CRITICAL STRAIN ENERGY OF DISTORTION IN SOIL AT LIMITING YIELD

By J. E. Jennings,* B.Sc.(Eng.), S.M., M.I.C.E., M.(S.A.)I.C.E.

and

P. F. Kirchmann,** M.Sc.(Eng.), Stud. M.(S.A.)I.C.E.

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Current practice in soil mechanics theory and testing makes use of the Mohr-Coulomb hypothesis of strength. In applying this hypothesis, "strength" has been accepted as the ultimate stress observed at failure, commonly referred to as the peak point value in shear testing. The practice has led to certain difficulties, as for example a lack of agreement in the values of the friction angle, ϕ, as observed from the slope of the rupture envelope in the Mohr diagram and, from the inclination of the failure planes in the test specimen. Further attention is given to this and other difficulties in a subsequent section of this paper, but at this stage it may be stated that they have led many soil mechanics workers to view current interpretations and applications as empirical procedures. In the attempt to devise a theoretical basis employing a new criterion of failure, which it is hoped will give better agreement with observed soil behavior, two propositions are made in terms of generalized theory of strength of materials as follows:

(1) Any soil which is not allowed to undergo significant change of soil properties as a result of stress application, will reach a condition of yielding when the strain energy of distortion reaches a critical value for the particular soil. This critical energy will be defined in terms of a surface represented by a parabaloid of revolution which is symmetrical about the $\sigma_1 = \sigma_2 = \sigma_3$ axis in the principal stress space having co-ordinate axes σ_1, σ_2 and σ_3.[1]

(2) The stress condition represented by stresses on the paraboloid will be defined as the condition of yielding. For stresses in excess of this condition the soil behavior will depart significantly from a linear relationship between the logarithm of the mean effective stress and the strain.

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* Professor of Civil Engineering, University of Witwatersrand, Johannesburg, and Visiting Professor in Soil Engineering, Massachusetts Institute of Technology, 1961-1962.

** Assistant Engineer, Electricity Supply Commission, Johannesburg.

[1] The subscripts to the principal stresses in this co-ordinate system bear no relation to the relative magnitudes of the stresses.

It will be shown that both of the above propositions have a theoretical background but that such derivation requires several assumptions. Hence, following precedent with previous theories, these two statements have preferably been made as propositions: They are then examined to see what theoretical bases they might be considered to have, and finally, they are investigated experimentally with a set of careful laboratory tests.

Limiting Strain Energy of Distortion

Rendulic (1938), in considering the various possible forms of limiting stress surfaces, suggested that one of these might be a paraboloid of revolution as follows:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = a^2 \{(\sigma_1 + p) + (\sigma_2 + p) + (\sigma_3 + p)\} \quad (1)$$

where "a" is a constant and "p" is in an intrinsic pressure giving rise to cohesion, i.e., for any particular value of void ratio and with constant stress history, "p" may also be considered a constant. An almost identical surface has been proposed more recently by Stassi-D'Alia (1959)[2]

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3) + (\sigma_3 - \sigma_1)^2 = 2(1 - 1/q)\, \sigma_c\, (\sigma_1 + \sigma_2 + \sigma_3) + \frac{2}{q}\, \sigma_c^2 \quad (1a)$$

where σ_c[3] is the unconfined compressive strength and q is the ratio of unconfined compressive strength to unconfined tensile strength[3] of the material. Both expressions (1) and (1a) are similar in form.

Equation (1) may be derived by extending the Huber-von Mises equation[4] if two assumptions be made, namely that the elastic modulus is a linear function of the mean principal stress in the material and that yielding will take place when the energy of distortion reaches a critical value which is constant for the material. The Huber-von Mises equation is as follows:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = \frac{6V_dE}{1 + \mu} \quad (2)$$

where V_d is the elastic strain energy of distortion, E is Young's

[2] Equation (1a) is Stassi-D'Alia's equation modified to the convention compression positive.
[3] As defined later in this paper, the unconfined compressive and tensile strengths are the strengths when the lateral confining pressures are zero.
[4] Timoshenko (1951) ascribes this concept to M. T. Huber in 1904 and to R. von Mises in 1913.

modulus and μ is Poisson's ratio. If these properties are constants then the right hand side of the equation is constant and the expression represents a right cylindrical surface with axis $\sigma_1 = \sigma_2 = \sigma_3$ in the principal stress space σ_1, σ_2, σ_3. This cylinder circumscribes the hexagonal Tresca surface representing the maximum shear strength theory of the nineteenth century: the Huber-von Mises theory may therefore be viewed as an extension of the maximum shear strength theory. Further, since the terms on the right hand side are all concerned with elasticity constants, it is clear that this equation can only apply if there are real meanings to these terms. Therefore, the "yielding" which takes place when stress conditions reach the critical surface must be of the nature of an "elastic limit" or "yield point," as understood in the behavior of a ductile material such as steel.

