JOURNAL *of the*

BOSTON SOCIETY
OF
CIVIL ENGINEERS

116 YEARS

1848-1964

JULY - 1964

VOLUME 51 NUMBER 3

JOURNAL OF THE

BOSTON SOCIETY

OF

CIVIL ENGINEERS

Volume 51 JULY, 1964 Number 3

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PAPERS AND DISCUSSIONS

OF GENERAL INTEREST

Journal of Boston Society of Civil Engineers is indexed regularly by Engineering Index, Inc.

Second-Class postage paid at Boston, Mass.

Published four times a year, January, April, July and October, by the Society
47 Winter St., Boston, Massachusetts

Subscription Price $7.00 a Year (4 Copies)
$1.75 a Copy

All orders must be accompanied by check

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JOURNAL OF THE

BOSTON SOCIETY OF CIVIL ENGINEERS

Volume 51 | JULY, 1964 | Number 3

THE ANALYSIS AND DESIGN OF ANTENNA TOWER FOUNDATIONS

By H. M. Horn,* *Member*

(Presented at a meeting of the Structural Section, B.S.C.E., held on March 13, 1963.)

Introduction

It is almost impossible at the present time to read a newspaper without encountering some mention of exploration of outer space in the headlines. One of the most exciting and important events of this century has been the advent of the Space Age. The engineering profession can take justifiable pride in the important roles that it has played in the field of rocketry and in the conquest of space. One such role is performed by the engineer who is called upon to design foundations for towers which support the satellite and missile tracking systems. These systems are vital to the success of this country's space program and the defense of the free-world and, consequently, pose a great but welcomed challenge to the ingenuity of the foundation engineer.

The purpose of this paper is twofold, namely: (1) to outline and discuss the design considerations which are unique to foundations of support structures for highly accurate antenna tracking systems; and (2) to propose design approaches for some of the foundation problems which may be encountered.

The design approaches presented are based on the author's experiences in this area of foundation engineering. They are not unique to the author, and it is not proposed that the analytical procedures outlined necessarily allow the engineer to make an accurate prediction of foundation performance. In actual fact, relatively few comprehensive

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investigations of performances of antenna tower foundations have been made. Until sufficient performance data become available, it will not be possible to evaluate critically any design procedure proposed. Consequently, the engineer must always keep in mind that there is considerable uncertainty about the reliability of the theories, and the values of soil properties used in arriving at a design for a tracking antenna tower foundation.

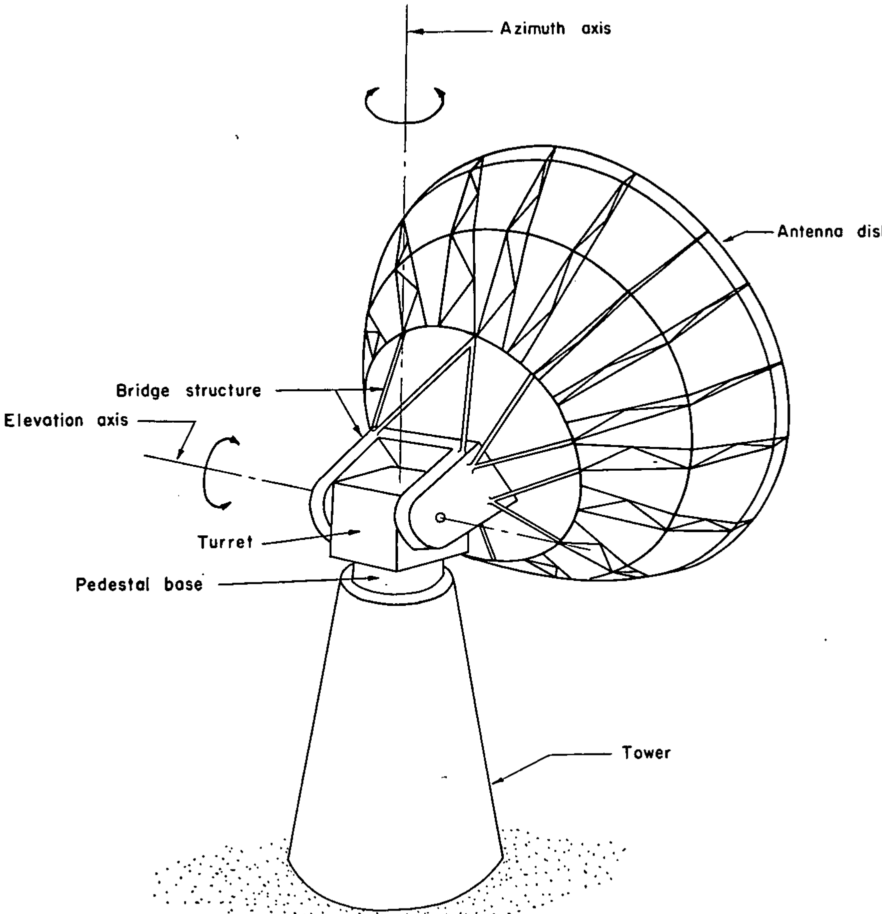
Characteristics of Antenna Tracking Systems

