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HYDRAULICS BY *ANALOG*

A Three-Part Series on the Use of Electronic Models in Hydraulic Engineering

PART 1. AN ELECTRONIC MODEL OF A PUMPING PLANT

By HENRY M. PAYNTER,* *Member*

PREFATORY REMARKS

ON November 5, 1958, a combined meeting of the Hydraulic Section and the Sanitary Section of the BSCE was held at the American Center for Analog Computing in Boston. A demonstration was given of the applications of high speed electronic models to the design and operation of engineering works related to storm drainage and sewage disposal problems in Metropolitan Boston.

This paper serves as the introductory part of the three-part series on this general subject. The latter two parts are intended for future issues of this JOURNAL.

Here we treat very briefly and somewhat dogmatically the case of a pumping plant of a fairly common type. Only the most salient steps are indicated, leading to the direct establishment of an operating model, comprised of standard commercially available computing components. However, once interconnected, such an electronic model can serve for many design and operating studies. When these are completed, the parts are totally "salvageable" and available for other uses.

For further details of the physical situation, the mathematical formulation, and the computing art, readers are referred to the many excellent existing books and papers, some of which have been indicated in the short Bibliography at the end of this paper. We shall assume

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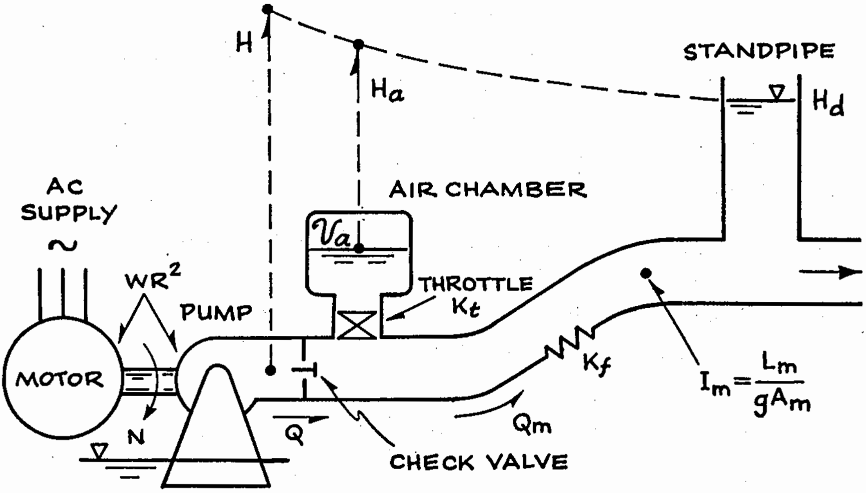
* Assistant Professor of Mechanical Engineering, Massachusetts Institute of Technology and Director, American Center for Analog Computing.

here that the mathematical formulation is correct, and will be concerned only with the steps necessary to make useful computer model studies. This particular problem was chosen primarily to demonstrate the benefits of modern machine computing but otherwise represents excessive over-simplification employed only for the sake of brevity; from the versatility and generality of the methods used, the extensions to actual cases should be apparent.

THE PHYSICAL SITUATION

Figure 1 depicts a commonly recurring type of centrifugal pump installation, supplying a standpipe with water, or other liquid, through a conduit, pipe, or force main. Typically, as shown, it might be equipped with a throttled air chamber to protect the discharge main from excessive pressure fluctuations, especially in the event of power interruption to the motor during operation and subsequent closure of the check valve. If there were no check-valve, the flow would reverse through the pump, ultimately reversing rotation and causing the pump to run as a turbine. Under such condition, the large reverse flow might flood the suction well and, in any event, would tend to drain the standpipe and other parts of the connected system.



PUMP INSTALLATION
FIG. 1.

Typical engineering studies are those concerned with effects on transient pressures in the discharge main of the following physical constants, among others:

(a) Check valve characteristics
(b) Air Storage: V_a
(c) Throttling loss: K_t
(d) Flywheel effect: WR^2

In particular, such investigations (which, of course, correspond to those made conventionally by graphical or numerical methods) are used to determine the economic size of the control features, such as the air storage and the flywheel effect, and the design values of such items as the throttling constant and check-valve parameters.

Table I outlines the physical situation in terms of the interrelationships between essential components and variables. In the computer model to be derived we are able to measure the values of these variables from instant to instant and are therefore able to study both transient and equilibrium conditions, for design and operating decisions.

TABLE I—COMPONENTS AND VARIABLES

	ELECTRIC SYSTEM	
AC Voltage (volts) : E_1 ↓		↑ I_1 : AC Current (amps)
	INDUCTION MOTOR	
Motor Torque (lb. ft.) : M_2 ↓		↑ N_2 : Motor Speed (rpm)
	MECHANICAL INERTIA	
Pump Torque (lb. ft.) : M_3 ↑		↓ N_3 : Pump Speed (rpm)
	CENTRIFUGAL PUMP	
Pump Head (ft.) : H_4 ↓		↑ Q_4 : Pump Discharge (cfs)
	CHECK VALVE	
Head Below Chamber (ft.) : H_5 ↑		↓ Q_5 : Check Valve Flow (cfs)
	AIR CHAMBER	
Upstream Head (ft.) : H_6 ↓		↑ Q_6 : Upstream Flow (cfs)
	FLUID INERTIA	
Gradient Head (ft.) : H_7 ↑		↓ Q_7 : Force Main Flow (cfs)
	FLUID RESISTANCE	
Standpipe Level (ft.) : H_8 ↑		↓ Q_8 : Downstream Flow (cfs)
	STANDPIPE	

The paragraphs below treat each of the components indicated and demonstrate how the basic relations can be used to establish a part-for-part electronic model. The computing components actually employed are described in Appendix A.

INDUCTION MOTOR CHARACTERISTICS

INPUT SIGNALS: Voltage E_1 and Speed N_2
OUTPUT SIGNALS: Current I_1 and Torque M_2

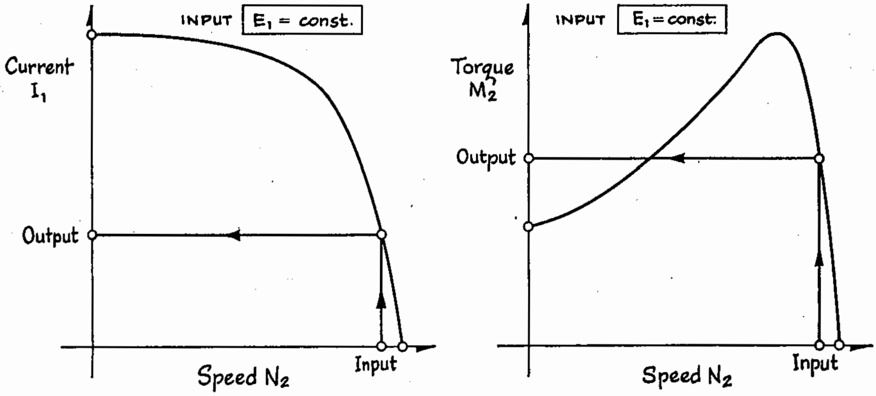


FIG. 2.