If the assumption is made that Young's modulus is a linear function of the mean principal stress σ_m, then:

$$E = a\,\sigma_m + b \qquad (3)$$

where "a" and "b" are constants. Soil behavior is more dependent on effective stresses than on total stresses and, hence, it is better to restate this assumption on terms of effective stresses as follows:

$$E = a\,\sigma_m' + b$$

$$= \frac{a}{3}(\sigma_1' + \sigma_2' + \sigma_3') + b \qquad (3a)^5$$

Substituting this in equation (2) gives

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = \frac{6V_d}{1+\mu} \left\{ \frac{1}{3} a(\sigma_1' + \sigma_2' + \sigma_3) + b \right\} \qquad (4)$$

To solve equation (4) in terms of the first proposition, two assumptions will be made:

(1) The Poissons ratio for the material remains constant and within the range of elastic possibility, i.e., μ will be between 0 and 0.5. Rauch (1957) has shown that this is approximately true for a soil provided that the stresses and strains remain within limits set by the approximately initial linear section of the stress-strain curve. This conclusion has also been reached by other full scale tests at the Road Research Laboratory in England.

(2) A limiting energy criterion may also be adopted for brittle materials

[5] The prime symbol, as in σ', in this paper will always refer to effective stress.

i.e., yielding will be reached when the elastic energy of distortion reaches a critical constant value. This assumption depends upon the assumption regarding the nature of E. It may be justified if the energy of distortion is viewed as some function of the area under the stress-strain curve and for the meanwhile it will be accepted that this depends upon some quantity such as the mean E over the range of the test.

If these two assumptions are accepted then equation (4) may be re-written for the stress conditions at yield.

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = c(\sigma_1' + \sigma_2' + \sigma_3') + d \qquad (5)$$

where "c" and "d" are constants.

The constants "c" and "d" may be solved in terms of two commonly expressed strengths, namely the strength in tension, σ_t, and the strength in compression, σ_c (compression considered positive).

For simple tension $\sigma_1 = -\sigma_t$ and $\sigma_2 = \sigma_3 = 0$

For simple compression $\sigma_1 = \sigma_c$ and $\sigma_2 = \sigma_3 = 0$

The two strengths will also be related by $\sigma_c = q\sigma t$

Substituting for c and d in equation (5), in terms of σ_c and q gives:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_c(1 - 1/q)(\sigma_1' + \sigma_2' + \sigma_3') + \frac{2\sigma_c}{q} \qquad (6)$$

Equation (6) expresses the stresses at yielding of the material, and is identical to that proposed by Stassi-D'Alia (1959) except the sign convention has been changed to make compression positive. The ratio $q = \sigma_c/\sigma_t$ is a measure of brittleness or, otherwise, ductility. It is also a measure of the position of the apex of the paraboloid of revolution with reference to the co-ordinate origin in the stress system σ_1, σ_2, σ_3. As q approaches 1.0 the material becomes more ductile and the paraboloid apex moves towards minus infinity, giving a cylinder of applied stresses as envisaged in the von Mises theory. For values of q exceeding 1.0 the paraboloid axis moves nearer to the origin and for q equals plus infinity (i.e., a material with zero tensile strength) the paraboloid apex is at the origin. For values of q greater than about 10, Stassi-D'Alia shows that equation (6) may be satisfactorily solved by placing q equal to plus infinity.

For brittle materials in which q is greater than about 10, equation (6) may be rewritten:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\sigma_c\,(\sigma_1' + \sigma_2' + \sigma_3') \quad (7)$$

This equation will apply to most soils since the compressive strengths are in general considerably greater than ten times the tensile strength.

Working in terms of effective stress in the triaxial test the principal stresses applied by the cell fluid pressure are equal, i.e., $\sigma_2' = \sigma_3'$ and equation (7) reduces to:

$$(\sigma_1'/\sigma_c - \sigma_3'/\sigma_c)^2 = (\sigma_1'/\sigma_c + 2\,\sigma_3'/\sigma_c) \quad (8)$$

If various values of (σ_3'/σ_c) are placed in this equation two solutions for (σ_1'/σ_c) result, one in which σ_1' is greater than σ_3' and the other with σ_1' less than σ_3'. These represent respectively the axial compression and axial extension tests. Figure 1 shows the solution of equation (8) plotted on a Mohr system of co-ordinates.[6] It will be observed that the strength in extension is about 15% greater than that in compression. A similar result has been found experimentally by Henkel (1959).

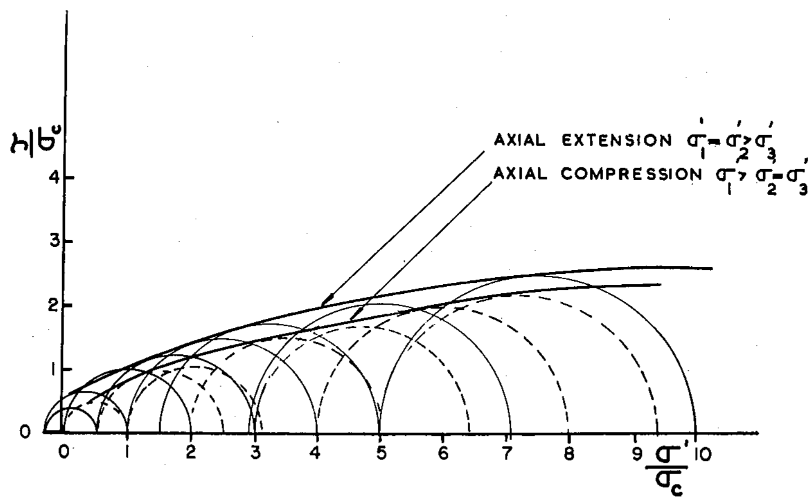

FIG. 1 THEORETICAL MOHR DIAGRAMS FROM EQUATION 8

[6] The form of envelope in Fig. 1 is not parabolic because the plotting axes are no longer in terms of principal stresses alone.

