THE NEW ACI CODE — ITS IMPLICATIONS AND RAMIFICATIONS

By Howard Simpson,* Member

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Introduction

In 1956 the American Concrete Institute revised its Building Code requirements for Reinforced Concrete. Some of the changes are of considerable importance. It is the purpose of this paper to discuss the more significant of these, with particular attention given to an evaluation of the magnitude of their effects and to precautions which should be observed in their application.

BEAMS

ULTIMATE STRENGTH DESIGN

Undoubtedly the most significant of the revisions is the addition of the single sentence in Section 601(b):

"The ultimate strength method of design may be used for the design of reinforced concrete members."

The latitude which this provision gives the designer can be more fully appreciated when it is realized that the Code does not specify what factors of safety or load factors must be employed when using the ultimate strength method. The Code Committee does append to the Code an abstract of the Report of the ACI-ASCE Joint Committee on Ultimate Strength Design,² thus tacitly recommending, but not requiring, its use.

A. Load Factors

Section A604 of the Appendix states in effect that the ultimate strength of members not subject to wind or earthquake loading should be the larger of the values obtained by the following formulas:

^{*}Associate Professor of Structural Engineering, Massachusetts Institute of Technology. Principal—Simpson, Gumpertz & Heger, Consulting Engineers, Cambridge, Massachusetts.

Similar equations are given for members subject to wind or earthquake loadings.

These equations enable the designer to take into account the fact that one can usually predict far more accurately the magnitude and distribution of dead loads than of live loads, and that there is a reduced probability that the maximum wind or earthquake loads would occur simultaneously with the maximum gravity loading.

If the live load equals or is less than the dead load, Equation (2) governs; that is, the computed ultimate strength must be at least 1.8 times the total load. If the live load is greater than the dead load, Equation (1) governs. Thus the required overall factor of safety of members without wind or earthquake loads will theoretically vary between 1.8 and 2.4, depending upon the ratio of live to dead load. If this ratio is 3, a value which is rarely exceeded in practice, the required ultimate strength is 2.1 times the total load.

B. Ultimate Strength Equations

Section A605(b) of the Appendix gives for the ultimate resisting moment of a singly reinforced concrete beam

The above value of p_{max} is about 12 percent less than that necessary to develop the maximum resisting moment of the concrete thus providing some protection against downward variations in concrete strength.

The effect of the ultimate strength method on the dimensioning of beams subject only to dead and live gravity loading will now be considered.

C. Comparison Between Elastic and Ultimate Strength Design

One of the most significant effects of the employment of ultimate strength design is the possible use of much smaller concrete sections, without the necessity for providing compression steel. Assuming a concrete strength of 3000 psi, the ACI elastic theory requires that the maximum resisting moment of the concrete at working loads shall not exceed

$$M = 236 \text{ bd}^2 \dots (4)$$

The corresponding equations for the ultimate strength theory are, for example:

for
$$\frac{L}{B} = 1$$
 : $M = 509 \ bd^2$ (5)

for
$$\frac{L}{B} = 3$$
: $M = 436 \text{ bd}^2 \dots (6)$

Note that Equations (5) and (6) give maximum working load moments, and hence can be compared directly with Equation (4). They were derived by substituting in Equation (3) $f'_{c} = 3000$ and the maximum permissible value of p, then dividing by the factors of safety required for the respective load ratios assumed.

The use of Equations (5) and (6) is equivalent to increasing the allowable concrete stress at working load from .45 f'_{o} to .77 f'_{o} and .69 f'_{o} , respectively. (Maximum steel stress assumed retained at 20,000 psi.)

