THE OVERTURNING FACTOR OF SAFETY FOR RETAINING WALLS

By Z. Getzler*

Safety against overturning is an obvious and necessary condition for the stability of gravity retaining walls. Nonetheless, as commented by Huntington (1957), ". . . there is no satisfactory and generally accepted procedure for computing the overturning factor of safety of retaining walls." The ordinary definition for this safety factor—ratio of resistance moment to overturning moment taken about the toe (or pressed edge of the foundation sole)—is adequate and satisfactory. However, many design procedures evaluate the resistance moment in an incorrect and unsafe way. A simple yet correct procedure is proposed in this paper.

Conditions with Linear Stress Distribution

Consider a symmetrical footing standing upon the surface of the ground or at the bottom of an excavation, with no contact surfaces other than the sole (bottom surface) of the foundation. The stresses and forces acting upon the footing can be replaced by a vertical force, V, which acts along the centerline and which includes the weight of the footing; a horizontal force, H, acting along the sole of the footing and balanced by shearing stresses at the sole; and an overturning moment, M, about the y-y axis of the sole (see Figure 1).

Taking the moments of these applied forces about the pressed edge, a, gives the *overturning factor* (not to be confused with the *overturning safety factor*):

$$\eta_a = VB/2M \qquad (1)$$

where B is the width of the footing in the plane of M, i.e., along the x-x axis.

The resultant, R, of the vertical stresses acting across the sole, is eccentric to the applied load, V, the eccentricity being:

$$e = M/V \qquad (2)$$

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The distance of the resultant force, R, from the pressed edge, a, is:

$$\xi = \frac{B}{2} - e = \frac{B}{2} - \frac{M}{V} = \frac{B}{2} - \frac{B}{2\eta_a} = \frac{\eta_a - 1}{\eta_a} \cdot \frac{B}{2} \qquad (3)$$

The distances e and ξ are, of course, functions of the overturning factor η_a.
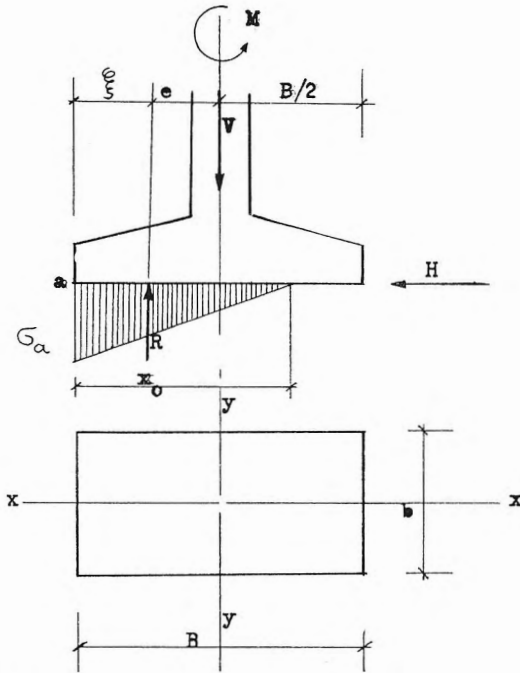

FIGURE 1.—MOMENTS AND FORCES ACTING ON A SYMMETRICAL FOOTING.

Computing the maximum contact pressure for the case $e < B/6$, i.e., the resultant lies within the "middle third," or the Kern, of the sole:

$$\sigma_a = \sigma_o \left(1 - \frac{6e}{B}\right) \qquad (4a)$$

where σ_o is the vertical pressure averaged over the entire area of the sole: $\sigma_o = V/Bb$. This case leads to a trapezoidal stress diagram. When $e > B/6$, the resultant lies outside the Kern, and it is commonly as-

sumed that no tensile stresses can develop. This case results in a tri-
angular stress diagram of width x_o and the maximum contact stress is:

$$\sigma_a = \frac{2V}{3\,\xi\,b} = \frac{4}{3}\,\sigma_o\,\frac{\eta_a}{\eta_a - 1} \tag{4b}$$

Figure 2, from Getzler (1959), lists the results obtained from
this well-known set of equations, for various assumed values of the
overturning factor, η_a. Several results appear immediately from these
equations. First, the quantities η_a, x_o, σ_a, etc., are interrelated, and
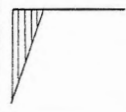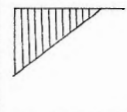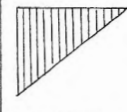therefore selection of a value for one quantity determines the values