It will be observed in the work up to this stage that there are two implicit conditions, namely that the soil behaves in an elastic fashion and that the limiting stress combinations correspond to a yield point, i.e., a point where the range of elastic behavior is exceeded.

The Condition of Yielding in a Soil

A range of elastic behavior in any material implies some definite relationship between stresses and deformations. This relationship should be independent of other variables such as time, modes of application of load, etc. The stress-strain relationship need not be linear, as for example with rubber or where hysteresis effects are important, the relationship may be circumscribed by requiring that loadings are undirectional. Where time effects are observed a further restriction may be applied by allowing sufficient time to elapse and only observing the stress and strain conditions at equilibrium. All of these restrictions are common in soil mechanics theory and interpretation, and hence there is no difficulty in visualizing a condition of yielding in a soil: it is to be anticipated, however, that the stress conditions at this point of yielding will be somewhat smaller than those applying at ultimate failure, currently used for shear strength measurements of soils, and generally referred to as the "peak point" criterion.

If such a relationship between stresses and deformations can be found, then yielding of the material may be defined as the point where the observed stress-strain curve departs from the curve predicted by the behavior relationship. The soils problem resolves itself firstly, into one of seeking a relationship which will also have application to the type of soil problem encountered in practice, and secondly, into finding a test procedure which will enable clear distinction to be made of the point at which the test results depart significantly from the predicted behavior.

The previous assumption that Young's modulus will be a linear function of mean effective stress allows an initial relationship to be derived for stress versus strain. Defining the instantaneous Young's modulus in the triaxial test also as the differential of deviator stress with respect to strain gives:

$$E = \frac{d\sigma_d}{d\epsilon} = a\sigma_m' + b \qquad (9)$$

If a drained triaxial test, with constant cell pressure σ_3' is carried out slowly to allow complete relief of pore pressure, then

$$d\sigma_d/d\epsilon = \frac{1}{3}a(\sigma_d + 3\sigma_3') + b$$

which by appropriate manipulation reduces to

$$d\epsilon = \frac{d\sigma_d}{\sigma_d + c}$$

(C is a function of a, b and σ_3' and is constant) which by integration gives

$$\epsilon = g \log (\sigma_m' + k) + h \qquad (10)$$

where g, k, and are constants. In the initial attempt to use equation (10) k will be taken as zero giving:

$$\epsilon = g \log \sigma_m' + h \qquad (10a)$$

If undrained triaxial tests be carried out with pore pressure measurements, similarly with constant cell pressure σ_3, and with slow rate of testing to allow full pore water pressure equalization, a similar result will be obtained if the assumption is made that Skempton's (1954) $A = \Delta\mu/\Delta(\sigma_1 - \sigma_3)$ is constant over the range concerned. This may be a doubtful assumption and the whole mathematics of the undrained case with various empirical relationships between A and ϵ requires further investigation. Equations (10) and 10a) giving a linear relationship between strain and the logarithm of the mean effective stress are so promising that they deserve investigation experimentally. If test results show that there is an initial linear relationship between log σ_m' and ϵ then the point of departure from the straight line will represent a limit of yielding.

From previous experience with the testing of soils it is known that there are two components of shear strength, namely cohesion and friction. Lambe (1960) has suggested that the strains required to mobilize cohesion are smaller than those required to mobilize friction, and hence we should expect that in a soil possessing both cohesion and friction, two laws of the type expressed in equations (10) and (10a) will apply. The first section of the ϵ: log σ_m' diagram will apply to the cohesion effect and the second section to the frictional effect. A curved transition between these two straight lines should be expected. The point of interest, namely the point of yielding for the soil as a whole, will be the end point of the second straight line as deduced generally in Figure 2.

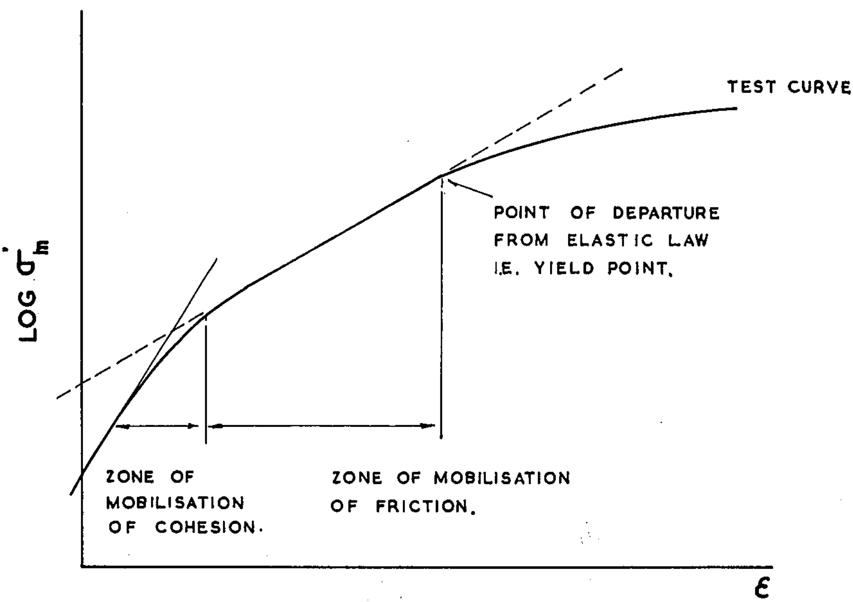

FIG. 2 THEORETICALLY DERIVED STRESS : STRAIN RELATIONSHIP FOR A SOIL.