It should be noted that the steel ratio, p, required to develop the concrete resisting moments given by Equations (5) and (6) is .030 for intermediate grade steel and .024 for hard grade steel. These "balanced design" steel ratios are considerably larger than the .0136 required to develop the smaller maximum resisting moments permit-

ted by the elastic theory. It is thus apparent that to avoid difficulties with steel placement, beams developing ultimate strength balanced design must be proportioned relatively wide and shallow. Shallow beams require larger steel areas, and are not usually economical; also, they may have excessive deflection. Furthermore, smaller concrete areas require more diagonal tension reinforcement. Therefore, except where it is necessary or desirable to severely limit beam depth, such as where headroom is restricted or in slab band construction, it will not ordinarily be practical or economical to take full advantage of the reduction in section dimensions permitted by ultimate strength design. If for the sake of argument, however, these problems are temporarily ignored, the ultimate strength method would permit, in the instance of a given beam designed for elastic balanced design, a 54 percent reduction in width or a 32 percent reduction in effective depth if L/B is equal to or less than unity, and a 46 percent reduction in width or a 26 percent reduction in depth if L/B equals 3. This is illustrated in Figure 1, together with the relative steel quantities required.

Since various considerations will usually dictate the use of sections which are under-reinforced according to the ultimate strength theory, the comparison of the two theories will now be extended to this type of section.

Defining M as the maximum allowable moment at working loads, the two curved lines in Figure 2 give the relation between M/bd^2 and p as obtained by dividing the expression for M_u in Equation (3) by the factors of safety 1.8 and 2.1, respectively (corresponding to L/B=1 and L/B=3). Intermediate grade steel and 3000 psi concrete are assumed. For comparison, the straight lines plot the elastic theory steel design equation

$$A_s = \frac{M}{f_s j d} \dots (7)$$

or, assuming j is constant at .87,

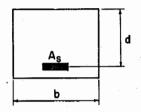
$$p = \frac{M}{bd^2} \times \frac{1}{.87f_s} \dots (7a)$$

These elastic theory curves are extended beyond the limits corresponding to elastic balanced design, and hence into regions where this theory requires compression steel. They are somewhat approximate in that they do not take into account the variation in j. Nevertheless,

they serve as a means for comparing the tensile steel areas required by the two theories for a given moment and concrete section. For example, at the point where an ultimate strength curve crosses the $f_s=20,000$ psi line, the area of tensile steel required by the ulti-

ELASTIC THEORY

p = .0136



ULTIMATE STRENGTH THEORY

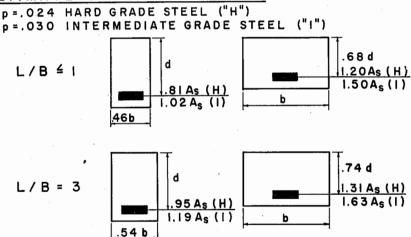
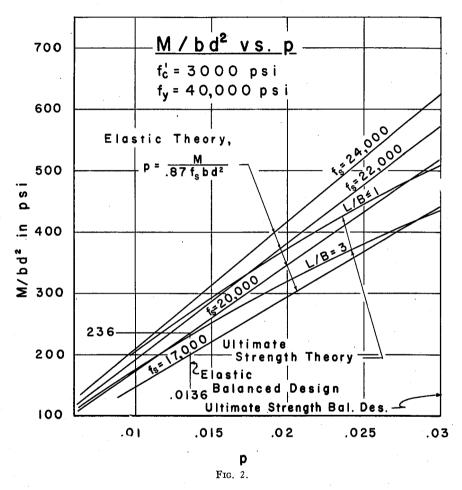


Fig. 1.—Comparison of "Balanced Design" Sections. All Sections Sustain the Same Working Load Moment.

mate strength theory equals that required by the elastic theory, if b and d are the same. Similarly, an intersection with the 22,000 psi line indicates a point at which the ultimate theory requires 20/22 as much tensile steel as the elastic theory. It should be kept in mind that the elastic theory also requires compression steel whenever p is greater than .0136.

Figure 2 shows that for intermediate grade steel and the values of M/bd^2 usually employed in elastic theory designs (equal to or less than 236 for 3000 psi concrete), ultimate strength design in effect permits a steel working stress of between 20,000 and 24,000 psi. For



the smaller ratios of L/B some increase over the currently used 20,000 psi working stress is allowed even when M/bd² is greater than 236.