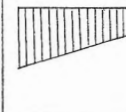|	$e > \frac{B}{6}$			$e = \frac{B}{6}$	$e < \frac{B}{6}$
η_a	1.0	1.5	2.0	3.0	> 3.0
ξ	0	$\frac{B}{6}$	$\frac{B}{4}$	$\frac{B}{3}$	$> \frac{B}{3}$
x_o	0	$\frac{B}{2}$	$\frac{3}{4} B$	B	B
σ_a / σ_o	∞	4.00	2.67	2.0	2.0
STRESS DIAGRAM					

FIGURE 2.—OVERTURNING FACTOR AND STRESSES IN ECCENTRICALLY LOADED FOOTING.

for the other quantities. For example, selection of σ_a on the basis of an
allowable bearing pressure automatically determines the overturning
factor η_a. Second, a value of $\eta_a = 1$ corresponds to infinitely large
stresses. This state-of-affairs is certainly inconsistent with the usual
concept of safety factor; i.e., a safety factor of unity should correspond
to limiting equilibrium, and hence to finite stresses.

It is apparent that the factor η_a does not provide a satisfactory
measure of the safety against overturning, although this factor is com-
monly used for just this purpose. The difficulty lies in the fact that
the moment of the soil stresses against the sole has not been considered.
The moment produced by these stresses must be subtracted from that

developed by the vertical force, V, in order to compute the true resistance moment. Once this is done, the point about which the moments are evaluated is of no consequence.

THE ULTIMATE RESISTANCE MOMENT

The resistance moment developed under any loading condition will thus depend upon the magnitude and distribution of the stresses against the sole of the footing. However, the resistance moment desired for evaluation of the overturning safety factor corresponds to a very specific condition: that which would exist if the loading and resisting conditions were changed until the overturning safety factor became unity. For this condition, the potential center of rotation lies not at the pressed edge (a condition which would imply infinite stresses), but along some line somewhat inside this edge. Between this line and the edge the bearing stresses are at the limit which the soil can withstand. Any further increase in applied moment would cause the footing to break into the ground along this edge, thus experiencing a large rotation.

The maximum possible value for the resistance moment is hence obtained when the stress at the pressed edge of the sole is just equal to the bearing capacity of the soil. Adopting the limit design approach, as is done, for example, in calculation of the ultimate capacity of a concrete beam, the limiting stress distribution would be as shown in Figure 3.** The bearing capacity of the soil must be of course that for the reduced footing width, x_{lim}. (For a study of the bearing capacity as a function of the inclination of the resultant and the eccentricity, see Meyerhof 1960.) Now, taking moments about the line of action of the vertical force, V:

$$M_{lim} = R_{lim} \cdot e_{lim} \tag{5}$$

$$R_{lim} = q_{lim} \cdot x_{lim} \cdot b \tag{6}$$

$$e_{lim} = \frac{B}{2} - \frac{x_{lim}}{2} \tag{7}$$

Hence the ultimate resistance moment becomes, where $R_{lim} = V$,

$$M_{lim} = V \cdot \frac{B}{2} \left(1 - \frac{\sigma_o}{q_{lim}} \right) \tag{8}$$

** If the limit pressure diagram is neither rectangular nor symmetrical, it may be assumed that the difference between the theoretical center of gravity of the diagram and the value $x_{lim}/2$ is negligible in relation to the couple arm, and hence the equation developed here will still give a satisfactory approximation.

and the true safety factor against overturning is:

$$\eta_{eff} = M_{lim}/M = \eta_a \left(1 - \frac{\sigma_o}{q_{lim}}\right) \tag{9}$$

This calculation indicates that the real overturning safety factor is a function of the ratio σ_o/q_{lim}, and that the use of η_a as a safety factor is justified only when $\sigma_o/_{lim}$ is very small; e.g., for light footings and strong soils. The diagram in Figure 4 shows the decrease of η_{eff} with relation to η_a with increasing average contact pressure.

This new and simple formula, eq. (9), takes into account the bearing capacity of the subgrade. Thus, two of the three required stability conditions are combined; i.e., those which affect the magnitude of the bearing stresses.

For non-cohesive soils, q_{lim} is itself a function of η_a, and a trial and error procedure is required. However, all trial and error procedure is avoided for cohesive soils; moreover, the new formula avoids the difficulties of the middle-third rule (see Terzaghi & Peck, 1948) which is too conservative for strong soils and is not sufficient for weak or compressible soils which may settle and tilt excessively as the result of large stresses at the pressed edge. The new formula regulates the design through the presence of the term σ_o/q_{lim}, which is related to the moment arm of the stresses acting across the sole. The small bearing capacity, q_{lim}, of soft soils leads to a significant shortening of the resistance arm, and thus to a clear distinction between the case of a soft soil and that of a strong soil.