The standard characteristics for a 60 cps, three-phase, squirrel-cage induction motor have the form indicated in Figure 2. For computing purposes, such characteristics may be generated very easily through a direct representation of the simplified equivalent circuit portrayed in Figure 3. The corresponding equations can be written:

$$L \cdot \frac{dI_1}{dt} = E_1 - E$$

L = Motor Inductance
= Reactance/2π (Frequency)

$$E = [RN_s]\ I_1/(N_s - N_2)$$

R = Motor Resistance

$$M_2 = \left[\frac{30}{\pi N_s}\right] \cdot E \cdot I_1$$

N_s = Synchronous Speed

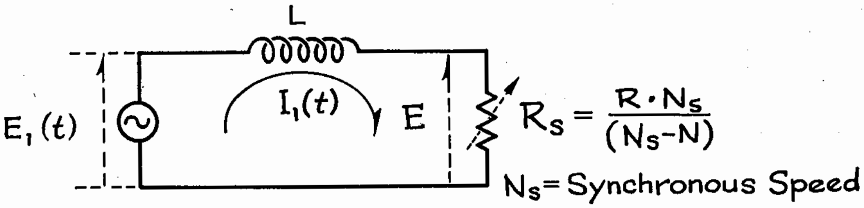


FIG. 3.

These equations may be directly instrumented in terms of the four basic computing components described in Appendix A. The resulting electronic model is given in block diagram form in Figure 4.

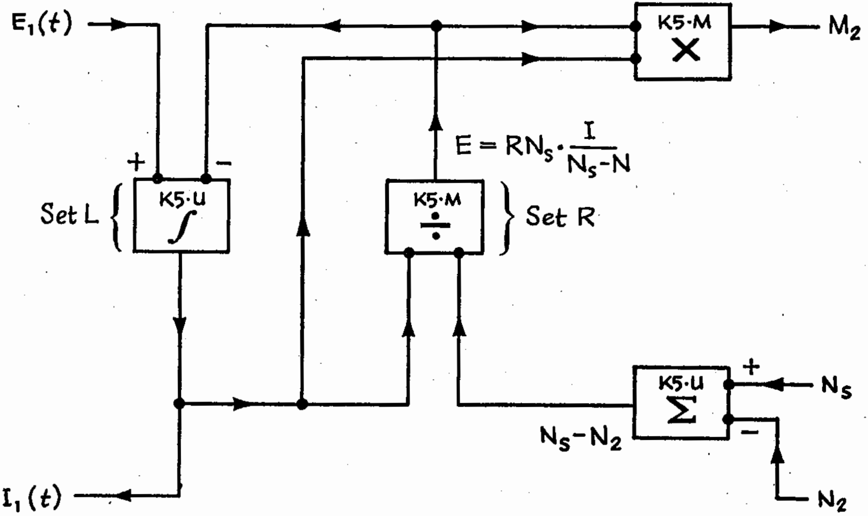


FIG. 4.

Actual measured characteristics for the computer model are indicated in Figure 5. These can be brought into conformity with any particular motor either by calculation of the motor constants R, L, and N_s, or by direct manipulation of the corresponding constants in the model.

FLYWHEEL EFFECT

INPUT SIGNALS: Motor Torque M_2 and Pump Torque M_3
OUTPUT SIGNALS: Motor Speed N_2 and Pump Speed N_3