Experimental Investigation of the Theory

Consolidated undrained and ordinary drained triaxial tests have been carried out on soaked samples of a specially prepared artificial soil. The test procedures recommended by Bishop & Henkel (1957) have been followed. The soil used was a mixture of kaolin and sand compounded to give specimens which would show reasonable failure planes for observation of the α-angles. The specimens were prepared at optimum water content of the soil by compacting to a predetermined density. Soaking of the samples was achieved by passing de-aired water upwards through the specimen in the triaxial cell until no further air bubbles could be seen in the emerging water. The properties of the soil and test conditions were as follows:

Particle specific gravity	2.64
Liquid limit	31.1%
Plastic limit	23.5%
Plastic index	7.6%
Optimum w.c.	16.1%
c_v for compacted soil over load increment 28-42 lbs/sq. in.	1.205×10^{-2} in²/min.
Rate of straining 1½ in. × 3 in. specimens:	
Undrained tests	2.3×10^{-3} in/min.
Drained tests	2.5×10^{-4} in/min.

Stresses and pore water pressures were observed at very small strain movements, particularly at the beginning of the tests. Careful corrections for area changes and rubber membrane resistance were made for all observations.

Figures 3 and 4 show typical stress-strain curves for the consolidated undrained and the drained triaxial tests. These are selected for values of σ_3, near to the average for each set of tests. The features of the theoretical curve Figure 2 can be clearly seen. The normal deviator stress-strain curves are also plotted and by transferring the yield value on the log $\sigma_m':\epsilon$ curve, the principal stresses σ_{1y} and σ_{3y} can easily be found. It was observed for each set of tests that the yield point became more difficult to distinguish at higher values of σ_3.

Figures 5 and 6 show the Mohr envelopes in terms of effective stresses drawn for the consolidated soaked undrained and soaked drained triaxial tests. Curved envelopes are found when the stresses at yielding are selected and straight line Mohr rupture envelopes result when the peak points of the $\sigma_d:\epsilon$ curves are used. Both envelopes are included in Figures 5 and 6.

Figure 7 shows all the results obtained for the angle of function, ϕ, plotted against the minor principal effective stress. These values are compared with the values observed from the α-planes. Reasonable agreement will be seen, confirming the observation that the rupture envelope is curved.

The test results illustrated in Figures 3 to 7 were obtained from only one series of carefully conducted tests made with the theoretical principles already described in mind. To test the theory, observations of stresses at very close strain intervals, particularly over the range $\epsilon = 0\text{-}3\%$, are required to demark the yield point clearly and, for completeness of any test series, at least six tests at different values of

