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OF GENERAL INTEREST

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EDWIN B. COBB
President
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SOME THOUGHTS RELATIVE TO THE SOCIETY

PRESIDENTIAL ADDRESS BY MILES N. CLAIR

(Presented at Annual Meeting of Boston Society of Civil Engineers, held on March 16, 1955.)

THE FOUNDERS of The Boston Society of Civil Engineers met* on April 26, 1848 "to consider the expediency of taking measures to form a Society, for social intercourse and professional improvement, to be composed of civil engineers and of gentlemen engaged in pursuits kindred to civil engineering." The first regular meeting was held on July 3, 1848 and the society was incorporated on April 24, 1851 by an Act of the General Court "for the purposes of promoting science and instruction in the department of Civil Engineering." The first article of the most recent Constitution, adopted June 12, 1910, reads:

ARTICLE I—OBJECTS

"The objects of this Society are: the professional improvement of its members, the encouragement of social intercourse among engineers and men of practical science, and the advancement of engineering. For the promotion of these objects, stated meetings of the Society shall be held and a library maintained for use of its members." The objectives of the group that met in 1848 and incorporated in 1851 appears to be covered by the constitution under which the Society functions. The Society operates as a legal entity under the Acts of Incorporation and its constitution and by-laws, with defined requirements for membership, officers, elections, meetings and amendments. The Officers consisting of a President, two Vice-Presidents, a Secretary, a Treasurer, and four Directors along with the three latest Past Presidents constitute the Board of Government which has the responsibility for the general management of the affairs of the Society. The Nominat-

*Journal of The Boston Society of Civil Engineers, Volume XXIII, Page 151.

ing Committee consists of three latest Past Presidents (not members of the Board of Government) and six elective members, three of whom shall be elected annually to serve two years. At the time of the Annual Election, the Nominating Committee makes nominations to fill vacancies in any office or in the Nominating Committee. Candidates for any office receiving the largest number of legal votes by letter ballot shall be elected. This information is available in the records of the society** but is repeated here to make clear the formal purposes of the Society and responsibility of the officers and the membership in forwarding these objectives. The Nominating Committee is of major importance since it nominates the officers and new members of the Nominating Committee. The members of the Society elect by secret ballot but essentially only approve the actions of the Nominating Committee.

It is the purpose of this paper to consider some aspects of the questions resulting from the fact that although the Society is 107 years old and financially in a better position than at any other time in its existence the membership is only as large as it was 20 years ago (figure No. 1). Are the objectives of the Society as given in the constitution being attained? Should they be reconsidered in view of changes in conditions since 1848 and 1910? What are the causes of the failure of the Society to grow in numbers and what can be done about it?

It would be foolish to deny that these questions in one form or another has been asked both inside and outside the Society. The fact that there are such questions is a healthy sign rather than a cause for concern. It shows that there is dissatisfaction with what the Society is doing or has to offer and it is a challenge to the officers, the Nominating Committee and the membership. The questions have risen before and will rise again.

The original statement of purpose of the Society places social intercourse first and professional improvement second while the present (1910) Constitution reverses this order. Whether or not there was any significance to the original order certainly some thought must have been given to reversing it. The third objective given in the present constitution, namely, "the advancement of engineering was not included in the original statement. The stated purpose of incorporation may well be considered to include the fourth and fifth objectives,

**Volume 38—1951, p. 306.

namely, "promoting service and instruction in the department of Civil Engineering."

If we consider the first objective as professional improvement and assume that this means a greater knowledge of engineering practices both technically and ethically then it would seem that the papers and discussions both before the main Society and the sections provide an opportunity for such improvement. It would appear, from the papers published, that the Society is fulfilling this objective.

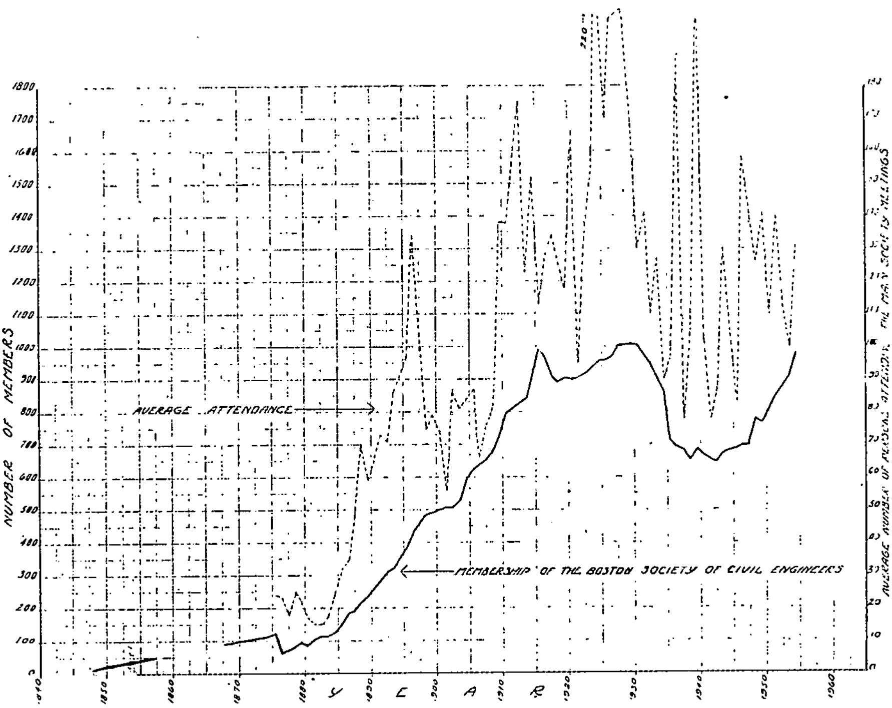

FIG. 1.

Social intercourse which apparently was of great importance to the Founder Group remained so for many years as is evidenced by the elaborate annual dinners, picnics and other trips. This objective rightfully is of prime importance since by meeting with a chance to relax and talk freely the members can come to know each other and this leads not only to exchange of ideas but contacts of value for business and employment which are particularly important to young engineers and engineers new in the area. The Society has changed from having dinners at every meeting to two or three per year at the present time with collations after the other meetings. The high cost of meals under present day conditions have led to this compromise. It is clear, however, that something has been lost in relation to intercourse between members and it may well be that the Society is not now fulfilling properly this objective. Consideration should be given to more dinner meetings or a more elaborate collation. Picnics with the ladies invited in conjunction with the inspection of some engineering project is also a pleasant way of getting members acquainted.

The third objective, namely, "the advancement of engineering" is a very broad objective and it is interesting to note that there is no specific mention of civil engineering in the objects of the Society as given by the present Constitution. This could hardly be an oversight and must mean that the intention was to have the Society cover the field of engineering in general. It is noteworthy that in 1848 the Society was to be composed of civil engineers and of gentlemen engaged in pursuits kindred to civil engineering. The present constitution states "members shall be civil, mechanical, mining, or electrical engineers, or other persons belonging to a technical profession." Membership is thus open to all engineers but a review of the roster of members does not show many outside of civil engineering or fields closely related.

The results of a study made by the Secretary, at the request of the Board of Direction, is given on page 380 of Volume 41 of the JOURNAL of the Boston Society of Civil Engineers. This indicates that the present membership is approximately as follows—

Architects	1.5%
Contractors & Construction	14.5
Electrical	0.5
Mechanical	1.5
Teachers & Research	7
Surveyors	9.5
Branches of Civil Engineering	65.5

The papers and talks given in these earlier years covered many subjects not purely civil engineering or closely related. In recent years the subjects have come closer to civil engineering and thus the effort toward the advancement of engineering may be considered as almost confined to civil engineering.

The work and reports of the special committees such as on the Sub-soil of Boston and Rainfall are important contributions to engineering and the community. More of these types of studies and reports are needed. It is also important that developments in other fields of engineering be brought to the attention of members and that a reversal of the present trend toward concentration on civil engineering subjects be given consideration.

Another possible interpretation of "the advancement of engineering" would be in relation to the public. The Society might well give more effort to educating the community on the importance of engineers and engineering advice in the government and where there are programs of expenditures for public works. A series of lectures sponsored by the Society for the information of the public should be considered.

The fourth objective of "promoting science" can be interpreted as similar to the advancement of engineering. The Society does provide a forum for discussion and has made available money such as the Desmond Fitzgerald Scholarship for research and study. Consideration should be given to means of providing additional funds for such studies in engineering and the sciences associated with civil engineering. Programs of meetings could well include papers on basic science.

The fifth objective of "instruction in the department of Civil Engineering" has received considerable attention in recent years through the Hydraulic, Surveying and Structural Lecture Series. All discussions, papers and talks may in a sense be considered as instruction but it is believed that the lecture series more closely fulfill the intent. The lectures have proven so popular to members and others that consideration should be given to repeating them at regular intervals and including other subjects with each of the Sections of the Society being the sponsor in succession.

The 1848 and 1910 objectives of the Society appears to be fulfilled in a reasonable manner, therefore, we should consider what present conditions may require.

We are in a period where there have come into being a large number of societies dealing with the sub-divisions of engineering. Members of the three principal fields of engineering, Civil, Mechanical and Electrical have a real and direct interest in these sub-divisions and specialties. The National Societies have attempted to keep the interest of their membership by the formation of local sections and by quarterly or other special meetings held in the various geographical areas. The local sections themselves hold area meetings of the sections. The efforts to get united action of engineering groups on matters of legislature and public interest as well as to economize on costs of quarters and notices of meetings has led to city and state association and organizations. Even in the Boston Society itself there is this sub-division in the organization of sections.

Most engineers have major interests that result in membership in more than one technical organization and minor interests that lead to the desire to attend some meetings of others. The result is that there now exists intense competition not only for membership but for the attention of the engineers and this is undoubtedly one of the reasons for difficulties encountered in attempts to increase membership. Additional reasons are probably the demands of the accelerated pace at which all our activities are being carried on particularly because of wars and threats of wars. The resulting mental and nervous fatigue is not conducive to attendance by members or maximum efforts of the officers of the Society. In order to meet this situation programs must be of more service and paid personnel must be utilized to a greater extent. In order to determine what to do we also must recognize our field of service to the engineering community. Although our membership includes engineers residing in many parts of the world and the United States, the larger number are from the Metropolitan Boston Area. This is the area of greatest concentration of Engineering Societies. What engineers in this area are eligible for membership? What have we to offer to these engineers?

The constitution states "The Society shall consist of Members, Honorary Members, Life Members, Junior Members, and Associates." Members shall be civil, mechanical, mining or electrical etc. Full membership in the oldest engineering society in the United States is thus open to engineers in anyone of the recognized fields of engineering with four years of professional experience or with two years of experience after receiving a technical degree. We provide a Society in which the younger engineers can have full membership. The membership fees also are lower than that of the national societies which is an important factor. The Section meetings are a means of direct contact with older members and a forum for early efforts at technical discussion and writing. The meetings of the five sections of the Society average five or six per year each. The meetings of the main Society average eight per year.

The Society as a local society is subject only to local control and is self supporting. Recognition of superior performance by important prize awards is possible without national competition.

Prize Awards include—

Desmond Fitzgerald Medal & Scholarship
Clemens Herschel Award
Section Prize Awards
Student Chapter Prize Award
John R. Freeman Fund Award

Scholarships are given to a student at Tufts University and at Northeastern University.

There are reading rooms at the Tremont Street Headquarters of the Society and an excellent collection of Civil Engineering Books and magazines.

The Society publishes a JOURNAL four times a year containing the papers delivered before the Society which is considered of such importance that subscriptions are held by many libraries and schools throughout the world.

Special publications are issued from time to time which are of interest to engineers locally and nationally.

It is apparent that a great deal is offered by the Society and available to Engineers in the Boston Area. The cause of lack of growth cannot, therefore, be failure of the Society to provide a service. The trouble may be (1) failure to keep members informed (2) the type of program of the meetings and (3) the fact that engineers do not know what is available to them by membership. Failure to keep members informed may result from the fact that most of the business of the Society is done by the Board of Direction. It would seem desirable to distribute to the members the minutes of each Board meeting or at least to publish them in the JOURNAL. A study of the distribution of interest or occupation of the members shows a large group concerned directly in construction and also another large group of surveyors. A review of the papers of recent years does not show a proportionate number of direct interest to these groups. The construction group is large enough to support a Section and consideration should be given to this as a means of being of more service.

The problem of Section programs and their relation to that of the main Society requires careful consideration. The service performed by the sections in providing a means of exchange of ideas by the presentation of papers and discussions for those particularly interested is of great importance. The programs often include speakers of international importance and the attendance is frequently greater than that of the main Society. It may well be questioned as to whether the sections are thus being of the greatest service from the standpoint of the whole society and of the individual member. In order to avoid the multiplicity of meetings there should be at least one meeting per year jointly with the main Society and possibly one jointly with one of the other sections on subjects of overlapping interest. The chairmen of the Sections are members of the Society Program Committee and their programs should also be the subject of review by that Committee so that there will not be duplications and so that the program of the Society as a whole will have some purpose and pattern. The main Society meetings certainly should deal with matters of general professional interest and developments of science. The Section meetings to be of greatest value must provide opportunity for the presentation of papers, talks and less formal discussion of the details of engineering practice by the younger and less articulate members.

The Chairmen of the Sections are now invited to attend the meetings of the Board of Government. It should be a requirement that they attend and report at each meeting on the activities of the Sections so that the Board will be kept fully informed of the activities and be in a position to advise and assist.

A basic problem is to attract the attention of those who should be members and to inform them of the value of the Society. This can be done by members personally and by special efforts of the Membership Central Committee as well as of the Publicity Committee. A Statement of the activities of the Society in concise printed form available for use by all in membership efforts would be of value. "By your works you shall be known" pin points undoubtedly the most potent means to advance the Society. The objectives of the founders and of the Constitution are still sound and apply today. The efforts of the Board of Government and the membership should be expanded with more attention to encouraging social intercourse, to cooperation with public bodies and other societies related to civil engineering and science, to encouraging research in engineering and to aiding engineering students. All these require increased funds which can come only from increases in gifts or grants, dues, returns from sales of publications and advertisements. Gifts or grants and legacies for application to the permanent fund should be encouraged from the members, friends and national foundations. A standing committee could well be created to give constant attention to this matter. Dues have not been changed since 1910 and should be raised at least $2.00 so as to provide additional funds without use of the principal of the Permanent Fund. Harrison P. Eddy, in his Presidential Address of March 1950, gave an excellent discussion of this matter. A change in the By-Laws to eliminate the entrance fee should remove a hurdle that often stops the younger men from applying. Sales of publications will increase if the material in the JOURNAL increases in interest. Advertisements could well support the JOURNAL if adequately exploited.

The Society is a great engineering society in terms of its contributions to engineering. The 107 years of existence of the Society and the large Permanent Fund are only of importance if the accumulated experience and wealth are used to provide even greater service to the engineering profession and the community. The possibilities of much greater service are inherent in its past and present and need for attainment only the wise and active attention of its officers and members.

EFFECT OF BAFFLE PIERS ON STILLING BASIN PERFORMANCE

By Donald R. F. Harleman,* *Member*

(Presented at a meeting of the Hydraulics Section of the Boston Society of Civil Engineers, held on November 3, 1954.)

Introduction

The hydraulic jump in a stilling basin is probably one of the most widely used and efficient methods of dissipating the energy of high velocity flows. In practically all dams and impounding structures, the hydraulic engineer must solve the problem of releasing water into a channel below the site without damaging the structure or channel by erosion and scour. The hydraulic jump not only dissipates a large quantity of energy by turbulence in the expansion but also converts most of the remaining energy from kinetic into potential form in the deeper flow downstream of the roller. Since the violent expansion of the flow is concentrated over a relatively short length of the stream, the jump itself would cause severe local erosion if bed protection were not provided beneath the expansion. It therefore becomes necessary to create and control the hydraulic jump in order to secure effective performance. Protection of the channel bed is obtained by a lining of concrete extending outward from the toe of the structure. This apron, together with sidewalls and perhaps an end still is called the stilling basin.

Fundamentally, the design of a stilling basin is based upon a knowledge of the velocity and depth of water entering the basin and upon a tailwater rating curve giving the relation between river stage and discharge. The elevation of the floor of the stilling basin with respect to the tailwater elevation can therefore be found from the known entrance conditions and the momentum equation of the hydraulic jump which gives the depth of water necessary for its formation. Local topography indicates whether it will be necessary to raise or depress the elevation of the spillway toe and stilling basin floor to obtain the desired depth. The other primary dimension of the basin, namely its length, is usually based on the experimental observation

*Assistant Professor of Hydraulics, Massachusetts Institute of Technology.

that the length required for the depth change and effective reduction of bottom velocities is of the order of five times the depth change in the jump. In the ideal design, therefore, the entire transition takes place within the stilling basin; however, economics frequently dictate a shorter basin, especially if the river bed is believed to have considerable resistance to scour and erosion. Complexities of the design become apparent when it is considered that a stilling basin designed to operate efficiently at the maximum discharge of the structure may result in a dangerous condition at lower discharges. This situation may arise if the tailwater rating curve is such that at lower discharges the depth in the basin becomes greater than that required to form a jump. A "drowned" jump is formed and the high-velocity jet entering the basin is not dissipated as effectively and may therefore reach the end still with velocities capable of causing scour. This situation can be remedied by the use of a sloping apron as in the stilling pool for Shasta Dam (1). Other difficulties may arise if certain operating conditions result in an asymmetrical flow into the stilling basin. Many hundreds of model studies have been made to verify and improve designs of stilling basins for hydraulic structures. These tests undeniably result in improved performance of the individual structure yet they contribute little to general design information. The designer of a structure which does not permit the time and expense of a model study is therefore likely to adopt the most conservative and perhaps the less satisfactory solution.

The research program upon which this paper is based is therefore concerned with the development of general design information for stilling basins with baffle piers. No attempt was made to reproduce a particular prototype situation, in fact, every effort was made to conduct the study in such a manner that the results would be of immediate use for design in a stilling basin in which the flow may be considered essentially two-dimensional. The results are presented in the form of dimensionless curves and since the primary forces involved are gravity and inertia, the Froude number of the flow at the beginning of the jump is used as the similarity parameter.

Theoretical Considerations

The basic theory of the hydraulic jump without baffle piers is well known and substantiated by experiments. The momentum analysis, originating with Belanger (1838), provides a rather simple and accurate relationship between the initial velocity and depth and the final depth of the jump. A similar analysis for the hydraulic jump with baffle piers can also be made. Referring to the definition sketch, Fig. 1, it is assumed that the flow is two-dimensional and that the

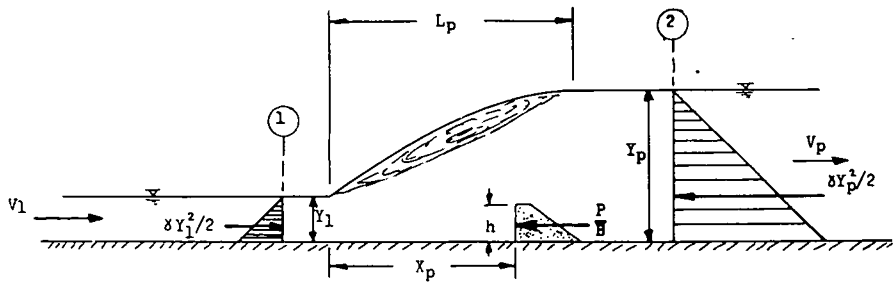

FIG. 1. DEFINITION SKETCH FOR ANALYSIS OF HYDRAULIC JUMP WITH BAFFLE PIERS

velocity at sections 1 and 2 does not vary in the vertical direction. The summation of forces acting on the free body of fluid contained between sections 1 and 2 must equal the rate of change of momentum $\rho q\ (V_p - V_1)$ where ρ is the density of the fluid flowing; q is the discharge per unit width, V_p is the average velocity at section 2 and V_1 the velocity at section 1. This analysis results in a modified form of the classical jump equation and shows the effect of the baffle piers on the characteristics of the hydraulic jump. The following forces must be considered in the summation on the body of fluid:

1. The forces acting on the vertical sections at 1 and 2 are due to the hydrostatic pressure distributions and are equal to the product of the average pressure and the unit area.

2. The forces acting along the horizontal floor of the channel between sections 1 and 2 are due to (a) the force which the piers exert on the fluid (equal and opposite to the force which the fluid exerts on the piers and (b) the viscous shear along the bottom which is small compared to the other forces and is neglected as in the original jump analysis. The momentum equation then becomes,

$$\frac{\delta Y_1^2}{2} - \frac{\delta Y_p^2}{2} - \frac{P}{B} = \rho q\ (V_p - V_1) \qquad [1]$$

where Y_1 is the depth at section 1, Y_p the depth at section 2 with the piers in place, P is the force exerted by the piers and B is the width of the channel. The continuity equation,

$$q = Y_1 V_1 = Y_p V_p \qquad [2]$$

together with the definition of the initial Froude number of the flow,

$$F_1 = \frac{V_1}{\sqrt{qY_1}} \qquad [3]$$

provide the additional relationships needed in the analysis. The velocities in equation [1] may be eliminated by means of eq. [2]; therefore, solving for P:

$$\frac{P}{\delta B} = \frac{1}{2}(Y_1^2 - Y_p^2) - \frac{q_2}{g}\left(\frac{1}{Y_p} - \frac{1}{Y_1}\right) \qquad [4]$$

Note that if the pier force $P = O$, $Y_p = Y_2$ the depth of the "free" jump without baffles and the classical depth relation is obtained, since,

$$O = \frac{1}{2}(Y_1^2 - Y_2^2) - \frac{q^2}{g}\left(\frac{1}{Y_2} - \frac{1}{Y_1}\right) \qquad [5]$$

and therefore,

$$\frac{Y_2}{Y_1} = \frac{1}{2}\left(\sqrt{8F_1^2 + 1} - 1\right) \qquad [6]$$

Subtracting eq. [5] from eq. [4]

$$\frac{P}{\delta B} = \frac{q^2}{g}\left(\frac{1}{Y_2} - \frac{1}{Y_p}\right) + \frac{1}{2}(Y_2^2 - Y_p^2) \qquad [7]$$

and dividing both sides by $Y_2^2/2$ and rearranging, gives:

$$\frac{P/B}{\delta Y_2^2/2} = \frac{2\,q^2}{g\,Y_2^3}\left(1 - \frac{Y_2}{Y_p}\right) + 1 - \frac{Y_p^2}{Y_2^2} \qquad [8]$$

but

$$\frac{2\,q^2}{g\,Y_2^3} = \frac{2\,q^2}{g\,Y_1^3\,(Y_2/Y_1)^3} = \frac{16F_1^2}{(\sqrt{8F_1^2 + 1} - 1)^3}$$

Finally, eq. [8] may be rewritten as follows:

$$\frac{P/B}{\delta Y_2^2/2} = 1 - \left(\frac{Y_p}{Y_2}\right)^2 - \left(\frac{Y_2}{Y_p} - 1\right)\frac{16\,F_1^2}{(\sqrt{8F_1^2 + 1} - 1)^3} \qquad [9]$$

From equation [9] the reduction in downstream depth due to a given force exerted by the baffle piers may be calculated. The left side of equation [9] equals the ratio of the force exerted by the baffle piers to the hydrostatic force due to the final depth Y_2 in the "free" jump. Since these two forces act to retard the flow and produce the momentum change it is a measure of the relative effectiveness of the piers. This term will subsequently be referred to as the "force ratio."

Attempts to continue the analytical development by evaluation of the pier force term have not been rewarding. The procedure would be to express P by means of the drag equation

$$P = C_D \rho \frac{V^2}{2} A$$

however, the coefficient of drag is a function of the pier shape and the pier spacing and the velocity is a function of the distance from the piers to the toe of the jump. Some recourse to experimental results must therefore be made and it was decided to evaluate P directly by means of a force measuring device.

Experimental Equipment and Procedure

It was desired that the experimental equipment permit the formation and control of hydraulic jumps having a considerable range of initial Froude numbers. A glass-walled, horizontal channel 18 inches

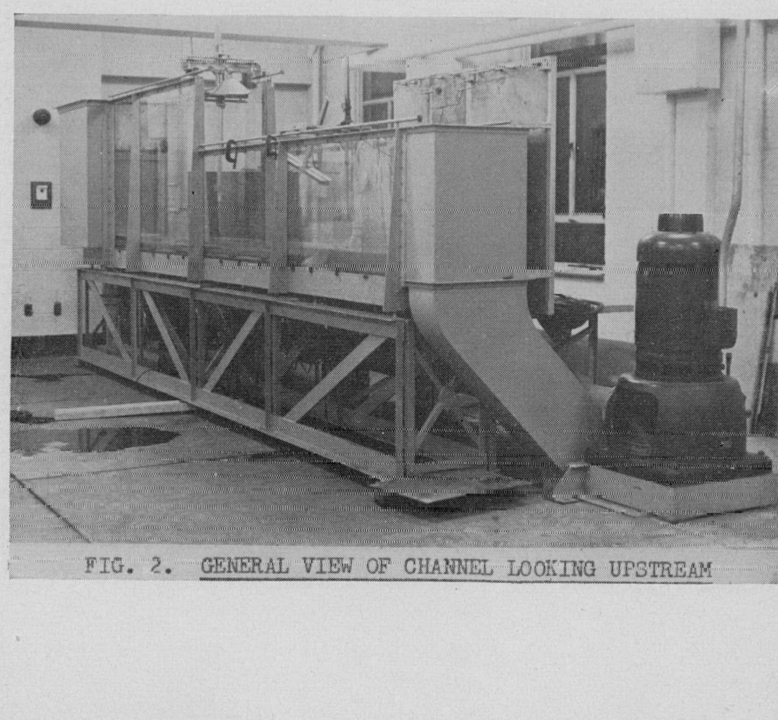

FIG. 2. GENERAL VIEW OF CHANNEL LOOKING UPSTREAM

wide and 18 feet long was chosen. Water is admitted to the channel from a head tank through an adjustable sluice gate and supercritical flows ranging in Froude numbers from 3 to 8 can be obtained. The downstream depth and hence the position of the jump is regulated by an adjustable tail gate. Provision was made for measuring depths by means of a point gage mounted on rails and also by means of approximately 60 piezometer connections in the bottom of the flume. The water discharges into a large reservoir and is returned to the head tank by means of a 1200 gpm centrifugal pump. An orifice meter in the 8″ discharge line provides a measurement of the discharge. A view of the channel looking from the downstream end is shown in Fig. 2. The baffle piers are mounted on a movable plate in the floor of the channel as shown in the schematic diagram in Fig. 3. The device

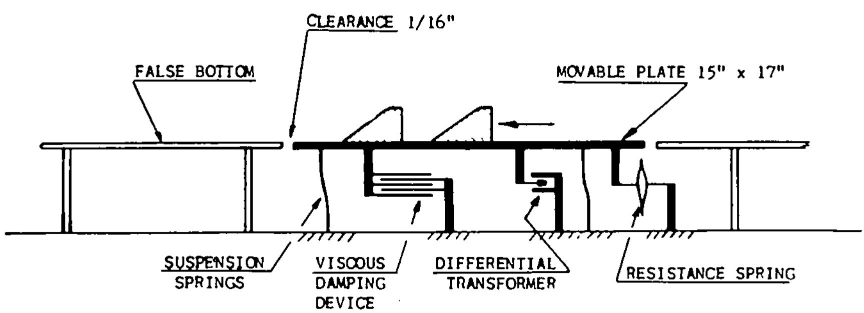

FIG. 3. DIAGRAM OF PIER FORCE MEASURING DEVICE

is designed to measure small displacements due to the application of horizontal forces resulting from the action of the flow on the piers. The instrument is composed of four basic components:

1. Suspension springs: These springs, of the leaf or cantilever type, are designed to carry the vertical load on the plate and to offer little resistance to horizontal motion.

2. Resistance spring: This spring offers the primary resistance to horizontal motion and therefore limits the horizontal deflection of the plate. The maximum allowable deflection was set at 0.03 inches and seven springs of varying stiffness were designed with each spring capable of measuring forces within a certain range. The total range of measurable forces is from a few hundredths of a pound to 20 pounds.

3. Viscous damping device: The damping device, consisting of two sets of intermeshed horizontal plates in a reservoir of glycerin, permits a fairly steady force reading which would otherwise be difficult due to the large turbulent fluctuations in the region of the jump.

4. Differential transformer: The small deflections of the floor plate were measured by a Schaevitz Differential Transformer. It consists of a movable magnetic core which is attached to the plate and three coaxial coils which are fixed to the flume proper. The center core is the primary of the transformer and is energized with an alternating current of 1000 cps. The two outer coils are the transformer secondaries and are wound in opposition so that with the core in the center or balance position the induced voltages will be equal and 180° out of phase. Any movement of the core from this balance position results in a net voltage which increases linearly with deflection of the core. The magnitude of the output voltage is read as a band width on a cathode ray oscilloscope.

The balance is calibrated (i.e. output voltage vs. force) by means of a pulley system which applies known horizontal forces to the baffle piers. Separate calibrations are necessary for each resistance spring.

Two types of baffle piers were used in the experiments; a standard rectangular pier which will be referred to as the "stepped pier" and the so-called "cavitation-free pier" developed by the Corps of Engineers. The cavitation-free pier is a semi-streamlined shape whose sides conform essentially to the separation streamlines of a flat plate held at right-angles to the flow thereby avoiding the extremely low pressures found in this region. This pier is recommended for use wherever high velocities and low submergence indicate a possibility of cavitation damage. The use of the rectangular or stepped pier in present practice is generally confined to low head spillways. It is apparent from equation [9] that pier streamlining of any sort reduces the effectiveness of the pier (through reduction of the drag coefficient) and that a pier so well streamlined as to be entirely free of cavitation would at the same time be of little value in the stilling basin. The Corps of Engineers design is a compromise between the opposing requirements of high form resistance and minimum pressure reduction. Figure 4 is a diagram showing the size and arrangement of the two piers used in this study. The eleven cavitation-free piers are arranged in two rows and the twelve stepped piers are placed in a single row. Ref. [2] gives the shape specifications for the cavitation-free piers.

The experimental procedure for one run consisted of adjusting the discharge and sluice gate opening to produce the desired initial Froude number in the supercritical flow portion. The downstream tailgate was then adjusted to place the hydraulic jump at the desired longi-

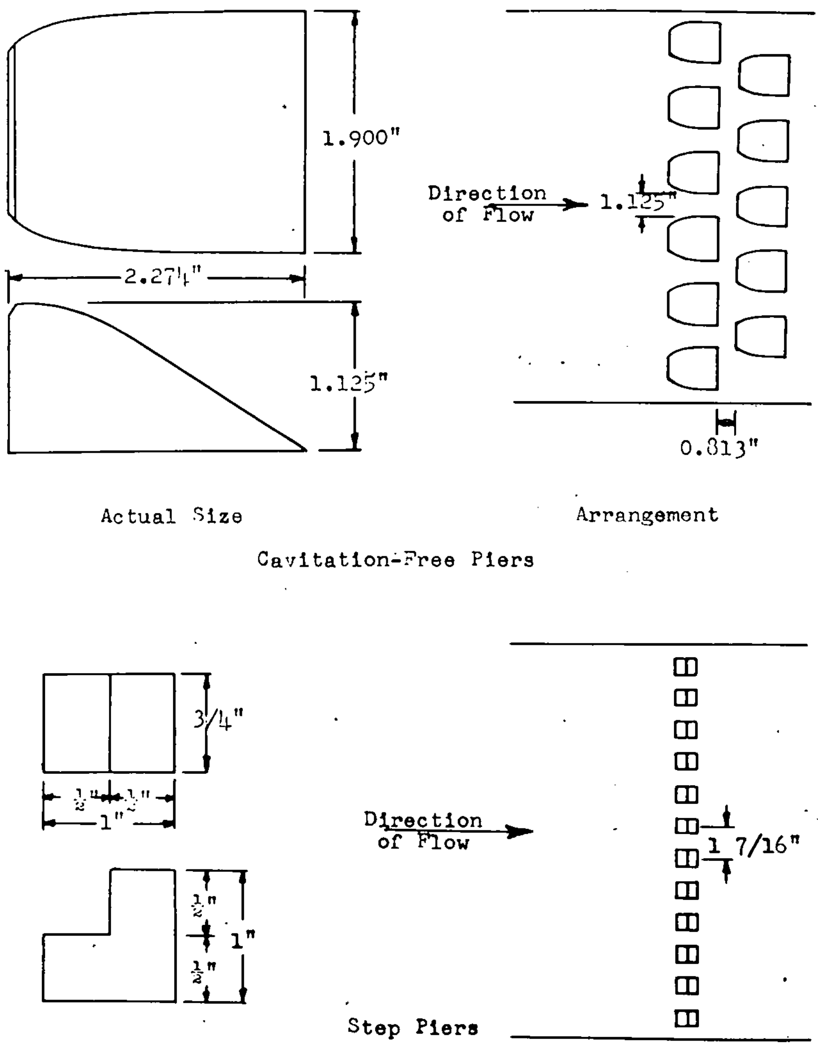

FIG. 4 SIZE AND ARRANGEMENT OF BAFFLE PIERS

tudinal position relative to the baffle piers. Data was then obtained for plotting the initial and final depths and the surface profile of the jump. In the region of supercritical flow and small depths, measurements were made with a point gage. Due to the extreme unsteadiness of the water surface in the vicinity of the jump roller, depth measurements in this region were determined from the piezometer connections in the bottom of the flume. Each pressure connection is attached to a water manometer arranged in a bank of 60 units. The rate of flow was obtained from the differential head of the orifice-meter manometer. Finally, the oscilloscope band width was recorded for determination of the force on the piers. Tests covering the range of Froude numbers were made and for each such condition, usually four runs were obtained by varying the position of the jump relative to the baffle piers. In addition, an identical program of control runs was carried out without baffle piers.

Experimental Results

In the analysis of the experimental data, all results are expressed in the form of dimensionless ratios: (See Fig. 1 for notation)

$$X_p/h = \frac{\text{distance from toe of jump to pier}}{\text{pier height}}$$

$$Y_p/Y_2 = \frac{\text{downstream depth with piers}}{\text{downstream depth without piers}}$$

$$L_p/L = \frac{\text{jump length with piers}}{\text{jump length without piers}}$$

In the tests on the stepped piers, the initial depth Y_1 varied from 85% to 112% of the pier height h and in the cavitation free piers from 75% to 100%. No appreciable effect was evident due to this variation. Figure 5 shows a series of photographs of the jump at various positions with respect to the piers. The appreciable reduction in jump length is apparent as the toe of the jump approaches the piers. As the tailwater depth is successively reduced, the full force of the jet impinges on the piers and no jump is formed. This is a graphic demonstration of the fact that under no circumstances can baffle piers cause the formation of a hydraulic jump without the presence of sufficient downstream depth.