This physical element manifests the effects of the rotary inertia

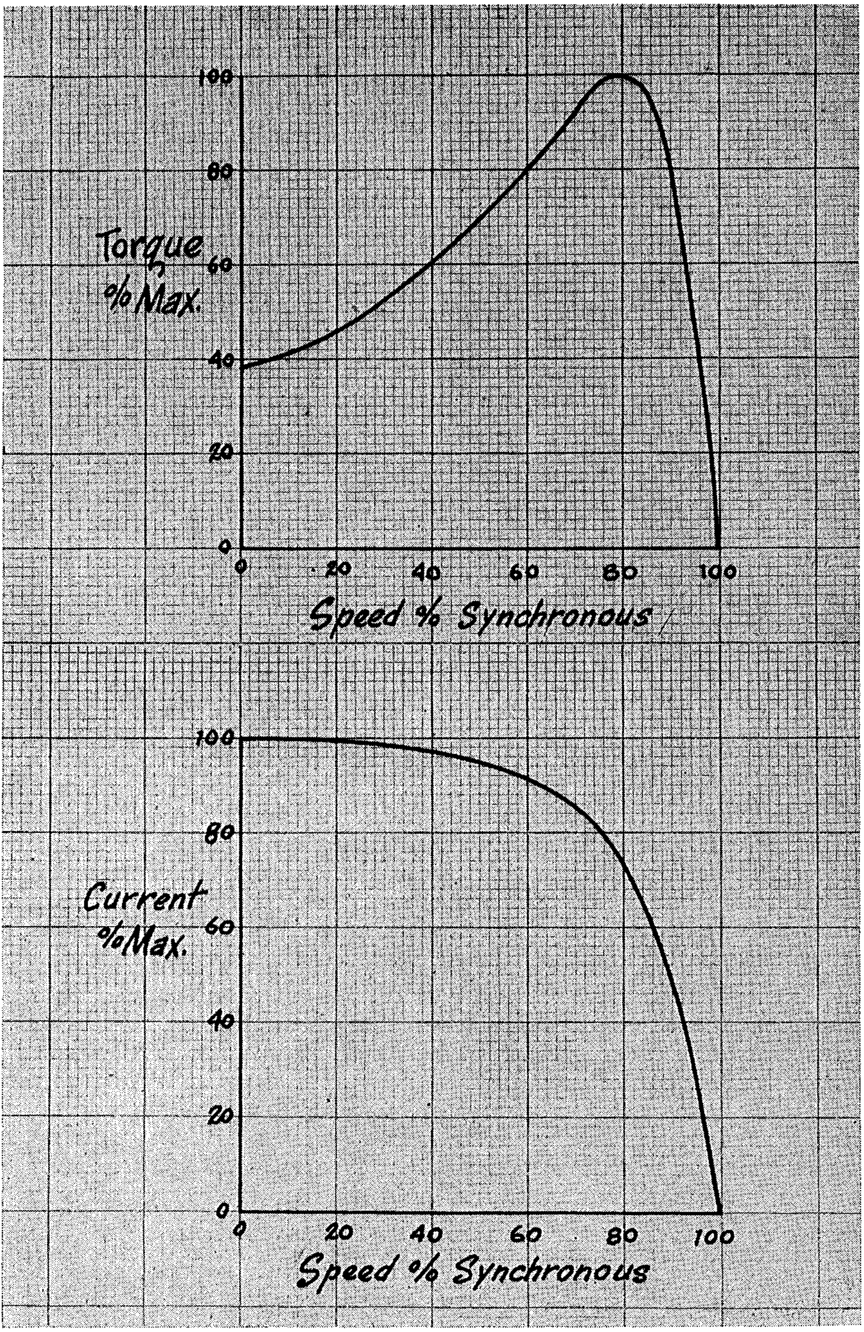

Torque
% Max.
100
80
60
40
20
0
0
20
40
60
80
100
Speed % Synchronous
Current
% Max.
100
80
60
40
20
0
0
20
40
60
80
100
Speed % Synchronous

FIG. 5.

of both motor rotor and pump impeller, together with the interconnecting shaft and any gearing. This involves the dynamical relation between speed variation (or acceleration) and the net accelerating torque, in the form:

$$\left[\frac{\pi WR^2}{30g}\right] \cdot \frac{dN}{dt} = M_2 - M_3$$

where $N = N_2 = N_3$ = Shaft Speed (in rpm)

WR^2 = Flywheel Effect (in lb ft^2)

g = Gravitational Acceleration (in ft/sec^2)

This relationship is instrumented by a single K5-U element used as a temporal integrator, as depicted in Figure 6.

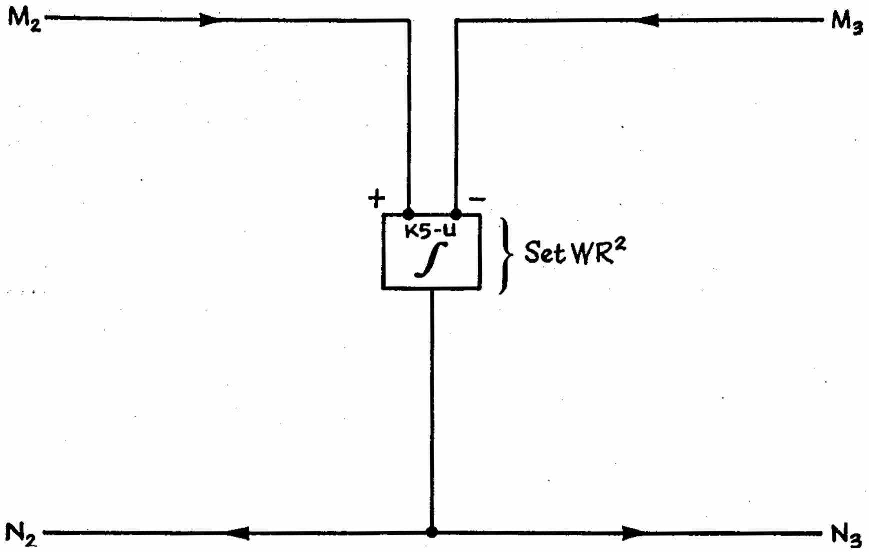


FIG. 6.

CENTRIFUGAL PUMP CHARACTERISTICS

INPUT SIGNALS: Pump Speed N_3 and Pump Flow Q_4
OUTPUT SIGNALS: Pump Torque M_3 and Pump Head H_4

The conventional characteristics of a typical medium head centrifugal pump would appear as sketched in Figure 7. Here, the variables N_3 and Q_4 serve as the inputs to produce as outputs, M_3 and H_4, as indicated.

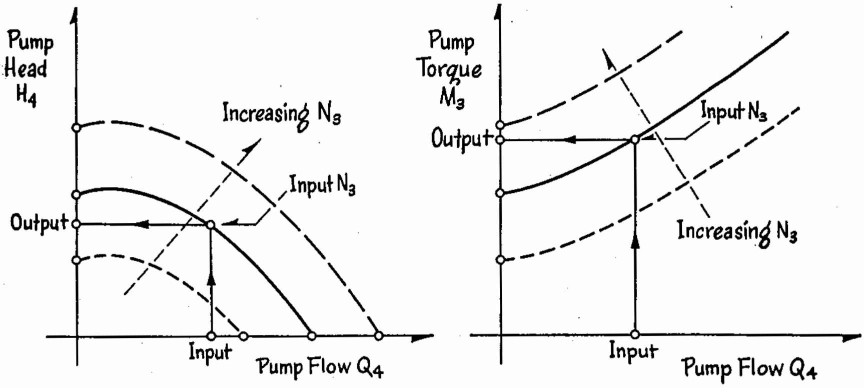


FIG. 7.

A very useful and practical approximation can be made to the characteristics of any fluid machine which is assumed to obey Euler similitude laws as the speed is varied. This representation can be expressed by the equations:

HEAD: $H = N\,(aN + bQ) - cQ^2$

TORQUE: $M = \left[\dfrac{\pi}{30w}\right] Q\,(aN + bQ) + dN^2$

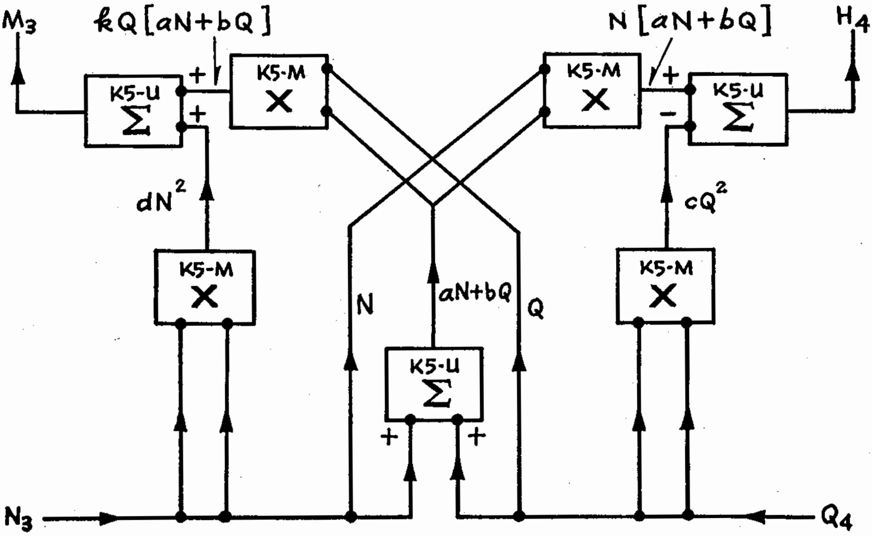


FIG. 8.