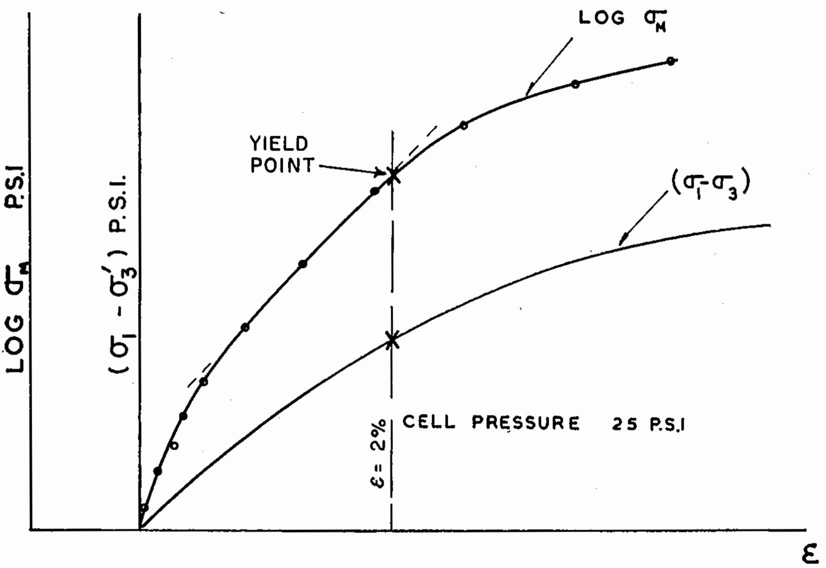

FIG. 3 TYPICAL STRESS : STRAIN DIAGRAM FOR CONSOLIDATED SOAKED UNDRAINED TEST

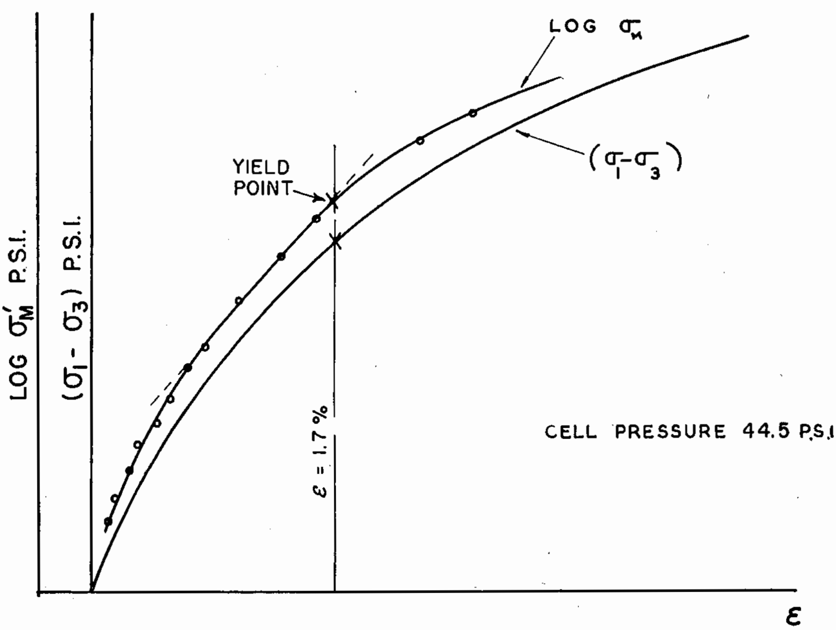

FIG. 4 TYPICAL STRESS STRAIN DIAGRAM DRAINED TEST

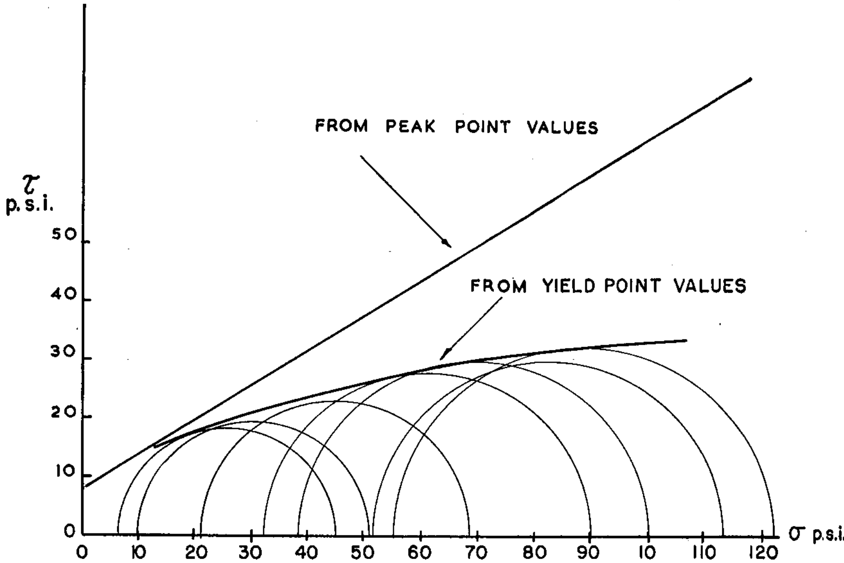

FIG. 5 MOHR ENVELOPES FOR CONSOLIDATED SOAKED UNDRAINED TESTS

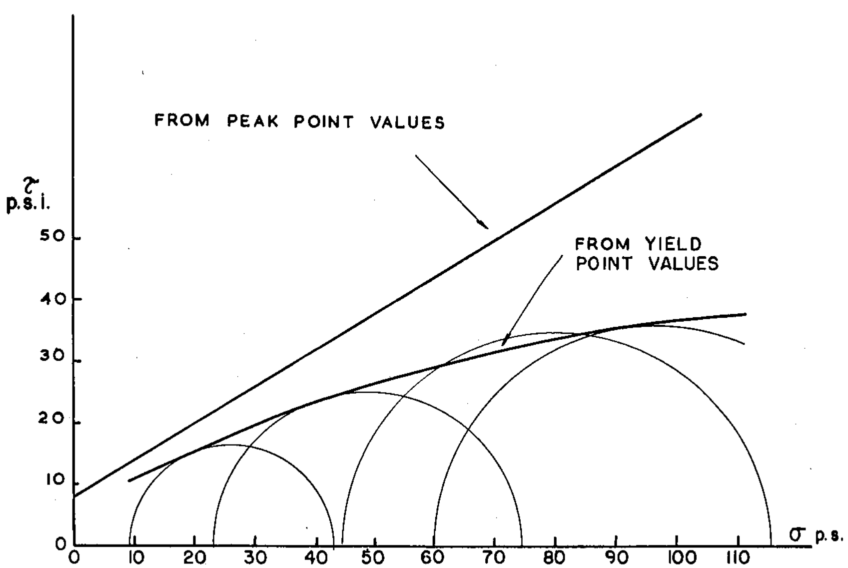

FIG. 6 MOHR ENVELOPES FOR DRAINED TESTS

chamber pressure, together with records of α-plane angles are necessary. Such detail is not usually observed in most triaxial tests. Nevertheless, re-plotting of a limited amount of test data from other laboratories appears to show quite clearly that the general law described by equation (10) is also satisfied for undrained triaxial tests with pore pressure measurements, both for normally consolidated samples, where σ_m' is decreasing, and for overconsolidated soils, where σ_m' is increasing during the test.

