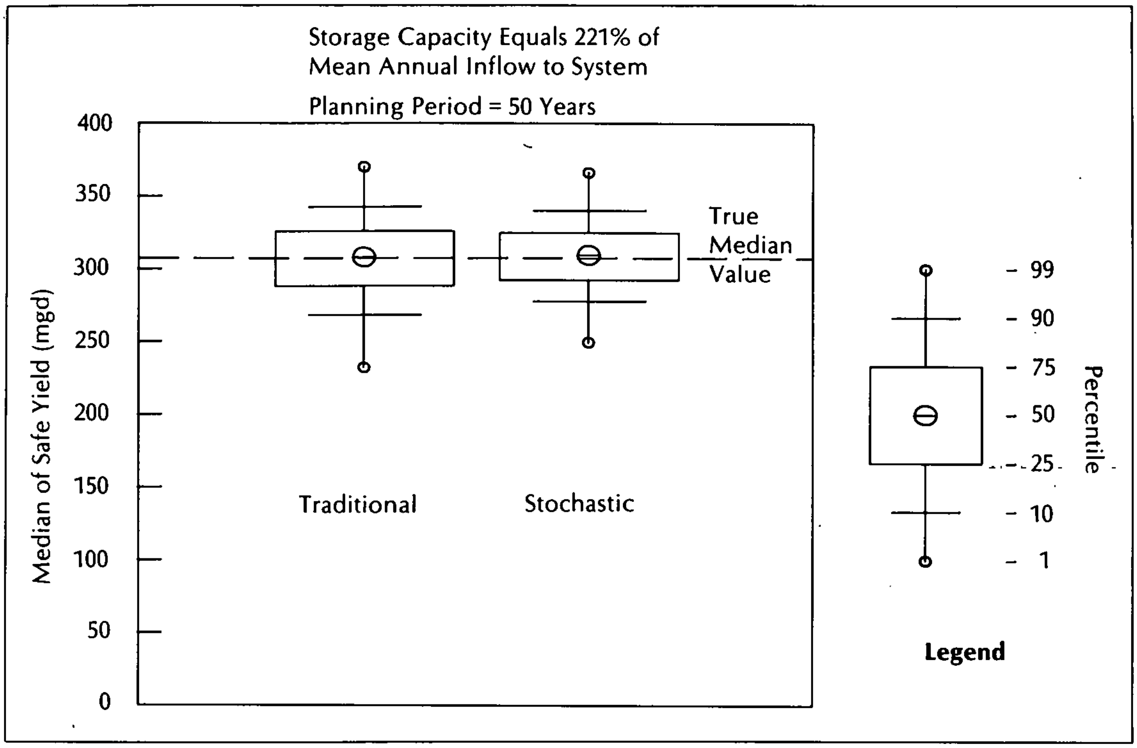

FIGURE 9. Box plots illustrating the variability of the median of "safe yield" estimates for the Quabbin-Wachusett reservoir system.

streamflow model in Equation 1 to each 50-year parent inflow sequence by estimating the parameters μ, σ, and ρ_1. The fitted AR(1) model is then employed to generate 50 annual streamflows that are in turn routed through the annual simulation model using the sequent peak algorithm. This procedure is repeated 1,000 times to generate a cumulative probability distribution of system yield.

This Monte-Carlo experiment provides a comparison of the precision of a "safe yield" estimate using the traditional approach and the stochastic approach. The results are summarized in the form of box plots in Figure 9. Each box plot simply describes the cumulative probability distribution of the median yield derived from the traditional approach and the stochastic approach. The median yield derived by the stochastic analysis is only slightly more precise (less variable) than the traditional "safe yield" estimate. Therefore, the stochastic analysis does not really improve the ability to estimate the median yield. This finding is consistent with a similar recent study by Staschus and Kelman on California's Central Valley Project.[13]

What is particularly interesting and revealing is the overall variability of the safe or median yield statistic. Figure 9 documents that the "safe yield" of the MDC/MWRA system could easily fall anywhere in the range 232 mgd to 370 mgd though its average value is approximately 300 mgd. This experiment reveals the remarkable uncertainty associated with the single "safe yield" estimate that is often quoted to be 300 mgd and considered as a guaranteed minimum yield.