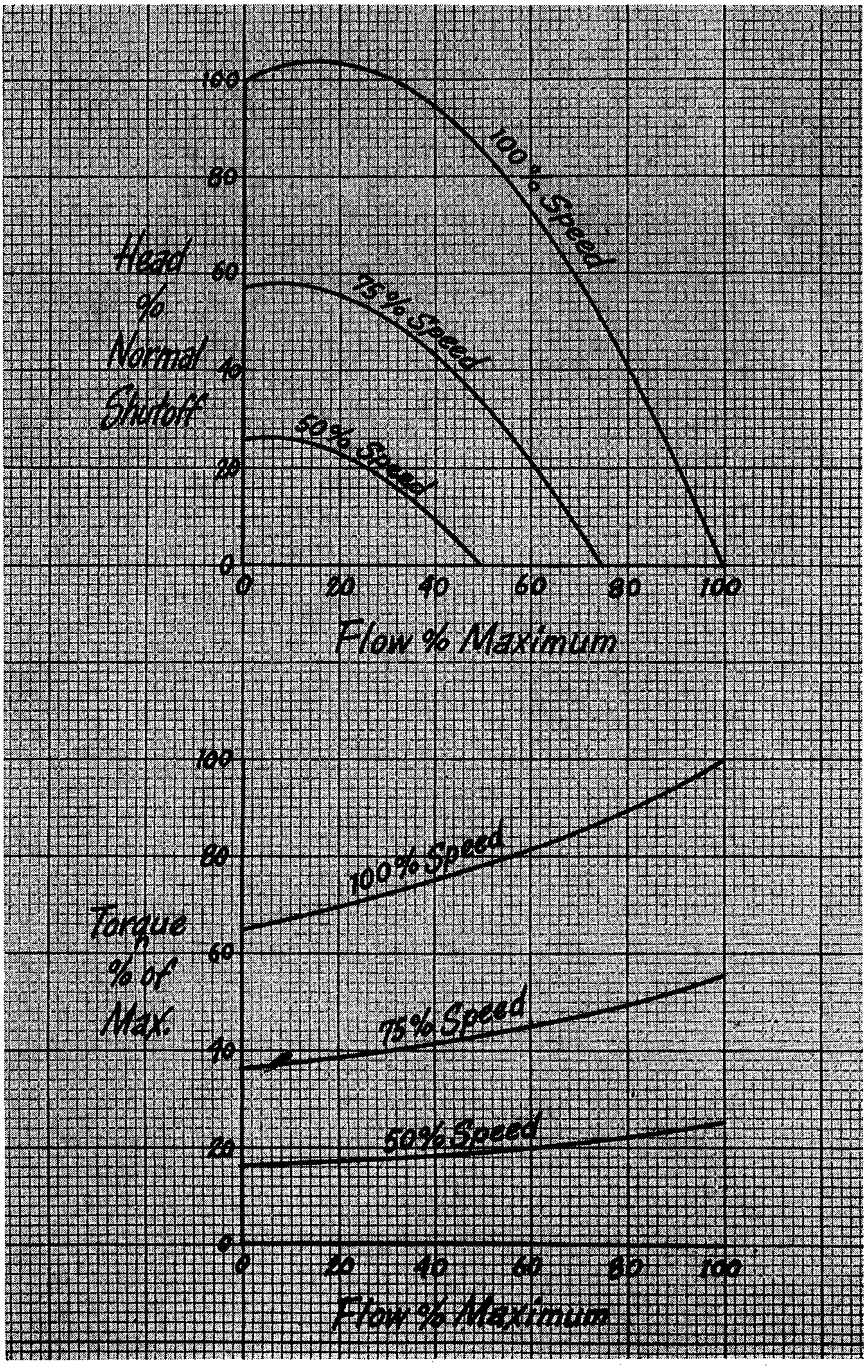

100
80
60
40
20
0
Head % Normal Shutoff
100 % Speed
75 % Speed
50 % Speed
0
20
40
60
80
100
Flow % Maximum
100
80
60
40
20
0
Torque % of Max.
100 % Speed
75 % Speed
50 % Speed
0
20
40
60
80
100
Flow % Maximum

FIG. 9.

where w is the fluid specific weight (lb/ft^3), and the constants a, b, c, d, depend on the particular type of pump. While these numbers generally can be related to the pump specific speed it is simplest to determine them by experimentation for each particular case.

Here, the term NQ (aN + bQ) represents the *whirl power* or reversible conversion: fluid energy $\rightleftharpoons$ mechanical energy. The terms on the extreme right of each equation represent fluid and mechanical losses respectively.

The execution, of these relations, in terms of the two components K5-U and K5-M described in Appendix A, is indicated in Figure 8. Actual computed characteristics are indicated in Figure 9, for speeds 100%, 75%, and 50% of motor synchronous speed. The reduced speed characteristics are, of course, of extreme importance during normal start-up and shut-down as well as for power failure studies.

Most such pumps are equipped with a manual or motor operated discharge valve. The throttling action of such a valve can be included very simply in the value of c in the above relations.

CHECK VALVE CHARACTERISTICS

INPUT SIGNALS: Pump Head H_4 and Chamber Head H_5
OUTPUT SIGNALS: Pump Flow Q_4 = Valve Flow Q_5

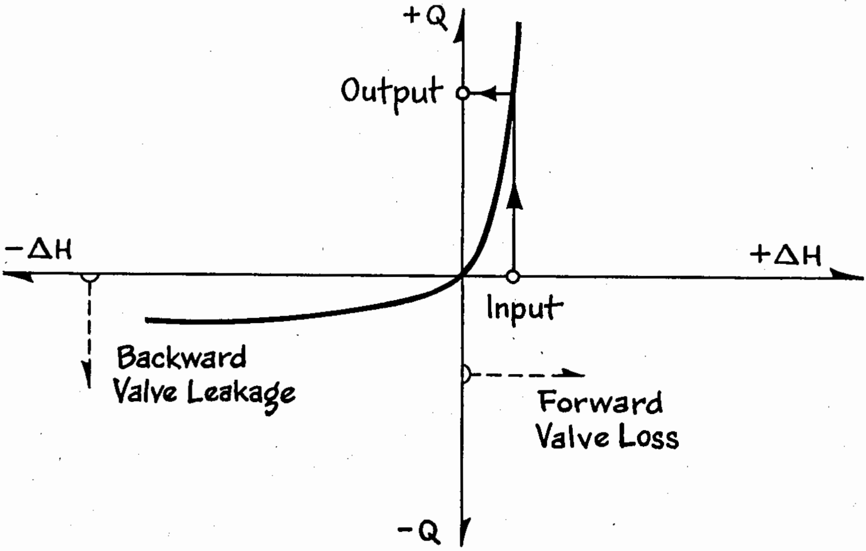


FIG. 10.