Figure 3 gives the corresponding curves for hard grade steel. It is at once apparent that in contrast to the ACI elastic theory, ultimate strength design permits full advantage to be taken of the greater

yield strength of hard grade steel. The ultimate strength curves lie everywhere above the $f_s=20,000$ psi line, and for the usual values of M/bd², the effective steel working stress is between 23,000 and 29,000 psi.

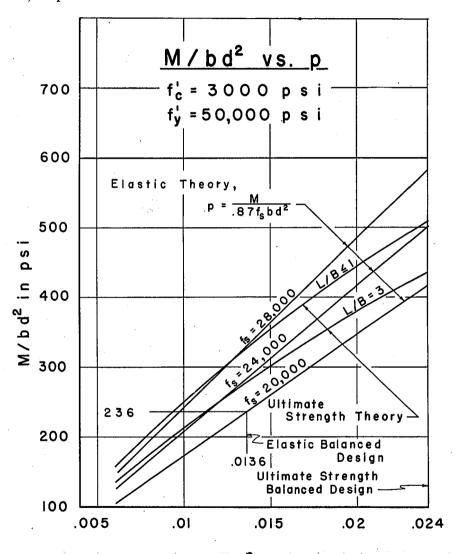


Fig. 3.

It can therefore be concluded that for a beam of a given depth, the ultimate strength theory usually will require less steel area and beam width than will the elastic theory, particularly if hard grade steel is used. Of course, it may not always be possible or economical to take maximum advantage of the permitted reduction in beam width, because of possible difficulties with steel placement and diagonal tension, etc., and because the steel area required by the ultimate strength theory increases as the beam width is decreased.

D. Recommended Precautions

The use of steel and concrete working stresses considerably larger than those with which we have built up a considerable backlog of experience requires increased care in design and construction. Some of the factors requiring particular attention are discussed below.

1. Concrete Strength

The effect of a downward deviation in concrete strength is potentially considerably more serious with the increased concrete working stresses which are, in effect, permitted by the ultimate strength method. This fact is recognized by the comparatively severe control requirements of paragraph A602(f) in the Appendix to the Code:

"Controlled concrete should be used and shall meet the following requirements. The quality of concrete shall be such that not more than one test in ten shall have an average strength less than the strength assumed in the design, and the average of any three consecutive tests shall not be less than the assumed design strength. Each test shall consist of not less than three standard cylinders."

2. Buckling of Compressive Steel

Steel located in the compressive face of a beam, even though not necessary for or considered in the design, can buckle at moments less than the calculated ultimate, thus disrupting and weakening the member. Such steel should be adequately restrained by stirrups or ties.

3. Diagonal Tension and Bond

No recommendations are given for the ultimate strength theory of diagonal tension and bond; it is assumed that for the present the customary elastic theory will be employed on an interim basis. But the computed ultimate bending moment usually cannot be developed without considerable yielding and plastic flow, during which the diagonal tension and bond strength must be enough to maintain the integrity of the section. Because of this, and because of the relative paucity of test data supporting the use of the customary allowable diagonal tension and bond stresses in beams with large steel percentages and working stresses, particular care must be exercised. Whitney and Cohen³ recommend the following precautions in addition to the more severe diagonal tension and bond provisions of the new Code:

"Web reinforcement shall be provided from the support to a point beyond the extreme position of the point of inflection a distance equal to either 1/16 of the clear span or the depth of the member whichever is greater even though the shearing stress does not exceed v_c ... Where required by this paragraph, the amount of web reinforcement at each section shall be [at least]:

- 1. Sufficient to carry 2/3 of the total shear where the unit stress exceeds v_e .
- 2. Sufficient to carry 2/3 of the total shear existing at the point of inflection, that is, the ratio of web reinforcement required at the point of inflection will be maintained back to the support . . ."