A dimensionless surface profile of the jump in three locations as obtained from both point gage and piezometer readings is shown in Fig. 6. In addition to the evident length reduction, a small reduction in the jump depth also is noticeable in this plot.

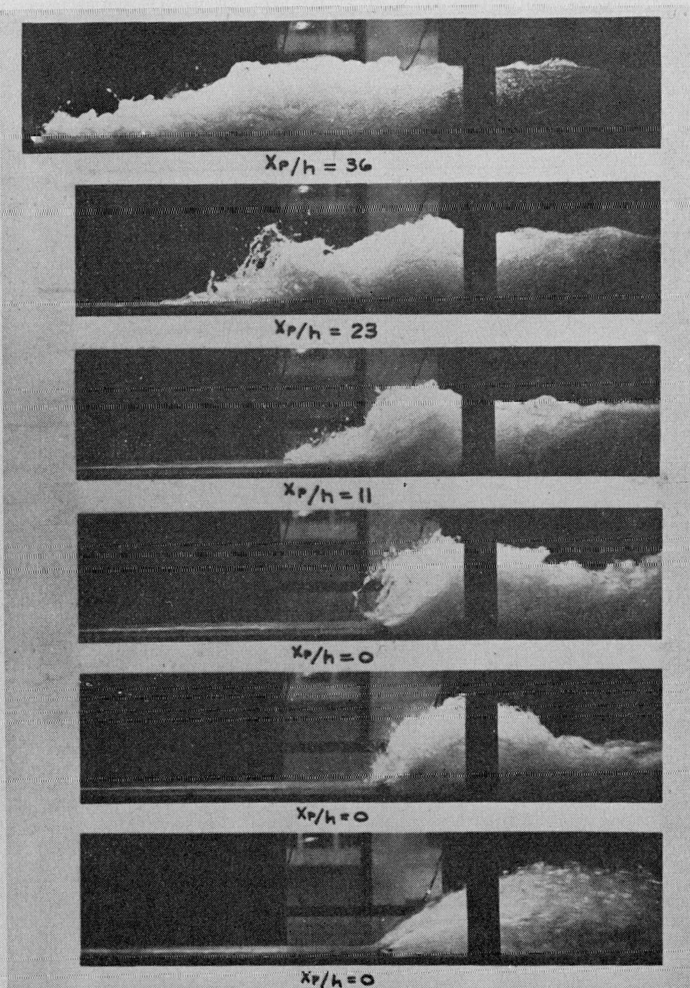

FIG. 5 JUMP PROFILES WITH STEPPED PIERS
$F_1 = 7.0$

The primary results of the experimental program are shown in Figures 7, 8, and 9:

Figure 7 shows the downstream depth reduction in terms of jump position and initial Froude number. At a Froude number of 7, the downstream depth for the two rows of piers varies from approximately 86% of the free jump depth when $X_p/h = 9$ to a negligible 98% when

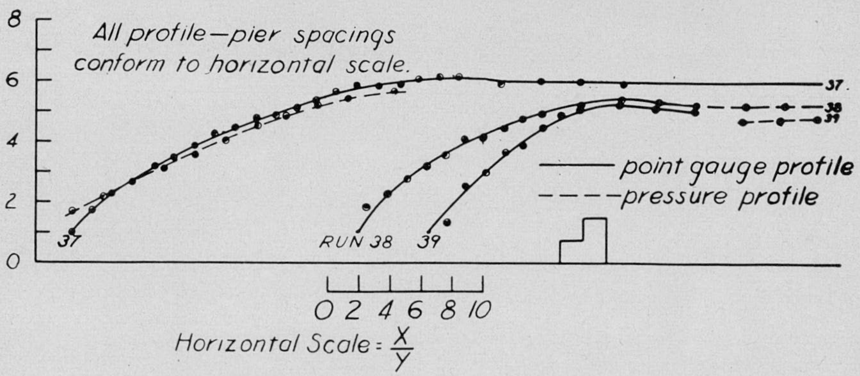

FIG. 6 DIMENSIONLESS JUMP PROFILES WITH STEPPED PIERS F_1 = 4.0

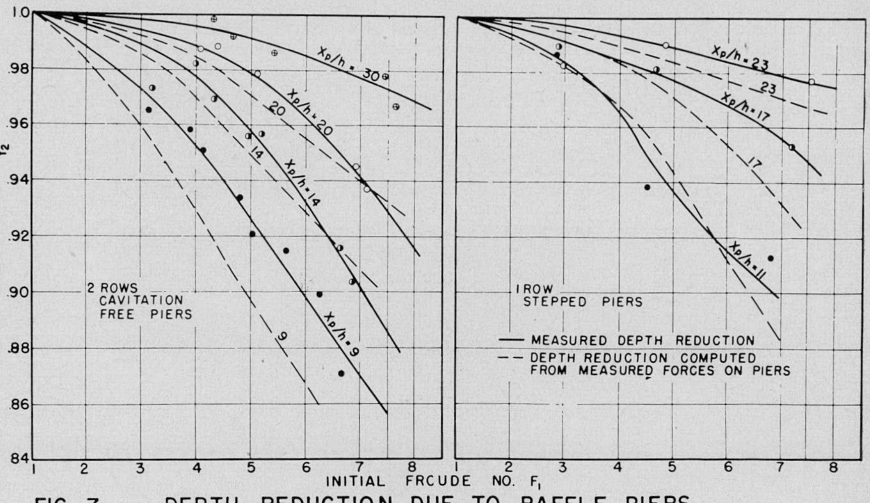

FIG. 7. DEPTH REDUCTION DUE TO BAFFLE PIERS

$X_p/h = 30$. The dashed curves represent downstream depth ratios computed from the measured force on the piers by means of equation [9]. The average agreement with the direct measurement of depths is between one and two per cent indicating a verification of the modified jump theory.

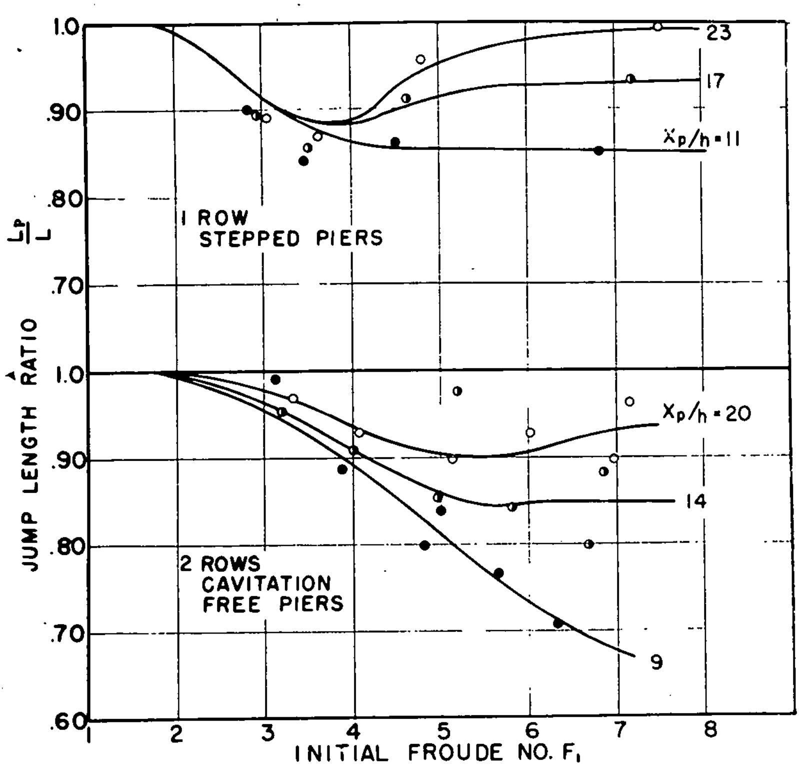

FIG. 8. JUMP LENGTH REDUCTION DUE TO BAFFLE PIERS

Figure 8 presents the summary of results on the change in the length of the jump due to the piers. Length reductions of as much as 70% can be obtained if the toe of the jump is very close to the piers.

Figure 9 shows the measured force ratios for the various runs from which the downstream depth ratio curves of Fig. 7 were computed. It is interesting to note that the maximum force obtained was only 20% of the force due to the downstream depth of the free jump, a further indication of the inability of piers to produce the total force necessary for the formation of the jump.

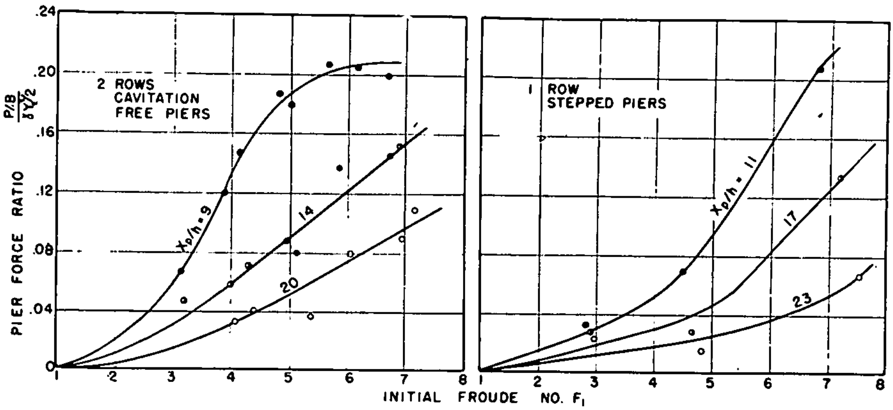

FIG. 9. RATIO OF PIER FORCE TO FORCE DUE TO DOWNSTREAM DEPTH

Application of Results in Stilling Basin Design

The foregoing results illustrate the effect of the baffle piers in length and depth reduction, but do not show the third and most important effect of piers; namely, stabilization of the jump in the stilling basin. This aspect can best be shown by consideration of a typical design problem.

A. Preliminary data for design of a stilling basin using cavitation-free baffle piers is assumed for illustrative purposes:

1. Conditions at entrance to stilling basin for design discharge of spillway or outlet:

   initial velocity $V_1 = 56$ ft/sec.
   initial depth $Y_1 = 2$ ft.
   therefore, initial Froude No. $F_1 = 7$ and the discharge per unit width $q = 112$ cfs/ft.

2. Tailwater condition:
   tailwater elevation = 100.0 at design discharge of 112 cfs/ft.

B. Design Procedure:

1. For complete protection of the river bed in a stilling basin without baffle piers, a length of $L = 5\ (Y_2 - Y_1)$ is generally

required, therefore from eq. [6] $Y_2/Y_1 = 9.4$ and $Y_2 = 18.8'$, and hence $L = 5(16.8) = 84'$.

2. The height of the baffle piers h is generally equal to or slightly greater than Y_1; take $h = 2.5'$.
3. If the first row of piers are placed at the midpoint of the stilling basin, then $X_p = 42'$ and $X_p/h = 17$. From Fig. 8 the reduced length ratio due to the piers is .88 and a stilling basin length $L_p = .88\ (84) = 74'$ could be used with a net saving of 10 feet. The piers remain 42 feet from the toe and are therefore located at 57% of the length of the apron.
4. From Fig. 7 with $F_1 = 7$ and $X_p/h = 17$, the ratio of downstream depths with and without baffle piers is .92. Due to uncertainties in the initial conditions at the entrance to the stilling pool, use of this small change is not justified and the elevation of the stilling basin floor should be determined using Y_2 or $(100.0 - 18.8) = 81.2'$.

C. Stability of the Jump in the Stilling Basin:

More important than the saving in length obtained is the fact that an analysis of the jump location can be made to determine the effect of an error or change in the estimated tailwater elevation at the design discharge.

1. Assume that the stilling basin has no baffle piers and that the tailwater elevation changes by 5% from $Y_2 = 18.8'$ to $18.0'$. The depth change for the free jump, eq. [6] can be written,

$$\frac{Y_2}{Y_1} = \frac{1}{2}\left(\sqrt{\frac{8\,q^2}{g\,Y_1^3} + 1} - 1\right)$$

It therefore becomes necessary to find the new value of Y_1 associated with $Y_2 = 18.0$ (q remains constant). Solving by trial: $Y_1 = 2.15'$. The jump must therefore move out into the stilling basin a distance equal to the length required for the supercritical flow to change in depth from 2.00 to 2.15′ on the horizontal floor. Assuming Manning's n for the apron, $n = .015$, a one-step backwater computation gives a distance of 58′ for this depth change. Since the length of the apron without piers is 84′, the toe of the jump would be at the 70% point on the apron, a condition which would result in severe scour downstream from the basin.

2. Assume that the same change in tailwater occurs with baffle piers as designed above.

To a first approximation, find the distance necessary for the jump to move in order for a 5% change in the magnitude of the downstream depth ratio to occur. A total reduced depth ratio of $.92 - .05 = .87$ is required since the jump in the original position had a reduced depth ratio of .92. Referring to Fig. 7 at $F_1 = 7$ and $Y_p/Y_2 = .87$, $X_p/h = 10$ is found. The jump, therefore, moves to a new X_p position such that $X_p = 10(h) = 25'$, resulting in a total movement (from the original $X_p = 42'$) of 17′ compared to 58′ without piers. Under this condition, the reduced length ratio is approximately .70 and $L_p = .70(84') = 59'$, therefore the entire length of the jump still occurs within the basin. Since the change in the initial depth of the supercritical flow was found to be 0.15′ in 58′, the change in the initial Froude number due to the movement of 17′ can be neglected.

Conclusions

1. The experimental results indicate that appreciable depth reductions due to baffle piers are possible only if the toe of the jump is within 10-20 pier heights of the piers. For stilling basin design under maximum discharge conditions, the piers cannot be placed close enough to the jump toe to provide any useful depth reduction without making the piers liable to serious cavitation damage. Even the so-called cavitation free piers would be expected to be damaged. It can be shown that in any stilling basin having an initial Froude number greater than 4, the piers would have to be located well into the forward half of the basin in order to make $X_p/h = 10$ and they would therefore be subjected to very high velocities and damage.

It is concluded that depth reductions due to piers be considered only as insurance against dangerous displacements of the jump into the basin due to errors or changes in the initial Froude number or tailwater depth.

2. The length reduction of the jump is larger than the depth reduction for similar initial conditions and certain savings in stilling basin length can probably be justified if the reduction is applied to the empirical length of the jump without piers. $L = 5(Y_2 - Y_1)$.

3. The maximum force exerted by the baffle piers is of the order

of 20 percent of the pressure force due to the downstream depth. Baffle piers cannot therefore cause the formation of a jump and sufficient depth must always be provided by controlling the elevation of the stilling basin floor with respect to the tailwater elevation.

4. The primary function of baffle piers is that of stabilization of the hydraulic jump against longitudinal displacements.

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ON SOME ASPECTS OF THE THEORY OF THIN ELASTIC SHELLS

By Eric Reissner*

Introduction. We consider in the following some aspects of the problem of structural analysis of thin-shell construction when use is made of such shapes as the hyperbolic paraboloid or the conoid. Constructions using shapes such as these appear to be increasing in frequency [7].

We begin by reviewing membrane theory of thin shells, in the form developed in particular by Pucher [11]. We add to the existing methods of solution of the equations of this theory a method of expansions in powers of a non-dimensional rise of the shell. We then complement the known equations of statics by a system of stress-strain relations. This system of stress-strain relations is obtained by minimum energy methods and differs significantly from relations recently given by Ban [2]. Determination of displacements corresponding to a state of stress is shown to be amenable to the same method of expansion in powers of shell rise as the solution of the equations of statics. Systematic omission of all but the first terms in these expansions furnishes a system of equations suitable for the analysis of *shallow* membranes. The problem of determining stresses and displacements in a hyperbolic paraboloid shell due to its own weight is solved as an example of application of this theory.

The second part of this paper begins with a statement of Marguerre's general theory for shallow shells [10]. This general theory considers bending action as well as membrane action and nonlinear (finite-deflection) effects as well as linear effects. Shallow membrane theory as deduced from general linear membrane theory is shown to be a special case of general shallow-shell theory.

The use of general shallow-shell theory is illustrated with the help of two basic problems of the hyperbolic paraboloid shell under the influence of its own weight. The first problem concerns the possible buckling of such a shell. An explicit buckling formula is obtained as well as an expression for the wave length in buckling. The second

*Associate Professor, Massachusetts Institute of Technology.

problem concerns the effect of support conditions which prevent the existence of a pure membrane state in the shell. It is shown how to simplify the equations of the theory by suppression of unessential terms and how to establish the existence of narrow-edge-zone (or boundary layer) effects. It is shown that in these edge zones there arise bending stresses of the same order of magnitude as the known membrane stresses for the special support conditions which allow a pure membrane state.

While the results obtained for the illustrative examples are felt to be of interest by themselves, it is thought that part of the value of these examples consists in showing the way towards a rational analysis of other related problems.

I. General Linear Membrane Theory

1. Nature of membrane action A thin shell is called a membrane shell if its load carrying capacity involves bending to a negligible extent only. Whether or not a shell is a membrane shell depends not only on the stress-free form of the shell but also on the nature of the applied loads.

An originally flat plate is a limiting form of a shell. If all loads on the plate are parallel to its plane then no bending is involved and we might call the plate a membrane (although in the case of the plate this word is not generally used in this connection). For loads perpendicular to the plane of the plate bending resistance is essential, at least for moderate loads.

Experience of an experimental as well as of a theoretical nature indicates that a shell prefers to act as a membrane if it can do so without special effort. Of course, what is meant by the words "without special effort" is worthy of a more technical discussion than we shall give here. We merely say that the situation is similar to what is encountered in the analysis of frameworks where bending and torsional stresses are often (but by no means always) of a secondary nature.

2. Geometry of the shell. Let us assume that we have given the shape of a thin shell by means of an equation for its middle surface. Taking Cartesian coordinates we write this middle surface equation in the form

$$z = z(x, y) \tag{2.1}$$

To illustrate, and for future reference, for hyperbolic paraboloid shells equation (2.1) becomes

$$\frac{z}{c} = \frac{x}{a}\frac{y}{b} \tag{2.2}$$

or

$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2} \tag{2.3}$$

depending on the direction of the axes x and y.

For conoidal shells,

$$\frac{z}{c} = \frac{x}{a} f\left(\frac{y}{b}\right) \tag{2.4}$$

and in particular, if the cross section of the conoidal shell is parabolic,

$$\frac{z}{c} = \frac{x}{a}\left[1 - \left(\frac{y}{b}\right)^2\right] \tag{2.5}$$

Further calculations are conveniently carried out with the help of simple vector analysis. We write the radius vector to points of the middle surface in the form

$$\underset{\sim}{r} = x\underset{\sim}{i} + y\underset{\sim}{j} + z\underset{\sim}{k} \tag{2.6}$$

where *i, j, k* are unit vectors in the direction of the coordinate axes.* We deduce from (2.6) the equations of two tangent vectors *s* and *t* to the surface,

$$\underset{\sim}{s} = \frac{\partial \underset{\sim}{r}}{\partial x} = \underset{\sim}{i} + \frac{\partial z}{\partial x}\underset{\sim}{k} \tag{2.7}$$

$$\underset{\sim}{t} = \frac{\partial \underset{\sim}{r}}{\partial y} = \underset{\sim}{j} + \frac{\partial z}{\partial y}\underset{\sim}{k} \tag{2.8}$$

The vectors *s*dx and *t*dy describe an elementary element of area on the shell surface. This element is not rectangular, but its projection on the x,y-plane is rectangular. The vectors *s* and *t* are not unit vectors. Their lengths, s and t, are given by

$$s^2 = |\underset{\sim}{s}|^2 = 1 + z_x^2 \,, \quad t^2 = |\underset{\sim}{t}|^2 = 1 + z_y^2 \tag{2.9}$$

where for brevity we write $\partial z/\partial x = z_x$ and $\partial z/\partial y = z_y$. The angle θ enclosed by the vectors *s* and *t* is not, in general, a right angle. We have from vector algebra $s \cdot t$ = st $\cos\theta$ and therewith

$$\cos\theta = \frac{z_x z_y}{\sqrt{1+z_x^2}\,\sqrt{1+z_y^2}} \tag{2.10}$$

---

*Vectors are characterized here by a wavy underline (in the equation) or by italics (in the text).

The element of surface area dA is given by dA = st sinθ dxdy. With the help of (2.9) and (2.10) this expression becomes

$$dA = \sqrt{1+z_x^2+z_y^2}\; dx\,dy \qquad (2.11)$$

3. *Equilibrium of pseudo-stress resultants.* It is convenient to write for the forces acting on the faces s dx and t dy of the elements of the shells,

$$d\underset{\sim}{F}_x = \underset{\sim}{N}_x\, dy \;, \quad d\underset{\sim}{F}_y = \underset{\sim}{N}_y\, dx \qquad (3.1,2)$$

The quantities N_x and N_y are not stress resultant vectors in the usual sense since they are not forces per unit of length of the parametric curves. Stress resultant vectors in the usual sense are N_x/t and N_y/s.

The resultant vectors N_x and N_y are next written in component form as follows,

$$\underset{\sim}{N}_x = N_{xx}\,\underset{\sim}{s} + N_{xy}\,\underset{\sim}{t} \qquad (3.3)$$

$$\underset{\sim}{N}_y = N_{yx}\,\underset{\sim}{s} + N_{yy}\,\underset{\sim}{t} \qquad (3.4)$$

Absence of nontangential stress resultant components characterizes the shells under consideration as membranes.

The quantities N_{xx}, N_{xy}, and N_{yy} may be called pseudo-stress resultants. Actually, N_{xy} and N_{yx} are stress resultants in the usual sense but N_{xx} and N_{yy} are not. Rather, $(t/s)N_{xx}$ and $(s/t)N_{yy}$ are stress resultants in the usual sense. Note that while N_{xy} and N_{yx} represent shearing stresses only, the quantities N_{xx} and N_{yy} represent both, shear and normal stresses.

In addition to the pseudo-stress resultants defined by (3.3) and (3.4) we define a pseudo-load intensity vector q by the statement that q dxdy is the external load on the element dA of the shell. We write

$$\underset{\sim}{q} = q_x\underset{\sim}{i} + q_y\underset{\sim}{j} + q_z\underset{\sim}{k} \qquad (3.5)$$

The pseudo-load intensity vector q and the actual load intensity vector p are related by the formula q dxdy = p $\sqrt{1 + z_x^2 + z_y^2}$ dxdy.

In order to obtain the equation of force equilibrium for elements of the shell we consider the changes of N_x and N_y from one edge of the area element to its opposite. In this way we obtain the force equation

$$\frac{\partial \underset{\sim}{N}_x}{\partial x} + \frac{\partial \underset{\sim}{N}_y}{\partial y} + \underset{\sim}{q} = 0 \qquad (3.6)$$

The condition of moment equilibrium is most conveniently written by the use of the vector product formula for moments. In this manner we have

$$\underset{\sim}{s} \times \underset{\sim}{N}_x + \underset{\sim}{t} \times \underset{\sim}{N}_y = 0 \tag{3.7}$$

From (3.7) follows, since $s \times s = t \times t = 0$, that

$$N_{yx} = N_{xy} \tag{3.8}$$

and this is the only consequence of (3.7).*

The vectorial force equation (3.6) may be reduced to scalar form in at least two ways. We here carry out this reduction by first writing (3.3) and (3.4) in the alternate form

$$\underset{\sim}{N}_x = N_{xx}(\underset{\sim}{i} + z_x \underset{\sim}{k}) + N_{xy}(\underset{\sim}{j} + z_y \underset{\sim}{k})$$

$$\underset{\sim}{N}_y = N_{yx}(\underset{\sim}{i} + z_x \underset{\sim}{k}) + N_{yy}(\underset{\sim}{j} + z_y \underset{\sim}{k})$$

and by separately setting equal to zero the i, j and k component equations. In this manner we obtain as basic equations of the statics of membrane shells

$$\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} + q_x = 0 \tag{3.9}$$

$$\frac{\partial N_{yx}}{\partial x} + \frac{\partial N_{yy}}{\partial y} + q_y = 0 \tag{3.10}$$

$$\frac{\partial(N_{xx}z_x + N_{xy}z_y)}{\partial x} + \frac{\partial(N_{yx}z_x + N_{yy}z_y)}{\partial y} + q_z = 0 \tag{3.11}$$

Since $N_{yx} = N_{xy}$ we have just as many equilibrium differential equations as we have unknown stress quantities. In this sense the problems of membrane theory are *statically determinate.* An obvious exception is the case for which $z_x = 0$ and $z_y = 0$, that is the case of the flat plate. In this latter case (3.9) and (3.10) are the equilibrium equations of generalized plane stress which are not statically determinate and (3.11) reduces either to an identity, when $q_z = 0$, or to an impossibility, when $q_z \neq 0$.

The earliest reference known to the writer where the above form of the equations of membrane theory is given is a dissertation by Pucher [11]. The same form is also used, apparently independently by Lafaille [9] and Aimond [1]. For shells of revolution other forms

*Two of the three scalar equations which would correspond to the vector equation (3.7) are automatically satisfied through the assumption that N_x and N_y are tangential to the shell surface.

of the membrane equations are more convenient. A discussion of this important class of studies, the first one of which dealt with the spherical membrane [12] is, however, not within the scope of the present paper.

4. *Use of Airy's stress function.* The solution of the equilibrium equations (3.9 to 3.11) is usually effected by reducing them to one single differential equation as follows. Equations (3.9) and (3.10) are satisfied identically by setting

$$N_{xx} = \frac{\partial^2 F}{\partial y^2} - \int q_x \, dx \qquad (4.1)$$

$$N_{yy} = \frac{\partial^2 F}{\partial x^2} - \int q_y \, dy \qquad (4.2)$$

$$N_{xy} = -\frac{\partial^2 F}{\partial x \, \partial y} \qquad (4.3)$$

where F is Airy's stress function. Introduction of (4.1) to (4.3) into (3.11) gives a differential equation for F, of the following form

$$\frac{\partial^2 z}{\partial x^2}\frac{\partial^2 F}{\partial y^2} - 2\frac{\partial^2 z}{\partial x \, \partial y}\frac{\partial^2 F}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2}\frac{\partial^2 F}{\partial x^2} = q \qquad (4.4)$$

The load term q is given by

$$q = -q_z + z_x q_x + z_y q_y + z_{xx}\int q_x \, dx + z_{yy}\int q_y \, dy \qquad (4.5)$$

Equations (4.4) and (4.5) are the fundamental equations of membrane theory as this theory has been utilized in structural analysis. A good deal has been written on methods of solving the partial differential equation (4.4), by exact or approximate means. We mention in particular a paper by Tester [16] on exact solutions for various loading cases of the hyperbolic paraboloid shell and a paper by Flügge [5] on the replacement of (4.4) by a finite difference analogue, which can then be solved by relaxation techniques.

5. *Some mathematical observations.* The differential equation (4.4) being linear we know that its solution may be composed of a particular solution F_q plus the general solution F_h of the homogeneous equation,

$$F = F_h + F_q \qquad (5.1)$$

We shall have some comments both on F_h and F_q.

Homogeneous equation. We omit the subscript h for simplicity's sake and consider the equation

$$z_{xx}F_{yy} - 2\, z_{xy}F_{xy} + z_{yy}F_{xx} = 0 \qquad (5.2)$$

The general solution of (5·2) involves arbitrary functions. The way these arbitrary functions occur, depends on the character of the differential equation. Before explaining the meaning of the word character let us take three examples which are typical for what may happen.

Example 1. For a shell with shape given by

$$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2} \tag{5.3}$$

equation (5·2) becomes

$$a^{-2}F_{yy} + b^{-2}F_{xx} = 0 \tag{5.4}$$

The general real solution of (5.4) is known to be

$$F = f\left(\frac{x}{a} + i\frac{y}{b}\right) + \bar{f}\left(\frac{x}{a} - i\frac{y}{b}\right), \tag{5.5}$$

where $i = \sqrt{-1}$ and where $\bar{f}$ is the conjugate of the function f. The solution (5.5) involves one arbitrary function only.

Example 2. For

$$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2} \tag{5.6}$$

equation (5.2) becomes

$$a^{-2}F_{yy} - b^{-2}F_{xx} = 0 \tag{5.7}$$

The general real solution of (5.7) is

$$F = f\left(\frac{x}{a} - \frac{y}{b}\right) + g\left(\frac{x}{a} + \frac{y}{b}\right) \tag{5.8}$$

and this solution contains two arbitrary functions.

Example 3. For

$$\frac{z}{c} = \frac{x^2}{a^2} \tag{5.9}$$

equation (5.2) becomes

$$F_{yy} = 0 \tag{5.10}$$

and the solution of this is

$$F = f(x) + y\,g(x) \tag{5.11}$$

We have again two arbitrary functions but this time of one and the same argument. This fact represents an important difference of the solution (5.11) compared to the solution (5.8).

According to the theory of partial differential equations example 1 represents an equation of *elliptic type,* example 2 represents an equation of *hyperbolic type,* and example 3 represents an equation of *parabolic type.* This classification has to do roughly with the form in which the arbitrary functions appear in the general solution. The form in which arbitrary functions appear in the general solution is of predominant importance for the decision as to what represents a physically reasonable system of boundary conditions or support conditions for the actual determination of the solution of a specific problem.

We do not here proceed further with the discussion of this particular aspect of membrane theory other than to state the following general criterion. The behavior of the solution of (5.2) at a point x,y depends on whether

$$z_{xx} z_{yy} - (z_{xy})^2 \begin{cases} > 0 \\ = 0 \\ < 0 \end{cases} \tag{5.12}$$

The first of these cases is called the elliptic case, the second the parabolic case and the third the hyperbolic case. In what follows we shall be primarily concerned with the hyperbolic case. For discussions of this case, which go beyond what we can say here, reference is made to the paper by Lafaille [9] and to recent articles by Flügge [4, 6].

We add two further examples, one of them because we wish to return to it later from a different point of view, and the other because it appears to us worthy of record.

Example 4. For the hyperbolic paraboloid

$$\frac{z}{c} = \frac{x}{a} \frac{y}{b} \tag{5.13}$$

equation (5.2) becomes

$$\frac{\partial^2 F}{\partial x \, \partial y} = 0 \tag{5.14}$$

and the solution of this is

$$F = f(x) + g(y) \tag{5.15}$$

Example 5. For the parabolic conoid

$$\frac{z}{c} = \frac{x}{a}\left(1 - \frac{y^2}{b^2}\right) \tag{5.16}$$

equation (5.2) becomes

$$2y\frac{\partial^2 F}{\partial x \partial y} - x\frac{\partial^2 F}{\partial x^2} = 0 \qquad (5.17)$$

According to the criterion (5.12) equation (5.17) is of the hyperbolic type, except along the line $y = 0$ where it is of parabolic type. An observation of interest to the teacher of mathematics is the fact that (5.17) is a first order partial differential equation in $\partial F/\partial x$, and examples of such first order equations are infrequent in the applications. We may solve (5.17) in two steps. The first order equation for $\partial F/\partial x$ has the general solution

$$\frac{\partial F}{\partial x} = f\left(\frac{x^2}{y}\right) \qquad (5.18)$$

With the help of this result we find that F is given in terms of two arbitrary functions f_1 and f_2,

$$F = x f_1\left(\frac{x^2}{y}\right) + f_2(y) \qquad (5.19)$$

where the function f in (5.18) is given in terms of f_1 through the relation $f(\xi) = f_1(\xi) + 2\xi f_1'(\xi)$.

Let us finally note that the same method of solution is applicable when the shell surface is given in more general form by

$$z = x\, Y(y) \qquad (5.20)$$

For this case we have

$$-2\, Y'(y)\frac{\partial^2 F}{\partial x \partial y} + x\, Y''(y)\frac{\partial^2 F}{\partial x^2} = 0 \qquad (5.21)$$

and this again is a first order differential equation for $\partial F/\partial x$.

Particular Solutions. The task of obtaining particular solutions of the nonhomogeneous equation is usually no more difficult than the task of obtaining the general solution of the homogeneous equation. However, the integrations to be performed may lead to somewhat involved expressions and this in turn may lead to difficulties with the evaluation of stress-strain relations. We shall do no more here than illustrate the situation for a relatively simple example and propose a procedure which in many instances will permit a better insight into the stress and deformation picture than might otherwise be obtained.

For our example we choose the problem of determining a particular solution for the effect of weight of the hyperbolic paraboloid shell of uniform wall thickness. We have from (4.4) with $z/c = (x/a)(y/b)$, as differential equation

$$-\frac{2c}{ab}\,\frac{\partial^2 F}{\partial x\,\partial y} = q \tag{5.22}$$

Let p_0 be the weight of the shell per unit of area. This weight acts in the negative z-direction. The load term q in its general form is given by (4.5). We here have $q_x = q_y = 0$ so that $q = -q_z$. In view of the fact that q_z was defined as the z component of the load per unit of the *projected* area (see equation (3.5)), we have

$$q_z = -p_0\sqrt{1+Z_x^2+Z_y^2} \tag{5.23}$$

For the hyperbolic paraboloid we have $1 + Z_x^2 + Z_y^2 = 1 + (c/a)^2(y/b)^2 + (c/b)^2(x/a)^2$ and the differential equation to be solved is

$$\frac{\partial^2 F}{\partial x\,\partial y} = -p_0\frac{ab}{2c}\sqrt{1+\left(\frac{c}{a}\right)^2\left(\frac{y}{b}\right)^2+\left(\frac{c}{b}\right)^2\left(\frac{x}{a}\right)^2} \tag{5.24}$$

In order to obtain the pseudo-stress resultants N_x, N_y and N_{xy} we need not determine F itself. Rather, we find $\partial F/\partial x$ and $\partial F/\partial y$ by integration and then differentiate the result. In this way we find after considerable calculations

$$N_{xy} = -\frac{\partial^2 F}{\partial x\,\partial y} = p_0\frac{ab}{2c}\sqrt{1+\left(\frac{cy}{ab}\right)^2+\left(\frac{cx}{ab}\right)^2} \tag{5.25}$$

$$N_x = \frac{\partial^2 F}{\partial y^2} = -p_0\frac{y}{2}\log\left[\frac{cx}{ab}+\sqrt{1+\left(\frac{cy}{ab}\right)^2+\left(\frac{cx}{ab}\right)^2}\right]+f''(y) \tag{5.26}$$

$$N_y = \frac{\partial^2 F}{\partial x^2} = -p_0\frac{x}{2}\log\left[\frac{cy}{ab}+\sqrt{1+\left(\frac{cy}{ab}\right)^2+\left(\frac{cx}{ab}\right)^2}\right]+g''(x) \tag{5.27}$$

and these formulas are equivalent to those given by Tester [16] for the special case $a = b$.*

6. *Development in powers of nondimensional shell rise.* Instead of proceeding in the foregoing manner it will often be of advantage to use the following approach. We know that in the range of interest we have $(y/b)^2 \leqslant 1$ and $(x/a)^2 \leqslant 1$. Suppose we also know that the shell is relatively shallow. By this we mean that the rise c is relatively small compared to the spans a and b. Under these circumstances we may develop the square root in (5.24) in powers of c, as follows

$$\frac{\partial^2 F}{\partial x\,\partial y} = -p_0\frac{ab}{2c}\left\{1+\frac{1}{2}\left[\left(\frac{cy}{ab}\right)^2+\left(\frac{cx}{ab}\right)^2\right]+\dots\right\} \tag{6.1}$$

*Here and in the remainder of Part I we have written N_x and N_y instead of N_{xx} and N_{yy}

The integration now is as simple as possible. We get immediately

$$F = f(y) + g(x) - p_o \frac{ab}{2c} xy \left\{ 1 + \frac{1}{6} \left[\left(\frac{cy}{ab} \right)^2 + \left(\frac{cx}{ab} \right)^2 \right] + \ldots \right\} \quad (6.2)$$

and from this

$$N_x = f''(y) - p_o \frac{c}{ab} \frac{xy}{2} + \ldots \quad (6.3)$$

$$N_y = g''(x) - p_o \frac{c}{ab} \frac{xy}{2} + \ldots \quad (6.4)$$

Equations (6.2) to (6.4) agree as they should with what would be obtained from (5.25) to (5.27) by development in powers of c.