The check valve is provided to prevent reverse flow through the pump during an emergency loss of power. Typically, a swing-check would be used in such an installation, having a head-flow characteristic as depicted in Figure 10.

For most design and operating purposes, the detailed characteristics are not so important as the ascertainment of the head loss which can be tolerated for forward flow, when the check is opened, and the leakage reverse flow permissible under closed conditions. Thus a simplification in modeling is justified, which results in the block diagram of Figure 11 and the computed characteristics of Figure 12.

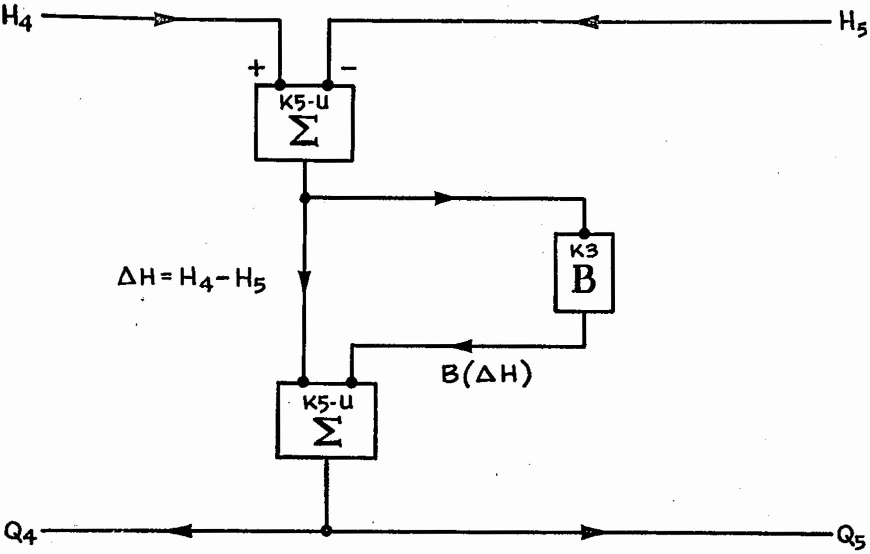


FIG. 11.

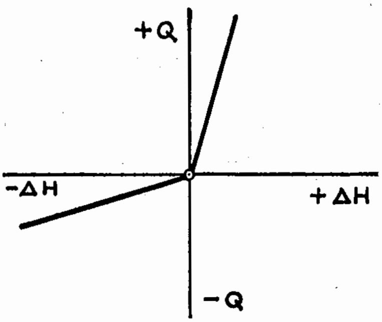


FIG. 12.

Here, the loss characteristic is represented as two straight lines, with vertex at the origin: $\Delta H = 0, Q = 0$.

AIR CHAMBER CHARACTERISTICS:

INPUT SIGNALS: Check Valve Flow Q_5 and Upstream Flow Q_6
OUTPUT SIGNALS: Chamber Head H_5 = Upstream Head H_6

The relationships governing the behavior of a throttled air chamber are determined in terms of the variables and parameters indicated in Figure 13.

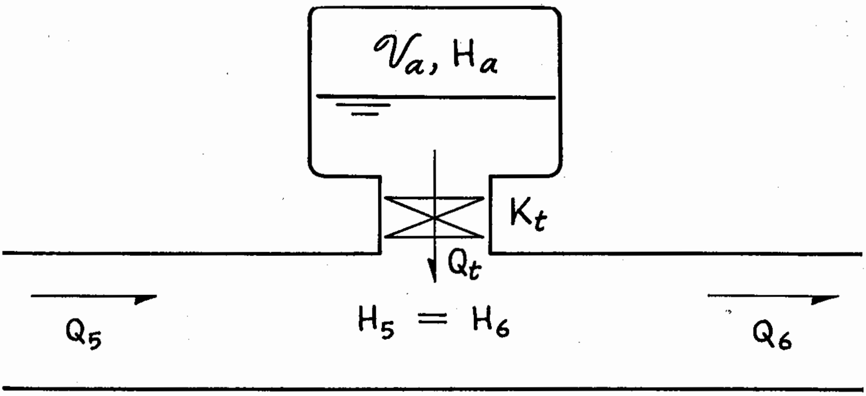


FIG. 13.

Thus the performance of any such system may be closely predicted in terms of the relations:

CHAMBER OUTFLOW: $Q_t = Q_6 - Q_5$
AIR VOLUME: $V_a = V_o + \int^t Qt\,dt$
AIR PRESSURE HEAD: $H_a = [H_o V_o]/V_a$
CHAMBER HEAD: $H = H_a - K_t|Q_t|Q_t = H_5 = H_6$

where V_o = Steady State Air Volume (in ft^3)
H_o = Corresponding Pressure Head (in ft)
V_a = Instantaneous Air Volume (in ft^3)
H_a = Corresponding Pressure Head (in ft)
K_t = Throttling Loss Factor (in ft/cfs^2)

Again these relations are simply and directly realized in terms of the three basic units (K3-V, K5-M, K5-U) indicated in Figure 14.

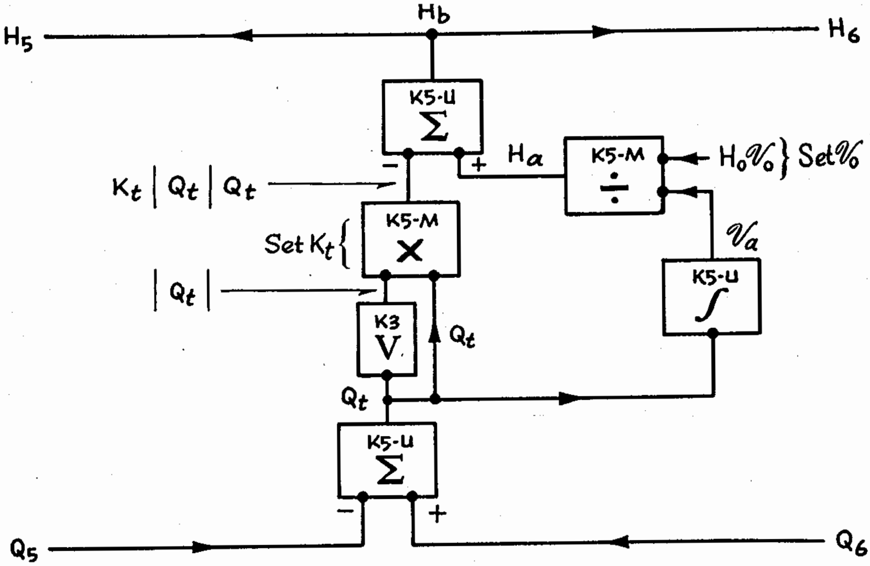


FIG. 14.