The "safe yield" statistic is more variable than the *theoretical maximum yield statistic* because the "safe yield" statistic accounts for variability in the mean, variance and autocorrelation of the observed streamflows (see Equation 1), whereas the theoretical maximum yield statistic only accounts for the variability associated with the mean annual streamflow.

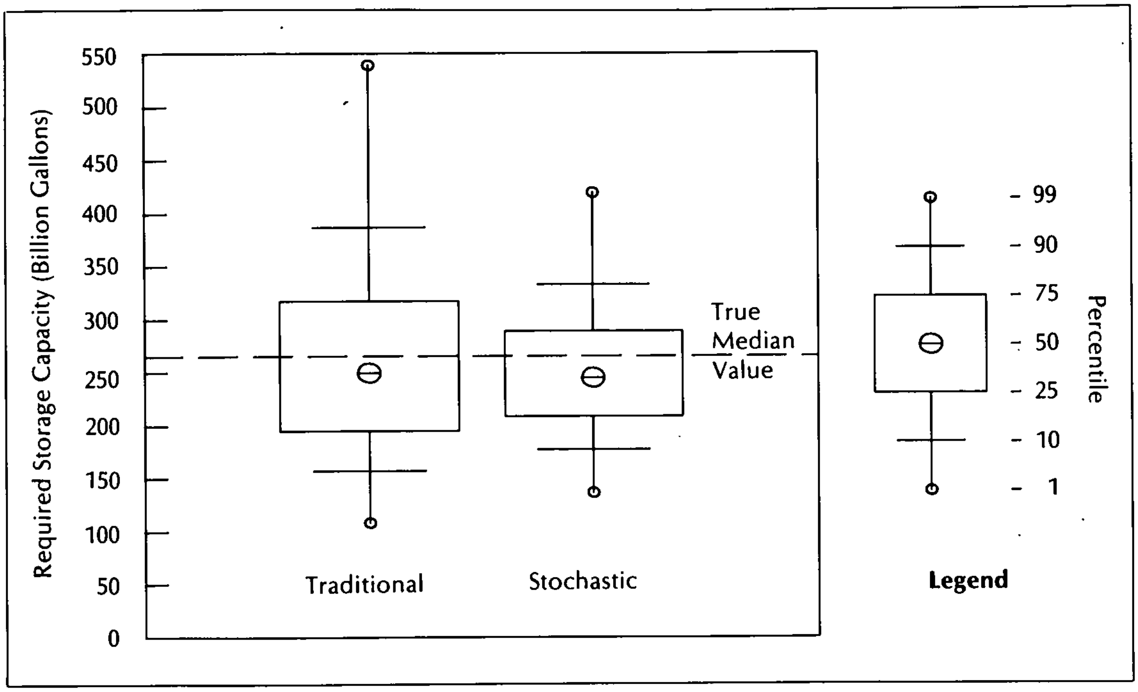

FIGURE 10. Box plots illustrating the variability of the required Quabbin-Wachusett reservoir storage capacity to deliver a yield of 300 mgd over 50 years without failures.

Monte-Carlo Experiment: Variability of Design Storage Capacity. The previous experiment illustrated that when the stochastic hydrologist employs the correct model, estimates of the median yield are about as precise as if the traditional period of record analysis is employed. Recent research has shown that stochastic streamflow models can dramatically improve the precision (reduce the variability) of design storage capacity estimates.[20,21] Consider the inverse problem faced by the original design engineers: Given a system "safe yield" of 300 mgd, what is the minimum storage capacity of the Quabbin-Wachusett reservoir system that would be required to deliver that yield, with no failures, over a 50-year period?