Equation (9), together with an analysis of safety against sliding, is proposed as a standard requirement for the stability analysis of retaining walls and other similar structures resting upon cohesive soils, and with trial-and-error procedure for non-cohesive soils.

Since the problem is essentially one of bearing capacity, it would seem appropriate to require a safety factor of 1.5 to 3, depending upon the character of the structure and the certainty with which the strength of the soil is known.
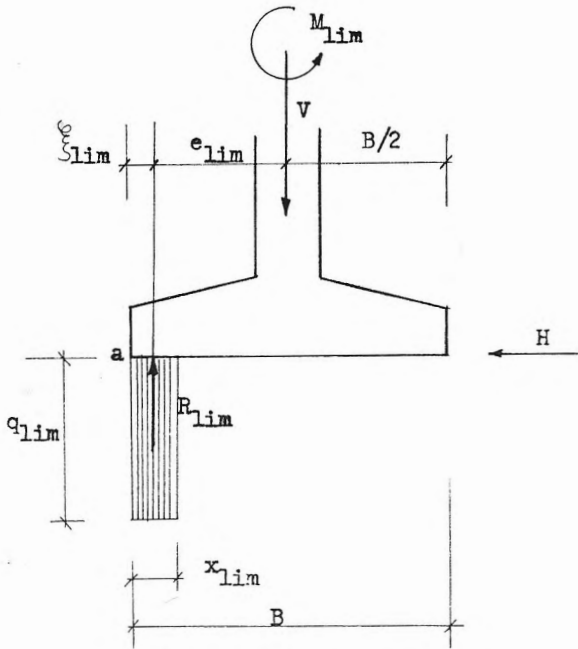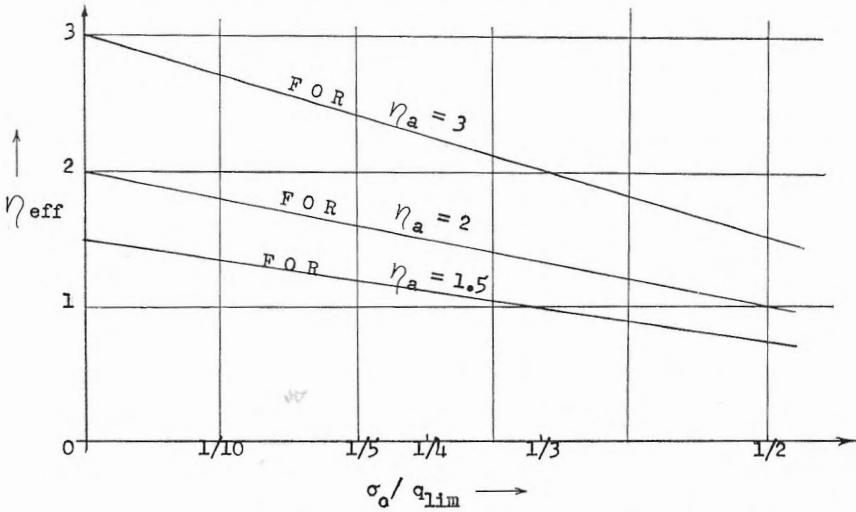
FIGURE 3.—THE LIMIT EQUILIBRIUM.

FIGURE 4.—DECREASE OF EFFECTIVE FACTOR OF SAFETY AGAINST OVERTURNING FOR DIFFERENT OVERTURNING FACTORS η_a AND DIFFERENT CONTACT PRESSURES σ_o.

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List of Symbols

a — pressed edge of the foundation sole.

b — width of the sole perpendicular to the plane of the moment.

B — width of the sole in the plane of the moment.

e — eccentricity.

e_{lim} — eccentricity in limit equilibrium.

H — horizontal force acting along the sole.

M — overturning moment acting about the y-y axis of the sole.

M_{lim} — overturning moment in limit equilibrium.

q_{lim} — limit soil stress (bearing capacity for the reduced sole width).

R — resultant of the vertical stresses acting on the sole.

R_{lim} — resultant of the vertical stresses in limit equilibrium.

V — vertical force acting along the centerline of the sole, includes the weight of the footing.

η_a — overturning factor.

η_{eff} — effective (true) factor of safety against overturning.

x_o — pressed contact width of the sole (in the plane of M).

x_{lim} — pressed contact width of the sole in limit equilibrium.

ξ — distance of the resultant force, R, from the pressed edge.

ξ_{lim} — distance of the resultant force, R, from the pressed edge in limit equilibrium.

σ_o — average contact pressure against the sole.

σ_a — maximum contact pressure at the pressed edge of the sole.