Just as in this example it will be possible in many other similar cases to develop the solution in powers of a dimensionless parameter involving rise and span of the shell. In the present case the development may be considered to be in powers of c/a or of c/b, either one of these quantities being a parameter of the right sort.

7. *Formulas for shallow shells.* The foregoing results become particularly simple if c/a and c/b are sufficiently small to disregard all but the first term on the right of (6.1) so that the differential equation which remains is

$$\frac{\partial^2 F}{\partial x \, \partial y} = -p_o \frac{ab}{2c} \quad (7.1)$$

It would seem that this approximation is quite adequate provided that we limit ourselves to shells for which

$$\frac{c}{a}, \frac{c}{b} < \frac{1}{6}. \quad (7.2)$$

With c so restricted the first neglected term in (6.1) is less than 3 per cent of what is retained. All other neglected terms are less significant still.

What this simplification amounts to in more general terms is the systematic omission of quadratic terms of the slope components $\partial z/\partial x$ and $\partial z/\partial y$ in comparison with unity. The theory obtained in this manner is the *theory of shallow shells,* in the present instance the *linear membrane theory* of such shells.

8. *Stress strain relations of membrane theory.* As soon as all or some of the edge conditions of a shell problem involve the displacements of points of the edge of the shell does it become necessary to have a means of determining the state of displacement corresponding to an equilibrium state of stress. For elastic materials to which this

report is effectively restricted the state of displacement is related to the state of stress through the stress strain relations. The formulation of stress-strain relations is a well-known matter as long as attention is restricted to true stresses, normal and tangential, and to rectangular elements of area and of volume. For the theory of membrane shells which is discussed here it has been found convenient, so far as the equations of statics were concerned, to deviate from the standard normal and tangential stress representations and from rectangular elements of area. This brings with it two difficulties as soon as an attempt is made to study deformations of membranes. The first difficulty has to do with the formulation of stress-strain relations which express the same physical facts as a set of such relations in conventional terminology. The second difficulty has to do with the geometrical question of expressing the strain quantities to which one is led in terms of appropriate displacement quantities. It appears that these questions were considered only quite recently. A paper by Ban [2] presents formulas relating the pseudo-stress resultants N_x, N_y and N_{xy} to components of displacement u, v and w in the direction of x, y and z, respectively, under the assumption that the material of the shell is isotropic and obeys the generalized Hooke's Law. We have not been able to follow Ban's derivations in every respect and have therefore considered the problem in a manner quite different from Ban's. In this way a new system of formulas has been obtained, the derivation of which may be found in the Appendix of this paper. Our formulas are as follows:

$$\begin{aligned}(1+Z_x^2)\left[(1+Z_x^2)N_x+2Z_xZ_yN_{xy}+(1+Z_y^2)N_y\right]&\\ -(1+\nu)(1+Z_x^2+Z_y^2)N_y &= Eh\sqrt{1+Z_x^2+Z_y^2}\,(U_x+Z_xW_x)\end{aligned} \tag{8.1}$$

$$\begin{aligned}(1+Z_y^2)\left[(1+Z_y^2)N_y+2Z_xZ_yN_{xy}+(1+Z_x^2)N_x\right]&\\ -(1+\nu)(1+Z_x^2+Z_y^2)N_x &= Eh\sqrt{1+Z_x^2+Z_y^2}\,(V_y+Z_yW_y)\end{aligned} \tag{8.2}$$

$$\begin{aligned}2(1+\nu)(1+Z_x^2+Z_y^2)N_{xy}+2Z_xZ_y\left[(1+Z_x^2)+2Z_xZ_yN_{xy}\right.&\\ \left.+(1+Z_y^2)N_y\right] &= Eh\sqrt{1+Z_x^2+Z_y^2}\,(U_y+V_x+Z_xW_y+Z_yW_x)\end{aligned} \tag{8.3}$$

We may note that in (8.1) to (8.3) we have three simultaneous first order partial differential equations for the three functions u, v, and w, in terms of N_x, N_y and N_{xy} as given by the equations of equilibrium. It is to be expected that for many problems the best way to solve these equations is again through development in powers of a parameter as in the example of the determination of stresses in the hyperbolic paraboloid shell under its own weight. The further pursuit of this particular example through consideration of the appropriate stress-strain relations would in fact seem to be an appropriate starting point of work of this kind.

We conclude this section by stating the form of the stress-strain relations which are obtained if we systematically neglect second and higher power terms in the shell slope functions z_x and z_y. The appropriate equations are seen to be

$$N_x - \nu N_y = Eh(u_x + z_x w_x) \qquad (8.4)$$

$$N_y - \nu N_x = Eh(v_y + z_y w_y) \qquad (8.5)$$

$$2(1+\nu) N_{xy} = Eh(u_y + v_x + z_x w_y + z_y w_x) \qquad (8.6)$$

and these are, as we shall see, special cases of the corresponding equations in the general theory of shallow shells.

9. Displacements of hyperbolic paraboloid shell due to own weight according to shallow membrane theory. We consider again a shell with middle surface equation

$$z = c\,\frac{x}{a}\,\frac{y}{b} \qquad (9.1)$$

under the influence of its own weight p_o per unit of surface area. Under the assumption of shallowness we have the following expressions for stress resultants

$$N_{xy} = p_o \frac{ab}{2c}, \quad N_x = f''(y), \quad N_y = g''(x) \qquad (9.2,3,4)$$

where f and g are arbitrary functions.

Let us assume that the shell which we consider spans the rectangular area

$$0 \leqslant x \leqslant a, \quad 0 \leqslant y \leqslant b \qquad (9.5)$$

and that this shell represents one quadrant of a structure. We shall assume that the edge members along the edges $y = 0$ and $x = 0$ have negligible stiffness in directions normal to their axes. This means that we prescribe as boundary conditions the following,

$$x = 0, \quad 0 < y \leqslant b; \quad N_x = 0 \qquad (9.6)$$

$$y = 0, \quad 0 < x \leqslant a; \quad N_y = 0 \qquad (9.7)$$

These conditions lead to the result that N_x and N_y vanish throughout,

$$N_x = 0, \quad N_y = 0. \tag{9.8,9}$$

While the edge members at $x = 0$ and $y = 0$ receive horizontal loads only the edge members at $x = a$ and $y = b$ receive both horizontal and vertical loads. The vertical loads on these edge members are the ones which hold up the weight of the shell. Let V_x and V_y be the intensities of these vertical loads. We have

$$V_x = \left(N_{xy} \frac{\partial z}{\partial y}\right)_{x=a} = p_o \frac{a}{2} \tag{9.10}$$

$$V_y = \left(N_{xy} \frac{\partial z}{\partial x}\right)_{y=b} = p_o \frac{b}{2} \tag{9.11}$$

As an over-all check we note that $V_x b + V_y a = p_o ab$ which is as it should be.

In order to determine the deformation of the shell we introduce (9.2), (9.8) and (9.9) into the stress-strain relations (8.4) to (8.6). These now assume the following form

$$\frac{\partial u}{\partial x} + \frac{c}{a}\frac{y}{b}\frac{\partial w}{\partial x} = 0 \tag{9.12}$$

$$\frac{\partial v}{\partial y} + \frac{c}{b}\frac{x}{a}\frac{\partial w}{\partial y} = 0 \tag{9.13}$$

$$\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{c}{a}\frac{y}{b}\frac{\partial w}{\partial y} + \frac{c}{b}\frac{x}{a}\frac{\partial w}{\partial x} = \frac{2(1+\nu)}{Eh} p_o \frac{ab}{2c} \tag{9.14}$$

The first two of these equations can be integrated directly, giving,

$$u + \frac{c}{a}\frac{y}{b} w = U(y) \tag{9.15}$$

$$v + \frac{c}{b}\frac{x}{a} w = V(x) \tag{9.16}$$

where U and V are two arbitrary functions. We substitute from (9.15) and (9.16) into (9.14). This gives

$$U'(y) + V'(x) - \frac{2c}{ab} w = \frac{2(1+\nu)}{Eh} p_o \frac{ab}{2c} \tag{9.17}$$

We now solve for w, u, and v and obtain the following general expressions for these quantities, in terms of two arbitrary functions U and V, setting further $E/2(1+\nu) = G$,

$$W=-\frac{R}{Gh}\left(\frac{ab}{2C}\right)^2+\frac{ab}{2C}\left[U'(y)+V'(x)\right] \qquad (9.18)$$

$$u=U(y)-\frac{C}{a}\frac{y}{b}W \qquad (9.19)$$

$$v=V(x)-\frac{C}{b}\frac{x}{a}W \qquad (9.20)$$

The form of these general expressions imposes limitations on the nature of the displacement boundary conditions which may be prescribed. We consider two illustrative examples.

Example. 1. Assume that the membrane is supported by two stiff beams such that their deflections due to the loads distributions V_x and V_y may be neglected. We then have as boundary conditions

$$x=a,\ 0\leqslant y\leqslant b;\quad W=0 \qquad (9.21)$$

$$y=b,\ 0<x<a;\quad W=0 \qquad (9.22)$$

In order to satisfy (9.21) and (9.22) we must have

$$U'(y)+V'(a)=\frac{R}{Gh}\frac{ab}{2C} \qquad (9.23)$$

$$U'(b)+V'(x)=\frac{R}{Gh}\frac{ab}{2C} \qquad (9.24)$$

and this indicates that both $U'(y)$ and $V'(x)$ must be constants. Accordingly

$$U'(y)+V'(x)=\frac{R}{Gh}\frac{ab}{2C} \qquad (9.25)$$

and therewith

$$W=0 \qquad (9.26)$$

We have thus the interesting result that if the vertical displacements are zero along two adjacent edge beams they are zero throughout the shell.

We now turn to the determination of the displacement components u and v. With $w = 0$ and with U and V linear we have

$$u=C_{11}y+C_{10},\quad v=C_{21}x+C_{20} \qquad (9.27)$$

where c_{10} and c_{20} are independent rigid body displacements and where according to (9.25) c_{11} and c_{21} are related as follows

$$C_{11}+C_{21}=\frac{R}{Gh}\frac{ab}{2C} \qquad (9.28)$$

We are considering the present shell as one quadrant of the entire structure. This suggests as further boundary conditions the following symmetry conditions

$$x = a, \quad 0 \leqslant y \leqslant b; \quad u = 0 \tag{9.29}$$

$$y = b, \quad 0 \leqslant x \leqslant a; \quad v = 0 \tag{9.30}$$

The first of these conditions would make $c_{11} = c_{10} = 0$. The second would make $c_{21} = c_{20} = 0$. Since according to (9.28) we cannot have both c_{11} and c_{21} equal to zero this means that the present solution does not permit satisfaction of the symmetry conditions (9.29) and (9.30).

It is not surprising that the present solution cannot satisfy the displacement symmetry boundary conditions (9.29) and (9.30). The reason for this is that the force boundary conditions (9.6) and (9.7) along the edges $x = 0$ and $y = 0$ because of the character of the differential equation of the problem, automatically set up the equivalent force boundary conditions along the edges $x = a$ and $y = b$, and these force boundary conditions are not compatible with the displacement conditions (9.29) and (9.30).

Example 2. We now assume that the edges $x = a$ and $y = b$ of the membrane are kept from moving horizontally, but not from moving vertically, that is, we prescribe as displacement boundary conditions equations (9.29) and (9.30). Introduction of u and v from (9.19) and (9.20) into (9.29) and (9.30) gives

$$U(y) - \frac{cy}{ab}\left[-\frac{p_0}{Gh}\left(\frac{ab}{2c}\right)^2 + \frac{ab}{2c}\left\{U'(y) + V'(a)\right\}\right] = 0 \tag{9.31}$$

$$V(x) - \frac{cx}{ab}\left[-\frac{p_0}{Gh}\left(\frac{ab}{2c}\right)^2 + \frac{ab}{2c}\left\{V'(x) + U'(b)\right\}\right] = 0 \tag{9.32}$$

The solution of these two first order differential equations for U and V is of the form

$$U(y) = C_1 y^2 + Ay \tag{9.33}$$

$$V(x) = C_2 x^2 + Bx \tag{9.34}$$

where

$$\frac{1}{2}(A - B) - C_2 a = \frac{-p_0}{Gh}\,\frac{ab}{4c} \tag{9.35}$$

$$\frac{1}{2}(B - A) - C_1 b = \frac{-p_0}{Gh}\,\frac{ab}{4c} \tag{9.36}$$

so that two of the four constants in (9.33), (9.34) are arbitrary.

Introduction of (9.33) and (9.34) into (9.18) gives for the vertical deflection w,

$$W = -\frac{p}{Gh}\left(\frac{ab}{2C}\right)^2 + \frac{ab}{2C}(A+B) + \frac{ab}{C}(C_1 y + C_2 x) \qquad (9.37)$$

or, with c_1 and c_2 from (9.35) and (9.36)

$$\begin{aligned} W = &-\frac{p}{Gh}\left(\frac{ab}{2C}\right)^2\left[1-\frac{x}{a}-\frac{y}{b}\right] + \frac{ab}{2C}(A+B) \\ &+ \frac{ab}{2C}(A-B)\left(\frac{x}{a}-\frac{y}{b}\right) \end{aligned} \qquad (9.38)$$

In particular, the deflection of the corner (0,0) with respect to the corner (a, b) is of magnitude

$$W(0,0) - W(a,b) = \frac{2p}{Gh}\left(\frac{ab}{2C}\right)^2 \qquad (9.39)$$

This should be compared with the corresponding (much larger) deflection which would follow if instead of the shell we had a flat plate of the same thickness supported along two adjacent edges.

II. General Shallow Shell Theory

10. Basic equations of shallow shell theory. A system of differential equations which permits the analysis of stresses and deformations of shallow shells, including nonlinear and bending effects, has been given by K. Marguerre [10]. This system of equations includes as special cases linear membrane theory of shallow shells and also the nonlinear theory of finite bending of originally flat plates. In view of the nonlinearity of this system of shell equations, it may also serve as the starting point for the analysis of possible elastic instability or buckling of shells.

In the application of this theory the restriction of shallowness, that is the negligibility of z_x^2, z_y^2 and $z_x z_y$ in comparison with unity, must always be kept in mind. We saw how for the special case of linear membrane action knowledge of an exact system of equations permitted the explicit determination of corrections to the results for the shallow membrane. We do not at present have available similar procedures for the general case. As a general rule, however, it may be stated that shallow shell theory will be more than accurate enough as long as $z_x, z_y \leqslant 1/8$ and often accurate enough for practical purposes as long as $z_x, z_y \leqslant 1/2$.

The system of equations is of the following form

$$\frac{\partial N_{xx}}{\partial x}+\frac{\partial N_{xy}}{\partial y}+q_x=0 \tag{10.1}$$

$$\frac{\partial N_{xy}}{\partial x}+\frac{\partial N_{yy}}{\partial y}+q_y=0 \tag{10.2}$$

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}+\frac{\partial\left[(Z_x+W_x)N_{xx}+(Z_y+W_y)N_{xy}\right]}{\partial x} + \frac{\partial\left[(Z_x+W_x)N_{xy}+(Z_y+W_y)N_{yy}\right]}{\partial y}+q_z=0 \tag{10.3}$$

$$\frac{\partial M_{xx}}{\partial x}+\frac{\partial M_{xy}}{\partial y}-Q_x=0 \tag{10.4}$$

$$\frac{\partial M_{xy}}{\partial x}+\frac{\partial M_{yy}}{\partial y}-Q_y=0 \tag{10.5}$$

$$\epsilon_x=\frac{\partial u}{\partial x}+\frac{\partial Z}{\partial x}\frac{\partial W}{\partial x}+\frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2=\frac{N_{xx}-\nu N_{yy}}{Eh} \tag{10.6}$$

$$\epsilon_y=\frac{\partial V}{\partial y}+\frac{\partial Z}{\partial y}\frac{\partial W}{\partial y}+\frac{1}{2}\left(\frac{\partial W}{\partial y}\right)^2=\frac{N_{yy}-\nu N_{xx}}{Eh} \tag{10.7}$$

$$\gamma=\frac{\partial u}{\partial y}+\frac{\partial V}{\partial x}+\frac{\partial Z}{\partial y}\frac{\partial W}{\partial x}+\frac{\partial Z}{\partial x}\frac{\partial W}{\partial y}+\frac{\partial W}{\partial x}\frac{\partial W}{\partial y}=\frac{N_{xy}}{Gh} \tag{10.8}$$

$$M_{xx}=-D\left[\frac{\partial^2 W}{\partial x^2}+\nu\frac{\partial^2 W}{\partial y^2}\right] \tag{10.9}$$

$$M_{yy}=-D\left[\frac{\partial^2 W}{\partial y^2}+\nu\frac{\partial^2 W}{\partial x^2}\right] \tag{10.10}$$

$$M_{xy}=-(1-\nu)D\frac{\partial^2 W}{\partial x\,\partial y},\quad D=\frac{Eh^3}{12(1-\nu^2)} \tag{10.11}$$

In these equations the various symbols have the following significance

N_{xx}, N_{yy}, N_{xy}	tangential stress resultants
Q_x, Q_y	transverse stress resultants
M_{xx}, M_{yy}, M_{xy}	bending couples
u, v, w	displacement components in x, y and z directions
q_x, q_y, q_z	components of load intensity in x, y and z directions

To the degree of approximation involved in the use of this theory the difference between tangential stress resultants and stress resultants in the direction of x and y is negligible. The situation is *not* analogous in regard to transverse stress resultants and stress resultants in the direction of z. Rather, stress resultants in the direction of z are given by the expressions.

$$V_x = Q_x + (Z_x + W_x) N_{xx} + (Z_y + W_y) N_{xy} \tag{10.12}$$

$$V_y = Q_y + (Z_x + W_x) N_{xy} + (Z_y + W_y) N_{yy} \tag{10.13}$$

while effective *edge* stress resultants, as in the theory of plates, are obtained from the above in the form $V_x + \partial M_{xy}/\partial y$ and $V_y + \partial M_{xy}/\partial x$.

11. Reduction to two simultaneous equations. As long as the load intensity components q_x and q_y are given functions of x and y it is possible to reduce the above system of equations to two simultaneous equations for an Airy stress function F and for the displacement component w. We set, as in linear membrane theory,

$$N_{xx} = \frac{\partial^2 F}{\partial y^2} - \int q_x \, dx \tag{11.1}$$

$$N_{yy} = \frac{\partial^2 F}{\partial x^2} - \int q_y \, dy \tag{11.2}$$

$$N_{xy} = -\frac{\partial^2 F}{\partial x \, \partial y} \tag{11.3}$$

and make use of a compatibility relation which can be shown to be

$$\begin{aligned} \frac{\partial^2 \epsilon_x}{\partial y^2} + \frac{\partial^2 \epsilon_y}{\partial x^2} - \frac{\partial^2 \gamma}{\partial x \, \partial y} = {} & 2 \frac{\partial^2 Z}{\partial x \, \partial y} \frac{\partial^2 W}{\partial x \, \partial y} + \left(\frac{\partial^2 W}{\partial x \, \partial y} \right)^2 \\ & - \frac{\partial^2 Z}{\partial x^2} \frac{\partial^2 W}{\partial y^2} - \frac{\partial^2 Z}{\partial y^2} \frac{\partial^2 W}{\partial x^2} - \frac{\partial^2 W}{\partial x^2} \frac{\partial^2 W}{\partial y^2} \end{aligned} \tag{11.4}$$

Introduction of (10.6) to (10.8) into this compatibility relation gives, for shells with constant Eh, the first of the two simultaneous equations in the following form

$$\begin{aligned} \nabla^2 \nabla^2 F = {} & \frac{\partial^2}{\partial y^2} \left(\int q_x \, dx \right) + \frac{\partial^2}{\partial x^2} \left(\int q_y \, dy \right) - \nu \left(\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} \right) \\ & + Eh \left\{ 2 \frac{\partial^2 Z}{\partial x \, \partial y} \frac{\partial^2 W}{\partial x \, \partial y} + \left(\frac{\partial^2 W}{\partial x \, \partial y} \right)^2 - \frac{\partial^2 Z}{\partial x^2} \frac{\partial^2 W}{\partial y^2} - \frac{\partial^2 Z}{\partial y^2} \frac{\partial^2 W}{\partial x^2} - \frac{\partial^2 W}{\partial x^2} \frac{\partial^2 W}{\partial y^2} \right\} \end{aligned} \tag{11.5}$$

The second of the system of two simultaneous equations is obtained by introducing (10.8) to (10.10), (10.4) and (10.5) into the equilibrium equation (10.3). The result is, when D = constant,

$$D\,\nabla^2\nabla^2 W = q_z + \left(\frac{\partial^2 Z}{\partial x^2} + \frac{\partial^2 W}{\partial x^2}\right)\frac{\partial^2 F}{\partial y^2} + \left(\frac{\partial^2 Z}{\partial y^2} + \frac{\partial^2 W}{\partial y^2}\right)\frac{\partial^2 F}{\partial x^2} - 2\left(\frac{\partial^2 Z}{\partial x \partial y} + \frac{\partial^2 W}{\partial x \partial y}\right)\frac{\partial^2 F}{\partial x \partial y} - \frac{\partial}{\partial x}\left[\left(\frac{\partial Z}{\partial x} + \frac{\partial W}{\partial x}\right)\int q_x\,dx\right] - \frac{\partial}{\partial y}\left[\left(\frac{\partial Z}{\partial y} + \frac{\partial W}{\partial y}\right)\int q_y\,dy\right] \tag{11.6}$$

We observe the following facts

(1) membrane theory is the special case of (11.5) and (11.6), given when D = 0

(2) linear membrane theory follows from (11.6) by omitting all terms with w. In this manner equation (11.6) is reduced to equations (4.4) and (4.5)

(3) Flat-plate theory follows from (11.5) and (11.6) by setting z = 0.

The number of applications of special forms of the above theory is very great. We may mention as of some interest in connection with the main subject of this paper some earlier work on shallow spherical shells [13] and some very recent work on helicoidal cantilever shells [15]. In both these cases the theory is conveniently transformed, before use, into polar coordinate form.

12. *Equations for hyperbolic paraboloid shell.* To illustrate the possibilities of the general theory of shallow shells we consider again a shell with middle surface equation given by

$$z = c\,\frac{x}{a}\,\frac{y}{b} \tag{12.1}$$

We shall further assume for simplicity that we have loads in z-direction only, that is

$$q_x = 0, \quad q_y = 0 \tag{12.2}$$

The two basic differential equations of the theory now assume the form

$$\nabla^2\nabla^2 F = Eh\left\{\frac{2c}{ab}\,\frac{\partial^2 W}{\partial x \partial y} + \left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 - \frac{\partial^2 W}{\partial x^2}\,\frac{\partial^2 W}{\partial y^2}\right\} \tag{12.3}$$

$$D\nabla^2\nabla^2 W = q_z - \frac{2c}{ab}\,\frac{\partial^2 F}{\partial x \partial y} + \frac{\partial^2 W}{\partial x^2}\,\frac{\partial^2 F}{\partial y^2} + \frac{\partial^2 W}{\partial y^2}\,\frac{\partial^2 F}{\partial x^2} - 2\,\frac{\partial^2 W}{\partial x \partial y}\,\frac{\partial^2 F}{\partial x \partial y} \tag{12.4}$$

Before considering the solution of boundary value problems pertaining to this system of equations we list separately for ready reference certain special cases of these equations.

1. *Linear membrane theory*

$$\frac{2C}{ab}\frac{\partial^2 F}{\partial x \partial y} = q_z \tag{12.5}$$

2. *Linear bending theory*

$$D\nabla^2\nabla^2 W = q_z - \frac{2C}{ab}\frac{\partial^2 F}{\partial x \partial y} \tag{12.6}$$

$$\nabla^2\nabla^2 F = Eh\frac{2C}{ab}\frac{\partial^2 W}{\partial x \partial y} \tag{12.7}$$

3. *Nonlinear membrane theory*

$$\left(\frac{2C}{ab} + 2\frac{\partial^2 W}{\partial x \partial y}\right)\frac{\partial^2 F}{\partial x \partial y} - \frac{\partial^2 W}{\partial x^2}\frac{\partial^2 F}{\partial y^2} - \frac{\partial^2 W}{\partial y^2}\frac{\partial^2 F}{\partial x^2} = q_z \tag{12.8}$$

$$\nabla^2\nabla^2 F - Eh\left\{\frac{2C}{ab}\frac{\partial^2 W}{\partial x \partial y} + \left(\frac{\partial^2 W}{\partial x \partial y}\right)^2 - \frac{\partial^2 W}{\partial x^2}\frac{\partial^2 W}{\partial y^2}\right\} = 0 \tag{12.9}$$

One of the most interesting questions from a theoretical point of view is the following. For what range of values of suitably chosen parameters is the physical phenomenon adequately covered by one of the three special cases and for what range is it necessary to use the complete system (12.3) and (12.4). We have previously formulated and answered this same question in connection with the problem of axi-symmetrical deformations of spherical shells [14].

13. Simply supported shell with uniform load. As an example for a completely stated problem we consider the hyperbolic paraboloidal shell with uniform load $q_z = p_o$. We assume that the edges $x = 0$, a and $y = 0$, b have moment-free support, that the edge stiffening members are rigid in the direction of their axes and have negligible bending resistance in planes tangent to the shell,

$$\begin{array}{ll} x=0,a & \left\{ \begin{array}{ll} W=0, \quad M_{xx}=0 & (13.1) \\ N_{xx}=0, \quad \epsilon_y=0 & (13.2) \end{array}\right. \\ 0 \leq y \leq b & \end{array}$$

$$\begin{array}{ll} y=0,b & \left\{ \begin{array}{ll} W=0, \quad M_{yy}=0 & (13.3) \\ N_{yy}=0, \quad \epsilon_x=0 & (13.4) \end{array}\right. \\ 0 \leq x \leq a & \end{array}$$

Expressed in terms of w and F these conditions are

$$\left.\begin{matrix} x=0,a \\ y=0,b \end{matrix}\right\} \quad w = \nabla^2 w = F_{xx} = F_{yy} = 0 \qquad (13.5)$$

It is readily seen by inspection that a solution which satisfies all these conditions and also the differential equations (12.3) and (12.4) is

$$w=0, \quad F=-p_0 \frac{ab}{2c} x y \qquad (13.6)$$

This means that the linear membrane theory solution derived previously in equations (9.2) and (9.26) is actually a solution of the more complete system of equations incorporating nonlinear and bending effects. That this is so is closely connected with the boundary conditions which have been assumed. For instance, replacement of the conditions $N_{xx}(a,y) = N_{yy}(x,b) = 0$ by the conditions $u(a,y) = v(x,b) = 0$ no longer allows this simple solution. In fact, so far as linear membrane theory is concerned, this change leads to an incorrectly posed problem. This, however, is not the case if bending and/or nonlinear effects are taken into account.

Before proceeding with other aspects of the above solution we note the following interesting fact. The solution (13.6) appears to be valid for *all* values of c, as long as $c \neq 0$. On the other hand when $c = 0$ we have a flat plate and the load is carried exclusively by transverse bending. One would expect that as c increased from zero bending action would *gradually* be converted into membrane action. This, however, seems not to be the case, as one concludes by considering the linear version of the problem as expressed by equations (12.6) and (12.7). It may be assumed that the solution for this linear problem has to be unique, and we have then as this unique solution the simple membrane solution (13.6). The only explanation which can be given here for this paradoxical-sounding result is that the form of the boundary conditions on F is such as to lead to these conclusions.

14. Buckling of hyperbolic paraboloid shell with uniform load. We now consider the solution (13.6) of the boundary value problem given by the differential equation (12.3) and (12.4) and the boundary conditions (13.5) from the point of view of whether other, alternate solutions of the nonlinear problem are possible. We do not here intend to treat this problem in its entirety but limit ourselves to a consideration of the classical stability theory approach. To this end we set in the differential equations (12.3) and (12.4),

$$W = 0 + w, \quad F = -p_o \frac{ab}{2c} xy + \Phi \qquad (14.1)$$

and linearize in terms of the incremental functions w and Φ. In this manner we obtain the following system of differential equations,

$$\nabla^2 \nabla^2 \Phi = Eh \frac{2c}{ab} \frac{\partial^2 w}{\partial x \partial y} \qquad (14.2)$$

$$D \nabla^2 \nabla^2 w = -\frac{2c}{ab} \frac{\partial^2 \Phi}{\partial x \partial y} + p_o \frac{ab}{c} \frac{\partial^2 w}{\partial x \partial y} \qquad (14.3)$$

The boundary conditions for w and Φ are

$$\left.\begin{matrix} x = 0, a \\ y = 0, b \end{matrix}\right\} \quad w = \nabla^2 w = \Phi_{xx} = \Phi_{yy} = 0 \qquad (14.4)$$

We note that the present problem is similar to in character but less simple than the classical problem of buckling of flat plates under shear. The latter problem follows as a special case if we write $p_o\, ab/c = 2\,\tau h$ and then set $c = 0$ in all other terms.

In order to obtain critical values of p_o we may proceed as in the simpler flat-plate case by assuming

$$w = \sum \sum A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \qquad (14.5)$$

$$\Phi = \sum \sum B_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} + B_{oo} xy \qquad (14.6)$$

and by suitably operating on an infinite determinantal equation which results upon substitution of (14.5) and (14.6) in (14.2) and (14.3).

15. Qualitative consideration of stability equations. We note that for the rectangular simply supported plate the critical shearing stress τ_c is given by

$$\tau_c = K\left(\frac{a}{b}\right) \frac{E}{1-\nu^2} \left(\frac{h}{b}\right)^2 \qquad (15.1)$$

where K varies from 7.75 for $a/b = 1$ to 4.40 for $a/b = \infty$. Application of this formula to the present problem, that is omission of the Φ term in the differential equation for w would lead to formula

$$p_{oc} = K \frac{2E}{1-\nu^2} \frac{c}{a} \left(\frac{h}{b}\right)^3 \qquad (15.2)$$

in view of the fact that we have $\tau = \frac{1}{2}\, p_0\, ab/ch$. This formula, however, is not even qualitatively correct, as we will show. In other words the present stability problem is essentially dependent on the coupling of in-plane and out-of-plane action.

Rather than carrying out the required calculations for the given boundary value problem we modify the boundary conditions in such a way that a simpler method of solution becomes possible. We temporarily introduce new axes ξ and η, rotated 45 degrees with reference to the old axes,

$$\xi = (x+y)/\sqrt{2}, \quad \eta = (y-x)/\sqrt{2} \tag{15.3}$$

We have with these new independent variables ξ and η the following relations

$$\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}=\frac{\partial^2}{\partial \xi^2}+\frac{\partial^2}{\partial \eta^2}, \quad \frac{\partial^2}{\partial x \partial y}=\frac{\partial^2}{\partial \xi^2}-\frac{\partial^2}{\partial \eta^2} \tag{15.4}$$

and the differential equation (14.2) and (14.3) assume the form

$$\left(\frac{\partial^2}{\partial \xi^2}+\frac{\partial^2}{\partial \eta^2}\right)^2 \Phi = Eh\frac{2c}{ab}\left(\frac{\partial^2}{\partial \xi^2}-\frac{\partial^2}{\partial \eta^2}\right) W \tag{15.5}$$

$$D\left(\frac{\partial^2}{\partial \xi^2}+\frac{\partial^2}{\partial \eta^2}\right)^2 W = \left(\frac{\partial^2}{\partial \xi^2}-\frac{\partial^2}{\partial \eta^2}\right)\left(-\frac{2c}{ab}\Phi + p_0\frac{ab}{c}W\right) \tag{15.6}$$

Equations (15.5) and (15.6) have solutions of the form

$$W = A\cos(\alpha\xi)\cos(\beta\eta) \tag{15.7}$$

$$\Phi = B\cos(\alpha\xi)\cos(\beta\eta) \tag{15.8}$$

where A, B, α and β are constants. Introduction of these expressions into (15.5) and (15.6) gives

$$\left[D(\alpha^2+\beta^2)^2+(\alpha^2-\beta^2)\,p_0\frac{ab}{c}\right]A-(\alpha^2-\beta^2)\frac{2c}{ab}B=0 \tag{15.09}$$

$$Eh\frac{2c}{ab}(\alpha^2-\beta^2)A+(\alpha^2+\beta^2)^2B-0 \tag{15.10}$$

For a nontrivial solution the determinant of this system must vanish. This condition restricts the admissible values of p_0 to

$$p_0\frac{ab}{c} = D\frac{(\beta^2+\alpha^2)^2}{\beta^2-\alpha^2}+4Eh\left(\frac{c}{ab}\right)^2\frac{\beta^2-\alpha^2}{(\beta^2+\alpha^2)^2} \tag{15.11}$$

We note that it is the second term on the right which is responsible for the coupling of the Φ-effect to the w-effect. For given values of α and β this second term increases the possible values of p_o, that is exerts a stabilizing influence.