Note that incorporation of the loss characteristic permits the exploration of various throttle sizes (i.e. values of K_t) and their effects on performance. Similarly the size of the air chamber may be varied by altering the value of V_o.

FLUID INERTIA CHARACTERISTICS

INPUT SIGNALS: Upstream Head H_6 and Gradient Head H_7
OUTPUT SIGNALS: Upstream Flow Q_6 = Main Flow Q_7

If the compressibility of the water in the discharge main is neglected, which is a reasonable assumption for the present situation, with an air chamber at one end and a standpipe at the other end, the acceleration of the mass of water in the force main demands that:

$$\left[\frac{L_m}{gA_m}\right] \cdot \frac{dQ_m}{dt} = H_6 - H_7$$

where: $Q_m = Q_6 = Q_7$ = Flow in the Main (cfs)
L_m = Equivalent Developed Length of the Main (ft)
A_m = Equivalent Cross-Sectional Area of the Main (ft^2)
g = Gravitational Acceleration (ft/sec^2)

For the general case of a nonuniform pipe, the inertial factor, L_m/A_m, is given by:

$$L_m/A_m \equiv \int_0^L ds/A(s)$$

This fluid inertia effect may then be included merely by realizing the above relationship in terms of a single K5-U operator as indicated in Figure 15. The value of L_m/A_m for any installation is altered by decade switch settings.

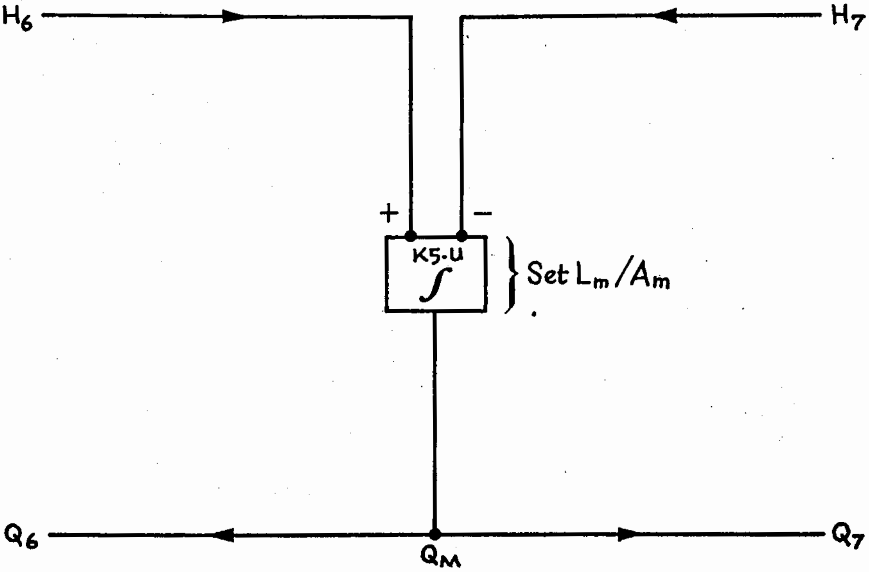


FIG. 15.

FLUID RESISTANCE CHARACTERISTICS

INPUTS: Pipe Discharge Q_7 and Standpipe Level H_8
OUTPUTS: Gradient Head H_7 and Pipe Discharge Q_8

This element represents all the pipe friction in the discharge main. The output flow Q_8 is, of course, the same as the input flow Q_7. If we can assume fully turbulent flow in a relatively rough pipe, then the head loss across the entire length of pipe is given by:

$$H_7 = H_8 + K_f|Q_7|Q_7$$

where the resistance coefficient K_f can be defined by the expression:

$$K_f = \frac{1}{2g} \cdot \frac{fL_m}{D_m A_m^2}$$

f = Pipe Darcy Friction Factor
D_m = Equivalent Pipe Diameter
and A_m, L_m, and g are as defined previously.

The value of K_f can be determined either by calculation or by observation for any pipe or composite series of pipes.

The corresponding block diagram for this element is indicated in Figure 16 with its performance as indicated in Figure 17. For those

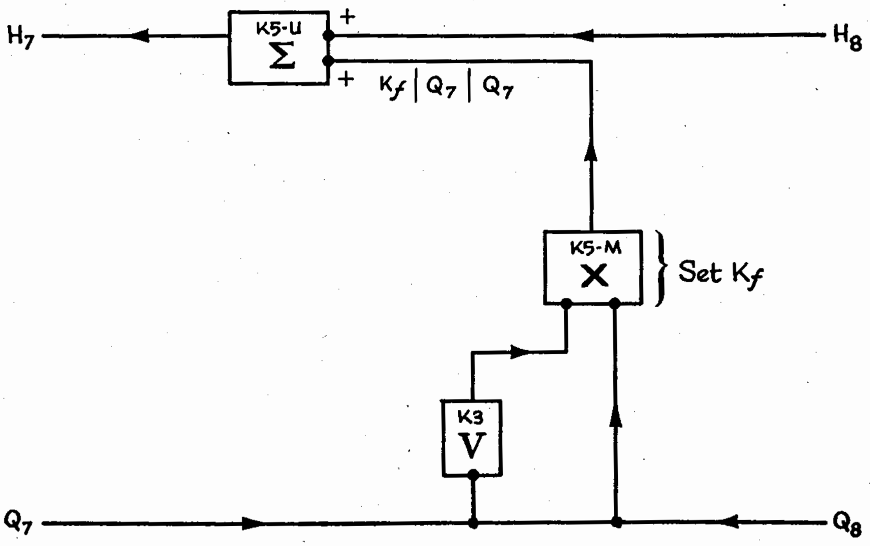


FIG. 16.

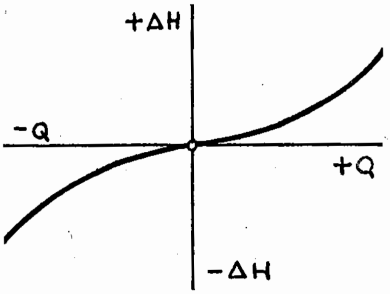


FIG. 17.

systems in which the forward loss constant K_f^+, is not the same as the backward loss constant K_f^-, it is possible to represent this effect simply and directly on the model.