Re-examination of the Mohr Coulomb Theory

Up to the beginning of the twentieth century the theory of failure most commonly used was the "Maximum Shear Theory." In this theory failure takes place on planes of maximum shear stress inclined at 45° to the principal planes. This theory holds well for highly cohesive ductile materials but is not valid for brittle materials where the angle of inclination of the failure planes to the plane of major principal stress is usually considerably greater than 45°.

Mohr's hypothesis (1914) states that failure depends upon the stresses on the slip planes and failure will take place when the obliquity of the resultant stress exceeds a certain maximum value. He stated that "the elastic limit and the ultimate strength of materials are dependent on the stresses acting on the slip planes." Mohr showed that the stresses acting on the three principal planes could be represented by circles shown in Figure 8. The stresses on any plane within the body must then lie within the shaded area of Figure 8. The obliquity of stress τ/σ equals the inclination of the line through the origin and the point (σ, τ). If the circles of stress in Figure 8 represent a condition of failure then the tangent to the largest circle represents the condition of maximum obliquity. Failure takes place along those planes on which the stresses are represented by points A and B. These stresses act on planes which are parallel to the direction of the intermediate principal stress. Hence the diameter of the largest Mohr circle and the values of stresses represented by points A and B are independent of the intermediate principal stress, σ_2.

The Mohr-Coulomb equation for soils results from this hypothesis. As proposed by Taylor (1948), it implies that the soil may possess an intrinsic internal stress p_i, as follows:

$$\begin{aligned}\tau_f &= (p_i \tan\phi) + \sigma \tan\phi \\ &= c + \sigma \tan\phi \qquad (11)\end{aligned}$$

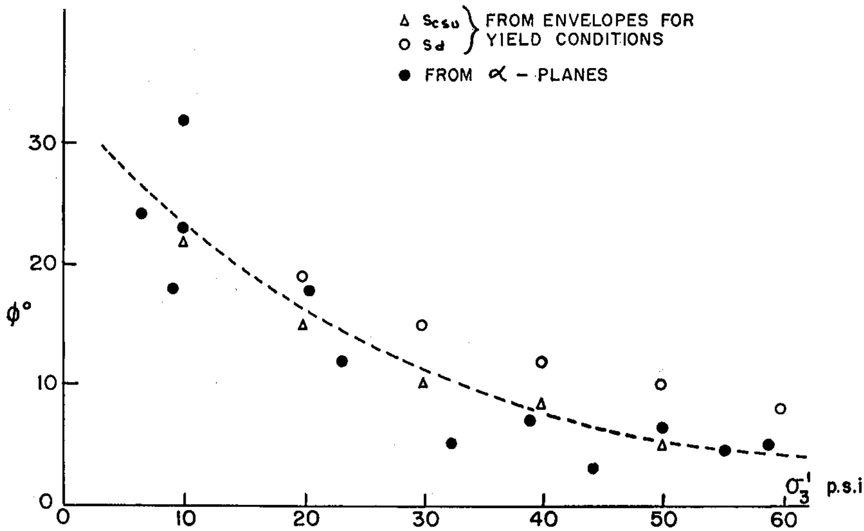

FIG. 7 COMPARISON OF ANGLES OF FRICTION OBTAINED BY VARIOUS METHODS

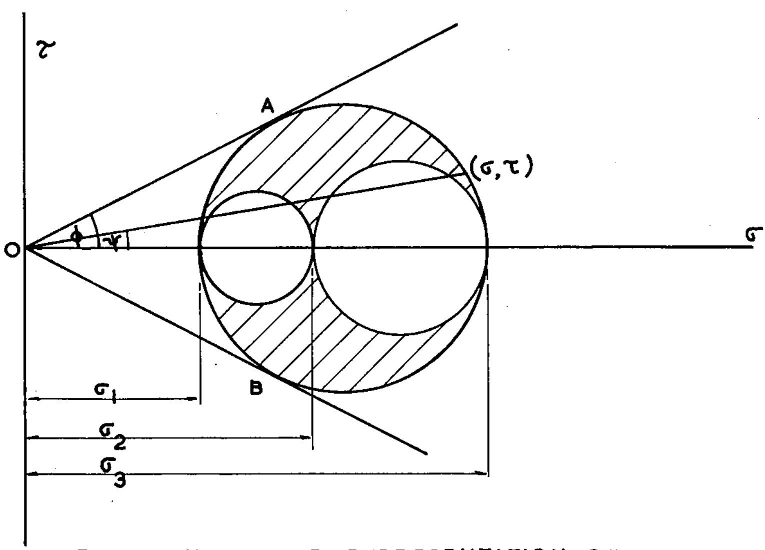

FIG. 8 THE MOHR REPRESENTATION OF STRESSES IN A THREE DIMENTIONAL STRESS SYSTEM.

where τ_f is the shearing stress on the plane of failure at failure

= Shear strength of the soil

σ is the normal stress on the plane of failure at failure

ϕ is the angle of internal friction

p_i is the intrinsic internal stress in the material

c is the cohesion of the material (or the shear strength when $\sigma = 0$)

The intrinsic pressure p_i for a soil is not a constant but depends also upon the void ratio of the soil. Hvorslev (1937) modified the Mohr-Coulomb equation to take account of variation in cohesion. His work still provides the most satisfactory method for determining the true angle of friction ϕ_e and true cohesion c_e.