It is instructive to compare the consequences of the above results with the consequences of the equation without second term on the right. We set

$$\beta = \pi / l_\eta, \quad \alpha = \pi / l_\xi \tag{15.12}$$

and see that the smallest possible value of p_o occurs if $\alpha = 0$ or $l\xi = \infty$. As choosing a finite value of $l\xi$ does no more than modify what amounts to the factor K in the stability equation (15.1) we set $\alpha = 0$ for simplicity's sake. Then

$$p_o \frac{ab}{c} = \frac{\pi^2 D}{l_\eta^2} + \left(\frac{c}{ab}\right)^2 \frac{4Eh}{\pi^2} l_\eta^2 \tag{15.13}$$

In the absence of the second term of the right p_o decreases continuously as the wave length $l\eta$ increases. If we identify the wave length $l\eta$ with the edge width b we obtain, without the second term, the incorrect result

$$p_{oc} \frac{ab}{c} = \frac{\pi^2}{12} \frac{E}{1-\nu^2} \frac{h^3}{l^2} \tag{15.14}$$

which except for a numerical factor is the same as the also inappropriate equation (15.2). Note that by choosing $l\eta$ as a suitable numerical multiple of b we can make (15.14) and (15.2) agree with each other exactly.

Retention of the second term on the right changes these results in a very essential way. We see that now there is a definite finite value of $l\eta$ which makes p_o smallest. This value is given by

$$l_\eta^4 = \frac{\pi^4 D}{4Eh} \left(\frac{ab}{c}\right)^2$$

or

$$l_\eta = \frac{\pi}{2\sqrt[4]{3(1-\nu^2)}} \sqrt{ab} \sqrt{\frac{h}{c}} \tag{15.15}$$

For sufficiently small values of h/c the half wave length $l\eta$ is appreciably smaller than the geometrical mean of the edge lengths of the shell. This means that when $h/c \ll 1$ then the present formula has physical significance going beyond that for the simple set of special boundary conditions which we initially assumed. What would seem to happen when $h/c \ll 1$ is that the shell "crinkles" into waves in the direction of the existing compression. The value of the load at which this crinkling takes place is obtained by introducing (15.15) into (15.13) as follows

$$p_{oc}\frac{ab}{C} = \frac{\pi^2 D 2\sqrt{Eh}}{\pi^2\sqrt{D}}\frac{C}{ab} + \left(\frac{C}{ab}\right)^2\frac{4Eh}{\pi^2}\frac{\pi^2\sqrt{D}}{2\sqrt{Eh}}\frac{ab}{C}$$

or, after some cancellations,

$$p_{oc} = \frac{2E}{\sqrt{3(1-\nu^2)}}\frac{h^2C^2}{a^2b^2} \quad . \qquad (15.16)$$

A comparison of formula (15.16) with the plate-theory formula (15.2) shows that the plate formula does, in fact, depend on the various linear dimensions of the shell in a manner which differs from the shell theory formula. The shell theory formula shows a more pronounced dependency on shell rise c and a less pronounced dependency on the shell thickness h than the plate theory formula.

Formula (15.16) for the critical load intensity may alternately be written so as to give the critical value of the shearing stress produced by the load p_o. We have

$$\tau_o = \frac{N_{xy}}{h} = \frac{p_o}{h}\frac{ab}{2C}$$

and therewith

$$\tau_{oc} = \frac{E}{\sqrt{3(1-\nu^2)}}\frac{hC}{ab} \qquad (15.17)$$

We may again compare this buckling formula with the corresponding formula (15.1) for the simply supported flat plate. It is seen that except for a difference in numerical coefficients the shell formula differs from the plate formula by the fact that a thickness square factor is replaced by the product of wall thickness and shell rise.

16. Analysis of edge effects for hyperbolic paraboloid shell with uniform load. We now return to the completely stated boundary value problem (12.3), (12.4) and (13.1) to (13.4). We ask the following questions. How is the stress distribution in the shell changed from the simple membrane condition if instead of assuming that the edge stiffners are rigid in the direction of their own axes we assume that these stiffners have finite or even negligibly small axial stiffness. We will here for simplicity consider the latter case, that is restrict attention to the case where the edge conditions $(\epsilon_y)_{x=0,a} = 0$ and $(\epsilon_x)_{y=0,b} = 0$ are replaced by the condition $N_{xy} = 0$ along all four edges of the shell.

We then have the following system of boundary conditions, expressed in terms of w and F.

$$x=0,a;\quad w=\nabla^2 w=F_{yy}=F_{xy}=0 \tag{16.1}$$

$$y=0,b;\quad w=\nabla^2 w=F_{xx}=F_{xy}=0 \tag{16.2}$$

Since we are interested in the way in which the simple membrane solution $F=-p_o(ab/2c)xy$ is changed by the modification of the boundary conditions we set

$$F=-p_o\frac{ab}{2c}xy+\Phi$$

and consider the differential equation problem in terms of w and Φ.

$$\nabla^2\nabla^2\Phi=Eh\left\{\frac{2c}{ab}\frac{\partial^2 w}{\partial x\partial y}+\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2-\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2}\right\} \tag{16.3}$$

$$\begin{aligned}D\nabla^2\nabla^2 w=-\frac{2c}{ab}\frac{\partial^2\Phi}{\partial x\partial y}+p_o\frac{ab}{c}\frac{\partial^2 w}{\partial x\partial y}\\+\frac{\partial^2 w}{\partial x^2}\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2 w}{\partial y^2}\frac{\partial^2\Phi}{\partial x^2}-2\frac{\partial^2 w}{\partial x\partial y}\frac{\partial^2\Phi}{\partial x\partial y}\end{aligned} \tag{16.4}$$

$$x=0,a;\quad w=\nabla^2 w=\Phi_{yy}=0,\quad \Phi_{xy}=p_o\frac{ab}{2c} \tag{16.5}$$

$$y=0,b;\quad w=\nabla^2 w=\Phi_{xx}=0,\quad \Phi_{xy}=p_o\frac{ab}{2c} \tag{16.6}$$

We now take account of the following considerations.

(1) The deviation from the simple membrane condition may be restricted to relatively narrow edge zones.

(2) We need transverse edge shear to support the load and this means bending is not negligible, at least in edge regions.

(3) Nonlinear terms and possibly other terms in the differential equation may be of negligible importance in the range of loads and dimensions which are of interest.

In order to see to what extent the above points apply we use a method of dimensional analysis. We set

$$x=\lambda\xi,\quad y=\lambda\eta \tag{16.7}$$

$$\Phi=\Phi_o f(\xi,\eta),\quad w=w_o g(\xi,\eta) \tag{16.8}$$

where the length λ and the amplitude factors Φ_o and w_o are chosen such that f and g and their derivatives with respect to ξ and η are at most of order unity. Introduction of the new variables ξ, η, f and g into (16.3) and (16.4) leaves the following system of equations

$$\frac{\Phi_0}{\lambda^4}\nabla^2\nabla^2 f = Eh\left\{\frac{2C}{ab}\frac{W_0}{\lambda^2} g_{\xi\eta} + \frac{W_0^2}{2\lambda^4}K(g,g)\right\} \qquad (16.9)$$

$$\frac{DW_0}{\lambda^4}\nabla^2\nabla^2 g = -\frac{2C}{ab}\frac{\Phi_0}{\lambda^2} f_{\xi\eta} + p_0\frac{ab}{C}\frac{W_0}{\lambda^2} g_{\xi\eta} - \frac{W_0\Phi_0}{\lambda^4}K(f,g) \qquad (16.10)$$

where

$$\nabla^2 = (\ )_{\xi\xi} + (\ )_{\eta\eta},\qquad K(f,g) = 2f_{\xi\eta}g_{\xi\eta} - f_{\xi\xi}g_{\eta\eta} - f_{\eta\eta}g_{\xi\xi} \qquad (16.11)$$

The boundary conditions (16.5) and (16.6) become

$$\xi = 0, \frac{a}{\lambda};\quad g = g_{\xi\xi} = f_{\eta\eta} = 0,\ f_{\xi\eta} = \frac{\lambda^2}{\Phi_0}p_0\frac{ab}{2C} \qquad (16.12)$$

$$\eta = 0, \frac{b}{\lambda};\quad g = g_{\eta\eta} = f_{\xi\xi} = 0,\ f_{\xi\eta} = \frac{\lambda^2}{\Phi_0}p_0\frac{ab}{2C} \qquad (16.3)$$

We now dispose of the three parameters λ, Φ_0 and w_0 in a manner which helps to simplify the problem.

Since $f_{\xi\eta}$ is to be of order unity we set

$$\frac{\lambda^2}{\Phi_0}p_0\frac{ab}{2C} = 1 \qquad (16.14)$$

Since we want in-plane action coupled with bending action we further set

$$\frac{\Phi_0}{\lambda^4} = Eh\frac{2C}{ab}\frac{W_0}{\lambda^2},\qquad \frac{DW_0}{\lambda^4} = \frac{2C}{ab}\frac{\Phi_0}{\lambda^2} \qquad (16.15)$$

Solution of these three simultaneous equations gives

$$W_0 = \frac{p_0}{Eh}\left(\frac{ab}{2C}\right)^2,\qquad \Phi_0 = p_0\sqrt{\frac{D}{Eh}}\left(\frac{ab}{2C}\right)^2 \qquad (16.16)$$

$$\lambda^4 = \frac{D}{Eh}\left(\frac{ab}{2C}\right)^2,\qquad \lambda = \frac{\sqrt{ab}}{2\sqrt[4]{3(1-\nu^2)}}\sqrt{\frac{h}{C}} \qquad (16.17)$$

We now introduce these values of w_0, Φ_0 and λ into the differential equations (16.9) and (16.10). We find that if this is done that there remains only one dimensionless parameter in these differential equations, as follows,

$$\nabla^2\nabla^2 f = g_{\xi\eta} + \mu\frac{1}{2}K(g,g) \qquad (16.18)$$

$$\nabla^2\nabla^2 g = -f_{\xi\eta} + \mu\left[2g_{\xi\eta} - K(f,g)\right] \qquad (16.19)$$

The parameter μ is,

$$\mu = \frac{p_o}{\sqrt{EhD}}\left(\frac{ab}{2c}\right)^2 = \frac{\sqrt{3(1-\nu^2)}}{2}\,\frac{p_o}{E}\left(\frac{ab}{ch}\right)^2 \tag{16.20}$$

The possible magnitude of this parameter μ becomes clearest if we recall our earlier determination of a critical load intensity p_{oc} as given by equation (15.16). We see that

$$\mu = \frac{p_o}{p_{oc}} \tag{16.21}$$

Accordingly, as long as we keep the load intensity p_o well below the buckling load intensity p_{oc} we shall have $\mu \ll 1$.

We now have established a *range of applicability of linearized theory* given by the order of magnitude relation

$$\mu = \frac{\sqrt{3(1-\nu^2)}}{2}\,\frac{p_o}{E}\left(\frac{ab}{ch}\right)^2 \ll 1 \tag{16.22}$$

In this range the differential equations to be solved are

$$\nabla^2\nabla^2 f = g_{\xi\eta}, \quad \nabla^2\nabla^2 g = -f_{\xi\eta} \tag{16.23}$$

We may if we wish calculate nonlinear corrections to the results of this linear theory by expanding the solutions of the complete system (16.18) and (16.19) in powers of the parameter μ.

We next consider the magnitude of the characteristic length λ. We see that as long as $h/c \ll 1$ we have that λ is small compared to $\frac{1}{2}(ab)^{1/2}$. This means that as long as the shell thickness h is small compared to the shell rise c we have the existence of characteristic edge regions, of width small compared with the lateral dimensions of the shell. When h is of the same order of magnitude as c then this is no longer the case. We are then dealing with a problem for which the shell behaves like a flat plate. As long as $h/c \ll 1$ we have $a/\lambda \gg 1$ and $b/\lambda \gg 1$, (assuming that a and b are of the same order of magnitude). We are then further tempted to assume that the stresses along the pair of sides $\xi = 0$ and $\eta = 0$ are effectively independent of the stresses along the pair of sides $\xi = a/\lambda$ and $\eta = b/\lambda$, and that so far as the sides $\xi = 0$ and $\eta = 0$ are concerned the other two sides might as well be at infinity. In this way we arrive at the following simplified system of boundary condition for the solutions of the system (16.23).

$$\xi = 0,\ 0 < \eta; \quad g = g_{\xi\xi} = f_{\eta\eta} = 0, \quad f_{\xi\eta} = 1 \tag{16.24}$$

$$\eta = 0,\ 0 < \xi; \quad g = g_{\eta\eta} = f_{\xi\xi} = 0, \quad f_{\xi\eta} = 1 \tag{16.25}$$

The considerations leading to these boundary conditions are clearly not rigorous. Their justification depends on properties of the solution which must still be established. The properties to be established state that outside the edge zones of width λ the modifications of the simple membrane state are effectively nonexistent. For this to be so, the solutions of the system of equations (16.23) subject to the boundary conditions (16.24) and (16.25) must have the property that as ξ or η tend to infinity all second derivatives of the solution functions f and g must tend to zero.

We make the following further observations.

(1) Bending stresses σ_B which occur in the edge zones are of the same order of magnitude as the membrane shearing stress $\tau_0 = \frac{1}{2} p_0\, ab/ch$. This is seen as follows. We have $\sigma_B = \frac{1}{2} Eh\, w_{xx} = \frac{1}{2} Eh\, (w_0/\lambda^2)\, g_{\xi\xi}$ where $g_{\xi\xi}$ is of order unity. According to (16.16) and (16.17) we have further

$$\frac{Eh}{\lambda^2} W_0 = \frac{Eh}{ab}\,\frac{c}{h}\, 4\sqrt{3(1-\nu^2)}\,\frac{p}{Eh}\left(\frac{ab}{2c}\right)^2 = \sqrt{3(1-\nu^2)}\, p_0\, \frac{ab}{ch} \qquad (16.26)$$

so that in fact σ_B is of the same order as τ_0.

(2) The same type of analysis can be carried out for other conditions of support which prevent existence of the simple membrane state throughout. For example, instead of absence of restraint against bending moments we may consider the other limiting case of built-in edges. Or, instead of replacing the condition of axially rigid edge stiffners by the condition of axially nonresistant edge stiffners, as done here, we may determine the effect of varying the magnitude of the axial edge member stiffness on the distribution of stresses in the shell.

(3) The present order of magnitude analysis indicates that when p_0/p_{0c} is of order unity that then the nonlinear terms in the differential equations are no longer negligible. This means that the possibility must be considered that the buckling load p_{0c} according to classical stability theory cannot in actuality be reached, the structure becoming unstable for a load p_0 which is a fraction only of p_{0c}. An analogous situation is known in regard to the problem of buckling of cylindrical and spherical shells [8].

APPENDIX

Derivation of stress-strain relations of linear membrane theory. We derive our stress-strain relations without direct recourse to geometrical considerations. Our method consists in an application of the principle of minimum complementary energy together with the notion of the Lagrangean multiplier.

The principle of minimum complementary energy, as it is used here, may be stated in the following form. *Among all states of stress which satisfy the equilibrium equations in the interior of body and all prescribed surface stress conditions, the state of stress which also satisfies the stress-strain (displacement) relations in the interior and all prescribed displacement boundary conditions on the part S_d of the surface is characterized by the variational equation*

$$\delta \Pi \equiv \delta \left\{ \int W(\sigma,\tau) dV - \int_{S_d} \underset{\sim}{p} \cdot \underset{\sim}{\bar{u}}\, dS \right\} = 0 \qquad (A,1)$$

The function $W(\sigma, \tau)$ is a stress energy function, the form of which is such that in three-dimensional Cartesian coordinates the stress-strain relations are,

$$\epsilon_x = \frac{\partial W}{\partial \sigma_x}, \ldots, \gamma_{yz} = \frac{\partial W}{\partial \tau_{yz}} \qquad (A,2)$$

p is the unknown surface load intensity vector on the part S_d of the surface, and $\bar{u}$ is the prescribed displacement vector on S_d.

For a three-dimensional isotropic material obeying the generalized Hooke's Law the function W is of the form

$$W = \frac{1}{2E}\left\{ \sigma_x^2 + \sigma_y^2 + \sigma_z^2 - 2\nu(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z) + 2(1+\nu)(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2) \right\} \qquad (A,3)$$

In order to obtain an analogous expression for membranes, in terms of pseudo-stress resultants (N_{xx}, N_{yy}, $N_{xy} = N_{yx}$ we proceed as follows. We may first consider a rectangular element of area in the middle surface of the membrane. Let N_1, N_2, $N_{12} = N_{21}$ be the actual stress resultants acting over the faces of this element. Since the material of the membrane is assumed to satisfy the same stress-strain relations as those leading to (A,3) we have

$$\begin{aligned} W &= \frac{1}{2Eh^2}\left\{ N_1^2 + N_2^2 - 2\nu N_1 N_2 + 2(1+\nu) N_{12}^2 \right\} \\ &= \frac{1}{2Eh^2}\left\{ (N_1 + N_2)^2 + 2(1+\nu)(N_{12}^2 - N_1 N_2) \right\} \end{aligned} \qquad (A,4)$$

The function W in (A,4) is to be transformed by expressing N_1, N_2, N_{12} in terms of N_{xx}, N_{yy}, N_{xy}. One finds as transformation formulas the following relations which express appropriate static equilibrium conditions

$$\sqrt{1+Z_x^2+Z_y^2}\,(N_1+N_2)=(1+Z_x^2)N_{xx}+2Z_xZ_yN_{xy}+(1+Z_y^2)N_{yy} \qquad (A,5)$$

$$N_{12}^2-N_1N_2=N_{xy}^2-N_{xx}N_{yy}$$

$$W=\frac{1}{2Eh^2}\left\{\frac{[(1+Z_x^2)N_{xx}+2Z_xZ_yN_{xy}+(1+Z_y^2)N_{yy}]^2}{1+Z_x^2+Z_y^2}+2(1+\nu)(N_{xy}^2-N_{xx}N_{yy})\right\} \qquad (A,6)$$

Equation (A,6) appears to have been obtained first by Lafaille [9].

For what follows it is convenient to assume that all boundary conditions for the stressed membrane are displacement boundary conditions. The expression for the complementary energy of the membrane is then

$$\Pi=\iint Wh\,dA-\int \underset{\sim}{N}\cdot\overline{\underset{\sim}{u}}\,ds \qquad (A,7)$$

where dA is given by (2.11) and ds is an element of arc along the boundary curve of the membrane. We further need to know explicitly the value of the line integral only along a line x = const., all corresponding other results following by analogy. Along a line x = const. we have

$$\underset{\sim}{N}ds=\underset{\sim}{N}_x dy=(N_{xx}\underset{\sim}{s}+N_{xy}\underset{\sim}{t})dy$$
$$=[N_{xx}(\underset{\sim}{i}+Z_x\underset{\sim}{k})+N_{xy}(\underset{\sim}{j}+Z_y\underset{\sim}{k})]dy \qquad (A,8)$$

With

$$\underset{\sim}{u}=u\underset{\sim}{i}+v\underset{\sim}{j}+w\underset{\sim}{k} \qquad (A,9)$$

we have further

$$\underset{\sim}{N}\cdot\overline{\underset{\sim}{u}}\,ds=[N_{xx}\bar{u}+N_{xy}\bar{v}+(Z_xN_{xx}+Z_yN_{xy})\bar{w}]dy \qquad (A,10)$$

Introduction of (A,6) and (A,10) into (A,7) gives

$$\Pi=\iint\frac{1}{2Eh}\left\{\frac{[(1+Z_x^2)N_{xx}+2Z_xZ_yN_{xy}+(1+Z_y^2)N_{yy}]^2}{\sqrt{1+Z_x^2+Z_y^2}}+2(1+\nu)(N_{xy}^2-N_{xx}N_{yy})\sqrt{1+Z_x^2+Z_y^2}\right\}dx\,dy$$
$$-\int[N_{xx}\bar{u}+N_{xy}\bar{v}+(Z_xN_{xx}+Z_yN_{xy})\bar{w}]dy\Big)_{x=\text{const.}}$$
$$-\ \ldots \qquad (A,11)$$

We now apply the principle of minimum complementary energy in the following manner. We take account of the equilibrium conditions (3.9) to (3.11) in evaluating $\delta\Pi = 0$ by adding them, suitably multiplied and integrated, to (A,11) as follows.

$$\begin{aligned}
\delta\Big[\iiint\Big\{ & \frac{[(1+z_x^2)N_{xx} + 2z_x z_y N_{xy} + (1+z_y^2)N_{yy}]^2}{2Eh\sqrt{1+z_x^2+z_y^2}} \\
& + \frac{1+\nu}{Eh}(N_{xy}^2 - N_{xx}N_{yy})\sqrt{1+z_x^2+z_y^2} \\
& + \lambda_1\Big[\frac{\partial N_{xx}}{\partial x} + \frac{\partial N_{xy}}{\partial y} + q_x\Big] \\
& + \lambda_2\Big[\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_{yy}}{\partial y} + q_y\Big] \\
& + \lambda_3\Big[\frac{\partial(N_{xx}z_x + N_{xy}z_y)}{\partial x} + \frac{\partial(N_{xy}z_x + N_{yy}z_y)}{\partial y} + q_z\Big]\Big\}\,dx\,dy \\
& - \int\Big[N_{xx}\bar{u} + N_{xy}\bar{v} + (z_x N_{xx} + z_y N_{xy})\bar{w}\Big]_{x=c} dy \\
& - \cdots \qquad \cdots\Big] = 0
\end{aligned} \qquad (A,12)$$

The quantities λ_n in (A,12) are the Lagrange multipliers and because of their presence it is legitimate to consider the variations of the pseudo-stress components N_{xx}, N_{yy}, N_{xy} arbitrary.

Because of the arbitrariness of N_{xx}, N_{yy} and N_{xy} we have three Euler equations for the variational equation (A,12). These three Euler equations are exactly the stress-strain relations (8.1) to (8.3) except that the place of u, v and w is taken by λ_1, λ_2 and λ_3. The quantities λ_1, λ_2 and λ_3 may, however, be identified with u, v and w. By consideration of the line integral in (A,12) it is established that this identification is possible for the specific line x = const. in (A,12). As the line x = const. is arbitrary it is concluded that the identification is possible everywhere and this then establishes the validity of the stress-strain relations (8.1) to (8.3).

The earliest reference known to the author concerning use of the principle of complementary energy for the establishment of stress-displacement relations is a paper by Trefftz [17]. It may be worth mentioning that the object in Trefftz' paper and in the present paper differs in the following respect. While the earlier work uses complementary energy for the derivation of approximate two-dimensional results for a three-dimensional problem, the present work uses complementary energy for the derivation of exact two-dimensional results for a two-dimensional problem.

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PHYSICAL AND ENGINEERING PROPERTIES OF PLASTICS

BY ALBERT G. H. DIETZ,* *Member*

(Presented at a meeting of the Structural Section of the Boston Society of Civil Engineers, held on December 8, 1954.)

Plastics are used today in considerable quantities in building construction, and there is lively interest among architects, engineers, builders and plastics producers concerning the possibilities of still greater utilization of these relatively new materials of construction.

One of the oldest uses of plastics in building is as adhesives. Plastics provide the completely waterproof glues for waterproof plywood and other waterproof glued construction. Plastics made possible better varnishes and better lacquers, better drying oils, and other finishes today widely used in buildings. Plastics combined with wood, paper and fabrics provide building boards of superior qualities. Large quantities of plastics are used in flooring. Upholstery and wall coverings are made of tough sheets, coated fabrics, and woven plastics. Transparent and translucent plastics are used in such places as schoolhouse windows and street lights where breakage and maintenance costs are high. They are used for skylights and illumination where their ready formability makes them especially adaptable. All of these applications came about because of certain physical properties and combinations of properties to be found in plastics.

TYPES OF PLASTICS

Before considering the physical and engineering properties of plastics it will be well to summarize the principal types.

All plastics can be put into two more or less definite classes: the thermosetting resins or plastics and the thermoplastic. Thermosetting materials pass through a soft plastic stage once and then harden irreversibly, i.e., they cannot again be softened by heat or any other means. Thermoplastics soften upon heating and harden upon cooling, and can be softened and hardened any number of times. This must be kept in mind when selecting a plastic for any given purpose,

*Professor of Building Engineering and Construction and Director, Plastic Research Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts.

particularly if changes in temperature are to be expected during the life of the part.

Among the thermosetting plastics are the following principal types:

Phenol formaldehyde and related products made by the interaction of phenol (or similar chemicals) and formaldehyde. This is one of the oldest and most versatile plastics and utilizes a large variety of fillers to give a wide range of properties. This group is usually called the phenolics.

Urea formaldehyde and melamine formaldehyde. These are chemically similar although melamine formaldehyde is harder and is more durable upon exposure, especially to moist conditions. These plastics are in some respects similar to phenolics but have much better color range. They are also more expensive. Often they are grouped as urea and melamine.

Unsaturated polyesters. These are usually used as the base resin for reinforced plastics, especially in applications of interest to building. Unlike the phenolics, ureas, and melamines, they give off no water as a byproduct during cure or hardening.

Expoxies. Like the polyesters these are mainly used as a base for reinforced plastics.

Alkyds. This group is of particular interest in molded electrical insulating parts but has not found wide application in building.

The principal thermoplastics are:

Cellulosics. Cellulose is reacted with various acids and others to produce cellulose nitrate, cellulose acetate, cellulose acetate-butyrate copolymer, and ethyl cellulose.

Acrylics. These are the familiar transparent plastics used in aircraft windows and similar applications.

Vinyl and vinylidene chloride. Flexible films for raincoats and shower curtains, electrical insulation, "mothballs" for ships, rigid pipe, floor tile and upholstery are among the applications.

Polyethylene. This waxy-feeling grayish flexible material is widely used for tubing, flexible cups and dishes, films for packaging, squeeze bottles and the like. Its chemical inertness at ordinary temperatures is excellent.

Fluorinated plastics, especially the completely fluorinated and fluoro-chloro ethylenes are among the most inert materials known, resistant to extremely corrosive conditions, maintain their properties at high and low temperatures, and are highly moisture-repellent.

Nylons. As textiles these are well known, but they are also widely used for moldings where toughness, resistance to abrasion and wear, and resistance to moderately elevated temperatures are important.

Silicones. Based upon silicon rather than carbon as the central atom, these materials are among the best in resistance to ozone, ultraviolet and infrared, electrical discharges, high and low temperatures, and in moisture-repellence.

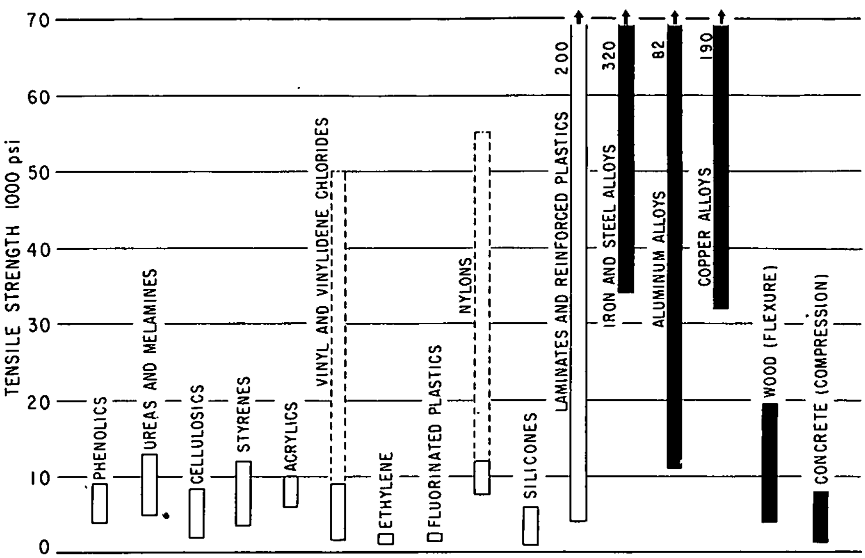
Among the most promising building applications are the *laminates and reinforced plastics* in which sheet materials like paper, cotton cloth, wood, asbestos cloth and paper, and a wide variety of woven fabrics and mats made of continuous or staple glass fibers are combined with plastics to provide decorative or utilitarian laminates and reinforced plastics having a very large range of strength properties, toughness, and other engineering attributes of particular interest in building. These are already in wide use, and promise to become still more important.

Physical Properties

The range of properties of plastics, including laminates, within their own limits is as great as the range of properties of metals, and is much greater than that of either concrete or timber. This is brought out in Figure 1 in which the ranges of tensile strengths of the principal classes of plastics, and of plastics-based laminates and reinforced plastics, are compared with iron and steel alloys, aluminum alloys, copper alloys, wood, and concrete. Depending upon the formulations and reinforcing materials employed, plastics may be extremely weak or extremely strong with a range approximately 200 to one.

Elastic and Plastic Behavior

To some degree plastics behave in an elastic manner; that is to say, deformation under load is directly proportional to the load, and when the load is removed the deformation disappears. In engineering design steel, concrete and timber are assumed to behave elastically and as long as stresses are kept within limits, steel certainly behaves elastically and concrete and timber do to a large extent. Some plastics, especially the thermosets, are also essentially elastic within appropriate stress limits. Deformation is proportional to the load applied and disappears quickly when the load is released.



TENSILE STRENGTH OF PLASTICS AND VARIOUS OTHER MATERIALS.

FIG. 1.—COMPARISON OF TENSILE STRENGTHS OF THE PRINCIPAL PLASTICS WITH TENSILE STRENGTHS OF FERROUS ALLOYS, ALUMINUM ALLOYS, COPPER ALLOYS, CONCRETE AND WOOD.

Plastic behavior, in contrast to elastic, involves the flow or creep of materials under load, so that the deformation depends not only on the load but on the rate at which it is applied and on its duration. When the load is removed the material may eventually recover all or part of the deformation. Concrete and timber are plastic to some degree, and deformation, especially under high stress, is likely to increase with time. The same is true of metals at high stresses and at elevated temperatures.

Many plastics, especially the thermoplastics, exhibit time-dependent plastic behavior. The effect is greater at elevated temperatures and may be quite marked at temperatures found in buildings, such as hot water lines, roofs and walls exposed to the sun, and similar elevated-temperature conditions. These materials become stiffer and stronger as the temperature drops and eventually most of them become brittle, some at temperatures in the vicinity of zero Fahrenheit, others at much lower temperatures. Their strength, and the amount of deformation before failure occurs also depend on the rate at which they are loaded. If loaded quickly they carry higher loads but stretch or otherwise deform less before failure than when loaded slowly. The effects of temperature and rate on the load-deformation behavior of such materials are shown diagrammatically in the stress-strain curves of Figure 2.

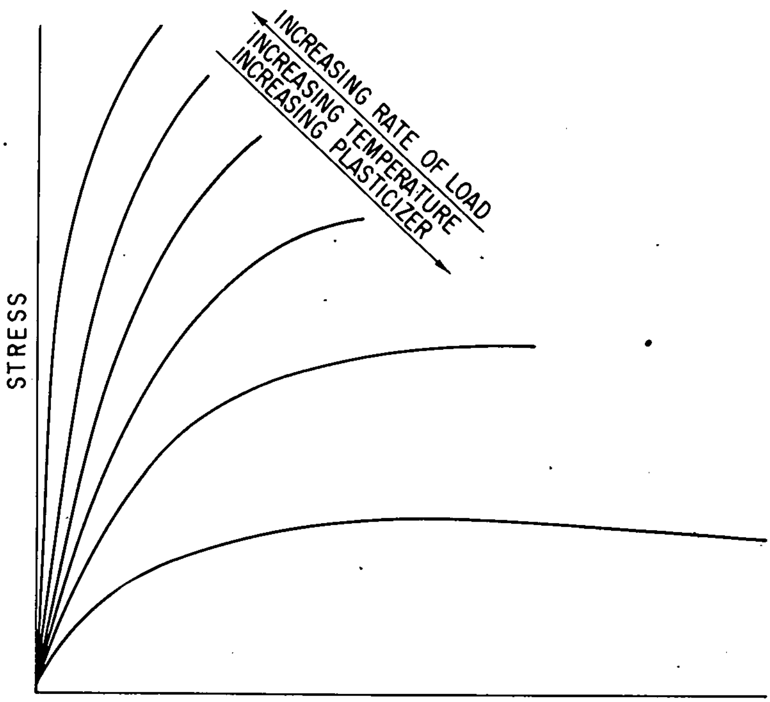


FIG 2.—EFFECT OF TEMPERATURE, PLASTICIZER CONTENT, AND RATE OF STRESS APPLICATION UPON STRESS-STRAIN BEHAVIOR.

Materials which exhibit plastic behavior are subject to creep. When loads are imposed these materials tend to continue to deform as time passes and the degree of creep depends on the range of stresses and temperatures encountered. If stresses are low enough and temperatures are moderate, creep is small and may be of no particular importance, as brought out in the lowest curve of Figure 3 which shows a typical low-stress creep curve at moderate temperatures in which creep eventually dies off to zero. This curve also shows that if the load is removed (dotted line) the material eventually returns to its original undeformed condition. The second curve shows a condi-

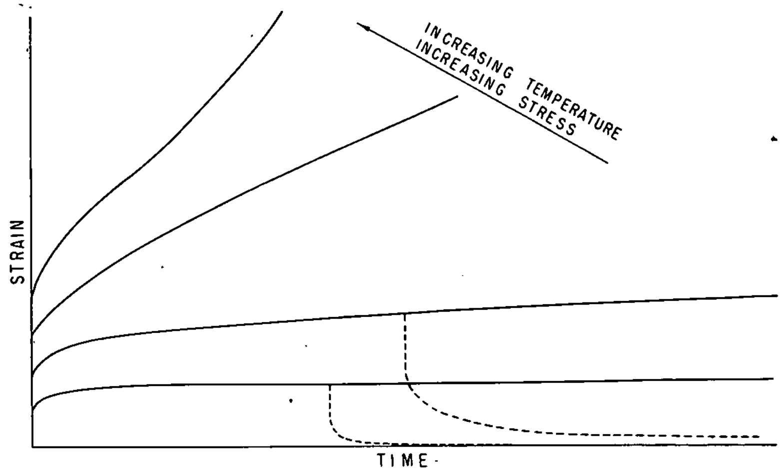


FIG. 3.—CREEP AND CREEP RECOVERY AS AFFECTED BY STRESS LEVEL AND TEMPERATURE.

tion of higher stress in which creep continues to increase slowly with time. If continued long enough this may lead to failure. If unloaded (dotted line), most of the deformation is recovered, but some residual deformation remains. As stresses or temperatures increase, creep also increases and may lead to rapid distortion and failure as in the upper curve of Figure 3. Plastic pipe, for example, which might be entirely satisfactory for cold water at normal pressures could well be completely unsatisfactory as a hot-water line.

Although engineers, builders and architects are familiar with the fact that the strengths of materials are dependent upon temperature, stress, and duration of load, for many construction materials these factors are of relatively minor significance. In the case of many plastics, particularly thermoplastics, the relationship is of major significance and must be taken into account. If it is, the plastics can be employed satisfactorily, but if it is not, failures may occur. Appropriate stress levels and factors of safety must therefore be employed. For materials exhibiting no sharply defined yield points or elastic limits, the working stresses are more dependent upon the degree of creep which can be tolerated than upon yield points or breaking stresses.