"This addition is not intended to apply to small T-beams forming part of a joist floor construction. The use of bent-up bars for diagonal tension reinforcement is desirable and should be used where practical."

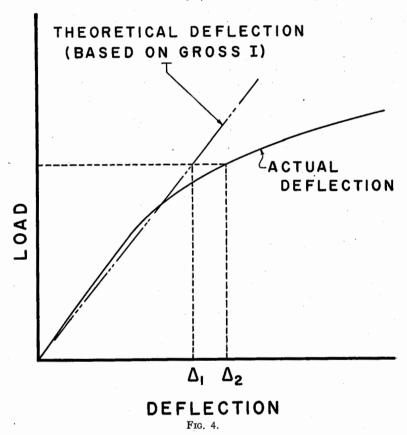
In order to resist longitudinal tension due to volume changes Whitney and Cohen also recommend that at least half of the positive reinforcement lap with at least one-third of the negative reinforcement on the opposite face for a distance not less than the depth of the beam.

If high tensile steel and bond stresses exist simultaneously at working loads, there is a possibility that bond creep may eventually cause an objectionable opening up of transverse cracks in the concrete adjoining the tensile steel. Until a sufficient backlog of experience is built up, it would be wise to avoid a combination of high tensile and bond stresses at working loads.

4. Deflections

The use of smaller sections means more instances where deflection must be checked. Furthermore, the usual assumptions made for

deflection calculations may not be sufficiently accurate. Elastic deflections are usually calculated on the basis of the gross moment of inertia of the section; because of the increased number of concrete cracks which exist at higher steel stresses (even though they may be almost invisible to the naked eye) the use of the gross moment of



inertia may be considerably in error. For example, in Figure 4, the actual deflection \triangle_2 is much larger than the calculated deflection, \triangle_1 . Also, the usual allowances⁴ for creep may be too small for sections with high concrete working stresses.

5. Elastic and Plastic Flexibility

Most structures are subject to some differential settlement. This settlement, while not usually considered in design, is sometimes of

sufficient magnitude to develop considerable plastic deformation (yield hinges). According to the principles of limit design, members which can sustain this plastic deformation without failure suffer no reduction in their ability to sustain gravity loads. But smaller, heavily reinforced beams, while more flexible elastically, cannot sustain as much plastic rotation before failure.⁵ Therefore high percentages of reinforcement should be used only in structures on good foundations, or where a careful evaluation is made of the possible magnitudes of the differential settlements and their effects. In the latter case, the section rotation angle at ultimate moment must be calculated^{5,6,7} and compared with the requirements of the particular application.

Another undesirable characteristic of beams with high percentages of reinforcement is the possibility that they may give insufficient warning prior to bending failure. Large deflections under overloads are desirable; heavily reinforced beams in general have smaller section rotation angles at ultimate moment and therefore less deflection prior to failure.

6. Fatigue

Although this subject is not discussed in the Code, it is important that suitable limits be placed on the stresses at working loads in members subject to fatigue loading, such as elevator machinery supports.

E. T-Beams

True T-beams will occur only very rarely if the ultimate strength design method is used, since there ordinarily will not be enough room for sufficient steel to bring the neutral axis at failure into the stem. Formulas for T-beams are presented in Section A607 of the Appendix.

LIMIT DESIGN

The Code does not as yet permit the use of limit design, which is the taking into account of the redistribution of bending moments and forces in a structure as failure approaches. Section A601(b) states:

"It is assumed that external moments and forces acting in a structure will be determined by the theory of elastic frames."

SHEAR (DIAGONAL TENSION) AND BOND

There have been a number of failures recently in certain rigid frame structures^{8,9} which were apparently designed in accordance with the ACI Code. These failures occurred near the point of inflection

under shears less than that permitted by the concrete alone. There is still some doubt concerning the exact cause of these failures, although they were apparently due to tensile stress caused by volume changes in restrained members. The Portland Cement Association has undertaken a test program which promises to throw additional light on this matter. Meanwhile, however, certain additions and changes have been effected in the shear and bond provisions which, it is believed, will eliminate this type of failure in the future.