Kirchmann (1961) has re-examined Mohr's (1914) work and finds that:

(1) He stated that while the straight line envelope is usually assumed, this is merely a simplification of what actually takes place. "It is obvious that every material with a high all round pressure will enter a condition in which the shearing stress on the slip planes will approach a constant and be independent of the size of the normal compressive stress. We can conclude that the envelope is concave to the σ axis and it approaches a line parallel to this axis." Mohr concluded that ϕ is not a constant.

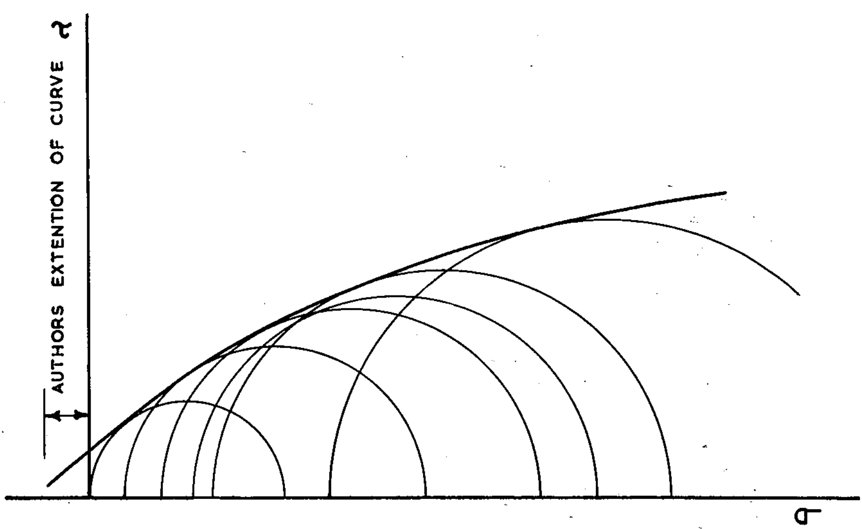

FIG. 9 MOHR'S CONCEPT OF THE RUPTURE LINE USING ELASTIC LIMITS AS FAILURE CRITERION

He gave measurements of the inclination of slip planes at the elastic limits for marble and sandstone. These indicate that ϕ becomes smaller as the all round pressure increases. Mohr's figure illustrating these effects is shown as Figure 9.

(2) While Mohr states that his hypothesis applies for both ultimate strength and elastic limits as definitions of failure of the material, he only tested it by using elastic limits from triaxial tests on marble and sandstone. It might therefore be presumed that he had in mind a hypothesis of failure applying specifically to the strength of materials at the elastic limit or the yield point at which the first slip lines just appear. The first appearance of slip lines is well below ultimate failure of most materials.

Conclusion

If it is accepted that there is promising agreement between the theory and experiment described in the preceding sections of this paper, it will be useful to re-state and re-examine the assumptions which have been made as follows:

(1) The Young's Modulus for any soil may be expressed as a linear function of the mean effective stress, equation (3a). Taylor (1948) shows that this is approximately true for sandy soils in so far as the initial chamber pressure is concerned. However, in undrained tests on over-consolidated soils and with all drained tests, σ_m' increases during the shearing process and hence one would expect the normal $(\sigma_1\text{-}\sigma_3):\epsilon$ curve to have a convex shape. With normally consolidated soils, tested undrained with pore pressure measurements, σ_m' decreases during the test. This gives the expected concave stress-strain curve and the different behavior in the previous cases remains unexplained. This point may be a refinement requiring further explanation when more thought can be given to the precise nature of Youngs modulus and its relationship to applied stress conditions in very brittle and very weak materials such as soils.

(2) Yielding of soil takes place at a critical and constant value of the strain energy of distortion. This is a hypothesis having precedent in the Huber-von Mises criterion. It may also be difficult to visualize a constant value of this energy for a material with a changing Young's modulus, unless the energy is viewed as some portion of the area under the stress-strain curve. In view of the hypothetical nature of this assumption it can only be tested experimentally.

(3) Poissons ratio is a constant for the soil over the range concerned and falls within the elastically possible values 0-0.5. This assumption is reasonably verified by other experimental work.

(4) The soil behaves in an elastic fashion up to the yield point providing limitations are applied relating to undirectional loading and sufficiency

of time of test to eliminate time effects due to pore pressure dissipation or equalization. The elastic law is of a nonlinear type with strain as a straight line function of the logarithms of mean effective stress. This assumption has a theoretical basis for drained test conditions but the basis for undrained tests is still obscure. Nevertheless, experimental examination of available results are very promising.

These four assumptions lead directly to the curved envelope in the Mohr plot and to the paraboloid of limiting stress at yield in the principal stress space. The final and most important experimental test is whether the angles of friction measured on the rupture envelopes agree with the angles measured from the α-planes. Figure 7 indicates good agreement and further work of this nature is required before the theory can be generally accepted.