Fillers and Plasticizers

Plastics are greatly modified in their properties by the judicious addition of fillers and plasticizers. Fillers are used primarily with thermosetting materials and plasticizers primarily with certain thermoplastics. For example, a pure thermosetting material like phenolic is hard to mold, is fairly costly, and is quite brittle. If wood flour is added, it is much more moldable, it is cheaper, it shrinks less in the mold, and it gives a better all around product. So for general-purpose moldings, materials like phenolics are loaded with wood flour. Similarly, if high electrical properties are required, it is customary to add mica. For high heat resistance asbestos fiber is added. Chopped fabric or chopped tire cord markedly increases the toughness and the strength. White pure alpha cellulose is added to the light-colored thermosets as, for example, in the widely-used dishes made of molded melamine. Clay and other inorganic fillers may be added to reduce cost, increase temperature and flame resistance, and to provide better stability and lessened shrinkage. The same basic resin, therefore, may be given a wide range of properties depending upon the type of filler that is added. In Figure 1 the ranges of strength properties shown for the phenolics, urea, and melamine resins are the result of adding different fillers in varying quantities.

Plasticizers are commonly used with thermoplastics, some of which in their unmodified state are too brittle and too hard to be molded at temperatures below their decomposition temperatures. Others, although moldable, may be too hard and brittle for their intended uses. Plasticizers make them more flexible at ordinary temperatures, that is, plasticizers have much the same effect at ordinary temperatures as heat has in increasing plasticity, flexibility, and, frequently, toughness. Strength is generally decreased at the same time. So by varying the plasticizer content, the same plastic can be varied from hard, rigid, and brittle to soft, distensible, and tough. The cellulosics and polyvinyl chloride are particularly good examples of the use of plasticizers to provide a wide range of hardness, strength, and flexibility, as shown in Figure 1.

Orientation

When certain plastics are drawn into fine filaments their strength rises markedly. This phenomenon is utilized in the manufacture of textile materials. Although textiles are beyond the scope of this conference, it is interesting to see in Figure 1 how the vinyls and nylon, for example, are increased in tensile strength when drawn into filaments. Many of the other high-strength textile materials are obtained in a similar manner.

Copolymers

Several basic types may be built into the same molecular chain. When this occurs a wide range of properties may be obtained, depending on the types and proportions of basic plastics employed. Many such copolymers exist. The range of properties shown in Figure 1 for styrene and the acrylics stems largely from copolymerization. The cellulosics and vinyls similarly can be copolymerized in addition to being plasticized.

Stiffness

Compared with traditional structural materials, the stiffness of plastics as measured by modulus of elasticity is generally low. As shown in Figure 4, the thermosetting materials, that is, phenolics, urea and melamine, are in the same general range as wood and concrete, but the rest of the plastics, other than the laminates and reinforced plastics, rank lower than all of the traditional structural materials. All of the plastics rank well below all of the metallic alloys. Magnesium, not shown because it is not customarily found in buildings, ranks slightly higher than the upper limits of the plastics.

The modulus of elasticity of plastics requires some explanation. For most construction materials, the modulus of elasticity is simply defined as the ratio of stress to strain at some specified portion or point of the stress to strain curve as determined by a standard test. Variations from this value are not great under ordinary conditions. The same holds true of those plastics which exhibit little or no creep. If creep occurs, however, strain is no longer directly proportional to stress, the plastics part sags or otherwise distorts under load with time and the stiffness, or modulus of elasticity, in effect decreases. The modulus of elasticity, therefore, varies with time and ought perhaps to be called a modulus of plasticity. By taking this into account and by using correspondingly reduced moduli, plastics parts can be designed for long-time loading and long-time allowable deflections.

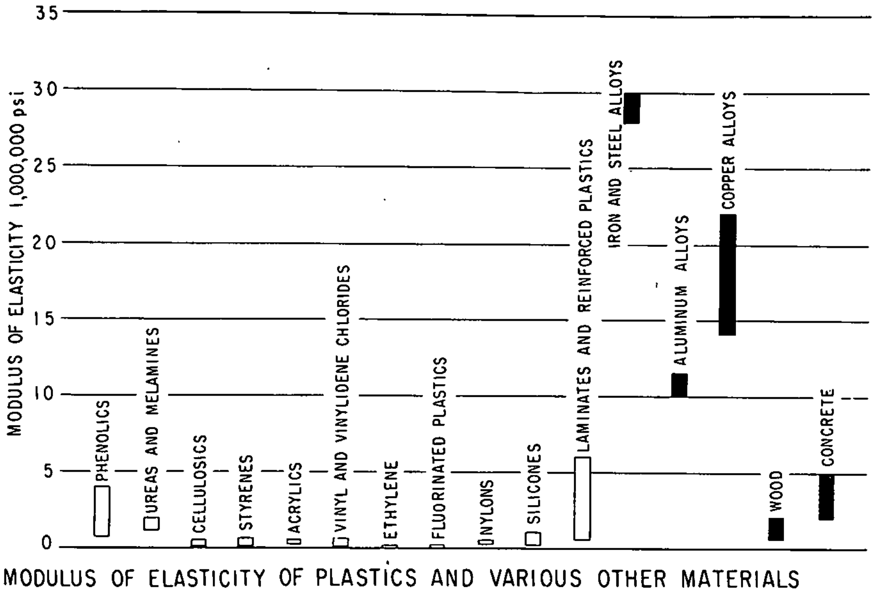


FIG. 4.—MODULUS OF ELASTICITY OR STIFFNESS OF THE PRINCIPAL PLASTICS COMPARED WITH FERROUS ALLOYS, ALUMINUM ALLOYS, COPPER ALLOYS, CONCRETE AND WOOD.

Laminates and Reinforced Plastics

Where large sheets or large molded objects or high strength are required, the answer is usually found in laminates and reinforced plastics. These are materials in which plastics have been combined with sheet or fibrous materials to produce a composite which has properties unavailable in either constituent alone.

The term "laminate" usually refers in the trade to a sheet material which has been made under high pressure and high temperature, and the term "high-pressure laminate" is often used to differentiate this class of materials from reinforced plastics, formerly called "low-pressure laminates." Paper, fabric, wood and other sheet materials are combined with thermosetting resins, usually phenolic or melamine or both. The usual procedure is to impregnate the sheet stock, dry, assemble the dried sheets, and press under temperatures ranging up to 350°F and pressures ranging from 1000 to 3000 psi. The building field uses large quantities of these materials for table tops, counter fronts, furniture in general, and for wall covering and other applications where their strength, toughness, resistance to marring, and resistance to water, alcohol and other commonly found solvents, make them particularly useful because maintenance costs are low, and refinishing costs are largely eliminated.

Reinforced plastics refer to plastics employing fibrous reinforcements, almost always some form of glass fiber, combined with a liquid resin which is converted to a solid by catalysts and hardeners with or without heat or a moderate degree of pressure. Because the heat and pressure requirements are moderate, it is possible to make large compound-curved parts on relatively simple molds as contrasted with the tool steel molds and heavy presses required for ordinary molding and for high-pressure laminates. Familiar examples of reinforced plastics are auto bodies, the corrugated sheet beginning to be used in large quantities in buildings, boat hulls, and a large variety of other similar applications.

The fibers in reinforced plastics are in the form of mats or of woven fabrics. Many types of weaves, including plain cross weave, satin or crow's-foot weaves, and unidirectional weaves in which practically all of the filaments are aligned in one direction, are employed in reinforced plastics. The same fabrics, of course, can be and are used with high-pressure laminates.

The engineering design of laminates and reinforced plastics introduces some interesting and complex problems faintly similar to the problems of reinforced concrete design. In the design of metal structures and of plain concrete, it is assumed that the materials are homogeneous and have the same properties in all directions, that is, they are isotropic. When plastics are combined with fabrics, especially unidirectional fabrics, the strength properties are strongly influenced by the directional properties of the fabrics, and such familiar constants as modulus of elasticity and Poisson's ratio become variables depending on direction. Strength, of course, is also greatly influenced by direction. The same plate may easily vary ten to twentyfold in these properties, depending on the direction chosen with respect to the weave of the fabric. Surprising and complex interrelationships of stress occur within and between the layers of the laminate. The engineering designer must be wary and alert. Figure 5 shows how some of these properties may vary with respect to the direction of weave.

The extremes of strength properties shown in Figures 1 and 4 are reflections of these directional effects. At the low end, the properties are largely those of the resin alone; at the high end are found rods

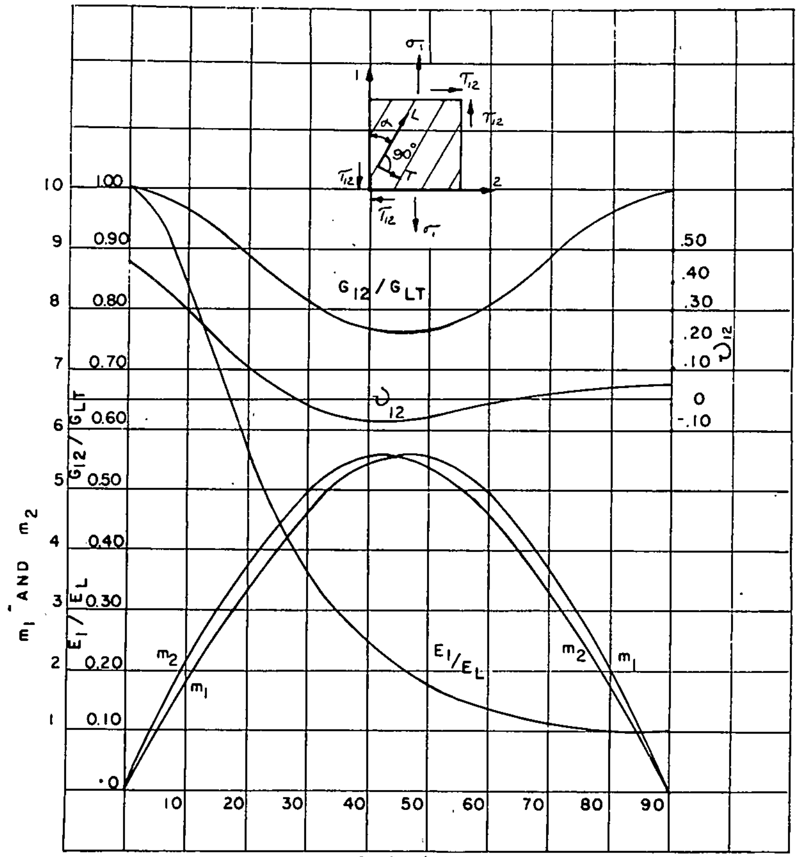


FIG. 5. EFFECT OF DIRECTION OF WEAVE UPON ELASTIC CONSTANTS OF REINFORCED PLASTICS AND LAMINATES. DIRECTION PARALLEL TO FIBER IS 0°.

containing a very high proportion of glass filaments all carefully aligned in one direction.

The variety of reinforcing materials available, and the freedom with which these may be arranged to meet the loads imposed, gives the engineering designer wide latitude in tailoring his materials to meet the requirements. Furthermore, engineering and architectural designers have practically unlimited freedom in selecting shapes and forms to enclose their spaces and in choosing their structural elements in the most advantageous manner.

Sandwich Constructions

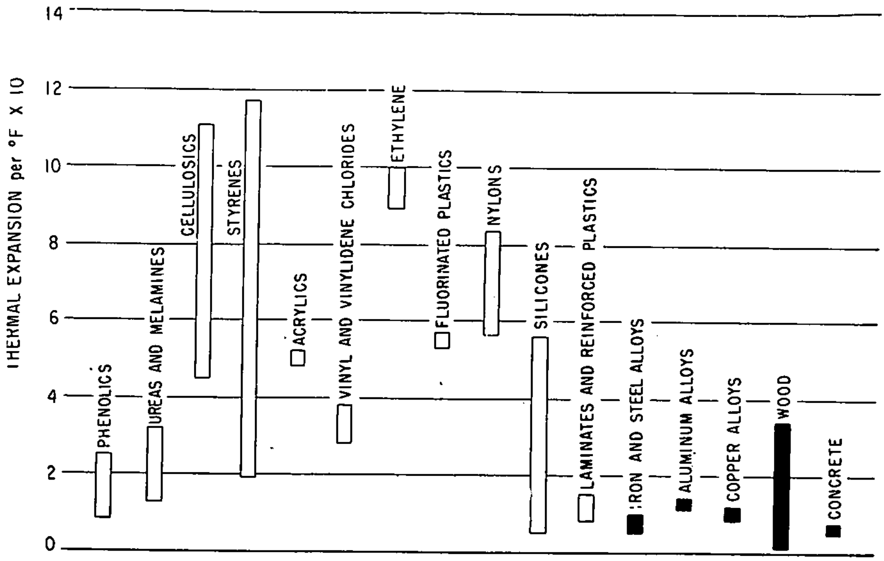
In sandwich constructions two thin, hard, strong facings are combined with a relatively thick, lightweight, weak core to provide combined lightness, strength, and rigidity. The facings provide the strong elements but because of their thinness would buckle under bending and compressive stresses unless supported laterally by the core which must be stiff enough to provide this support. The core must also resist the shear stresses occurring in the sandwich as it bends. The adhesive bond between facings and core must resist shear and tensile stresses between these components.

Many combinations of materials can be employed in sandwiches. The building industry already uses sandwiches composed of insulating board cores and cement-asbestos facings, metal facings and lightweight concrete cores, and various others. Plastics enter into this picture to provide structural and decorative facings of high-pressure laminates or of reinforced plastics, cores of foamed plastics or of cellular grids such as honeycombs of resin-impregnated paper, and the high-strength engineering adhesives to bond all sorts of facings and cores together.

Thermal Expansion

In building design it is frequently necessary to take into account the thermal expansion and contraction of materials. This is particularly true of such items as metal roofs, gutters, and pipe lines. The large metal window frames in glass-faced buildings present a particular problem. Among traditional materials, aluminum and copper and their alloys have higher coefficients than the iron-based alloys. Wood has a low coefficient parallel to the grain but quite high perpendicular to the grain, especially in dense woods like yellow pine and birch. Swelling and shrinking because of moisture changes usually mask out thermal expansion perpendicular to the grain. Concrete also comes and goes because of both thermal and hygroscopic effects.

It may come as a surprise to many that the thermal coefficients of expansion of plastics are high, as shown in Figure 6. There is a widespread notion that plastics are stable materials thermally, when actually they expand and contract considerably more than metals. In the design of plastics parts, especially if they are to be used in conjunction with other materials, this feature must be kept in mind. It is not insurmountable, but it is something to allow for in design.



THERMAL EXPANSION OF PLASTICS AND VARIOUS OTHER MATERIALS.

FIG. 6.—THERMAL EXPANSION OF PLASTICS COMPARED WITH FERROUS ALLOYS, ALUMINUM ALLOYS, COPPER ALLOYS, CONCRETE AND WOOD.

Thermal Conductivity

Like most other nonmetallic materials, the thermal conductivity of plastics is low, although it varies from plastic to plastic as shown in Figure 7, in which plastics are compared among themselves and with wood and concrete at the left-hand side of the figure, and compared as a group with iron, copper, and aluminum alloys at the right. Few of the plastics approach the heat insulating value of wood perpendicular to the grain (lowest value of wood) but all are vastly better insulators than any of the metals.

Durability

Corrosion. Resistance to corrosion and to attack by a wide variety of solvents is one of the most attractive features of many of the plastics. At least one plastic material can be found to resist practically any corrosive condition to be encountered in building. The fluorinated plastics are particularly outstanding and are used in the highly corrosive chemical industries. Polyethylene is highly resistant

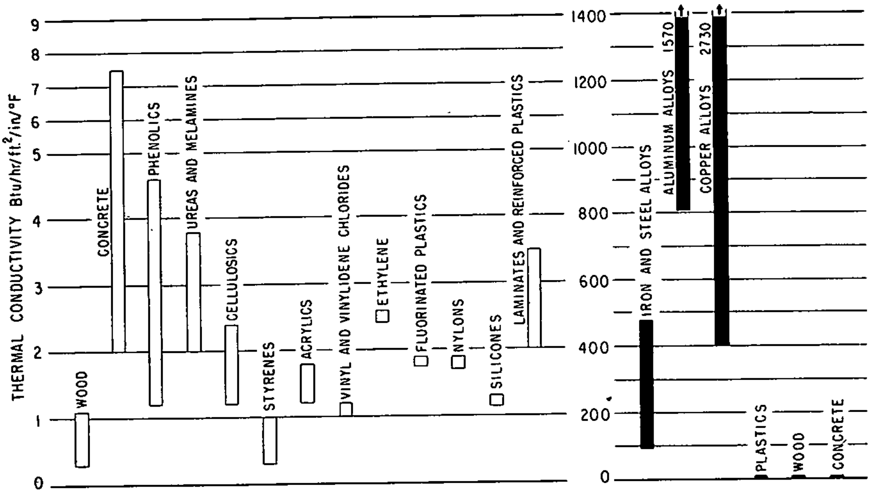


FIG. 7.—THERMAL CONDUCTIVITY OF PLASTICS COMPARED WITH OTHER MATERIALS.

to attack by a wide variety of corrosive materials at temperatures below its softening point. The silicones are inert and, like the fluorinated plastics, highly moisture-repellent. Moisture absorption by other plastics varies from practically zero to moderate. Other plastics are selectively attacked by classes of solvents, so that the choice for any given condition should take this selectivity into account. For corrosive conditions normally encountered in buildings, the plastics as a whole are excellent.

Heat. All plastics can be destroyed by fire. They may burn easily or they may be self-extinguishing, depending on the basic resin and the fillers and plasticizers compounded with them. Building designers recognize that resistance to fire is to a large extent a matter of proper design as well as the proper selection of materials. Buildings constructed of incombustible materials have been disastrous losses in fires, whereas other buildings of combustible materials have stood up well. Nevertheless, it must be kept in mind that all plastics can be destroyed by fire, and that some of them burn easily of their own accord once they are ignited.

Maximum temperatures to which plastic should be exposed are not easy to establish because use conditions are extremely variable. Maximum recommended temperatures for more or less continuous exposure are given in Figure 8, in which plastics are compared with

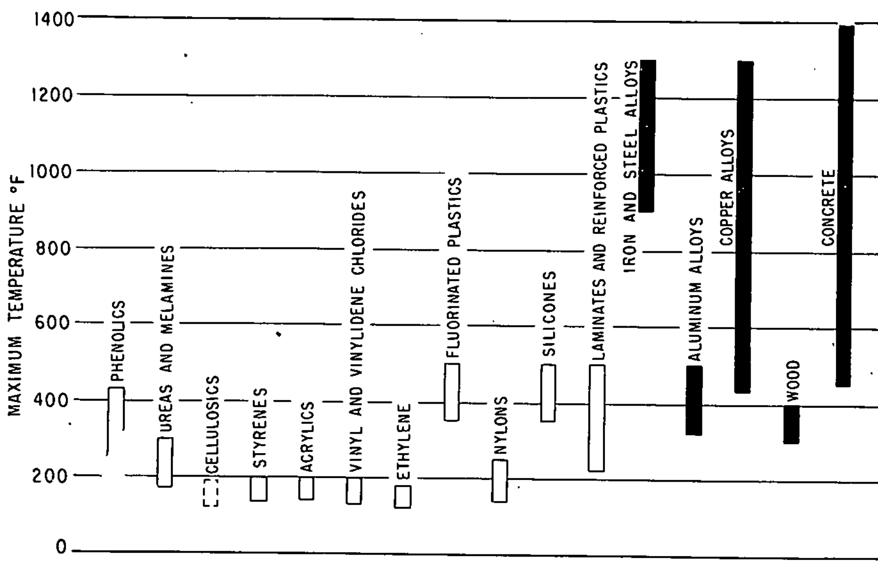


FIG. 8.—APPROXIMATE TEMPERATURE LIMITS FOR PLASTICS COMPARED WITH OTHER MATERIALS.

other building materials. The temperatures given for metals and concrete are the temperatures at which strength drops to approximately 50 percent. The wood temperatures are based more on the chance that charring will occur than on loss of strength because wood does not become weaker upon heating. Concrete temperatures are uncertain because when concrete is used as fireproofing for steel its insulating value rather than its strength is important. On the other hand, some lightweight concrete load-bearing structures may easily lose more than half their strength at fairly low temperatures.

Among the plastics the silicones and the fluorinated plastics are outstanding in temperature resistance. Laminates may be highly resistant if the basic resin is resistant and the reinforcement is an inorganic material like glass or asbestos. Other plastics, especially the thermoplastics, are sensitive to heat and the recommended temperatures are based upon creep and distortion rather than ignition temperatures. Manufacturers of the cellulosics, because of the range of basic types and of plasticizer contents, hesitate to recommend specific maximum temperatures. Thermosetting plastics, as is to be expected, are generally superior to the thermoplastics.

Irradiation of some plastics has been shown to raise their heat distortion temperature markedly. Polyethylene bottles, for example, which collapse or distort badly at 150-175°F retain their shape at temperatures between 300 to 400°F after irradiation. A cross-linking action occurs binding the molecules together in a structure similar to thermosetting plastics and thereby raising their resistance to distortion by heat.

Weathering. Resistance to weathering over a long period of time is one of the most uncertain aspects of plastics. Whereas there is reasonably good certainty that indoor exposures should have relatively little effect on most plastics, outdoor conditions may affect many of them adversely. Many of the large-volume plastics in use today were emerging from the laboratory ten to twenty years ago, and changes in formulations are being made continually. For many plastics, therefore, there is no record of outdoor exposure extending over long periods of time.

A few transparent plastics, of which the acrylics are the best example, have been used for aircraft glazing and other outdoor applications for periods ranging up to twenty years. Their performance is therefore relatively well known and they can be expected to give a good account of themselves in buildings. The phenolics have also had a fairly long history of outdoor exposure. Loss of gloss and dulling of colors can be expected to occur upon weathering, but properly formulated phenolics have otherwise given a good account of themselves outdoors for periods ranging to twenty years or more. Long histories are lacking for most other plastics, but the chemical nature of the fluorinated plastics and the silicones indicates that they should have outstanding long-time weather resistance. High cost stands in the way of large-scale use of these materials. Certain of the vinyl and vinylidene compounds, especially when formulated with carbon black and other utraviolet light-excluding pigments, show promise on the basis of exposure histories ranging up to ten years.

The lack of completely reliable accelerated weathering tests, which are difficult to set up because of the complexities and vagaries of actual weather, is a serious obstacle to the adoption of promising new plastics as they appear. In building, the customary units of time are twenty to fifty years (although it may be argued that practically all buildings are obsolete before that time) and building designers hesitate to use materials which have not proven themselves in actual use. This attitude, while perhaps a deterrent a rapid progress, is understandable, and plastics manufacturers must recognize that it exists.

Building Trends

Several major trends in building design and construction are bound to have a decided influence upon the possible uses of plastics in building. Perhaps the most important is the trend toward shop-fabricated units assembled quickly in the field to reduce expensive field time and field labor. Large shop-fabricated building panels, particularly for walls but also for floors and roofs, are becoming common not only in dwelling houses but in industrial and commercial buildings. The stressed-skin principle is already in use, especially in housing, and the application of load-carrying sandwiches is beginning to grow in this field after the pioneering efforts of the aircraft engineers. A major problem still calling for completely satisfactory solution, is the caulking and sealing of the joints between these panels. The highly durable, though expensive, plastics might help to solve this problem.

Mechanical and electrical equipment, and built-in features of all kind, are being shop-fabricated to a greater and greater extent to reduce field labor and erection time. Portability, including minimum weight and resistance to damage in transit, becomes important. The lightness and flexibility of design of plastics, coupled with good strength and impact resistance, can make them attractive for all of these applications.

Figure 9 shows an aluminum exterior wall panel being put into position on a multi-story office building. In this particular instance the panel was backed with cast-in-place, light insulating material and then was plastered on the inside. A sandwich with built-in insulation and finished on the inside with a material like decorative laminate should be feasible here. The sealing of the joint becomes a major item, as indicated previously.

Figure 10 shows a small, completely shop-built house with wall, floor, and roof panels of waterproof plywood facings bonded to a honeycomb core of resin-impregnated paper. The house can be folded, transported on a trailer, and erected on a grid of foundation beams in a matter of hours.

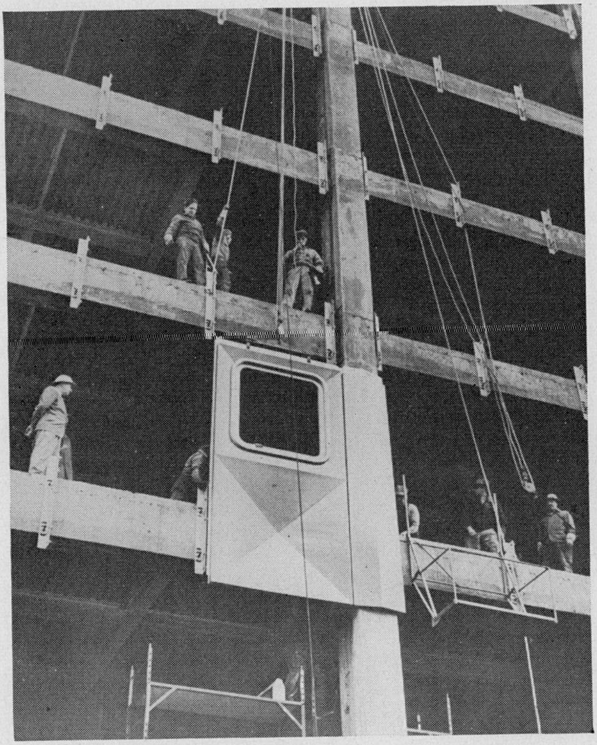


FIG. 9.—ALUMINUM EXTERIOR WALL PANEL BEING ERECTED.

A second major trend is toward large clear spans to avoid internal supports. This provides freedom for industrial operations and allows for easy rearrangement of office space. Even in dwellings there is a trend toward roof construction spanning from outside wall to outside wall to eliminate bearing partitions and to allow for free arrangement of space. Flat slabs, cantilever construction, and long spans are consequently becoming common in buildings. There is also a trend toward the highly efficient structural shapes such as shells, domes, arches and vaults for the spanning of very large spaces. Space frames also are efficient for the enclosing of large spaces. Here it is entirely possible for plastics, although they may not be used for the primary load-bearing members such as arches, to be fitted into the structure in the form of secondary load and light-transmitting units. The readiness

FIG. 10.—SHOP-FABRICATED HOUSE EMPLOYING SANDWICH PANEL WALLS, ROOF, AND FLOOR.

with which many plastics can be formed into complex shapes recommends them for these purposes in competition with other materials not so readily formed.

The ready formability of materials like reinforced plastics recommends them for the making of complex forms for grid and cell construction of reinforced concrete. Such forms have already been used for precast concrete slabs, and this use could readily be extended, particularly where complex shapes are required and a large number of reuses of the forms is called for. A major problem exists, for example, in the forming of large-span, thin concrete domes. A strong deterrent to the greater use of these efficient structural shapes is the cost of forming. The falsework ribbing and sheathing required for the forms of a dome such as this can easily constitute half the total cost of the dome. All of this must be erected and then taken down, much of it as scrap. It is conceivable that segmental forms could be built and rotated about a central axis as the dome is cast, thereby reducing the amount of forming material required. Possibly plastics could provide the tough curved surfaces of such forms.

Figure 11 illustrates some of the possibilities in reinforced concrete grid construction. This Italian example by Pier Luigi Nervi shows imaginative engineering in the arrangement of ribs and cells for the most efficient handling of stresses in vaulted roofs. The cost

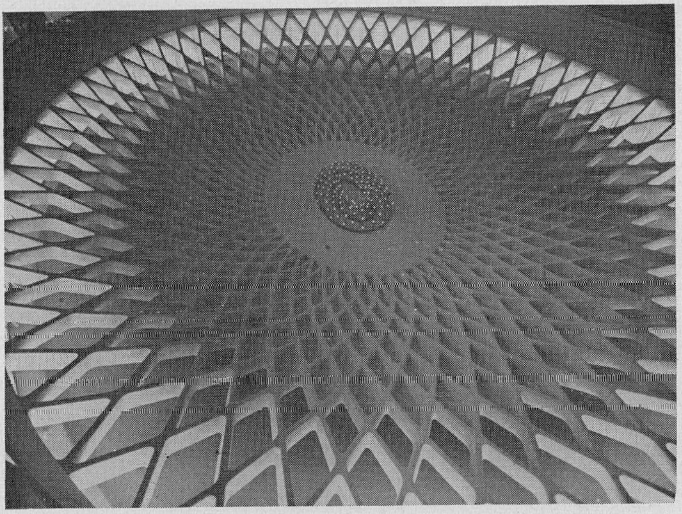


FIG. 11.—RIBBED CONCRETE DOMED CEILING.

of forming by usual methods would be considered prohibitive in the United States. Here, again, reinforced plastics or similar materials might be useful, provided there were enough reuses of the forms. Experience has shown that such forms can be reused in large number of times.

A third major trend, coupled with open plans and minimum interior obstructions, is the greatly increased use of transparent and translucent walls to admit maximum sunlight and to provide a feeling of spaciousness. Transparent and translucent panels, particularly if they can combine light transmission with structural qualities, are highly desirable for such buildings. Sandwich panels having a combination of structural attributes and light transmission are being developed for these purposes. Such panels can carry at least part of the superimposed floor and roof loads and they are sufficiently strong to withstand wind pressures. In this connection the ready formability of plastics should allow for the development of structurally efficient shapes providing light-transmittance and, at the same time, able to provide the necessary shading to prevent undue overheating of interiors by bright sunlight. The solar load can become a major problem in buildings with light-transmitting walls. Outward heat loss is another problem for which double-walled units could help to provide a solution.

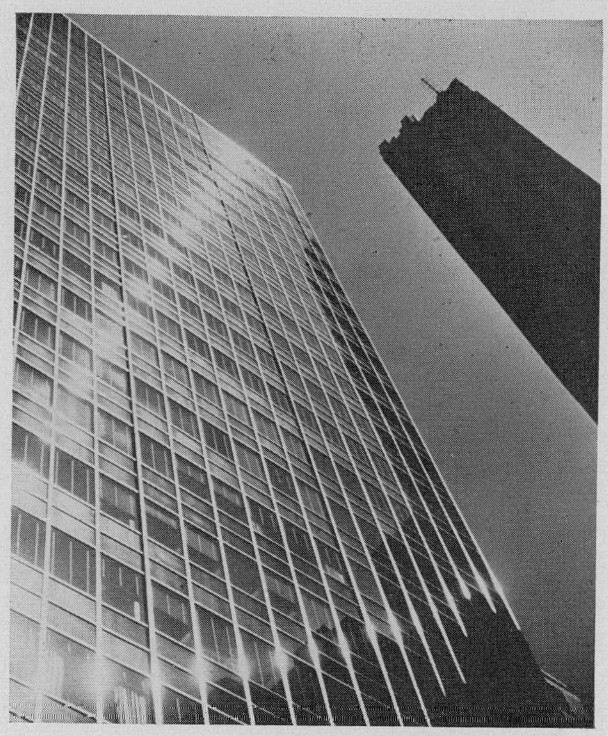


FIG. 12.—MULTI STORY BUILDING WITH GLASS AND METAL EXTERIOR SKIN

Figure 12 shows a recently-built glass-walled building, typical of the trend toward open walls. Heat-absorbing glass, blinds, and various other devices are used to lessen the solar load when walls are exposed to the direct sun. Nevertheless, the heat load imposed can become severe. The readiness with which transparent and translucent plastics can be formed suggests that by properly shaping the units in an exterior wall much of the load could be reduced by high-angle reflection and shading.

A fourth major trend is the increasing importance of mechanical and electrical equipment for such purposes as air conditioning, illumination, power, and the multiplicity of services required in a modern building. This is coupled with the necessity for flexibility to accompany the flexible planning associated with large open areas. When space is rearranged the mechanical and electrical services must also be rearranged. Flexible piping, conduit, cable, and ducts are required. Continuous electrical outlets are desirable. Luminous ceilings are often called for. The open planning and lightness of construction involved pose problems of thermal insulation and particularly of acoustical control, which must often be combined with overall illumination in ceilings. To all of these problems plastics can make a decided contribution if they are used with imagination and understanding of their possibilities and limitations.

Radar Enclosure

A large radar enclosure recently built of reinforced plastic may serve in a small way to illustrate an application in which the formability and properties of this material were used to meet the requirements of the problem. (Figure 13). The rotating motion and the general shape of the radar made a spherical dome-shaped enclosure the logical choice. This shape is efficient structurally and aerodynamically considering the fact that it must withstand winds from any direction. Its exposed position atop a building in a large open area made wind loading the most important structural consideration. Electrical requirements ruled out any metal in the enclosure.

From an analysis of wind loading based on the best available wind-tunnel data, it became apparent that reinforced plastics could readily withstand the imposed stresses. The surface was divided into triangular segments to approximate the shape of a sphere, and molded reinforced plastic units, each incorporating several triangular segments, were fabricated. At the edges of the units downstanding stiffening lips were formed. Reinforcing ribs were also formed along the lines of the basic triangles. Glass fiber mat was employed for the surfaces, and additional strength was achieved at the lips and ridges with woven glass fabric strips molded into these thickened sections.

The units were field assembled without undue difficulty, and a high degree of transluscence was achieved together with structural strength. Here, therefore, was a combination of enclosure, structure, and light transmission in which plastics played the essential role. Incidentally, the dome withstood the 1954 hurricanes without damage. It is now being moved to a still more exposed location.

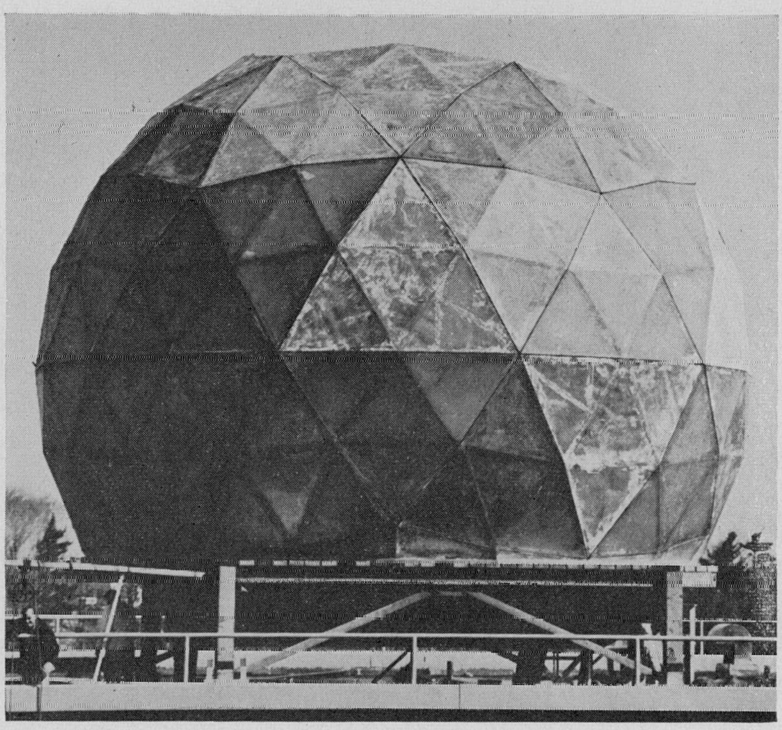


Fig. 13.—Dome-Shaped Radar Enclosure of Reinforced Plastics.