Section 801(d) requires that when the shearing stress exceeds that permitted for the concrete alone, web reinforcement shall be provided for a distance equal to the depth, d, of the member beyond the point theoretically required.

Section 801(e) states:

"Where continuous or restrained beams or frames do not have a slab so cast as to provide T-beam action, the following provisions shall apply. Web reinforcement shall be provided from the support to a point beyond the extreme position of the point of inflection a distance equal to either 1/16 of the clear span or the depth of the member, whichever is greater, even though the shearing unit stress does not exceed v_c. Such reinforcement shall be designed to carry at least two-thirds of the total shear at the section. Web reinforcement shall be provided sufficient to carry at least two-thirds of the total shear at a section in which there is negative reinforcement."

Section 807 requires that when web reinforcement is necessary, the amount shall be not less than 0.15 percent of the spacing multiplied by the beam width. This is equivalent to saying that when the shearing stress exceeds the amount that can be taken by the concrete, the assumed excess shear must not be less than 30 psi.

The following statement has been added to Section 902(a):

"At least one-third of the total reinforcement provided for negative moment at the support shall be extended beyond the extreme position of the point of inflection a distance sufficient to develop by bond one-half the allowable stress in such bars, not less than 1/16 of the clear span length, or not less than the depth of the member, whichever is greater."

FLAT SLABS

Chapter 10 on Flat Slabs has been completely rewritten. Some of the more significant changes are discussed herewith.

DESIGN BY ELASTIC ANALYSIS

A. Calculation of Design Moments

The 1951 Code permitted the design negative moment to be taken at a distance .073 L + .57 A from the column centerline, where A was usually one-half the column capital diameter. This provision sometimes resulted in too low a design moment for flat plate floors. The new Code states that the critical negative moment should be computed at a distance A from the support centerline, where A has been redefined so as to be usually equivalent to one-half the column capital diameter plus the depth of the drop panel (if any) plus one-half the slab thickness. The new provision is approximately equivalent to the old when the capital diameter is .225 L, the drop panel thickness L/80, and the slab thickness L/40, but gives more conservative results for shallower or no drop panels, or when smaller or no column capitals are used.

In the 1951 Code, a formula was given for the required minimum sum, M_{\circ} , of the maximum positive and average maximum negative bending moments. This formula gave larger values than the formula specified for the empirical method of design. Because of the more conservative assumption for the location of the critical sections, there is no longer a minimum requirement for M_{\circ} when the elastic analysis method is used.

B. Apportionment of Moments

A new table is provided which gives the percentage of total moment at any section to be distributed to the column and mid strips. No distinction is drawn between slabs with and without drop panels.

DESIGN BY EMPIRICAL METHOD

A. Limitations

The maximum variation in successive span lengths has been changed from 20 percent of the shorter span to 20 percent of the longer span. Also, columns can now be offset as much as 10 percent of the span in the direction of the offset.

B. Minimum Slab Thickness

In Section 1004(d) two rather involved equations are now given for the minimum thickness of slabs with and without drop panels. These equations were each derived from a strength calculation for a

slab of certain assumed minimum proportions. M_{\circ} was calculated from the empirical equation given in Section 1004(f) (discussed below). The design coefficients were taken as .50 for the slab without drops (which is the tabulated coefficient at the first interior column) and .38 at the edge of the drop for the slab with drop panels. The latter coefficient is an average value obtained from studies, based on the tabulated coefficient of .56 for the moment at the centerline of the first interior column.

C. Calculation of M_o

The revised equation for M_o,

$$M_o = 0.09 \text{ WLF} \left[1 - \frac{2c}{3L} \right]^2$$

differs from the old one in the inclusion of the factor F, which is equal to

$$1.15 - c/L$$

where c is usually equal to the column capital diameter, or to the column diameter if capitals are not used. The purpose of this provision is to provide additional protection when the column capitals are small or absent.