At the same time as testing the possible limitations which may be contained in assumptions (1) to (4) given above, it is only fair to examine the theory in relation to the present applications of the Mohr-Coulomb theory as follows:

(1) The Mohr-Coulomb theory takes no account of the value of the intermediate principal stress. Habib (1953), in tests which allowed this stress to vary between its two extreme limits found that strength increased with increase in the intermediate principal stress. The proposed theory takes account of this.

(2) In the Mohr-Coulomb theory there should be no difference in strength for a material failing in axial extension or axial compression. Henkel (1959) found that strengths in extension were about 15% greater than those in compression for equal normal stress on the failure plane. This is shown theoretically in Figure 1.

(3) Gibson (1953) and others have observed that shearing of soils, in particular with drained tests, external work is performed by or on the soil and that this is reflected in the result as a component part of the shear strength. The idea of external work contributing to a basic property such as a strength is difficult to accept in any general theory of failure of materials. In the proposed theory no such external work component is included because strength is defined by a yield point below which volume changes are compatible with Poisson's ratio effects.

(4) The angle of internal friction, ϕ, as found from the various Mohr-Coulomb empirical relationships used in soil mechanics seldom agrees with the real angle of internal friction for the material as observed from the inclinations of slip planes or from some other special form of test devised to yield the "true" parameters. This has been used as the ultimate test of the proposed theory and good agreement has been found.

(5) The Mohr Hypothesis does not satisfactorily explain the rupture of

brittle materials such as glass and cast-iron when failure takes place in tension. In this case the surface of rupture is perpendicular to the tensile stress and has a smooth, conchoidal appearance as compared with rough inclined surface when the same material is failed in compression. The curved rupture lines found with the theory proposed here offers considerable promise in considering such cases. It also offers promise for explanation of the vertical failure planes observed when concrete is tested between frictionless plattens.

On balance, the theory and experiments described in this paper appear to deserve attention by research workers in soil mechanics. If the basis is correct then opportunity is offered to eliminate some of the empiricism which at present surrounds shear strength theory and application. The strains involved at yielding are small, of the order 1-3%, which are much lower than those found with current applications of the "peak point" criterion of shear failure, and these may be more compatable with the limitation of deformation required in many practical structures.

The full implications of the proposed theory on practical soil mechanics problems has not yet been examined properly. At the outset, it is to be expected that friction angles and design strengths are likely to be lower than those found with current shear test interpretation. This will probably be of advantage in those cases where structural deformations must be limited, as for example in the design for foundation bearing capacity or in problems involving active or passive pressure, where structural movement must be kept to a minimum. However, in soil mechanics there is a further group of problems where deformation is not of great consequence and where the engineer is more concerned with ultimate or failure conditions. The most important example in this group is the problem of slope stability. Here it may be quite permissible also to take into account the shear strength component which is due to volume change of the soil. These are problems which will require further investigation and fortunately there are a number of field records available for re-examination.

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REFERENCES

BISHOP, A. W., AND HENKEL, D. J. (1957) "The Measurement of Soil Properties in the Triaxial Test." Edward Arnold (London, 1957) (i) Appendix 4 (ii) Appendix 1 (iii) p. 52-63.

GIBSON, R. E. (1953) "Experimental Determination of the True Cohesion and True Angle of Friction in Clays." Proceedings of the Third International Conference on Soil Mechanics and Foundation Engineering. Switzerland (1953) Volume I, p. 126.

HABIB, P. (1953) "Influence of the Intermediate Principal Stress on the Shearing Resistance of Soils." Proceedings of the Second International Congress on Rheology (1953).

HENKEL, D. J. (1959) "The Relationships Between the Strength Pore Water Pressure and Volume Change Characteristics of Saturated Clay." Geotechnique, Sept. 1959.

HVORSLEV, M. J. (1937) "The Shearing Resistance of Remoulded Cohesive Soils." Summary of "Uber der Festigkeitseigenschaften gestorter bindiger Boden." Ingeniorvidenskabeliga Skrifter, A. No. 45 Copenhagen (1937).

KIRCHMANN, P. F. (1961) "The Criterion of Failure in Soils." Unpublished M.Sc.(Eng.) thesis, University of the Witwatersrand.

LAMBE, T. W. (1960) "A Mechanistic Picture of Shear Strength in Clay." ASCE Research Conference on Shear Strength of Cohesive Soils, Boulder, Colorado.

MOHR, O. (1914) "Abhandlungen aus dem Gebiete der technischen Mechanic." W. Ernst u Sohn (Berlin, 1914) p. 200-230.

RAUCH, H. P. (1957) "The Significance of Poisson's Ratio in the Determination of Stress and Settlement in Soils." Witwatersrand University, M.Sc. Thesis (1957).

RENDULIC, L. (1938) "Eine Betrachtung zur Frage der Plastischen Grentzzwstande." Der Bauingenieur, Vol. 19, p. 159-164.

SKEMPTON, A. W. (1954) "Proe Pressure Coefficients A and B." Geotechnique, Dec. 1954.

STASSI-D'ALIA, F. (1959) "A Limiting Condition of Yielding and its Experimental Confirmation." Industria Grafica Nazionale (Palermo).

TAYLOR, D. W. (1948) "Fundamentals of Soils Mechanics." John Wiley, p. 319.

TIMOSHENKO, S., AND GOODIER, J. N. (1951) "Theory of Elasticity" McGraw Hill, p. 149.