Summary

This paper has attempted to set forth the ranges of strength, stiffness, thermal properties, corrosion resistance, and durability of plastics in comparison with the familiar metals, wood, and concrete used in building. It has briefly touched upon the engineering aspects of laminates, reinforced plastics and sandwiches. It has attempted to show the major trends occurring in building design and to indicate how plastics fit into these trends. Many of these items, briefly mentioned, will be amplified in other papers.

One conclusion should be clear. Plastics possess their own combinations of advantages and limitations which set them off from other materials. Attempts to use them like other materials are likely to produce unhappy results and are certain to fail to use them most advantageously. Properly and imaginatively used, with an intelligent understanding of their properties, they can contribute significantly to the advance of building technology and design.

THE DEVELOPMENT OF STRUCTURAL FLIGHT TESTING

BY VICE ADMIRAL FREDERICK M. TRAPNELL,* U.S.N.(Ret.)

(Presented at a joint meeting of the Boston Society of Civil Engineers and the Northeastern Section A.S.C.E., held on October 27, 1954.)

Flight Testing has been going on ever since there were airplanes. Formal structural flight testing, however, dates only from the time when the Military Services began procurement of planes on contract for specialized purposes, about 40 years ago. I can take you back only about 30 years, at which time the quaint and picturesque had given way to the scientific approach and to airplanes that were soundly engineered. In retrospect, the test procedures seem simple and primitive; but they were rugged.

The contract with the airplane builder required the construction of the machine to specifications and the demonstration of overall strength in flight by those maneuvers which produce design limit stresses. They were mainly high diving speeds and high wing loads during pull-out in the various conditions of loading under which it was to be used in service.

The test pilot was usually a character of some definite kind. They varied all the way from the hell-for-leather type uninhibited by any semblance of imagination, to the shrewdest and cagiest, who knew what he was doing every second and needed the dough. But most were very competent young men, devoted to their flying and to the responsibility of flight testing and endowed with outstanding courage and self possession. A number of well known pilots specialized in this work and some are still alive. His job was to make the required number of dives in various loading conditions pulling out at the required wing load each time. His equipment included an accelerometer, which was essential to the test and a parachute.

In the case of the Navy; the demonstration was observed by the Flight Test Section, then based in Washington, which subsequently was to take over the new plane for complete tests. In the case of fighters and dive bombers the plane was climbed to its service ceiling and from there dived vertically for 10 or 15 thousand feet. A true vertical dive was required and the pull out was to be at not less than

*Consultant, Grumman Aircraft Engineering Corp.

the design acceleration, usually 7g and naturally not much over it. During this dive a Navy pilot followed down, in some proven aircraft, and if he reported the dive as not truly straight down it had to be repeated. The dive had to be repeated if the acceleration attained as read off the gage on the pilot's instrument board from the maximum-reading pointer after landing did not show the required reading. If any sort of partial or incipient failure occurred the matter had to be corrected and that particular dive repeated.

These procedures depended to a great extent on individual judgment and left much room for wrangling, controversy and even bitterness. Some structural demonstrations which might have been completed in 5 or 6 dives took 10 or 15 due to repeated minor structural failures or to deficiencies in pilot technique. Through the years there were many major failures some in the high speed part of the test and some in the wing load part.

There were also some remarkable "Overshoots" which demonstrated wing strength far beyond that expected. In the early Curtiss "Hawk," which was a wonderful airplane for its day, old Luke Christopher pulled 12.5g acceleration without killing himself. The airplane was stretched, bent and sprung considerably out of shape, but could be landed; and Luke had to go to the hospital and have his internals put back where they belonged.

Most demonstrations were conducted in a very satisfactory manner and with a high degree of precision. Occasionally the pilot added a flourish which was good for the builder's public relations because there was always a large number of ground observers. I remember following Jim McAvoy down in the first real dive bomber built for the Navy. He was carrying 1000 lb. bomb, a very large item in those days; and I half expected to watch the airplane go to pieces before he reached terminal velocity or certainly during pullout. But Jim did not hedge in the least on the straightness of his dive or on the acceleration during pullout; furthermore he continued his pullout on into a complete loop with the bomb on board, and the spectators thought they had seen everything. So did I.

Years ago Harvey Ogden brought to us for a test a Curtiss "Hawk" which was fitted with a new and much larger engine than we had had previously. After completing all of his dives in a very satisfactory manner, I asked him, in the idle conversation, whether he thought this engine and propeller would withstand the overspeeds that would occur at wide-open throttle in terminal dives. Harvey's answer was "Sure, I'll show you!" I did not like this because we were not at the moment concerned specifically with engines and propellers. But he insisted, and went up and did a terminal velocity dive with full power on, pulling out over the field where it was apparent that extreme over-speed of engine and propeller had occurred, without damage.

From the very beginning, ground static load tests have been employed to prove the structural strength of all parts of the aircraft, before any limit loads are ever applied in flight. The trouble with such static tests is that in spite of all research, the precise distribution of air loads cannot be known or duplicated exactly as it varies widely under surface deflections. Such tests are absolutely essential, but they cannot be relied upon for the final proof. They used to be done by mounting the airplane, inverted, in the shop and loading the wings with the appropriate number of sand bags to duplicate the air loads.

Now the procedure is considerably more elaborate and precise. A large number of hydraulic jacks, working at carefully regulated pressures, are employed to duplicate air loads so far as they can be calculated. These tests are ultimately carried to destruction of the various parts of the static test airplane.

Aircraft performance and the associated structural strength problems have progressed, like everything else, only perhaps more so. The objectives of structural flight testing remain the same: basically that the designed strength of every part of the airplane must be tested and proved, under all design conditions, before production of the model is justified. Every possible precaution and device must be employed to reduce the possibility of losing the prototype machine or the pilot's life. Even if we grant the pilot a reasonable chance to escape from a disastrous failure, the loss of a typical prototype airplane is a calamity which can be expressed only in many millions of dollars and many hundreds of thousands of man-hours of engineering and shop work. It is a calamity also to the effectiveness of our national defense.

Since World War II it has not been possible to go through the orderly procedure of designing and building a prototype airplane, testing it and making changes, and only then, with complete assurance of its characteristics, setting up production on a large scale. If that were done, service planes would be hopelessly out of date. Complexity has so extended the time required for design and construction, while progress has continued to accelerate, that production must be begun before the prototype is in the air. If this were not done we'd be out of the race. It is a calculated risk, justified by the capabilities of present day engineering and all the research behind it. But it tends more than ever, to place a critical value on the prototype airplane and its flight tests.

In regard to the question of time scales for production of new aircraft, Grumman has recently accomplished a near-miracle, in putting a new prototype airplane in the air within 15 months of its inception. Twice this length of time is much more usual.

Structural flight testing has progressed with the performance of airplanes in general and, of course, has been forced into more complex patterns as the structure grew complex. If I seem to be obsessed with "complexity," I feel it is justified. Nothing could be more misleading than the simple, slick-looking exterior of a modern jet fighter. Every single cubic inch of its guts is packed with machinery, instruments, fuel, or flames. Each and every visible component of its structure is a profound aerodynamic problem in airflow, lift, drag, interference, stability, and load distribution. The interior of each is an equally involved engineering problem in strength, elasticity, vibration and flutter characteristics, which must be solved at the minimum possible weight, and every aspect of these problems is interrelated with almost every other. There are nearly enough trained people in the country to build these things, but to an ex-Naval officer it is not obvious who is going to continue to pay for them indefinitely.

We arrived at this streamlined complexity by generally steady progress but there were several discontinuities, sudden steps in progress which were reflected in structural flight test procedures. The first of these steps was the transition from externally braced airplanes to internally braced or cantilever monoplanes. The old externally braced airplanes, mostly biplanes, with the deep truss for sustaining wing loads, were very rigid. We never thought of flutter, or wing elasticity with its associated changes in load distribution. They were strong and light and dependable, but were not fast. In testing simple structures of that nature it was necessary only to measure speed (in dives) and acceleration during the pull out. The speed was limited by the high drag of the airplane and the acceleration was controlled by the pilot.

The terms "acceleration" and "wing load" are synonymous except that the former includes all parts of the airplane and is normal to flight path. In basic physics it is contripetal acceleration associated with curvature of the flight path, and is developed by increasing the wing lift by some factor greater than the 1g required by gravity for straight level flight. It appears in horizontal turns of course but is most clearly visualized in recovery from dives. What we are concerned with is the increased loads throughout the aircraft but principally in wings, tail, and control surfaces.

In the early days about all that was done was to develop this load to the design maximum at the highest attainable dive speed for fighters and the highest allowable for others. The pilot watched his visual accelerometer and had to reach or exceed the required figure but for safety's sake he did not go much above it. This accelerometer left its maximum reading on the dial until reset after the flight. That was checked and the entire aircraft was carefully inspected for wrinkles, pulled rivets, or any other indication of strain before proceeding with the next dive. On the whole, non-structural parts such as cowlings, canopies, fairings and access plates gave more troubles than the carefully engineered main structure. Naturally the pilot had practiced and gradually built up toward the limit speeds and accelerations before he was ready to make the formal proof test.

The next item that became available in the way of instruments was a V-G recorder which gave a continuous simultaneous reading of speed and acceleration. It was not visible to the pilot but could be read immediately after the flight to determine what limits had been reached.

We also developed a hand-made accelerometer which broadcast a radio signal when a certain load factor was reached. It had two elements which gave different tones. A ground observer could watch the dive and pullout with head phones on and check the accelerations reached in two steps, as they occurred and were broadcast as a low note and a high note. It had the rather gruesome advantage that, in case of a disastrous failure, there was some information available as to approximately what acceleration had caused it. Otherwise, there was usually none, although V-G records have survived crashes.

The development of high strength aluminum alloys permitted the construction of cantilever wings. So monoplanes immediately put the biplane out of business forever. The internally braced monoplane saved no weight but it jumped the speed and is universal now. With it came many new problems mostly in connection with the stiffness as distinct from strength. These problems are mainly vibration and flutter, and changes in load distribution due to structural deflection, or "aeroelasticity" as it is generally termed.

A flag is completely flexible and it flutters at all times. A cantilever wing may look stiff but in fact it is not and will flutter at a high enough speed. If a movable control surface is attached to its trailing edge (without proper balance) you have practically a flag.

I had a disagreeable experience with the first cantilever tail surface delivered to the Navy which was on a souped-up and cleaned-up service fighter. It was a beautiful looking job, strong enough statically, and gave fine control. But I had to jump out on the third dive because the elevators, which were improperly balanced-dynamically, fluttered themselves to ribbons and nearly shook the plane to pieces.

The second interruption to smooth progress was a great leap into the "wild blue yonder" (as the Air Force calls it) with the advent of jet propulsion.

The most obvious thing about a jet is that there is less drag—because there is no external cooling problem and the engine is small in diameter. But more important is the fact that we have vastly greater power available for high speed. The comparison may be shown in two sets of curves on Figure No. 1 which refers to a propeller driven

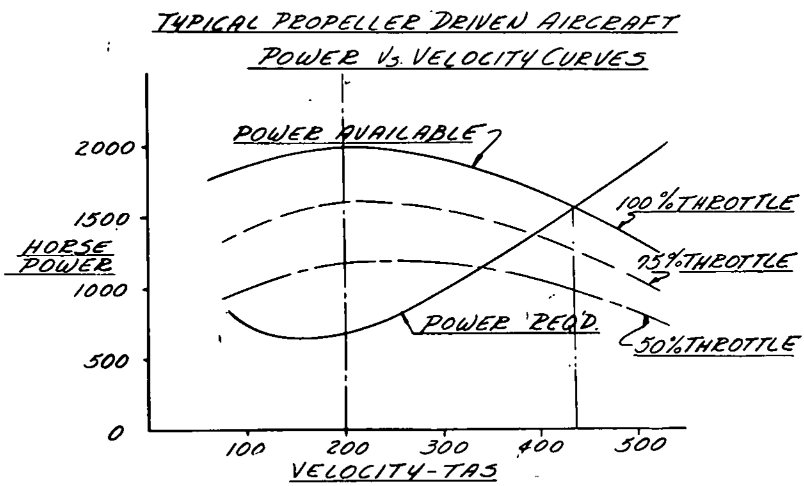


FIG. 1.

airplane. One curve shows the power actually available to pull the airplane in level flight after all losses have been deducted. In other words it's propeller thrust times speed. Propeller efficiency changes cause it to drop off at high speeds. Another curve shows the power required to move the airplane along at all speeds. It is the actual drag in pounds multiplied by the speed. Where these curves intersect is the actual top speed in level flight.

Figure No. 2 shows the same curves for a jet. The jet engine is essentially a constant thrust machine. The power available, being the product of thrust and speed, rises directly with speed to extraordinary

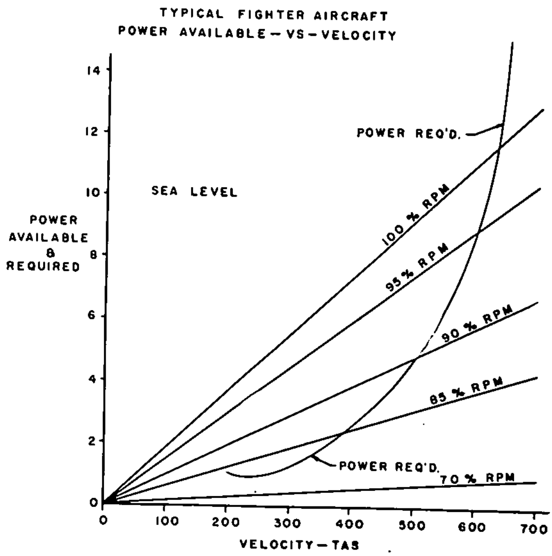


FIG. 2.

values compared to the constant power reciprocating engine. By simple arithmetic the horsepower output is equal to the thrust at 375 MPH. At 750 it is doubled. At the same time the overall drag of the airplane is lower and the required power less. Consequently the intersection, which determines maximum level speed, is higher.

The third great step in progress was the increase in speeds past the magic number of Mach 1, the speed of sound. This was made possible by the steady development of jet engines to greater and greater thrust and by a vast amount of research on methods of reducing transonic and supersonic drag.

Airflow which is entirely subsonic around an airplane follows a definite normal pattern and is subject to detailed analysis, based on all the old classical research.

Airflow which is entirely supersonic around an airplane is also consistent and fairly predictable, as a result of intensive recent research. But the drag is much higher.

Between these two conditions there is a wide range from about Mach .75 to about Mach 1.05 where there is a mixture of the two types of flow all over the airplane. This is the "transonic" range and it is full of confusion and somewhat unpredictable uncertainties. It is here that the much discussed "buffetting" and erratic control occurs. But the really major obstacle to getting through to supersonic flight has been the sharp drag rise through this transonic range. After resorting to every possible means of reducing drag it still requires a lot of brute thrust to push through into the supersonic range. Once in this range the drag rise with increasing speed is somewhat gentler and speeds will certainly increase more rapidly in the next few years.

All of our recent jet fighters can be flown into the transonic range and dived into the supersonic range with only mild disturbances. The new ones will fly supersonic in level flight. The thrust available is going up and the drag is coming down mainly by making the wings and tails thinner and thinner. It looks almost as if we were about to have solid metal wings.

The magic clue to supersonic flight is to thin the wings down. Figure No. 3 shows you what this means structurally. The upper section is an old fashioned high lift wing. Note the depth available for the cantilever spars which heretofore have been the means of carrying wing loads into the fuselage. The middle section is a World War II high speed wing designed for laminar flow, low drag, and reasonably high lift. Speeds with this wing went above 400 MPH. The lower section is what we are faced with in order to fly supersonically. The spar is fast disappearing. The upper and lower skins must serve; the top and bottom pieces for almost the entire wing are milled from single aluminum alloy billets.

The static strength of this sort of wing is not too much of a problem but its elasticity is. Extraordinary efforts must be made to estimate its vibration and flutter characteristics, its natural frequencies and modes, and to so distribute the masses and the stiffness that flutter cannot be excited at any speed attainable by the aircraft. Obviously the wing must be heavy; and from looking at it closely, feeling it, or jumping on it you'd get no impression of flexibility. Yet its flexibility, spanwise, chordwise, and in tortion, are all of vital importance in design and the subject of extraordinary efforts in study

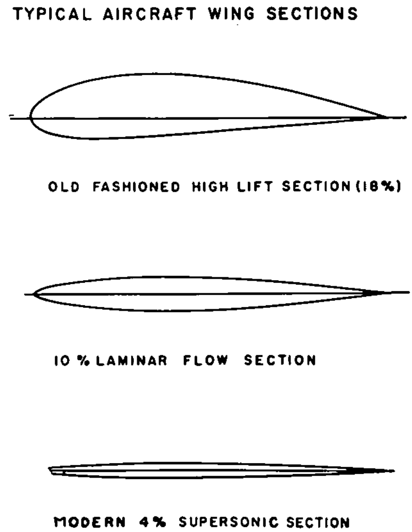


FIG. 3.

and in modern structural flight testing. These deflections produce variations in load distribution which may be favorable or may be highly unfavorable to ultimate wing strength. And they determine the speed at which the wing will flutter, in accordance with principles which are for beyond this discussion or my comprehension. The same considerations apply with equal force to tail surfaces. There is now in existence a rig for testing for flutter on the ground and is in constant use. It consists of a sled on which actual wings and tails may be mounted and run on a very long track. The sled is propelled by rockets and may reach speeds of almost 1000 mph.

Years ago it was decided that a maximum working load of 7g was the optimum upper limit. Greater strengths than this produced too heavy a structure; accelerations higher than 7g are unusable by the pilot, except for very brief intervals, because they block him out too severely. On the other hand experience has shown that if the flight load strength is reduced much below this figure there are too many failures in maneuvering airplanes. Although conditions are vastly different these days the 7g requirement, for working flight strength, still stands and is specified for military aircraft that must maneuver.

There are margins above this figure in the form of specified safety factors. The Navy specification, for instance, says that the yield point shall be not less than 115% of the working limit load (7g) and that the ultimate or failure point shall be 150% of the working limit. These margins are used sometimes inadvertently and sometimes deliberately in combat. In many cases airplanes have been "bent" without breaking. Strength in excess of 7g suggests that there is weight in the structure that might have been saved.

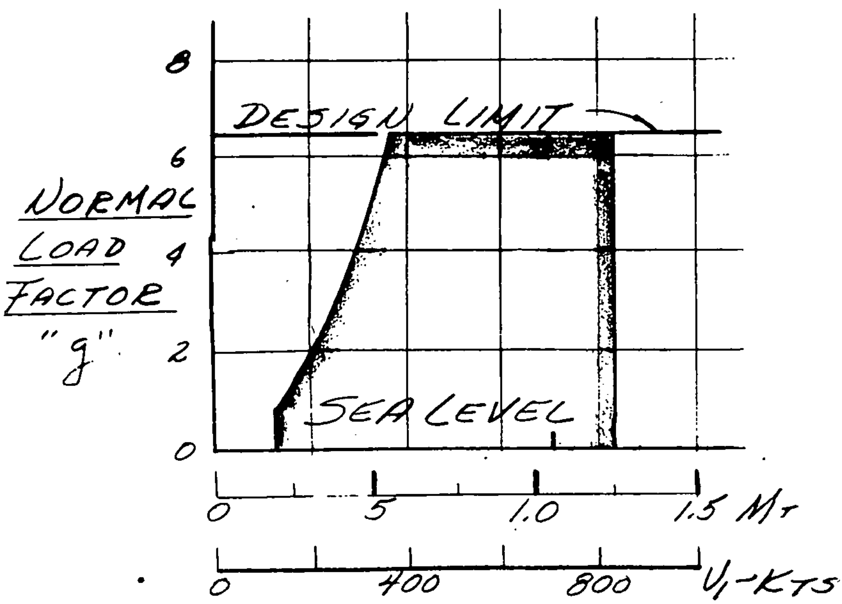


FIG. 4.—FLIGHT STRENGTH DIAGRAM; LOAD FACTOR VS. MACH NUMBER.

Figure No. 4 shows a typical flight strength diagram indicating the limits to which the airplane is designed and within which it must be tested. The coordinates are speed and acceleration. Above zero "g" the boundaries show the positive or upright working flight loads. Below zero, there are similar boundaries for negative or inverted—flight limits. The left boundary is the maximum attainable wing load at the low end of the speed range—and follows from the wing aerodynamic lift characteristics—that's the "stall line." The upper boundary is the 7 "g" limit load to which the aircraft was designed. The right hand boundary is the design limit speed, where dynamic pressure on such parts as windshields, canopies, tails, etc., may be critical. But of particular importance here is that this is the region of flutter problems, high dynamic pressures coupled with high velocities. These design limit airspeeds are attainable only in diving flight at high powers but they must be usable in combat planes. Structural flight tests must extend to these boundaries so that the limits of the entire area are explored. This is difficult to do in the matter of high speeds and some compromise must be made with safety. For instance, steep dives obviously cannot be continued below an altitude where recovery brings the flight path too close to ground level. They are made to low altitudes which allow a margin for errors in judgment but practically none for escape in case of a failure. Thus there is often some difficulty in combining the low altitude maximum design speed with maximum acceleration because of these physical limits to the air space available for such violent maneuvering.

This diagram is for low altitudes and most of the limits can be reached only at moderate or low altitudes. As the altitude is increased several things happen with reduced density and also with reduced temperature which increases Mach number in terms of actual speed. We have lower density and higher Mach numbers, both of which lower some of the attainable flight boundaries.

Structural problems practically disappear at extreme altitudes: available power is low; density and dynamic pressure are low; and high wing or flight loads cannot be developed. But any irregularity which shows up at high altitude will be vastly magnified at low altitude. For instance, a mild buffetting at say .95 Mach number at 40000 feet will certainly be severe at .95 Mach at 15000 feet and might well be destructive at .95 Mach at sea level. Perhaps it would be appropriate to point out that at .95 Mach number at 40000 feet the airspeed indicator would read only 295 knots and the true speed would be 550 knots. At sea level and .95 Mach No. the airspeed indicator would read 630 MPH and you'd actually be going 630 over the ground. These are basically density effects modified by effects of temperature on Mach number.

If you were concerned with getting from one place to another, the true airspeed is the criterion in the above comparison between two altitudes. If you are concerned with transonic irregularities in control or flight characteristics, the Mach number is the criterion. If you are concerned with the effect of such irregularities on the structure, or with flutter, or with the general airloads on the structure, the criterion is the airspeed meter reading, or indicated airspeed, as opposed to true airspeed.

For the above reasons, all flight tests in the high performance range, including structural flight tests, begin at high altitude. You "feel the plane out there first" partly for safety's sake and partly to insure that aerodynamic troubles don't immediately develop into structural troubles as they may well do at low altitudes. In general you can't proceed with tests to prove structural integrity until the aerodynamic and control characteristics have been completely tested and proved adequate. Otherwise it might not be possible to regulate the applied stresses with the precision which is essential.

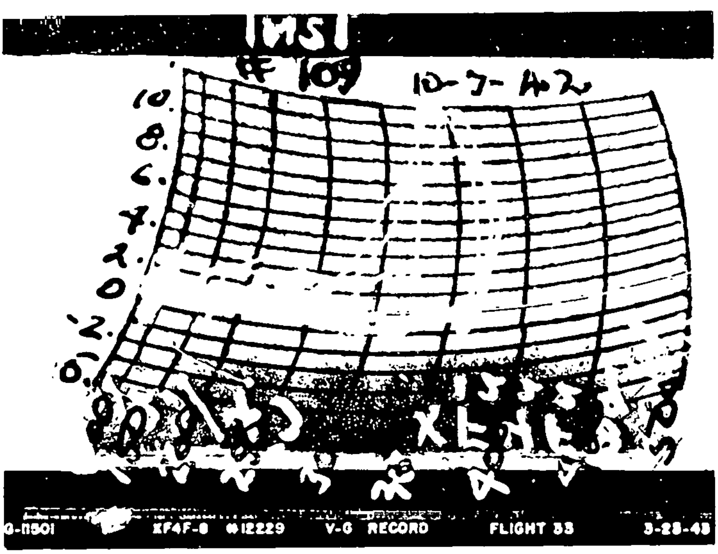


FIG. 5.—V-G RECORD SHOWING ACCELERATIONS BETWEEN 12 AND 13G. ACTUALLY ATTAINED.

Figure No. 5 is a remarkable V-G record. A V-G record is actually in the same form as the V-G diagram although at first the resemblance may not be apparent. In both, the coordinates are speed horizontally and acceleration vertically. There is no time element.

The trace merely shows that all these combinations of speed and acceleration occured during the flight.

In this case a young test pilot went out to practice and refine his technique for structural tests. He made one dive and tried to pull 6g but his cockpit accelerometer showed only 4 so he climbed back up and made another dive and pulled hard and he got only 4½ "g" on his gage. Then he climbed up up for a third dive and this time he really pulled her in hard but still he saw only 4½ "g" on the gage.

He came back dejected and allowed as how maybe he was "no good." Then someone noticed that the wings of the plane were in rather a bad shape, wrinkles and pulled rivets, so they took the V-G record out and this is what it revealed. His visual accelerometer had stuck at 4½. On the first dive he got 9.8 "g"; on the second he got 12 "g" and on the third he got 12.9 "g." I believe this is the highest ever attained without disaster and C. Alber is lucky to be alive, because the latter points were above design ultimate. The plane was a standard Navy fighter of World War II vintage.

Flight tests of an airplane are divided into four major phases: performance tests, stability and control tests, structural tests, and general suitability tests. The most important of the latter, in the case of Navy planes, are carrier suitability tests involving catapulting and arrested landings. The test program is very carefully planned to collect the most information with the least hazard and to get the maximum amount of other information before risking the plane in proof of its strength under limit loads. The program is executed by the aircraft manufacturer as part of his guaranteed contract performance.

Extensive instrumentation has become the basis of all modern structural flight test procedures. The first thing that is done, at the very beginning of construction, is to plan the instrumentation, and provide for it, as fabrication of the prototype proceeds. The extent and complexity of the instrument installation is formidable to say the least.

Strain gages are installed throughout the structure at such points as will measure compression, and tension (or bending) and shear in the main elements or in any fitting where theoretical analysis is difficult or inexact. These are wired, to an oscillograph of 36 channels. The oscillograph produces a record of the strain gage readings during maneuvers. The installation is calibrated on the ground by direct application of known forces to the structural members involved.

In the frequent cases, where vibration, flutter or severe buffet may require even more detailed investigation, strain gages and small high frequency accelerometers are installed at critical points and connected to an oscillograph. These latter instruments record frequency and the local acceleration due to the oscillation from which amplitude may be computed. Integrating accelerometers, which indicate amplitude directly, may also be used.

The most recent instrumentation job at Grumman provides an electrically driven "shaker" which vibrates the entire aircraft structure in order to excite any incipient oscillation or flutter. Its speed of operation is varied automatically in order that it may sweep through a range of frequencies covering the natural frequencies of the various parts of the structure. It is of course under control of the pilot. This installation further provides the unique feature of continuous radio telemetering of all strain gage readings, through the oscillograph, to a ground station. Here a group of engineers monitor the strain gage readings as they occur. These appear in the form of continuous traces which will reveal normal and excessive stresses or the oscillating stresses associated with flutter. The pilot is instantly warned of any unusual condition and will, of course, stop the shaker and discontinue the maneuver which produced the condition.

In addition to the oscillograph system, the test plane is equipped with a photopanel in which gage readings show the exact position of each control surface, control forces, vertical and transverse acceleration, the direction of the flight path in terms of angle of attack and yaw, the altitude and airspeed, and stop watch time. A motion picture camera, under the pilot's control, produces a continuous record of these readings during all significant maneuvers. Subsequent to the flight the film is laboriously read and interpreted to produce a plotted time history of each maneuver and everything that occurred therein.

In no other way can the flight characteristics be translated into useable engineering data. In the old days, before strain gages, photopanels, and oscillographs, the pilot used to have some of this instrumentation in the cockpit, and he did his best to read it and write the results on a knee pad. The pilot's job is far more exacting now but it also requires half a dozen instrument technicians and a couple dozen engineers to keep up with him.

The Navy has always had a special problem for structural flight testing in proving that its new designs were adequately strong for the rather rough procedures of carrier operations. Here we are concerned, for the first time, with horizontal accelerations applied by the catapult and the arresting gear, and with vertical acceleration too but in this case applied through the landing gear instead of through the wings. The same sort of measurements are taken with strain gages but the critical points are in landing gears and arresting hooks and their supporting structures. Accelerations are measured for correlation with the resulting local stresses. Approach speeds and rates of sink at contact are also carefully regulated over the entire design range and measured with various kinds of special instrumentation including radar. All sorts of landings are made into the arresting gear: high, low, fast, slow, off-center, and generally cock-eyed. Many minor failures occur, most of which are easily correctable, but the analysis of them is usually very detailed and thorough to determine for instance whether a beef-up is required or whether a change in shock absorbing or damping characteristics will do.

There are very few catapult structural failures. The forces are applied smoothly and consistently and the procedure is not actually as rough as it appears. All these carrier suitability tests are carried out at great length on shore, at the Naval Air Test Center, and then repeated on board ship. The shipboard operation is a very rigorous requirement that the land based aircraft does not have to meet. Designing a plane for the Air Force is considerably simpler than for the Navy unless severe penalties are to be taken in performance.

In present day structural flight test programs, the approach to all design limit conditions is a carefully regulated and controlled step-by-step build-up, constantly monitored and thoroughly analyzed at each step. The most significant design limits are as follows:

I. High speeds at all altitudes, attained by diving. At high altitudes the Mach number effects are critical: at low altitudes both Mach numbers and dynamic pressure are critical.

II. High wing and tail loads at all speeds.

III. Maximum rate of roll, obtained by full deflection of lateral controls at high speeds.

IV. Maximum rate of roll during high wing loading, the so-called "rolling pull-out." This is critical to the entire wing structure.

V. Maximum yaw attainable at high speed by application of a specified rudder force. This is critical to the fin and rudder.

One pilot usually carries out the entire program of build-ups and final proof of design strength. This takes a couple of months and sometimes more where there are several conditions of loading which must be proved and demonstrated. It is practically essential that he gain the experience with the airplane, during "build-ups," which will assure extreme precision as the limits are reached. It is a rough business. A rolling pull-out at low altitude at maximum attainable speed is a very violent maneuver; and a minor error in judgment will put you underground or an error in technique will pull the wings off. The high wing loads at low speeds frequently produce high-speed stalls because the speed is only comparatively low. Under these circumstances some modern jets will do three or four violent snap rolls before the pilot can collect his wits and get her straightened out. You may have noticed recently in the papers that Welch, of North American was killed in an F-100 "Super-Sabre" doing limit wing load pullouts at maximum speed. But on the whole the safety record to date has been such as to reflect extraordinary credit to the designers of these complex machines.

Present day test pilots are a far cry from the older school. They are half-engineer themselves and they have to be. They weren't born with the confidence required to carry out structural proof tests. They acquired confidence in themselves and in the planes by experience, and in the design by a sound understanding of the designer and his work.

Radio controlled automatic pilots have always been the dream of those individuals responsible for carrying out these tests. Pilotless aircraft under radio control have been flying regularly for 15 years or more—but it's going to be a long time yet before radio control can be depended on to fly a 10 or 20 million dollar prototype fighter. Somebody's electric razor might put it underground.

OF GENERAL INTEREST

PROCEEDINGS OF THE SOCIETY

MINUTES OF MEETING
Boston Society of Civil Engineers

DECEMBER 15, 1954.—A Joint Meeting of the Boston Society of Civil Engineers with the Sanitary Section, BSCE was held this evening at the American Academy of Arts & Sciences, 28 Newbury Street, Boston, Mass., and was called to order by President Miles N. Clair, at 7:00 P.M.

President Clair announced that the reading of the minutes of the previous meeting of November 17, 1954 would be published in a forthcoming issue of the JOURNAL and that the reading of the minutes would therefore be waived unless there was objection.

The President announced the death of the following members:—

D. Joseph Hennessey, who was elected a member June 20, 1950, and who died November 29, 1954.
Joseph A. Rourke, who was elected a member December 18, 1912, and who died December 11, 1954.

The Secretary announced the names of applicants for Membership in the BSCE and that the following had been elected to membership on December 13, 1954:—

Grade of Member—Arnold E. Howard, Francis H. McCarran, Jr., Angelo J. Polvere, Charles J. Rich, Bernard P. Skulte, Kenneth B. Wolfskill.

President Clair announced that this was a Joint Meeting with the Sanitary Section and called upon Ariel A. Thomas, Vice-Chairman of that Section to conduct any business necessary at this time.

President Clair then introduced the speaker of the evening, Frank L. Heaney of Fay, Spofford & Thorndike, who gave a most interesting talk on "Improvement to New Bedford's Sewerage Facilities, Including Intercepting Sewers and Pumping Stations".

Prepared discussions were presented by Prof. Gordon M. Fair, T. W. Williams, R. T. Jones, and Harold A. Thomas, Jr.

At the close of the meeting President Clair announced that a Collation would be served in the Lounge on the floor above.

Seventy-eight members and guests attended the meeting.

The meeting adjourned at 9:15 P.M.

ROBERT W. MOIR, *Secretary*

JANUARY 26, 1955.—A Joint Meeting of the Boston Society of Civil Engineers with the Surveying & Mapping Section, BSCE was held this evening at the American Academy of Arts & Sciences, 28 Newbury Street, Boston, Mass., and was called to order by President Miles N. Clair, at 7:00 P.M.

President Clair stated that the Minutes of the December 15, 1954 meeting would be published in a forthcoming issue of the JOURNAL and that the reading of the minutes therefore be waived unless there was objection.

The President called upon the Secretary to announce election of new Members and applicants for membership in the BSCE.

The Secretary announced the following had been elected to membership:—

Grade of Member—Carl G. Johnson, Lew K. Perley, Frederick Roach, Jr.*

Grade of Junior—Robert C. Miers.