D. Empirical Coefficients

The table of coefficients for empirical design has been considerably changed and enlarged for exterior panels and panels continuous across beams or walls.

E. Reinforcement

Tables and diagrams are now provided, giving details of steel lengths, bends and cut-offs. This information was calculated from elastic studies, and makes a very useful addition to this chapter.

F. Columns for Flat Slabs

An assumption implicit in the empirical equation for M_o is that the columns can be depended upon for a certain resistance to rotation. Unless sufficient column stiffness and strength are provided, use of the empirical coefficients may prove unsafe, particularly for large ratios of live to dead load. The new Code formally acknowledges this in the following provisions, which are based on extensive studies:

"1. The minimum dimension of any column shall be 10 in. For columns or other supports of a flat slab, the required minimum average moment of inertia, I_c , of the gross concrete section of the columns above and below the slab shall be determined from the following formula, and shall be not less than 1000 in. If there is no column above the slab, the I_c of the column below shall be twice that given by the formula with a minimum of 1000 in.

$$I_{e} = \frac{t^{3}H}{0.5 + \frac{W_{D}}{W_{L}}}$$

 $(W_D = total dead load on panel; W_D = total live load on panel.)$

"2. Columns supporting flat slabs designed by the empirical method shall be proportioned for the bending moments developed by unequally loaded panels, or uneven spacing of columns. Such bending moment shall be the maximum value derived from

$$(WL_1 - W_DL_2) \frac{1}{f}$$

 L_1 and L_2 being lengths of the adjacent spans ($L_2 = 0$ when considing an exterior column) and f is 30 for exterior and 40 for interior columns. (W = total dead and live load on panel.)

"This moment shall be divided between the columns immediately above and below the floor or roof line under consideration in direct proportion to their stiffness and shall be applied without further reduction to the critical sections of the columns."

COLUMNS

ELASTIC THEORY

A. Uncracked Sections

ACI 318-51 allowed columns for which the eccentricity ratio e/t is equal to or less than unity to be designed on the basis of the uncracked section. This sometimes resulted in a dangerously low factor of safety in cases where e/t was nearly one. The revised Code has reduced this limiting value of e/t to 2/3.

The formulas for the design of eccentrically loaded uncracked sections have been replaced by an interaction equation similar in form to that used for the design of steel columns:

$$f_a/F_a + f_b/F_b = 1$$

where

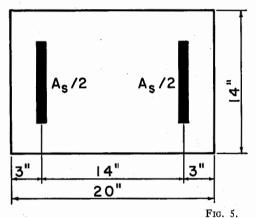
 f_a = nominal axial unit stress

 F_a = nominal allowable unit stress

f_b = bending unit stress (bending moment divided by transformed section modulus)

 F_b = allowable stress for pure bending

This equation gives results identical to those obtained from the old equations. In order to illustrate the proper application of the new



TIED COLUMN

f'_c= 3000 psi f_y= 40,000 psi A_s= 5.60 sq. in. p = 0.02 e = 5"

formula, the allowable capacity of the section shown in Figure 5 is

$$\begin{array}{l} f_a &= N/A_g = N/20 \text{ x } 14 = N/280 \\ F_a &= .8A_g \; (.225 \; f'_c + 16,000 \; p)/A_g \\ &= 18 \; f'_c + 12,800 \; p = 796 \; psi \\ I &= bt^3/12 + (n-1) \; A_s \; (14/2)^2 = 11,800 \; in.^4 \\ S &= 2I/t = 11,800/10 = 1180 \; in.^3 \\ f_b &= M/S = M/1180 \\ F_b &= .45 \; f'_c = 1350 \; psi \\ f_a/F_a + f_b/F_b = N/280 \; x \; 796 + M/1180 \; x \; 1350 \\ N/223 + 5N/1592 = 1 \; \; (N \; in \; kips) \\ N &= 131 \; kips \end{array}$$

B. Cracked Sections

calculated below.