FINAL COMPUTER MODEL

These performance relationships are programmed for solution by electronic computer following the block diagrams indicated in Figure 18. All the variables in such a computer representation correspond to actual plant variables and when the computer is properly interconnected it becomes, in effect, a flexible working model of the system.

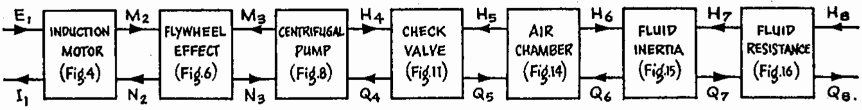


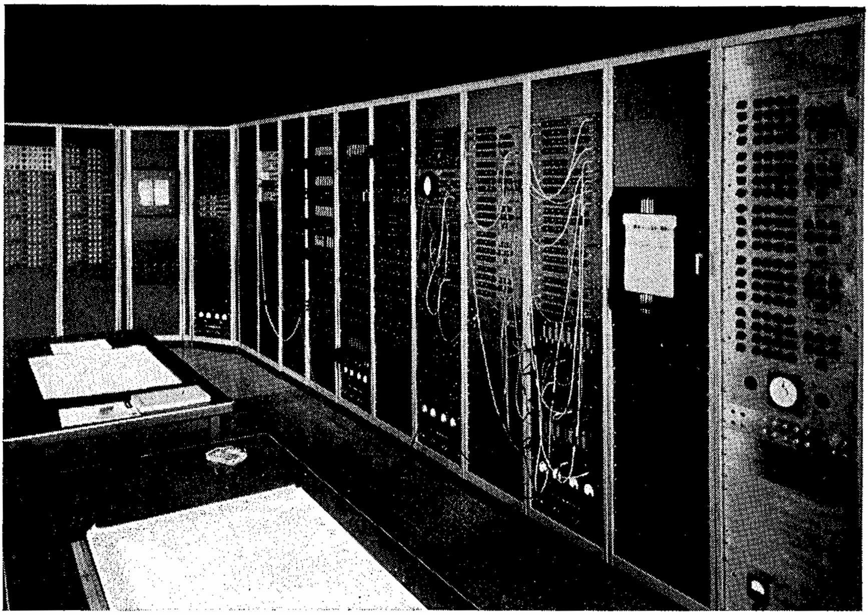
FIG. 18.

It is flexible in the sense that every physical characteristic in the actual system corresponds to an adjustable constant in the computer setup so that changes in design or in operating conditions can be explored fully. It is a working model because it reproduces both transient and steady-state phenomena present in the actual setup but frequently to different scales and of course in a different medium. For engineering studies of this sort, such an electronic model has many advantages, both for design studies and for simulated operating experience. When the particular studies are complete, such models can be disassembled, with complete and total salvageability. The next instalments in this series will indicate these advantages in more detail.

APPENDIX A

GENERAL NATURE OF ANALOG COMPONENTS

Recent advances in nuclear science, space travel, and automation testify dramatically to the value of computers in analysis, design, and development. The two major categories of computers are digital and analog. The first deals in numbers only; the latter, in continuous physical variables. In electronic analog computers, voltages are set up in direct correspondence with the pertinent physical quantities (such as speed, pressure, flow) of the problem to be solved. These voltages, the computer variables, are forced by obey relationships



closely approximating those of the problem to be solved. Time is generally the independent variable. The electronic components which establish the required relationships are principally amplifiers, potentiometers, resistors, diodes, and capacitors. However, those circuit elements are now commercially packaged so that no knowledge of electronics is required for successful use.

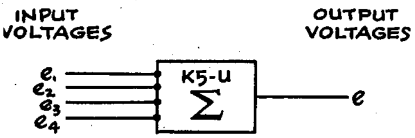
Such standard computing components form the major part of the equipment installed at AC/AC, the American Center for Analog Computing, which is a division of George A. Philbrick Researches, Inc. located in Boston.

The staff and equipment at this Center have enjoyed considerable experience in the application of these arts to a wide variety of physical, chemical, biological, economic, and engineering systems.

SUMMATION AND INTEGRATION

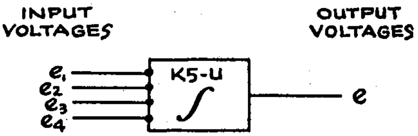
In this paper, these two operations are indicated by the blocks:

For Summation (Σ)



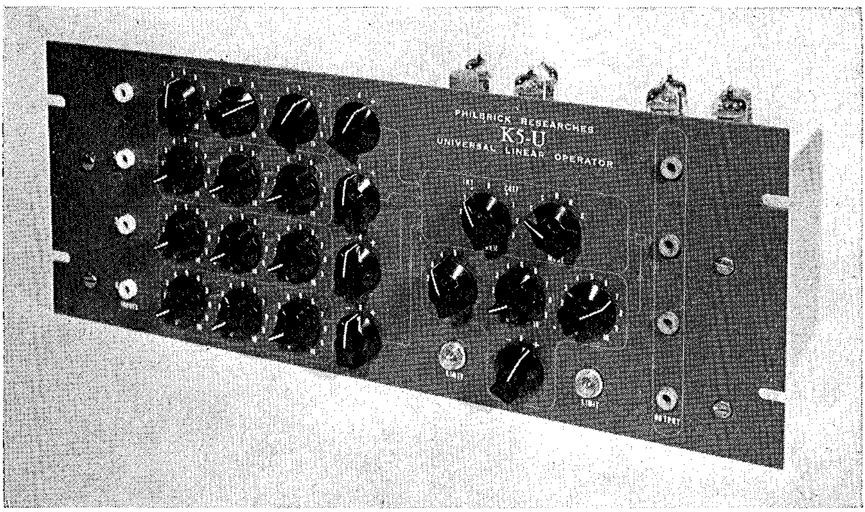
$$e = E + 10^m(a_1e_1 + a_2e_2 + a_3e_3 + a_4e_4)$$

For Integration ($\int$)



$$e = E + 10^m \int^t (a_1e_1 + a_2e_2 + a_3e_3 + a_4e_4)\,dt$$

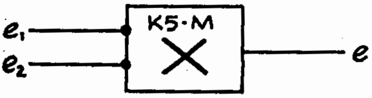
Both of these operations are embodied in the standard Philbrick Model K5-U Universal Linear Operator illustrated:

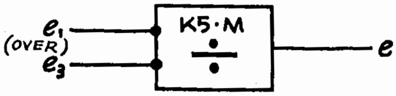


This unit combines inverting, proportioning, summing and integrating in a natural mathematical fashion. It provides set-run-hold conditions via perpetually reliable mercury relays. A combination of amplifiers maintain accuracy and stability, while coefficients and modes of operation are established by means of easily-set and easily-read decade switches.