President Clair requested the Secretary to present a recommendation of the Board of Government to the Society for action. The President stated that this matter was before the Society in accordance with provisions of the By-Laws and notice of such action published in the ESNE Journal dated January 17, 1955.

The Secretary presented the following recommendation of the Board of Government to the Society for initial action to be taken at this meeting.

MOTION "To recommend to the Society that the Board of Government be authorized to transfer an amount not to exceed $1500 from the Principal of the Permanent Fund to the Current Fund for current expenditures".

On motion duly made and seconded it was *VOTED* "that the Board of Government be authorized to transfer an amount not to exceed $1500 from the Principal of the Permanent Fund to the Current Fund for current expenditures".

President Clair stated that final action on this matter would be taken at the February 16 meeting of the Society.

President Clair announced that this was a Joint Meeting with the Surveying and Mapping Section and called upon Wilbur C. Nylander, Chairman of that Section to conduct any necessary business at this time.

President Clair then introduced the speakers of the evening, William F. Uhl, George R. Rich and VanCourt Hare of Charles T. Main, Inc.

Subject—"The St. Lawrence International Hydroelectric Development".

A discussion period followed after which the President announced that a collation would be served in the Lounge on the floor above.

One hunderd ninety-four members and guests attended the meeting.

The meeting adjourned at 8:25 P.M.

ROBERT W. MOIR, *Secretary*

* Transfer from Grade of Junior.

FEBRUARY 16, 1955.—A Joint Meeting of the Boston Society of Civil Engineers with the Structural Section, BSCE was held this evening at the American Academy of Arts & Sciences, 28 Newbury Street, Boston, Mass., and was called to order by President Miles N. Clair, at 7:00 P.M.

President Clair stated that the Minutes of the January 26, 1955 meeting would be published in a forthcoming issue of the JOURNAL and that the reading of the minutes therefore be waived unless there was objection.

The President called upon the Secretary to announce election of new members and applicants for membership in the BSCE.

The Secretary announced the following had been elected to membership:—

Grade of Member—Walter I. Keyes, Earl H. Page, George F. Parsons, Anne M. Whelan, Martin C. Murphy.

President Clair requested the Secretary to present a recommendation of the Board of Government to the Society for action. The President stated that this matter was before the Society in accordance with provisions of the By-Laws and notice of such action published in the ESNE Journal dated February 7, 1955.

The Secretary presented the following recommendation of the Board to the Society for final action to be taken at this meeting.

MOTION "To recommend to the Society that the Board of Government be authorized to transfer an amount not to exceed $1500 from the Principal of the Permanent Fund to the Current Fund for current expenditures".

President Clair stated that this was the final action on this matter.

President Clair announced that this was a Joint Meeting with the Structural Section and called upon Ruth D. Terzaghi, Chairman of that Section to conduct any necessary business at this time.

President Clair then introduced the speaker of the evening, Dr. Karl Terzaghi, Prof. of Civil Engineering, Division of Engineering Science, Harvard University, who gave a most interesting talk on "Evaluation of the Coefficients of Subgrade Reaction". The talk was illustrated with slides.

A discussion period followed after which the President announced that a collation would be served in the Lounge on the floor above.

One hundred seventy-nine members and guests attended the meeting.

The meeting adjourned at 8:35 P.M.

ROBERT W. MOIR, *Secretary*

MARCH 16, 1955.—The one hundred seventh annual meeting of the Boston Society of Civil Engineers was held today at the Hotel Vendome, 160 Commonwealth Avenue, Boston, Mass., and was called to order at 4:00 P.M., by President Miles N. Clair.

President Clair announced that the reading of the minutes of the Society meetings have been omitted during the year. The minutes of the December, 1954, January and February, 1955 meetings will be published in a forthcoming issue of the JOURNAL. The minutes of the April, May, September, October and November 1954 meetings to be declared approved as published.

The Secretary announced the following had been elected to membership:—

Grade of Member—Paul J. Berger, Christian M. Biersack, James R. Cass, Jr.,* John G. Chalas,* George F. Cuddy, Charles H. Folsom, William T. Gay, Jr., Arthur R. Hahn, Arthur Hebert,* George G. Hewit, Charles F. Kerr, Robert T. Koopman, Charles R. Kurz, William B. S. Leong, Robert S. Loomis,* Logan T. Murdoch, Peter S. Nelson, Victor G. Otto, David H. Owen, Euplio E. Rossi, Reuben Samuels,** Milton F. Sherman, James M. Simmons,* Francis W. Taylor.

The Secretary also announced the names of applicants for membership in the Society.

The Annual Reports of the Board of Government, Treasurer, Secretary and Auditors were presented. Reports were also made by the following committees: —Hospitality, Library, John R. Freeman, Subsoils of Boston, Membership, Advertising, Publicity and Registration Law.

It was *VOTED* "that these reports be accepted and placed on file".

The Annual Reports of the various Sections were read and it was *VOTED* "that the Annual Reports of the various Sections be accepted and placed on file".

President Clair stated that all foregoing reports would be published in the April issue of the JOURNAL.

The Report of the Tellers of Election, John F. Flaherty and George E. Townsend was presented and in accordance therewith the President declared the following had been elected officers for the ensuing year.

President—Edwin B. Cobb.

V-President (for two years) John G. W. Thomas.

Secretary (for one year) Robert W. Moir.

Treasurer (for one year) Charles O. Baird, Jr.

Directors (for two years) Thomas A. Berrigan, Ernest L. Spencer.

Nominating Committee (for two years) Allen J. Burdoin, Fozi M. Cahaly, Elliot F. Childs.

The retiring President, Miles N. Clair then gave his address entitled "Some

* Transfer from Junior.

* Transfer from Junior.
** Transfer from Student.

Thoughts Relative to the Society".

The meeting adjourned at 5:47 P.M., to re-assemble at 7:30 P.M., the Annual Dinner being held during the interim.

President Clair called the meeting to order at 7:30 P.M.

Following general remarks and the introduction of the newly elected President Edwin B. Cobb, and other guests at the head table President Clair announced that Honorary Membership in the Society had been conferred on one of the Society's distinguished members, in accordance with the vote of the Board of Government on February 7, 1955:

William Frank Uhl, who has been a member since November 15, 1911.

President Clair presented the newly elected Honorary Member with a certificate of Honorary Membership which reads as follows:

In recognition of his eminent attainments in the field of hydro-electric engineering

WILLIAM FRANK UHL

has been duly elected an

HONORARY MEMBER

By direction of the Board of Government February 7, 1955

Robert W. Moir (Seal) *Miles N. Clair*
Secretary President

President Clair stated that a number of prizes were awarded annually for worthy papers presented at the Society and Section meetings. The Secretary read the names of recipients and asked them to come forward and the President presented the Awards.

Award	*Recipient*	*Paper*
Desmond FitzGerald Medal	T. William Lambe	"The Improvement of Soil Properties with Dispersants".
Clemens Herschel Award	Howard P. Hall	"A Historical Review of Investigations of Seepage Toward Wells".
	Charles R. Williams Richard Dennis Leslie Silverman	"Symposium on Air Pollution".
Structural Section Award	John M. Biggs	"Buckling Considerations in the Design of Steel Beams and Plate Girders".
Transportation Section Award	John D. M. Luttman-Johnson	"Transportation Problems in Ceylon's Ports".
Surveying & Mapping Section Award	Ernest A. Herzog	"A Short History of Surveying".

President Clair introduced the guest speaker of the evening Dr. Carl S. Ell, President of Northeastern University, who gave a most interesting talk on "The Viking Countries". The talk was illustrated with colored slides.

At the conclusion of the address President Clair on behalf of the Society thanked Dr. Ell for a most enjoyable talk and then turned the meeting over to President-elect Edwin B. Cobb.

President Edwin B. Cobb presented retiring President Miles N. Clair with a certificate for services rendered and then adjourned the meeting at 9:55 P.M.

Two hundred thirty-five members and guests attended the dinner meeting.

ROBERT W. MOIR, *Secretary*

SANITARY SECTION

DECEMBER 1, 1954.—The meeting was called to order by Chairman Moore at 7:25 P.M. in the Society Rooms.

The following three most recent past

chairmen of the Section were nominated to be the Section Nominating Committee, as follows:

Wm. E. Stanley, Chairman
F. M. Cahaly
John S. Bethel, Jr.

The nominations were closed and the clerk directed to cast one ballot for the three nominees, who will bring in a slate of officers for the Section at the Section Annual Meeting on March 2, 1955.

Mr. Edwin B. Cobb then gave a paper entitled "Pollution Abatement Program at Newport, R. I." This interesting paper was prepared jointly by Mr. Cobb and Dean Coburn. It provoked considerable discussion.

Forty-seven were present at the meeting. Twenty-eight attended the dinner at Patten's Restaurant. The meeting closed at 8:30 P.M.

ARIEL A. THOMAS, *Secretary Protem*

MARCH 2, 1955.—The meeting was called to order at 7:02 P.M. by Chairman Edward W. Moore after an informal dinner at Patten's Restaurant. Thirty-two members and guests attended the meeting, and eighteen members and guests attend the dinner.

The Annual Report of the Executive Committee was read by the Clerk of the Section, and the report was accepted as read.

The Report of the Nominating Committee was read by Ralph Soule, substituting for the Chairman of the Nominating Committee, who was ill. The Nominating Committee's Report nominated the following people for the offices of the Sanitary Section for the coming year:

Chairman, Ariel A. Thomas
Vice-Chairman, Darrell A. Root
Clerk, John F. Flaherty
Member, Clair N. Sawyer
Member, Joseph C. Knox
Member, Harold A. Thomas, Jr.

The report of the Nominating Committee was accepted. The nominations were closed, and the Clerk cast one ballot for the nominees as presented.

The speaker of the evening was Clarence I. Sterling, Jr., Chief Sanitary Engineer and Deputy Health Commissioner of the Massachusetts Department of Public Health, and his subject was "Sanitary Engineering Problems in Alaska". Mr. Sterling presented a broad picture of the general conditions in Alaska, well illustrated with color slides, and discussed many sanitary engineering problems in connection with the climate of Alaska.

After a short discussion, the meeting was turned over to the newly elected Chairman, Ariel A. Thomas, who adjourned the meeting at 8:40 P.M.

DARRELL A. ROOT, *Clerk*

STRUCTURAL SECTION

DECEMBER 8, 1954.—A meeting of the structural section was held at 7:00 P.M. in the Society Rooms. Vice Chairman, C. Kray presided in the absence of Chairman Dr. Ruth Terzaghi.

Albert G. H. Dietz, Professor of Building Engineering and Construction, of the Department of Civil Engineering at Massachusetts Institute of Technology, was the speaker. His topic was "Plastics in Building". Professor Dietz told how the use of plastics in the structural field was increasing at a rapid rate, and how they are supplanting the more common metallic materials in numerous instances.

The typical plastics were classified in two main groups. Specimens of many kinds were shown and the characteristics of each explained. Specific uses, such as for piping in the oil and gas industry, were cited and the advantages of the plastics over the more common materials were listed.

Among the plastics discussed were silicones, vinyls, nylon and glass. Slides were used to illustrate the talk and explain such factors as strength and creep characteristics, resistance to corrosion, thermal conductivity, and appearance.

In closing, Professor Dietz pointed up the need for continued research to determine how these newer materials will react over long periods of time.

Thirty members attended the meeting which was followed by a lengthy question and answer period.

A. L. DELANEY, *Clerk*

JANUARY 12, 1955.—A meeting of the Structural Section was held at the Society Rooms. Chairman Dr. Ruth D. Terzaghi presided and opened the session at 7:10 P.M.

After the minutes of the previous meeting were read, a motion was made and seconded that the Chairman appoint a Nominating Committee to bring in a slate of officers for the Structural Section, to be elected for the coming year. The following members were appointed:

Prof. Charles Norris
E. C. Keane
Frank Cundari

Dr. Terzaghi introduced Mr. Albert P. Richards of the William F. Clapp Laboratories Inc., Duxbury, Mass. as the speaker. Mr. Richards' topic was "Biological Deterioration of Structural Materials in Marine Environments".

In the talk, which was illustrated, he discussed the various organisms which cause damage and how this damage influences the design of marine structures.

Special attention was given the organisms present in the vicinity of Boston, and ways and means of offsetting their destructive action were outlined.

Emphasis was placed on proper design, rigid specifications, and careful inspection as a means of combating their destructive influence.

The meeting proved very interesting and was attended by 38 members.

A. L. DELANEY, *Clerk*

HYDRAULICS SECTION

MAY 4, 1954.—The Hydraulics Section held a joint meeting with the Structural and Transportation Section at the American Academy of Arts and Sciences and the meeting was called to order at 7:30 P.M., by Chairman Ruth D. Terzaghi of the Structural Section. The speaker was Professor A. W. Skempton of the University of London, whose subject was "Civil Engineering 1500-1900; and Historical Sketch".

The speaker gave an interesting description of the development of civil engineering works in Europe, America and India; and included harbor developments, bridges, highways and irrigation projects. The talk was illustrated with slides.

A short discussion period followed, and a collation was held after the meeting. Attendance was 135.

JOSEPH C. LAWLER, *Clerk*

NOVEMBER 3, 1954.—A meeting of the Hydraulics Section was held in the Society Rooms this evening and was called to order promptly at 7:00 P.M., by Chairman of the Section, Professor Arthur T. Ippen. There being no objections, the minutes of the previous meeting were approved without reading.

There was a discussion of possible additions to the Boston Society of Civil Engineers library in the hydraulics field. There was also a discussion of possible future meetings. A tentative talk by Mr. C. W. Hubbard on turbine testing, plus a speaker from M.I.T. on valve testing, was announced for the next meeting. A meeting at the M.I.T. Hydrodynamics Laboratory was tentatively scheduled for May, 1955.

Dr. D. R. F. Harleman, Assistant Professor of Hydraulics at M.I.T., followed with a talk on "The Effect of Baffle Piers on Stilling Basin Performance". This topic has been the object of two experimental investigations at the M.I.T. Hydrodynamics Laboratory and was sponsored financially in part by the Freeman Fund Committee of the Boston Society of Civil Engineers. Professor Harleman's talk was followed by a long discussion period.

Thirty-four members of the Section were present at the meeting which adjourned at 8:45 P.M.

JOSEPH C. LAWLER, *Clerk*

FEBRUARY 2, 1955.—The Annual Meeting of the Hydraulics Section was held in the Society Rooms this evening. The meeting was called to order at 7:00 P.M., by Chairman Arthur T. Ippen. The minutes of the previous meeting were approved without reading.

The Nominating Committee presented the following slate of officers for the Hydraulics Section for the year 1955-56:

Chairman, Ralph S. Archibald
Vice Chairman, Joseph C. Lawler
Executive Committee, David R. Campbell, Dean F. Coburn, James W. Daily

Approval was voted of the Nominating Committee's report. No further nominations were made from the floor, and therefore, the clerk was instructed to cast one ballot for the slate nominated.

Mr. Clyde W. Hubbard, Hydraulic Engineer, Stone & Webster Engineering Corp., then spoke on "Field Tests on Rock Island Hydro-electric Plant". Mr. R. E. Nece, Instructor, Department of Civil and Sanitary Engineering, M.I.T., followed with a talk on "Hydraulic Performance of Check and Control Valves". A spirited question and discussion period followed these two interesting talks.

A total of 47 members were present at the meeting, which adjourned at 9:05 P.M.

JOSEPH C. LAWLER, *Clerk*

ADDITIONS

Members

Christian M. Biersack, Post Engr. Sec. Section A, Boston Army Base, Boston, Mass.
Charles H. Folsom, 70 Powder House Blvd., Somerville, Mass.
Arthur R. Hahn, Union Metal Mfg. Co., 1110 Little Bldg., Boston 16, Mass.
George G. Hewit, 662 Washington Street, Brookline, Mass.
Arnold E. Howard, Dept. of Natural Resources, 15 Ashburton Place, Boston, Mass.
Carl G. Johnson, 405 Grove Street, Melrose, Mass.
Walter I. Keyes, 195 Weston Street, Waltham, Mass.
Charles F. Kerr, 989 Washington Street, Newtonville, Mass.
Francis H. McCarran, Jr., 15 Morrison Road, Watertown, Mass.
Martin C. Murphy, 81 Parkland Avenue, Lynn, Mass.
Earl H. Page, 16 Eastern Avenue, Beverly, Mass.
Lew K. Perley, 59 Marion Street, Brookline, Mass.
Angelo J. Polvere, 571 Hyde Park Avenue, Roslindale, Mass.
Charles J. Rich, 14 Adams Street, Arlington, Mass.
Frederick Roach, Jr., 193 Savin Hill Avenue, Dorchester, Mass.
Bernard P. Skulte, 44 Cypress Street, Brookline 46, Mass.
Anne M. Whelan, 11 Beacon Street, Boston, Mass.

ANNUAL REPORTS

REPORT OF THE BOARD OF GOVERNMENT FOR YEAR 1954-1955

Boston, Mass., March 16, 1955

To the Boston Society of Civil Engineers:

Pursuant to the requirements of the By-Laws the Board of Government presents its report for the year ending March 16, 1955.

The following is a statement of the status of membership in the Society:

Honorary	8	Juniors	71
Members	888	Students	9
Associates	3		
		Total	979

Student Chapters	2
Summary of Additions	
New Members	57
New Juniors	22
New Students	2
Reinstatements	
Members	4
Summary of Transfers	
Juniors to Members	11
Summary of Loss of Members	
Deaths	12
Resignations	6
Dropped for non-payment of dues	21
Dropped for failure to transfer	3

Life members — 90.
Members becoming eligible today for Life Membership — 17.
Remission of dues — 1.
Applications pending on March 16, 1955 — 32.

Honorary Membership is as follows:

Frank M. Gunby, elected, February 15, 1950
Richard K. Hale, elected, February 15, 1950
Karl R. Kennison, elected, February 7, 1951
Charles W. Sherman, elected, February 19, 1947
Charles M. Spofford, elected, December 19, 1945
Howard M. Turner, elected, February 18, 1952
Karl Terzaghi, elected, March 3, 1952
William F. Uhl, elected, February 7, 1955

The following members have been lost through death:

Robinson Abbott, June 18, 1954
Harry P. Burley, Aug. 22, 1954
Karl T. Compton, June 22 ,1954
Joseph H. Fitch, Sept. 25, 1954
Vernald W. Fox, May 9, 1954
D. Joseph Hennessy, Nov. 25, 1954
Forrest J. Maynard — 1954
William N. Parsons, June 4, 1954
Joseph A. Rourke, Dec. 11, 1954
Walton H. Sears, Aug. 7, 1954
Frank H. Stuart, July 28, 1954
Dana M. Wood, May 10, 1954

Meetings of the Society

March 17, 1954.—Address of the retiring President, Chester J. Ginder. "Status of the Engineer."

April 13, 1954.—Joint Meeting with American Society of Civil Engineers, Northeastern Section. "Traffic Unlimited", John Kyle, Chief Engr., New York Port Authority.

May 19, 1954.—Joint Meeting with Surveying & Mapping Section, BSCE. "Problems Which New York City Faces in Expanding Its Water Supply Sources." Karl R. Kennison, Chief Engr., Board of Water Supply, New York City.

September 29, 1954.—Symposium—"Background and Highlights of the Interim Report of the Committee on Evaluation of Engineering Education." Dean Harold L. Hazen, Graduate School, Mass. Institute of Technology.

"The Humanistic-Social Content of the Engineering Curricula." William C. White, Vice-President, Northeastern University.

"Synthesis in the Education of the Engineer during Early Training Rather Than Later or Not at All." Dr. John B. Wilbur, Head, Dept. of Civil Engineering, Mass. Institute of Technology.

"Thoughts of an Engineering Education." Oscar S. Bray, Project Manager, Jackson & Moreland.

October 27, 1954.—Student Night. Joint Meeting with American Society of Civil Engineers, Northeastern Section. "Development of Structural Flight Testing", Vice Admiral Frederick M. Trapnell, USN(Ret.); Demonstration of Flight Safety Equipment Used in Jet Aircraft, Bruce Tuttle, Test Pilot and R. S. Moore, Safety Director at Grumman Aircraft Corp.

November 17, 1954.—"Engineering Ethics". Waldo G. Bowman, Editor of *Engineering News-Record.*

December 15, 1954.—Joint Meeting with Sanitary Section, BSCE. "Improvement to New Bedford's Sewerage Facilities, Including Intercepting Sewers and Pumping Stations." Frank L. Heaney, Fay, Spofford & Thorndike.

January 26, 1955.—Joint Meeting with Surveying & Mapping Section, BSCE. "St. Lawrence International Hydroelectric Development". William F. Uhl, Van Court Hare, George R. Rich of Chas. T. Main, Inc.

February 16, 1955.—Joint Meeting with Structural Section, BSCE. "Evaluation of the Coefficients of Subgrade Reaction." Dr. Karl Terzaghi, Harvard University.

Attendance at Meetings

Date	Place	Meeting	Dinner
March 17, 1954	Hotel Vendome	46	170
April 13, 1954	Northeastern University	58	72
May 19, 1954	American Academy of Arts and Sciences	66	*
September 29, 1954	American Academy of Arts and Sciences	110	*
October 27, 1954	Harvard University	274	273
November 17, 1954	American Academy of Arts and Sciences	91	*
December 15, 1954	American Academy of Arts and Sciences	78	*
January 28, 1955	American Academy of Arts and Sciences	194	*
February 16, 1955	American Academy of Arts and Sciences	179	*
Average Attendance—131.			

* Collation served in Lounge after meeting.

Sections

Twenty meetings were held by the Sections of the Society during the year. These meetings of the Sections offering opportunity for more detailed discussions continue to demonstrate their value to their members and to the Society. A wide variety of subject was presented and large attendance at these meetings has continued. The Annual Reports of the various Sections will be presented at the Annual Meeting and will be published in the April issue of the JOURNAL.

*Funds of the Society**

Permanent Fund. The Permanent Fund of the Society has a present value of $57,682.39. The Board of Government authorized the use of as much as necessary of the current income of this fund in payment of current expenses. By vote of the Society (as prescribed by the By-Laws) at the January 26, 1955 and February 16, 1955 meetings the Board of Government was authorized to transfer an amount not to exceed $1500 from the Principal of the Permanent Fund for Current Expenditures. It was not necessary to transfer any of the $1500 from the Permanent Fund for current expenditures.

John R. Freeman Fund. In 1925 the late John R. Freeman, a Past President and Honorary Member of the Society, made a gift to the Society of securities which was established as the John R. Freeman Fund. The income from this Fund is to be particularly devoted to the encouragement of young engineers. Mr. Freeman suggested several uses, such as the payment of expenses for experiments and compilations to be reported before the Society; for underwriting meritorious books or publications pertaining to hydraulic science or art; or a portion to be devoted to a yearly prize for the most useful paper relating to hydraulics contributed to this Society; or establishing a traveling scholarship every third year open to members of the Society for visiting engineering works, a report of which would be presented to the Society. Lawrence C. Neale, Member, was awarded the John R. Freeman Scholarship for year 1954-1955.

Edmund K. Turner. In 1916 the Society received 1,105 books from the library of the late Edmund K. Turner, and a bequest of $1,000, "the income of which is to be used for library purposes." The Board voted to use $50 of the income for the purchase of books for the library. The expenditure from this fund during the year was $46.80.

* Details regarding the values and income of these funds are given in the Treasurer's report.

Alexis H. French Fund. The Alexis H. French Fund, a bequest amounting to $1,000 was received in 1931, from the late Alexis H. French of Brookline, a former Past President of the Society. The income of the fund is "to be devoted to the Library of the Society." The Board voted to use $50 of the available income for the purchase of books for the library. The expenditure from this fund during the year was $43.32.

Tinkham Memorial Fund. The "Samuel E. Tinkham Fund", established in 1921 at the Massachusetts Institute of Technology by the Society, "to assist some worthy student of high standing to continue his studies in Civil Engineering," had a value of $2,608.86 on June 30, 1954. Frank E. Perkins of Brockton, Massachusetts a student in Civil Engineering, class of 1955, was awarded this Scholarship of $100 for the year 1954-1955.

Desmond FitzGerald Fund. The Desmond FitzGerald Fund, established as a bequest from the late Desmond FitzGerald, a Past President and Honorary Member of the Society, provides that the income from this fund shall "be used for charitable and educational purposes." The Board voted on April 5, 1954 to appropriate from the income of this fund the sum of $100 to be known as the Boston Society of Civil Engineers Scholarship in memory of Desmond FitzGerald, and to be given to a student at Northeastern University. This year it was voted "to adopt the recommendation of the Committee at Northeastern University, namely, that the Scholarship be awarded to Charles H. Flavin, Jr., Senior Civil Engineering Student. The presentation was made by President Clair at the Dean's List Dinner, June 8, 1954.

Clemens Herschel Fund. This fund was established in 1931, by a bequest of $1,000 from the late Clemens Herschel, a former Past President and Honorary Member of the Society. The income from this fund to be used for presentation of prizes for papers which have been particularly useful and commendable and worthy of grateful acknowledgment. The Board of Government voted on April 7, 1953 "to use this fund for Clemens Herschel Prizes only for the next few years until this fund is built up substantially." The expenditure made during the year from this fund was $25.95 for Prize Award.

Edward W. Howe Fund. This fund, a bequest of $1,000, was received December 2, 1933 from the late Edward W. Howe, a former Past President of the Society. No restrictions were placed upon the use of this money, but the recommendation of the Board of Government was that the fund be kept intact, and that the income be used for the benefit of the Society or its members. The Board of Government voted on April 5, 1954 "that a sum not to exceed $50 be appropriated from the Howe Fund to pay the balance of prizes awarded at the March 17, 1954 Annual Meeting."

William P. Morse Fund. This fund, a bequest of $2,000, was received April 8, 1949 from the late William P. Morse, a former member of the Society. No restrictions were placed upon the use of this money but the recommendation of the Board of Government was that the fund be kept intact and that the income be used for the benefit of the Society or its members. The Board of Government voted on April 5, 1954 "that no appropriation be made from the Morse Fund at that time." Upon recommendation of a Committee appointed by the President, the Board *VOTED* on January 24, 1955 "to use a sum not to exceed $100 from the income of this Fund for a Scholarship to a worthy student in Civil Engineering at Tufts College to be known as the Boston Society of Civil Engineers Scholarship in memory of William P. Morse".

AWARDS

Award	Recipient	Paper
Desmond FitzGerald Medal	T. William Lambe	"The Improvements of Soil Properties with Dispersants".
Clemens Herschel Award	Howard P. Hall	"A Historical Review of Investigations of Seepage Toward Wells".
	Charles R. Williams	"The Status of Knowledge of Air Pollution Today".
	Richard Dennis	"Measurement of Atmospheric Pollution".
	Leslie Silverman	"Modern Methods for the Control of Air Pollution".
Structural Section Award	John M. Biggs	"Buckling Considerations in the Design of Steel Beams and Plate Girders".
Transportation Section Award	John D. M. Luttman-Johnson	"Transportation Problems in Ceylon".
Surveying & Mapping Section Award	Ernest A. Herzog	"A Short History of Surveying".

Library

The report of the Library Committee contains a complete account of the Library Committee activities during the past year.

Committees

The usual special committees dealing with the activities and conduct of the Society were appointed. The membership of these committees is published in the JOURNAL and the reports of the committees will be presented at the Annual Meeting.

In addition to routine business of the Society the Board of Government took action as follows:

May 17, 1954.—Authorized obtaining a supply of B.S.C.E. pins, lapel buttons and tie bars to be retailed to the members at $5.00.

September 27, 1954.—Voted to conduct a series of Structural Lectures with cooperation of A.G.C., M.S.E.A., E. Mass. Assoc. of Pro. Engrs. and Land Surveyors as co-sponsors.

October 25, 1954.—Authorized ordering 500 copies of each Section of the Boring Data to be printed and held pending completion of the Series, the whole to be published in one volume at a future date.

November 15, 1954. Appointed a committee to cooperate with a committee from the Assoc. General Contractors toward arranging a series of lectures covering phases of contracting.

December 13, 1954.—Authorized printing and issuing a Certification of Membership to those requesting same.

January 24, 1955.—Voted "to establish a Scholarship from the income of the William P. Morse Fund in an amount not to exceed $100, to be presented to a worthy student in Civil Engineering at Tufts College, and to be known as the Boston Society of Civil Engineers Scholarship in memory of William P. Morse".

Your Board in conclusion wishes to express its appreciation of the excellent work done by the Officers of the Sections and by the Committees of the Society.

MILES N. CLAIR, *President.*

REPORT OF THE TREASURER

Boston, Mass., March 16, 1955

To the Boston Society of Civil Engineers:

The fiscal year of the Boston Society of Civil Engineers is from March 1, 1954 to March 1, 1955, and this treasurer's report is for that period of time.

The Boston Safe Deposit and Trust Company continues to be custodian of our securities and to be our investment counsel. The sale or purchase of all securities shown in this report have been made when recommended by the Boston Safe Deposit and Trust Company and approved by the Board of Government of the Boston Society of Civil Engineers. Table I is a record of Bonds and Stocks purchased, while Table II is a record of Bonds and Stocks sold.

A comparison of receipts and expenditures of the Current Fund are hereby shown below.

TABLE I

	1950-1951	1951-1952	1952-1953	1953-1954	1954-1955
Receipts					
Dues	$ 6,192	$ 6,643	$ 6,627	$ 7,085	$ 7,428
Other than dues	6,171	6,004	5,505	5,754	5,782
Total Receipts	$12,363	$12,647	$12,132	$12,829	$13,210
Total Expenditures	$14,772	$15,760	$14,576	$14,724	$15,814
Deficit	$ 2,409	$ 3,113	$ 2,444	$ 1,885	$ 2,604

The deficit, of $2,603.84, was again taken from the income to the Permanent Fund. The gross income to the Permanent Fund was $3,355.19 this year.

The book value of securities and bank deposits are shown below, comparing the last five years. The value of the Society's Library and other physical properties are not included.

TABLE II

	March 1 1951	March 1 1952	March 1 1953	March 1 1954	March 1 1955
Bonds	$35,549.23	$38,494.23	$39,475.48	$40,643.42	$ 39,617.01
Coop. Bank	9,764.13	9,984.59	10,221.96	5,425.68	5,559.44
Stocks	45,257.17	45,836.05	45,858.01	46,112.29	49,359.89
Cash	4,473.09	1,513.75	2,322.89	4,605.98	6,269.11
Totals	$95,043.62	$95,828.62	$97,878.34	$96,787.37	$100,805.45

It will be noted, that the book value of the Society's cash assets increased $4,018.08 during the year.

It is interesting to note a comparison of the Market value of the Society's Funds March 1, 1954 and March 1, 1955. A market value increase of $26,353.25.

TABLE III

	March 1, 1954	March 1, 1955
Bonds	$ 41,518.73	$ 40,152.48
Stocks	78,823.89	104,746.50
Cash	4,605.98	6,269.11
Coop. Bank	5,425.68	5,559.44
Secretary's Fund	30.00	30.00
Totals	$130,404.28	$156,757.53

The Publication Fund shown in Table III, used to publish "Contributions to Soil Mechanics", Volume I and II, indicate a healthy response. Already $2,270.50 of the original $5,000 investment has been realized.

Lawrence C. Neale, present Freeman Scholar, has received to March 1st $2,100.00 of his $4,200.00 scholarship.

The financial standing of the Society, as of March 1, 1955, is shown in the six accompanying tables.

Table I	Record of Investments—Purchased
Table II	Record of Investments—Sold
Table III	Distribution of Funds—Receipts and Expenditures
Table IV	Record of Investments—Bonds
Table V	Record of Investments—Stocks
Table VI	Record of Investments—Co-operative Bank

Respectfully submitted,

CHARLES O. BAIRD, JR., *Treasurer*

TABLE I—RECORD OF INVESTMENTS—PURCHASED

Bonds	Maturity	Interest Rate	Date Purchased	Cost	Par Value
U.S. of America Savings Bond Series K	1966	2.76 %	Aug. 5, 1954	$ 7,000.00	$ 7,000.00
Columbia Gas System Inc. Deb. Series D	1979	3.50 %	Sept. 10, 1954	2,066.17	2,000.00
Florida Power Corp. 1st Mtge.	1984	3.125%	Dec. 20, 1954	1,017.50	1,000.00
Totals				$10,083.67	$10,000.00

Stocks	Classification	Number of Shares	Date Purchased	Cost
Pacific Gas and Electric Co.	Common	43	March 11, 1954	$1,795.15
Continental Insurance Co.	Common	18	March 11, 1954	1,457.52
New England Electric System	Common	47	May 24, 1954	715.10
New England Electric System	Common	18	Oct. 14, 1954	270.00
Jewel Tea Co. Inc.	Common	20	Aug. 16, 1954	987.86
Jewel Tea Co. Inc.	Common	10	Sept. 10, 1954	455.04
Hartford Fire Insurance Co.	Common	2/4	Aug. 19, 1954	90.50
Totals				$5,771.17

TABLE II—RECORD OF INVESTMENTS—SOLD

Name	Date Sold	Book Value March 1, 1954	Amount Received	Profit
U.S. Savings Bond Series G	July 1, 1954	$ 7,000.00	$ 7,000.00	0.00
American Telephone and Telegraph Co.	Sept. 10, 1954	2,033.90	2,515.27	$481.37
Pennsylvania R.R. Gen. Mtge. Series A	Dec. 22, 1954	1,017.74	1,062.22	44.48
Puget Sound Power and Light Co. 1st Mtge.	Oct. 11, 1954	1,058.44	1,060.00	1.56
Hanover Bank	March 11, 1954	2,523.57	2,767.44	243.87
Hanover Bank	March 16, 1954	0.00	54.68	54.68
Totals		$13,633.65	$14,459.61	$825.96

Table III—Distribution of Funds—Receipts and Expenditures

	Book Value March 1, 1954 1	Interest & Dividends Cash 2	Interest & Dividends Credit 3	Net Profit or Loss at Sale or Maturity +4	Net Profit or Loss at Sale or Maturity −5	Transfer of Funds +6	Transfer of Funds −7	Book Value March 1, 1955 8
Bonds	$40,643.42	$1,470.07		$527.41		$10,083.67	$11,110.08	$39,617.01
Coop. Bank	5,425.68	30.00	$133.76					5,559.44
Stocks	46,112.29	4,041.38		298.55		5,771.17	2,523.57	49,359.89
Available for Investment	3,105.98					1,663.13		4,769.11
Total	$95,287.37	$5,541.45	$133.76	$825.96		$17,517.97	$13,633.65	$99,305.45

Columns 1 + 3 + 6 — 7 = 8

Funds	Book Value Mar.1, 1954	Allocation of Income—Profit and Loss: Income Col. 2 & 3	Allocation of Income—Profit and Loss: Net Profit Col. 4 & 5	Received	Expended	Book Value March 1, 1955
Permanent	$55,975.83	$3,355.19	$488.29	$ 690.00	$ 2,826.92	$ 57,682.39
John R. Freeman	29,872.52	1,790.57	260.58		2,219.06	29,704.61
Edmund K. Turner	1,060.19	63.55	9.25		51.02	1,081.97
Desmond FitzGerald	2,123.69	127.30	18.53		109.97	2,159.55
Alexis H. French	1,069.03	64.08	9.33		47.58	1,094.86
Clemens Herschel	1,119.07	67.07	9.77		30.40	1,165.51
Edward W. Howe	1,064.26	63.77	9.29		54.24	1,083.08
William P. Morse	2,086.06	125.02	18.20		8.32	2,220.96
Publication	605.47			1,810.23	145.50	2,270.20
Surveying Lecture	311.25	18.66	2.72		1.25	331.38
Structural Lectures	0.00			785.00	274.06	510.94
	$95,287.37	$5,675.21	$825.96	$ 3,285.23	$ 5,768.32	$ 99,305.45
Current Cash	1,500.00	2,603.84		13,209.98	15,813.82	1,500.00
Totals	$96,787.37	$8,279.05	$825.96	$16,495.21	$21,582.14	$100,805.45

Secretary's change fund of $30.00 should be added to show total cash.