For the purpose of taking into account the effect of plastic flow as ultimate load is approached (as is done in the elastic design of beams with compression steel), the new Code permits the modular ratio for the compressive reinforcement to be assumed at double the value given in Section 601. It is important to note that this value of modular ratio is to be used when computing the depth of the compressive stress block, as well as when taking the statical summation of the forces acting on the cross-section; otherwise, serious errors may result.

The allowable compressive stress in the concrete is given as .45 $f'_{\rm e}$, instead of as a function of eccentricity. This simplifies calculations, without encroaching unduly on the factor of safety.

ULTIMATE STRENGTH THEORY

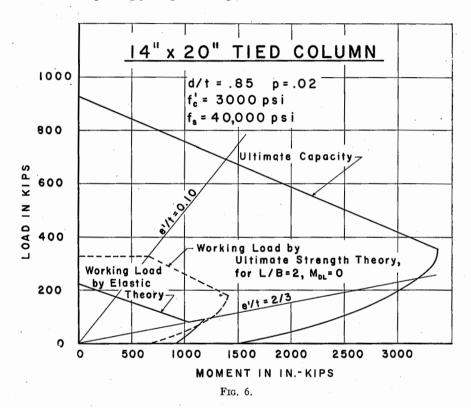
The ultimate strength theory offers a far more rational basis for column design than does the elastic theory. This has been reflected, in part, by the axially loaded column equation which has been in the Code for many years. The equations and load factors given in the Appendix now make it possible to design all columns with suitable factors of safety based on their ultimate strengths. The use of proper combinations of load factors, as provided in Section A604 of the Appendix, is especially important for columns, for which the critical loading condition is not always that in which all loads are at their maximum values.

The ultimate strength equations, while quite complex in appearance, are readily solved with the aid of published graphs.³ In consideration of the unavoidable accidental eccentricities always present in columns, axially loaded columns are designed as eccentrically loaded columns with eccentricities of .05 times the depth for spirally reinforced columns, and .10 times the depth for tied columns.

Figure 6 compares, for a typical column, the ultimate load with the allowable loads by the elastic theory and by the ultimate strength theory. The elastic theory appears to provide an excessive factor of safety at low values of moment, and a dangerously low value as pure bending is approached. (The low value is somewhat fictitious, however, since for this particular section, the ultimate strength equation gives overly conservative results at low values of P.)

The discontinuity in the elastic theory curve in Figure 6 is due to the change in assumptions at e/t = 2/3. In this case, the permissible increase in load due to the doubling of the compression steel mod-

ular ratio exceeds the decrease due to the cracked section assumption. This sudden change in capacity as e/t if increased may be either negative or positive, depending upon the section dimensions. In the case of columns with high steel ratios, the change is frequently an increase surprisingly large in magnitude.



PIPE COLUMNS

The formula given in the 1951 Code for columns consisting of steel pipe filled with concrete sometimes gave a lower allowable load than that obtained by omitting the concrete and using the column formulas specified by the American Institute of Steel Construction. This formula has been replaced by one based on an investigation sponsored by the Housing and Home Finance Agency, the report of which appears in Reference 10.

.Conclusion

The 1956 Code contains a number of important changes, and represents a major step forward in concrete design specifications. Nevertheless, there is still work to be done, particularly in connection with diagonal tension and shear.

The most significant of the new revisions is the provision permitting ultimate strength design. The use of this method will give designs which have more rational and consistent factors of safety. Considerable changes in section proportioning can result. There will be an increased demand for hard grade steel for beams.

Engineers employing ultimate strength design must proceed with caution. Here experience and good judgment will be more important than ever. Increased attention must be given to concrete quality, deflections at working loads, diagonal tension and bond, the possibility of fatigue loading or stress reversals, and foundation conditions. Greater consideration will have to be given to the economics of section proportioning, since the balanced design section will rarely be economical.

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