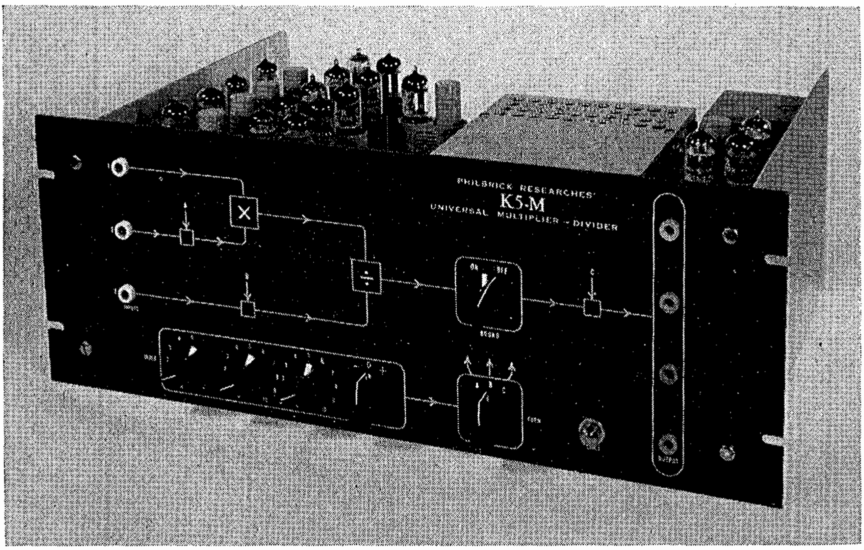
MULTIPLICATION AND DIVISION

In this paper, these two operations are indicated by the blocks:





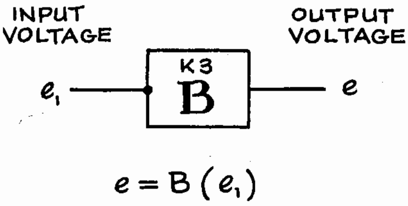
Both of these operations are embodied in the standard Philbrick Model K5-M Universal Multiplier-Divider illustrated:

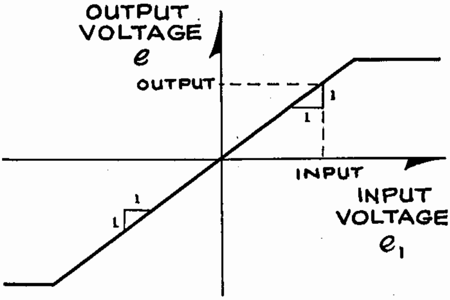


This component, in its most general usage, computes the product of two input voltages divided by a third. A constant voltage, adjusted by a triplet of decade switches, is additive to either type of input or to the output. Special cases taken in stride are the operations of squaring, reciprocating, and the evaluation of ratios and square roots. Accuracy, long-term, is of the order of 0.1%.

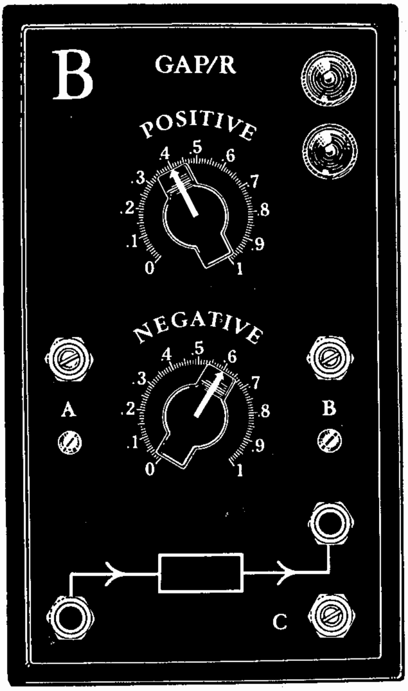
BOUNDING OR LIMITING

In this paper, this operation is indicated by the block:





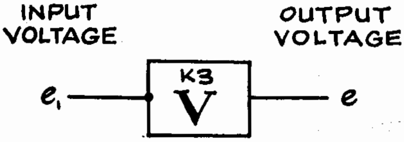
This operation is embodied in the standard Philbrick Model K3-B Bounding Component illustrated:



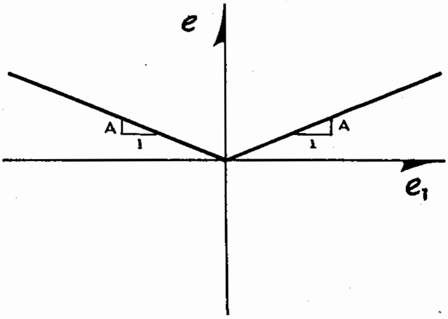
The output voltage from the K3-B is a limited version of the input voltage, in which the Positive and Negative bounds are individually adjustable. Each bound may be set linearly from zero up to a maximum of 50 volts. Except for being bounded, the output voltage follows the input and is not otherwise transformed.

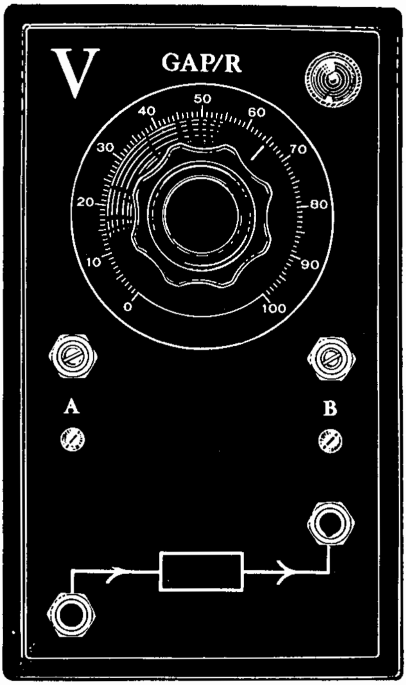
ABSOLUTE VALUE OR RECTIFICATION

In this paper, these two operations are indicated by the block:



$$e = A|e_1|$$





This operation is embodied in the Philbrick Model K3-V Absolute Value Component illustrated.

The K3-V Component, often called simply the "Vee," computes instantaneously the absolute value of the input. Adjustment of the 0-100 front dial varies the numerical scale factor A from zero to unity. The "full-wave rectifying" action of this unit has a number of applications in dynamics and controls.

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