# HISTORY AND BASIC THEORY OF PRESTRESSED CONCRETE

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Let us first consider the history of prestressed concrete and its present day application in the long lines method of prestressing concrete as we know it today. The first patent was taken out by P. H. Jackson, of San Francisco, in 1886, and in 1920 and 1925 R. H. Dill, of Alexandria, Nebraska, took out patents on the method of pretensioning. The development of prestressed concrete was slow because the initial approach was made using low carbon rods having a relatively short elongation within the proportional limit of the steel used. This meant that the effect of the prestressing force was partially nullified by the effect of creep and shrinkage of the concrete.

Later in Europe, Freyssinet and others approached the problem with the use of high carbon cold drawn wire having a relatively long elongation within the proportional limits of the steel. When allowance was made for creep and shrinkage of the concrete a high percentage of the initial stress in the wire was available for inducing compressive stresses in the concrete.

In Europe such names as Freyssinet, Magnel, Billig, Hewitt and Hoyer were common with the development of prestressed concrete, while in the United States such names as Preload, Roebling, Schorer and Billner have been associated with its development. Today its use has been demonstrated on a multiplicity of structures and now it remains for American engineers and contractors to exploit its possibilities here.

Prestressed concrete is simple in its application and design and I think that we may pause for a while and consider the basic theory of prestressed concrete.

If we consider a short compression member with a force "P" applied eccentrically to one of its axis, resolve the force into a concentric force "P" and a couple with a moment equal to "Pe", draw the stress diagram for the individual force and the couple, then draw the final stress diagram for the combined effect making the assumption that the distance of "e" is taken as a special case and is equal to b/6, then the total stress at either side will be

$$S = S_1 \pm S_2$$

$$S = \frac{P}{A} \pm \frac{Pec}{I}$$

$$S = \frac{P}{A} \pm \frac{Pe\,b^2}{6}$$

Irrespective of the magnitude of "P" if placed at outer edges of middle third, stress at end will always be "o".

Draw a side view of a part of a prestressed rectangular beam and again resolve the prestress force located at the third point into a concentric force "P" and a couple with a moment equal to "Pe". Show that the fibre stress at the top or bottom of the beam is computed from a similar equation as that used for the short compression member except that the effect of the external moment "M" is included.

We now have the general equation

$$fc = \frac{P}{A} \mp \frac{Pe\text{-}M}{I}\, y(tb) \qquad \text{(Equation 1)}$$

In order to clarify the sequence of the derivations from the basic formula Figure 1 can be used:

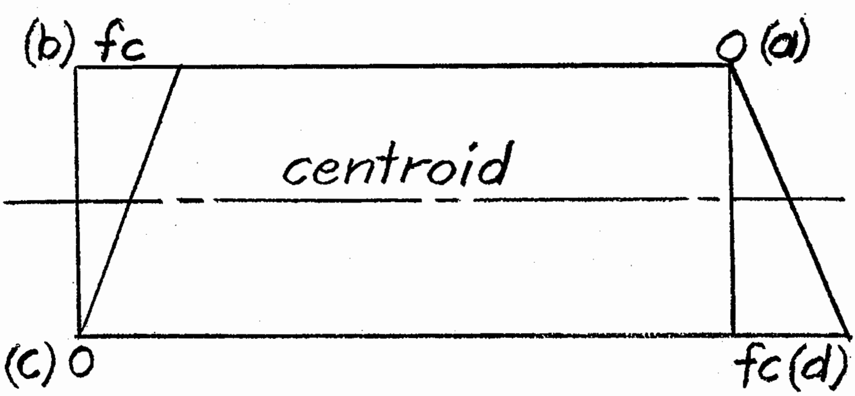


Figure 1.

*Determination of Eccentricity* condition (a) that no tension exists in the top fibres at the ends of the girder under critical loading conditions.

General Equation: $fc = \frac{P}{A} \mp \frac{Pe\text{-}M}{I} y\ tb$

Revised Equation for the set of loading conditions:

$$fc = \frac{P}{A} \mp \frac{Pe\text{-}M}{I} = 0\ yt\ b$$

$$0 = \frac{P}{A} - \frac{Pe}{I}\ yt$$

$$\frac{P}{A} = \frac{Pe\ (yt)}{I}$$

$$\frac{P}{A} = \frac{Pe\ yt}{I = Ar^2}$$

$$P = \frac{Pe\ yt}{r^2}$$

$$\frac{Pr^2}{yt} = Pe$$

$$e = \frac{Pr^2}{P\ yt}$$

$$e = \frac{r^2}{yt} \qquad \text{(Equation 2)}$$

"e" has been fixed so that top fibre will be = 0 from prestress alone.

Condition at (b) that the fibre stress "fc" be within that allowed by the specifications for the condition maximum "MT". Since in fixing the eccentricity "e" so that the stress in top fibre from prestress alone will be zero, the effect of prestress can be disregarded in computing "fc" at "b".

General Equation:

$$fc = \frac{P}{A} \mp \frac{Pe\text{-}M}{I}\ ytb$$

$$fc = \frac{MT}{I}\ yt \qquad \text{(Equation 3a)}$$

In the substitution of an actual problem the magnitude of "fc" will be a check on the design assumption for the size of the member selected and such changes as may be necessary should be made before proceeding farther.