In this case, the same problem as before is solved, but the storage capacity now becomes a random variable and the "safe yield" is considered a fixed value. The same Monte-Carlo experiment described in Figure 8 is performed and the results are illustrated using box plots in Figure 10. The variability associated with the required reservoir storage capacity is alarming, regardless of which procedure is employed. However, in this instance, the variability is considerably reduced when the correct stochastic streamflow model is employed. Other studies have shown similar gains even when different, yet reasonable, models are chosen.[20,21]

Perhaps the most revealing aspects of the experiments summarized in Figures 9 and 10 are not the gains attributed to the use of stochastic streamflow models, but rather the enormous variability associated with estimates of the design capacity of a reservoir system or its "safe yield" obtained from only 50 years of streamflow record. This variability is inherent in all water supply investigations and should be acknowledged, particularly in the context of a long-range planning study.

Summary

This study has sought to clarify the concept of "safe yield" within the context of planning the long-range surface water supply for eastern Massachusetts. Detailed monthly simulations of the MDC/MWRA water supply system using streamflow data for the period from 1930 to 1979 reveal that the existing reservoir system could have delivered approximately 300 mgd over that period if a drought management plan was implemented during dry periods. The monthly simulation model was also used to assess the incremental yields that could be sustained from the six alternate sources shown in Table 4. In short, the monthly simulation model is a useful tool for evaluating the yield of existing and proposed water supply system configurations.

Since 1969 the demand for water on the MDC/MWRA system has exceeded 300 mgd with projections for an additional demand of 120 mgd by the year 2020. In this long-range planning problem that seeks to determine the optimal alternative(s) to meet a demand of 420 mgd in the year 2020, it is essential to acknowledge a range of possible drought scenarios that could occur in the future. For this purpose, it was necessary to simulate the water supply system using alternative streamflow sequences similar in character to the observed streamflows, yet resulting in different drought scenarios than experienced over the historical period.

To simplify the analysis, yet still remain faithful to the system behavior, an annual water supply system simulation model was developed that mimics the storage-yield relationship resulting from a more complex monthly model. Figure 4 documents the excellent agreement between reservoir system volumes using the annual and the monthly system simulation models over the historical period of record. An annual stochastic streamflow model was fit to the historical streamflows and linked with the annual water supply system simulation model in order to generate a storage-reliability-yield relationship rather than simply a storage-yield relation produced by traditional analysis. The value of this exercise is illustrated in Figure 6 which describes the relationships among system yield, long-range reliability and planning horizon. Long-range reliability is defined as the probability the system can deliver a fixed yield, without failure, over a pre-specified planning horizon.

The stochastic analysis treats the "safe yield" of the MDC/MWRA system as a random variable instead of a fixed value as does the traditional analysis. "Safe yield" is shown to be a variable that depends on the choice of planning horizon and long-range reliability.

A Monte-Carlo experiment was performed to evaluate the precision associated with the "safe yield" estimate. It showed that the "safe yield" estimate could be as low as 232 mgd or as high as 370 mgd, though its average value is approximately 300 mgd. This result documents the uncertainty associated with the entire "safe yield" analysis and hopefully provides evidence to decision makers that engineers can only approximate "safe yield;" definitive statements are implausible.

The stochastic analysis presented is a *preliminary* one because it approximates the system behavior using only an annual model. A more detailed analysis would employ a monthly stochastic streamflow model linked to the monthly water supply system simulation model as was done in other recent water supply applications.[10,11,12,13,14,22,23] Nevertheless, the annual model employed mimics the system's monthly behavior fairly well. A more detailed stochastic simulation would probably generate similar results.

The preliminary storage-reliability-yield relationships developed for the MDC/MWRA water supply system provide a framework for evaluating the trade-offs among system costs and system reliability. Any "safe yield" value has associated with it construction costs, operation and maintenance costs, environmental impact costs and even user costs associated with drought management requirements. For each proposed system yield, the associated system costs must be weighed against the benefits derived from a safe (reliable) source of water. Figure 1 provides some historical guidance; previous system expansions have resulted in "safe yield" estimates well in excess of projected demands, especially during periods of growth in the demand curve. Given the substantial variability associated with both the estimate of "safe yield" and the projected demand, continuing the tradition of conservative system expansion similar to the expansion that occurred on completion of the Quabbin Reservoir is recommended.

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