Cash balance March 1, 1955

Investment Fund	$4,769.11
Current Account	1,500.00
Total	$6,269.11

TABLE IV—RECORD OF INVESTMENTS—BONDS

Bonds	Date of Maturity or Classification	Fixed or Current Interest Rate	Interest Received	Par Value	Mar. 1, 1955 Book Value	Mar. 1, 1955 Market Value
United States Savings Bond Series G	July 1, 1954	2½%	$ 87.50	$ 7,000.00*	$ 0.00	$ 0.00
United States Savings Bond Series G	Nov. 1, 1956	2½%	25.00	1,000.00	1,000.00	1,000.00
United States Savings Bond Series G	May 1, 1958	2½%	100.00	4,000.00	4,000.00	4,000.00
United States Savings Bond Series G	May 1, 1960	2½%	25.00	1,000.00	1,000.00	1,000.00
United States of America Treasury Bond	June 15, 1983	3¼%	260.00	8,000.00	8,000.00	8,492.48
United States of America Savings Bond Series K	Aug. 1, 1966	2.76%	96.60	7,000.00	7,000.00	7,000.00
American Telephone and Telegraph Company	Dec. 15, 1961	2¾%	41.10	2,000.00*	0.00	0.00
American Telephone and Telegraph Company Conv. Deb.	Dec. 10, 1965	3.75%	37.50	1,000.00	1,167.94	1,462.50
Columbia Gas System Inc: Deb. Series D	July 1, 1979	3.50%	35.00	2,000.00	2,066.17	2,100.00
Consumers Power Co. 1st Mtge.	Sept. 1, 1975	2⅞%	129.39	3,000.00	3,140.35	2,928.75
Florida Power Corp. 1st Mtge.	July 1, 1984	3.125%	15.63	1,000.00	1,017.50	1,011.25
Pennsylvania R.R. Gen. Mtge. Series A	June 1, 1975	4½%	47.38	1,000.00*	0.00	0.00
Province of Ontario	Sept. 1, 1972	3¼%	146.25	3,000.00	2,936.25	3,037.50
Public Service Electric & Gas Co. 1st & Ref.	June 1, 1979	2⅞%	115.00	4,000.00	4,097.50	3,870.00
Puget Sound Power and Light Company 1st Mtge.	Dec. 1, 1972	4¼%	38.72	1,000.00*	0.00	0.00
Southern Pacific 1st Series A, Oregon Lines	Mar. 1, 1977	4½%	270.00	4,000.00	4,191.30	4,250.00
Totals			$1,470.07	$39,000.00	$39,617.01	$40,152.48

*Sold

TABLE V—RECORD OF INVESTMENT—STOCKS

Stocks	Classification	Number of Shares	Dividend Received	Book Value March 1, 1955	Market Value March 1, 1955
American Telephone and Telegraph Company	Common	52	$ 468.00	$ 5,800.68	$ 9,587.50
Consolidated Edison Co. of New York Inc.	Common	50	120.00	2,556.12	2,437.50
Continental Insurance Co.	Common	68	190.50	3,483.73	6,766.00
General Electric Co. of New York	Common	150	230.00	2,341.47	7,725.00
Hanover Bank	Common	32⅝	0.00	0.00*	0.00*
Hartford Fire Insurance Co.	Common	18	51.00	930.25	3,645.00
Jewel Tea Co. Inc.		30	26.00	1,442.90	1,481.25
National Dairy Products Corp.	Common	100	155.00	1,154.74	3,825.00
New England Electric System	Common	198	155.48	3,106.60	3,514.50
North American Trust Shares	Juy 15, 1955	1500	471.00	5,342.00	12,465.00
Pacific Gas and Electric Co.	Preferred	100	150.00	2,704.89	3,650.00
Pacific Gas and Electric Co.	Common	100	220.00	3,387.28	4,800.00
Radio Corporation of America	Preferred	20	70.00	1,720.75	1,680.00
Southern California Edison Co. Ltd.	Preferred	40	80.00	1,161.22	2,210.00
Southern California Edison Co.	Common	45	90.00	1,374.25	2,216.25
Southern Railway Co.	Preferred	30	75.00	1,136.80	1,507.50
Standard Oil of New Jersey	Common	108	491.40	3,328.16	12,285.00
Texas Company	Common	104	390.00	2,956.32	9,620.00
Union Carbide and Carbon Corp.	Common	100	300.00	2,958.44	8,500.00
Union Pacific Railroad	Common	44	308.00	2,473.29	6,831.00
Totals			$4,041.38	$49,359.89	$104,746.50

* Sold

TABLE VI—RECORD OF INVESTMENTS—CO-OPERATIVE BANKS

Co-operative Banks	Classification	Number of Shares	Dividend Received	Book Value March 1, 1955
Suffolk Cooperative Federal Savings and Loan Association	Matured Certificate	5	$ 30.00	$1,000.00
Suffolk Cooperative Federal Savings and Loan Association Account No. S-631	Savings Account		133.76	4,559.44
Totals			$163.76	$5,559.44

REPORT OF THE SECRETARY

Boston, Mass., March 16, 1955

To the Boston Society of Civil Engineers:

The following is a statement of cash received by the Secretary and of the expenditures approved by the President, in accordance with the Budget adopted by the Board of Government.

FOR THE YEAR ENDING MARCH 16, 1955

	Account Number	Expenditures	Receipts
Office			
Secretary, Salary & Expense	(20)	$ 500.00	
Stationery, Printing & Postage	(22)	489.65	
Incidentals & Petty Cash	(23)	134.89	
Insurance & Treasurer's Bond	(24)	24.00	
Quarters, Rent, Lt. & Tel.	(26)	1,875.53	$ 800.00
Office Secretary	(28)	3,300.00	
Auditor's Charge	(30)	150.00	
Social Security	(31)	85.50	
Meetings			
Rent of Halls, etc.	(40)	235.00	
Stationery, Printing & Postage	(41)	42.70	
Hospitality Committee	(42)	520.95	416.06
Reporting & Stereopticon	(43)	—	
Annual Meeting, March, 1954	(44)	741.00	584.50
Sections			
Sanitary Section	(45)	27.67	
Structural Section	(46)	44.43	
Transportation Section	(47)	.90	
Hydraulics Section	(48)	15.90	
Surveying & Mapping Section	(50)	10.25	
Special Fund for Speakers	—	50.00	
Journal			
Editor's Salary & Expense	(60)	516.00	
Printing & Postage	(62)	5,448.18	
Advertisements	(64)	103.84	2,484.44
Sale of Journals & Reprints	(63)	—	1,302.67
Library			
Periodicals	(69)	90.50	
Binding	(70)	136.00	
Fines	(6)	—	4.96
Binding Journals for Members	(81)	14.00	14.00
Forward		$14,566.89	$5,606.63

SECRETARY'S REPORT (*Continued*)

	Account Number	Expenditures	Receipts
Brought Forward		$14,566.89	$5,606.63
Bank Charges	(82)	5.60	
Miscellaneous	(84)	267.99	100.60
Engineering Societies Dues and Charge for Journal Space	(88)	770.80	
Badges	(80)	212.54	75.00
Dues from BSCE Members	(10)	—	7,427.75
Transfer Income Permanent Fund			2,603.84
		$15,813.82	$15,813.82

Entrance Fees to Permanent Fund $690.00.

Fifty-seven New Members; 22 New Junior Members; 2 New Students; 11 Juniors transferred to Members.

The above receipts have been paid to the Treasurer, whose receipt the Secretary holds. The Secretary holds cash amounting to $30 included as payment under item 23 (Petty Cash) to be used as a fixed fund or cash on hand. $72.00 withholding tax and $32 Social Security, which is payable to the Collector of Internal Revenue in April, 1955 is not included in the above tabulation.

Respectfully submitted,

ROBERT W. MOIR, *Secretary*

REPORT OF THE AUDITING COMMITTEE

Boston, Mass., March 16, 1955

To the Boston Society of Civil Engineers:

We have reviewed the records and accounts of the Secretary and Treasurer of the Boston Society of Civil Engineers and we have compared the bank statement of securities held by the Boston Safe Deposit and Trust Company with the enumeration submitted by the Treasurer.

We have found them to be in order and to account accurately for the Society's Funds.

Respectfully submitted,

BYRON O. MCCOY

CHARLES H. NORRIS

REPORT OF THE EDITOR

Boston, Mass., March 1, 1955

To the Board of Government, Boston Society of Civil Engineers:

The JOURNAL was issued quarterly, in the months of April, July and October, 1954 and January, 1955 as authorized by the Board of Government on December 20, 1935.

During the year there have been published 21 papers presented at meetings of the Society and Sections.

The four issues of the JOURNAL contained 410 pages of papers and proceedings, 7 pages of Index and 62 pages of advertising, a total of 479 pages. An average of 1,410 copies per issue were printed.

The cost of printing the JOURNAL was as follows:

Expenditures

Composition and printing	$4,183.49*	
Cuts	985.82	
Wrapping, mailing & postage	278.87	
Editor	500.00	
Advertising Solicitor	103.84	
Copyright	16.00	
		$6,068.02*

Receipts

Receipts from sale of JOURNAL and reprints	$1,302.67	
Receipts from Advertising	2,484.44	
		$3,787.11

Net cost of JOURNAL to be paid from Current Fund $2,280.91*

*Includes $389.70 paid for printing 500 sets of Boring Data.

Respectfully submitted,

CHARLES E. KNOX, *Editor*

REPORT OF THE LIBRARY COMMITTEE

Boston, Mass., March 4, 1955

To the Boston Society of Civil Engineers:

The following is the report of the Library Committee for the year 1944-55.

Expenditures made during the year were as follows:

Subscriptions to Periodicals	$ 90.50
New Books	91.38
Bindings	136.00

Of the latter amount, $68.25 was for binding U.S.G.S. Water Supply Papers of which a section is being bound annually until eventually all are bound.

Two hundred and seven books were loaned during the year and a total of $4.96 was collected in fines.

The following books were purchased for the library:

Microbiology of Water and Sewage, 1952, Gainey & Lord
Industrial Wastes, 1953, Willem Rudolfs
Sewerage and Sewage Treatment, 7th Ed., 1953, Harold E. Babitt
Water Treatment for Industrial and Other Uses, 1951, Eskel Nordel
Public Health Engineering, Vol. II, The Food Contact, Earl B. Phelps
Engineering Mechanics, L. E. Grinter
Power Plant Engineering, F. R. Morse
Foundation Engineering, Peck, Hanson, Thornburn
Manual of Highway Construction Practices and Methods. Amer. Assoc. State Highway Officials
Engineering Contracts and Specifications, 3rd Ed. 1954. Robert W. Abbett
Proceedings of the Conference on Coastal Engineering, 4th Conference, Chicago, Illinois, 1953
Hydraulic Research, Vol. 9, 1954
Who's Who in Engineering, 1954

The following books were donated to the library:

Water Supply & Waste Water Disposal, Gordon M. Fair
Proceedings Minnesota International Hydraulic Convention, September 1953, by Metcalf & Eddy
Sewerage & Sewage Disposal, Metcalf & Eddy

At a meeting of the Board of Government held on October 25, 1954, one of the members recommended that the library be abandoned and its volumes turned over to the Northeastern University Library as a means of reducing cost to the Society. This recommendation was turned over to the Library Committee for study and report. It is the opinion of the Library Committee that it is in the best interest of the members of the Society to retain the library, that no reduction in annual costs would result unless the Society moved to smaller quarters, that the present organization of the library is satisfactory and that no changes are recommended.

Respectfully submitted,

JOHN F. FLAHERTY, *Chairman*

REPORT OF THE HOSPITALITY COMMITTEE

Boston, Mass., March 2, 1955

To the Boston Society of Civil Engineers:

The Hospitality Committee submits the following report for the year 1954-55.

Six regular meetings, a Student Night Meeting, an Annual Dinner and a joint meeting with the American Society of Civil Engineers were held during the year at the locations noted in the following table.

A collation was served by this Committee in the Lounge of the American Academy of Arts and Sciences at the meetings as noted.

A report was requested of the Hospitality Committee pertaining to the increasing deficit of the cost of these collations. A full committee meeting was held and the problem discussed at length and recommendations were submitted at the February 14th Board Meeting.

The attendance at regular meetings at the Academy averaged 120, an increase of 39 over the previous year (an increase of 50%).

SUMMARY OF MEETINGS AND ATTENDANCE

Date	Place	Meeting	Dinner
March 17, 1954	Hotel Vendome	46	170
April 13, 1954	Northeastern University	58	72
May 19, 1954	American Academy of Arts and Sciences	66	*
September 29, 1954	American Academy of Arts and Sciences	110	*
October 27, 1954	Harvard University	274	273
November 17, 1954	American Academy of Arts and Sciences	91	*
December 15, 1954	American Academy of Arts and Sciences	78	*
January 28, 1955	American Academy of Arts and Sciences	194	*
February 16, 1955	American Academy of Arts and Sciences	179	*

* Collation served in lounge after meeting.

The average attendance at all nine meetings was 131.

Respectfully submitted,

CARNEY M. TERZIAN, *Chairman*

REPORT OF COMMITTEE ON SUBSOILS OF BOSTON

Boston, Mass., March 16, 1955

To the Boston Society of Civil Engineers:

The Committee on Subsoils of Boston has continued its program of preparation of boring data for publication.

The report entitled, "Boring Data from Greater Boston: Section 5, West Roxbury and Brookline" which appeared in the JOURNAL for October, 1954, contained 3 maps and data from 539 borings.

Respectfully submitted,

DONALD W. TAYLOR, *Chairman*

REPORT OF MEMBERSHIP CENTRAL COMMITTEE

Boston, Mass., March 16, 1955

To the Boston Society of Civil Engineers:

The Membership Central Committee has held two meetings this year, one on Monday, May 24, 1954 and the other Friday, February 25, 1955.

Copies of the Brochure together with application forms have been furnished to prospective applicants by the Committee members. Application forms have also been made available at all Main Society and Section Meetings.

It is the concensus of opinion of your committee that much can be accomplished if each member of the Society will act to bring with them prospective applicants to the meetings and to assist those interested and qualified in becoming members.

At the end of the current year the new member and enrollment status is as follows:

Elected to Grade of Member	57
Elected to Grade of Junior	22
Elected to Grade of Student	2
Total new members	81

Applications on hand pending action — 32.

Your committee recommends that efforts be continued to augment the membership enrollment and in furtherance thereof that a Membership Central Committee be appointed for the ensuing year.

Respectfully submitted,

MURRAY H. MELLISH, *Chairman*

REPORT OF ADVERTISING COMMITTEE

Boston, Mass., March 11, 1955

To the Boston Society of Civil Engineers:

An effort has been made by the Advertising Committee to obtain a new solicitor following the resignation of George W. Hankinson in December, 1954.

Negotiations were made with the Tech-Rite Company to undertake the solicitation, but unfortunately in January the Tech-Rite Company declined to continue negotiations. At this writing there is no solicitor for the JOURNAL.

A draft of an agreement between the Boston Society of Civil Engineers and a prospective solicitor has been drawn up. This has been approved by the Board of Government with some suggested changes.

The following advertising has been carried in the JOURNAL during the year:

	April	July	October	Jan. 1955
Professional cards	32	33	34	34
Half page	2	2	2	2
¼ page	24	23	24	25
Full page	3	3	3	4
Inside front cover	0	0	0	0
Total pages of advertising	13	13	13	14

During the past year $2,475.60 was collected from advertisements, from which commissions totaling $95.00 were paid to the solicitor George W. Hankinson.

Respectfully submitted,

PAUL S. CRANDALL, *Chairman*

REPORT OF JOHN R. FREEMAN FUND COMMITTEE

Boston, Mass., March 16, 1955

To the Boston Society of Civil Engineers:

The John R. Freeman Fund Committee awarded a scholarship of $4200 per year to Mr. Lawrence C. Neale, Instructor in Hydraulic Engineering and Assistant Director of the Alden Hydraulic Laboratory of the Worcester Polytechnic Institute. Mr. Neale left in December and is to spend the year in the hydraulic laboratories of Europe.

At the request of the Sanitary Section, the Committee approved its program for a Seminar on Waste Water Treatment and Disposal as a suitable project for contribution by the Freeman Fund. This seminar is expected to take place next autumn.

Respectfully submitted,

HOWARD M. TURNER, *Chairman*

REPORT OF PUBLICITY COMMITTEE

Boston, Mass., March 16, 1955

To the Boston Society of Civil Engineers:

An annual report appropriately should be a review of the accomplishments of the past year assayed against previously defined objectives plus an outline of future plans. This annual report of the publicity committee will endeavor to follow this line. Unfortunately, to the best knowledge of this committee, there has never been any defined objective for the function of this committee.

Your committee therefore feels that it will be making a most significant contribution to the organization if it devotes the major portion of this report to bringing to your attention the need for such well defined objectives.

First, however, let us review the operation of the committee over the past year.

The Publicity Committee this year in the belief that it could do the Society a lot of good worked aggressively to spread abroad news of the Society's activities to its members and to non-members as well who might be interested. Prior to each principal monthly meeting of the Society the committee met at a noon luncheon to discuss and plan the publicity for that meeting. Attendance at the luncheon varied from four to six, indicating the importance the committee members attached to their work.

The Journal of the Engineering Societies of New England received weekly by the Society members is important but it does not arouse their interest sufficiently to bring them into the meetings in the number it should. Creating a desire to attend meetings is a selling job, pure and simple, which can be done by using methods familiar to publicity people. This will involve expenditures, therefore a budget, which so far as I know has never been provided. My first recommendation therefore is for a budget to build up and attract interest. None was available to your committee this year but if it had been it could have been used to excellent advantage, even as little as $100 or $200.

The Committee's work centered almost entirely on newspaper notices of the meetings and on pictures of our guest speakers with captions attached. When we began concentrating on pictures, we were successful in securing one or more Boston newspaper notices for every meeting we held. The second recommendation, therefore, is that we give major emphasis to distributing pictures of the speakers. This is an invaluable supplement, as it increases our chances of getting into the newspapers and, once in, it attracts more attention than would the story alone.

A feature story about one of our speakers in the Boston *Globe,* written by Ted Ashby, also did much to enhance the prestige of the Society, I believe.

For one of the meetings we used flyers—large, single sheet announcements in cartoon form—which the committee thought were appropriate and effective. At a later meeting we thought flyers again would be advisable to attract greater interest, but the cost to the Society was such that their use was eliminated. The Committee could readily have secured the money needed from among its members, but it would establish a precedent to plague future committees which might not be able to find the cash in the pockets of its members so readily, and the thought that a Publicity Committee member might be expected to contribute from his private funds could act as a deterrent to membership on the committee. Hence I repeat my recommendation for a small budget, especially where pictures are needed because it has happened on one or two occasions that we could secure but one picture of the speaker and several were needed so that each newspaper might have one, as a result of which the additional pictures had to be secured at the expense of the committee privately.

In addition, we feel that funds for direct mail would be well spent.

As to the future, we propose that the task of bringing the Society's activities to the attention of its members, to those in the engineering fields who are prospective members, to engineering students, and to the general public should be the function of a special Public Relations Committee. The activities of this committee would embrace not only publicity, but would include a variety of functions designed to increase the standing of this organization.

Some of the specific goals toward which the Public Relations Committee would work are the attraction of new members, inducement of youngsters to seek careers in civil engineering, furthering the education of promising students already enrolled in engineering courses, apprising the public of the contributions which civil engineers make to the economy and welfare of the community.

This work would naturally be done in conjunction with other committees of the Society such as the Membership, Program, and Awards Committee. It should not be thought that this recommendation implies that the Public Relations Committee should usurp the prerogatives of any other committee of this organization. Rather it is intended that the Public Relations Committee serve as the communications arm for these various committees, conveying in as many ways as possible the work and aims of the Society as a whole.

Various media of communications such as press, radio, and direct mail pamphlets would be handled by the Public Relations Committee for the benefit of all other committees.

The Boston Society of Civil Engineers is the oldest professional engineering Society in the country. It should also be the best, as it has been in the past. If our organization is to retain—or regain its virility, we must make a concerted effort to attract to our ranks young members of the profession who can ultimately assume the Society's leadership.

In order to attract this "new blood", we must give the indication that we are a highly worthwhile organization, that we have a progressive program through which these young men can elevate not only their own status but the status of the profession, and that we are progressive enough to let young men take positions of leadership.

This committee earnestly solicits that the organization take these recommendations under serious consideration. Instead of a Publicity Committee whose activities are confined to sending notices of meetings, we should establish a Public Relations Committee whose aim it would be to win for this organization and for the profession of civil engineering a new and greater appreciation on the part of its members; those with whom we come in contact in the pursuit of our business affairs; the students of our high schools, preparatory schools, and colleges; and the general public.

Respectfully submitted,

JOHN H. HESSION, *Chairman*

REPORT OF COMMITTEE ON REGISTRATION

Boston, Mass., March 16, 1955

To the Boston Society of Civil Engineers:

Your Committee on Registration of Engineers through its Chairman and other members has watched the proceedings of the Legislature as regards registration matters. The bills directly relating to Civil Engineers are as follows:

H-749 —Having a Surveyor on the Board of Registration. This bill was given approval by the committee on State Administration and referred to the House Committee on Ways and Means. No action as yet by that committee.

H-1989—Mandatory Registration of Surveyors. This bill was referred to the next annual session.

As an indication of the general interest in registration are the following bills:

H-703 —Registration of incorporated Master Plumbers. Leave to withdraw.

H-2243—Registration of real estate brokers. Next annual session.

H-752 —Registration of Social Workers. Next annual session.

The matter of bi-annual fees for registration was taken up with the Board of Registration but they were not interested.

Respectfully submitted,

A. B. EDWARDS, *Chairman*

REPORT OF THE EXECUTIVE COMMITTEE OF THE SANITARY SECTION

Boston, Mass., March 2, 1955

To the Sanitary Section, Boston Society of Civil Engineers:

Five meetings of the Sanitary Section were held during the year as follows:

March 3, 1954—Annual Meeting. The following officers and members of the Executive Committee were elected:

Edward W. Moore, Chairman
Ariel A. Thomas, Vice-Chairman
Darrell A. Root, Clerk
Joseph C. Knox, Member
John F. Flaherty, Member
Clair N. Sawyer, Member

A symposium on "Air Pollution" was presented by the following speakers: Charles R. Williams, Richard Dennis and Leslie Silverman.

Fifty members and guests attended the meeting and eighteen members and guests attended the informal dinner at Patten's.

June 2, 1954—This meeting was in lieu of the Annual Outing and consisted of a paper entitled, "Problem of a Metropolitan District System of Refuse Disposal Incinerators", presented by Charles A. Turner.

Thirty-eight members and guests attended the meeting and eighteen members and guests attended the informal dinner.

October 6, 1954.—Clair N. Sawyer presented the paper entitled "High Rate Sludge Digestion" prepared by Mr. Sawyer and Harold E. Schmidt, and there was a considerable discussion of this paper among the members present.

Sixty-seven members and guests attended the meeting and twenty members and guests attended the informal dinner.

December 1, 1954.—A paper entitled "Pollution Abatement Program at Newport, Rhode Island" presented by Edwin B. Cobb and prepared jointly by Mr. Cobb and Dean Coburn.

Forty-seven members and guests attended the meeting and twenty-eight members and guests attended the informal dinner.

A nominating committee consisting of William E. Stanley, Chairman, Fozi M. Cahaly and John S. Bethel, Jr., was elected and directed to present a slate of officers and members of the Executive Committee at the Annual Meeting of the Section to be held March 2, 1955.

December 15, 1954.—Joint Meeting with the Main Society.

Mr. Frank L. Heaney presented a paper entitled, "Improvements to New Bedford Sewerage Facilities Including Interceptor, Sewers and Pumping Stations".

Sixty-eight members and guests attended the meeting.

Total attendance for the five meetings was 280 and the average attendance per meeting for the year was 56.

Five meetings of the Executive Committee were held during the year.

The papers prepared for five of these meetings were presented to the Society for publication.

Respectfully submitted,

DARRELL A. ROOT, *Clerk*

REPORT OF THE EXECUTIVE COMMITTEE OF THE STRUCTURAL SECTION

Boston, Mass., March 9, 1955

To the Structural Section, Boston Society of Civil Engineers:

Eight meetings of the Structural Section were held during the year as follows:

April 14, 1954.—Dr. T. William Lambe of the Civil Engineering Department of M.I.T., presented a paper entitled "The Improvement of Soil Properties with Dispersants". Attendance 33.

May 4, 1954.—Joint Meeting of the Structural, Hydraulics and Transportation Sections. Professor A. W. Skempton, D. Sc. of the University of London, presented a paper entitled "Civil Engineering, 1500-1900; an Historical Sketch". Attendance 135.

October 13, 1954.—Professor John M. Biggs, Mass. Institute of Technology, presented a paper entitled "Buckling Considerations in the Design of Steel Beams and Plate Girders". Attendance 39.

November 9, 1954.—Professor Myle J. Holley, of Mass. Institute of Technology, presented a paper entitled "Relative Strength of Prestressed and Ordinary Concrete". Attendance 65.

December 8, 1954.—Albert G. H. Dietz, Professor of Building Engineering and Construction, M.I.T., presented a paper entitled "Plastics in Building". Attendance 30.

January 12, 1955.—Mr. Albert P. Richards of the William F. Clapp Laboratories, Duxbury, Mass., presented a paper entitled "Biological Deterioration of Structural Materials in Maritime Environments". Attendance 38.

February 16, 1955.—Joint Meeting with the Main Society. Dr. Karl Terzaghi, Professor of Civil Engineering at Harvard University, presented a paper entitled "Evaluation of the Coefficient of Subgrade Reaction". Attendance 79.

March 9, 1955.—Annual Meeting. The following officers were elected for the coming year:

Chairman	—	Casimir J. Kray
Vice-Chairman	—	Augustine L. Delaney
Clerk	—	John M. Biggs
Executive Committee	—	John G. Jarnis
		Richard W. Albrecht
		William A. Henderson

Dr. L. Don Leets, Professor of Geology and Seismologist in charge of the Harvard Seismograph Station, presented a paper entitled "Effects of Blasting Vibrations". Attendance 67.

The total attendance for the year was 694; average attendance 72.

Respectfully submitted,

AUGUSTINE L. DELANEY, *Clerk*

REPORT OF THE EXECUTIVE COMMITTEE OF THE HYDRAULICS SECTION

Boston, Mass., February 21, 1955

To the Hydraulics Section, Boston Society of Civil Engineers:

The following meetings were held during the year:

May 4, 1954.—Joint. Meeting of Hydraulics, Structural and Transportation Sections. Professor A. W. Skempton of the University of London spoke on "Civil Engineering 1500-1900; an Historical Sketch". Attendance 135.

November 3, 1954.—Dr. D. R. F. Harleman, Assistant Professor of Hydraulics at M.I.T., spoke on "The Effect of Baffle Piers on Stilling Basin Performance". Attendance 34.

February 2, 1955.—Annual Meeting of the Section and election of officers. The following were elected for the coming year:

Ralph S. Archibald, Chairman
Joseph C. Lawler, Vice Chairman
Clyde W. Hubbard, Clerk
David R. Campbell, Executive Committee
Dean F. Coburn, Executive Committee
James W. Daily, Executive Committee

Mr. Clyde W. Hubbard, Hydraulic Engineer, Stone & Webster Engineering Corp., spoke on "Field Tests on Rock Island Hydro-electric Plant", and Mr. R. E. Nece, Instructor, Dept. of Civil and Sanitary Engineering, M.I.T., spoke on "Hydraulic Performance of Check and Control Values". Attendance 47.

Total attendance for the year was 216. Average attendance was 72.

Respectfully submitted,

JOSEPH C. LAWLER, *Clerk*

REPORT OF THE EXECUTIVE COMMITTEE OF THE SURVEYING AND MAPPING SECTION

Boston, Mass., February 23, 1955

To the Surveying and Mapping Section, Boston Society of Civil Engineers:

The following meetings were held during the past year:

April 7, 1954.—Mr. Wilbur C. Nylander spoke on "The Middlesex Canal". Attendance 40.

May 19, 1954.—At a Joint Meeting with the Main Society Mr. Karl R. Kennison gave an illustrated report on "Problems that New York City Faces in Expanding its Water Supply". Attendance 66.

October 20, 1954.—John Clarkeson, President of Clarkeson Engineering Company, Inc., delivered a paper on "Use of Aerial Photogrammetry in the Location and Design of a Section of the Massachusetts Turnpike". Attendance 52.

January 26, 1955.—A Joint Meeting was held with the Main Society where papers were given by William F. Uhl, George R. Rich and VanCourt Hare of the firm of Uhl, Hall and Rich, an affiliate of Chas. T. Main, Inc., on "The Hydro-Electric Development of the St. Lawrence River". Attendance 194.

The total attendance was 352; average attendance 88.

Respectfully submitted,

L. T. SCHOFIELD, *Clerk*

PROFESSIONAL SERVICES
AND
ADVERTISEMENTS

The advertising pages of the JOURNAL aim to acquaint readers with Professional and Contracting Services and Sources of Various Supplies and Materials. You would find it of advantage to be represented here.

Volume 42 APRIL, 1955 Number 2

BOSTON SOCIETY OF CIVIL ENGINEERS

FOUNDED 1848

PROFESSIONAL SERVICES

INDEX TO ADVERTISERS

1848 BOSTON SOCIETY OF CIVIL ENGINEERS

SOCIETY JEWELRY AVAILABLE

10K Gold Pins, Watch Charms, and Lapel Buttons and 10K Gold faced Tie Bars bearing the Society's Emblem are now available. Make checks payable to Boston Society of Civil Engineers.

Boston Society of Civil Engineers
715 Tremont Temple
88 Tremont Street
Boston, 8, Mass.

Enclosed is my check for $............ for the purchase of:

Badge (pin)	**$5.00**
Badge (watch charm)	**5.00**
Badge (lapel button)	**5.00**
Tie Bar	**4.00**

Prices listed include 10% Federal Excise Tax.

Name ....
(please print)

Address ....

City and State ....
(where emblem is to be sent)

COMMITTEES

1955-1956

NOMINATING COMMITTEE

Past Presidents (Members of the Committee)

Harrison P. Eddy, Jr. — Thomas R. Camp — John B. Wilbur

James F. Brittain
Frank L. Lincoln
Henry A. Mohr
(Term expires March, 1956)

Allen J. Burdoin
Fozi M. Cahaly
Eliot F. Childs
(Term expires March, 1957)

SPECIAL COMMITTEES

PROGRAM

Edwin B. Cobb, *Chairman, ex-officio*

George R. Rich
Harold A. Thomas
Edward C. Keane
James Adam, Jr.
Donald W. Taylor
Fozi M. Cahaly
Robert W. Moir
Ariel A. Thomas
Casimir J. Kray
Ralph S. Archibald
Alexander J. Bone

PUBLICATION

Charles E. Knox, *Chairman*

Henry I. Wyner
Dean F. Coburn
Robert L. Meserve
Stephen E. Dore, Jr.
Ariel A. Thomas
Casimir J. Kray
Ralph S. Archibald
Alexander J. Bone

LIBRARY

George M. Reece, *Chairman*

Roland S. Burlingame
Henry R. Wheeler
Clifford S. Mansfield
Lorin M. Pittendreigh
Otto O. Pasanen
Llewellyn T. Schofield
Thomas C. Coleman
Myle J. Holley

HOSPITALITY

Roger C. Collette, *Chairman*

David H. Hamilton — Frank T. Smith, Jr. — Llewellyn L. Cross, Jr.

DESMOND FITZGERALD AWARD

Gordon M. Fair, *Chairman*

John B. Wilbur — John B. Babcock, 3rd.

SANITARY SECTION AWARD

Gordon M. Fair, *Chairman*

Kenneth F. Knowlton — William E. Stanley

STRUCTURAL SECTION AWARD

John B. Wilbur, *Chairman*

Albert Haertlein — Earle F. Littleton

HYDRAULICS SECTION AWARD

Gordon M. Fair, *Chairman*

Ariel A. Thomas — Frank L. Heaney

SURVEYING & MAPPING SECTION AWARD

John B. Babcock, 3rd, *Chairman*

Charles M. Anderson — Paul A. Dunkerley

TRANSPORTATION SECTION AWARD

John B. Wilbur, *Chairman*

John D. M. Luttman-Johnson — George W. Hankinson

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Irving B. Crosby
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Charles O. Baird, Jr., *Chairman*

Edwin B. Cobb — Donald W. Taylor

MEMBERSHIP CENTRAL COMMITTEE

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James L. Dallas
Frank L. Lincoln
James F. Brittain
Chester A. Moore
Robert A. Bierweiler
Edward W. Moore
George W. Hankinson
Percival S. Rice
John M. Biggs
Elliot F. Childs
Robert W. Moir

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Charles H. Norris — Thomas A. Berrigan

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Ferdinand A. Trautner
Herbert E. Fletcher, Jr.
Paul S. Crandall
Benjamin A. Lekesky
Joseph C. Knox
Albert Adelman
John C. Finn

JOINT LEGISLATIVE COMMITTEE

(formerly Committee on Registration Law)

Athole B. Edwards — Edward Wright

PUBLIC RELATIONS COMMITTEE

John H. Hession, *Chairman*

Joseph A. Dunn
Frank H. Dillaby
Frank Marcucella
Thomas C. Coleman
George W. Coffin