Since in prestressed concrete design it is possible to add and subtract opposite stresses or superimpose stress diagrams, the tension in the bottom fibres for the MT loading will be computed for later use in making stress diagrams.

$$fc = \frac{MT}{I} yb \qquad \text{(Equation 3b)}$$

Condition at (c) for any combination of design loading, no tension is permitted in the bottom fibres which would be critical near the middle of the span where the moment would be maximum.

General Equation:

$$fc = \frac{P}{A} \pm \frac{Pe\text{-}M}{I} y\ t_b$$

$$0 = \frac{P}{A} + \frac{Pe\text{-}MT}{I = Ar^2} yb$$

$$\frac{P}{A} = \frac{Pe\text{-}MT}{Ar^2} yb$$

$$\frac{Pr^2}{yb} - Pe = MT$$

$$P \left[ \frac{r^2}{yb} - e \right] = MT$$

$$P\ (\text{Final}) = \frac{MT}{\dfrac{r^2}{yb} + e} \qquad \text{(Equation 4)}$$

$$P\ (\text{Initial}) = \frac{\dfrac{MT}{0.8}}{\dfrac{r^2}{yb} + e} \qquad \text{(Equation 5)}$$

Condition at (d) under normal condition of loading the stress in the bottom fibre must be within that allowed by the specifications near the end of the member.

General Equation:

$$fc = \frac{P}{A} \mp \frac{Pe\text{-}M}{I} \text{ y t b}$$

$$fc = \frac{P}{A} + \frac{Pe}{I} \text{ yb}$$

$$fc = \frac{P}{A} + \frac{Pe}{Ar^2} \text{ yb}$$

$$fc = \frac{P}{A}\left[1 + \frac{eyb}{r^2}\right] \qquad \text{(Equation 6)}$$

In order to make diagrams for various combinations of loading conditions the stresses for the effect of the dead load of the girder will be computed for use later on in drawing stress diagrams.

$$fc = \frac{MG}{I} \begin{Bmatrix} \text{yt} \\ \text{yb} \end{Bmatrix} \begin{matrix} \text{compression} \\ \text{tension} \end{matrix} \qquad \text{(Equation 7)}$$

With the various equations the substitution of numerical values can be made and the stress diagrams can be drawn.

The long line method of precast pretensioning has been enlarged on and is today the method used in most of our plants throughout the country.

The first application of prestressed concrete that I recall was first used in bridge construction by the construction battalions of Julius Caesar. They made large barrels using curved staves around which bronze hoops were forced from opposite ends to provide circumferential compression. The barrels were used as pontoons for floating bridges for the transport of Caesar's army and military equipment in the conquest of Britain.

In April of 1886 P. H. Jackson, of San Francisco, obtained a patent to prestress artificial stone or concrete arches which could be used as floors for buildings or making sidewalks over excavations. This work was followed by several others in Europe, and in 1908 C. K. Steiner, an American, proposed to tighten reinforcing rods against green concrete.

Early attempts to develop a practical method of prestressing failed because of the lack of high tensile steel or because the developers lacked the knowledge of the shrinkage and plastic flow characteristics of concrete. Most of the prestressing force applied was nullified because the mild steel stretched about the same amount that the concrete decreased in length.

In 1925 R. H. Dill, of Alexandria, was the first to succeed in producing prestressed concrete members by the post-tensioning method. In Dill's method high tensile strength or hard steel was coated with a plastic substance to prevent bond with the concrete. After the concrete had hardened he induced stress in the steel by tightening nuts at the ends of the members.

In 1928 French engineer, M. Freyssinet, worked on the scheme and in 1939 he introduced a practical method for post-tensioning by means of double acting jacks and anchoring the cables at the ends by means of conical wedges. At the same time, Hoyer, of Germany, had developed a practical means of pretensioning by casting the concrete around wires that were already tensioned. After the concrete had hardened small units were made by cutting up the long continuous pieces.

At this point we ask ourselves how does prestressed concrete differ from ordinary reinforced concrete? The basic theory behind both schemes is to devise a method that will compensate for the low tensile strength of concrete and prevent cracking when such stress is produced.

Ordinary reinforcement helps this condition but it remains essentially inert until load is applied to the member. As the load is increased, there is an increase in the steel stress which is accompanied with an increase in the length of the bar. This increase could be greater than the concrete can withstand. Therefore, a hair crack develops in the tension area of the concrete. At the same time only about one-third of the concrete section is effective in resisting compressive stresses.

Prestressed concrete imposes preliminary internal stresses in the member before the working loads are applied in such a way to lead to a more favorable state of stress when the external loads are applied. In other words, eliminate tensile stresses by superimposing compressive stress by mechanical means. This now leaves the whole cross-section available for compression which may be demonstrated this way.

| *Reinforced Concrete* | *Prestressed Concrete* |
|---|---|
| $Mc = \frac{1}{2} fc \times \frac{3}{8} d1 \times \frac{7}{8} d1$ | $M^1c = \frac{1}{2} fc \times d^2$ |
| $= \frac{21}{128} d1^2 fc$ say $\frac{1}{6} d1^2 fc$ | $M^1c = \frac{1}{2} fc \times d^2 \times \frac{2}{3} d2$ |
| $Mc = \frac{1}{6} d1^2 fc$ | $M^1c = \frac{1}{3} fc \times d2^2$ |

For same size beam

M R/C 2500# concrete $M_c = M_c$

M (P.S.C.) 5000# concrete $M^1c = 4 M_c$

So you see that prestressed concrete is a more efficient use of the cross-section and the neutral axis is at the bottom of the beam.