Tracking antennas range widely in size and shape. At the present time, the largest movable antenna in this country is located at the National Radio Astronomy Observatory in Green Bank, West Virginia. This antenna, which was completed in 1962, has a dish that is 300 ft. in diameter and weighs 600 tons. The famed antenna at Jodrell Bank in England has a 250 ft. diameter dish weighing 750 tons. The author's experiences have been limited to antennas having dishes 60 ft., or less in diameter. However, he believes that problems associated with foundation design for highly accurate tracking antennas are largely independent of size.

The general features of a typical tracking system are shown in Fig. 1. The antenna dish, which is connected to a turret by means of a bridge structure, can be rotated usually about two axes; a vertical azimuth axis, and a horizontal elevation axis. The surface of the dish may or may not be solid, but generally consists of a mesh-like affair which is supported by a structural framework. The pedestal base, upon which the turret rotates, is fixed and rests on a support structure consisting of either a metal or a reinforced concrete tower.

Any angular displacement of the top of the tower results in an antenna aiming error. Only a very small error is tolerable during tracking operations. When the inaccuracy exceeds the tolerable limit, it is detected and an electro-mechanical servo-system applies a torque to the dish which moves the antenna's line of sight back towards the object being tracked. It is possible for the servo-system to overcompensate the original aiming inaccuracy, thus causing an error in the opposite direction. Such an error would be detected and a compensating torque applied to the antenna dish. This process is repeated until the tracking error is smaller than the tolerable limit.

Some antennas are provided with radomes which protect them from the elements, i.e., wind, rain, the rays of the sun, etc. Discussion in this paper will be limited to systems without radomes, as is illustrated in Fig. 1.



TRACKING ANTENNA COMPONENTS

FIGURE I

FOUNDATION DESIGN CRITERIA

A. *General*

The foundation of an accurate tracking system must satisfy all of the conventional requirements, that is, it must be stable under all of the loading conditions which will be imposed upon it, and it must not suffer excessive permanent total or differential settlement. In addition to these requirements, the foundation must act as an almost perfectly rigid platform for the antenna tower when acted upon by transient forces during tracking operations. It is the stringency of the rigidity requirements which makes antenna foundation design the challenge that it is.

B. *Loading Conditions*

Antenna foundations are subjected to both static and transient loads. Gravitational forces, which generally take the form of static loads, may cause differential settlement of the foundation, particularly if the compressibility or thickness of the underlying strata of soil or rock varies appreciably over the site. Differential settlement occurring after completion of construction usually proceeds at a relatively slow rate. Therefore, although such settlement causes angular displacement of the antenna about the elevation axis, the resulting aiming inaccuracy can usually be compensated for satisfactorily by periodic releveling of the antenna pedestal. However, releveling cannot be used to compensate for aiming inaccuracies resulting from transient loads, such as wind forces and inertial forces, because of their short duration. Accordingly, the critical characteristic of an antenna foundation is usually its rigidity when subjected to short-duration transient loads.

Tracking antennas are generally not expected to operate when acted upon by extremely strong wind. Consequently, two sets of design loading conditions are commonly specified, i.e., *survival loading conditions* and *operational loading conditions.*

Survival loading conditions comprise the most critical combination of forces, moments, and torques, insofar as stability is concerned, which can be expected to act upon the antenna system. Generally, survival loads are the result of high-velocity wind acting on the superstructure, with the design wind velocity depending on the climatic history of the site area. A survival wind velocity having a magnitude of 100 miles per hour might be considered typical. During survival conditions, the antenna is usually in a "stowed" position, that is, the antenna is directed at a point 90 degrees above the horizon. The survival forces, moments, and torques are computed on the basis of the wind acting on the system with the antenna in such a position, and are used in the analysis of foundation stability. Foundation stability, rarely, if ever, is a critical factor.

Operational loading conditions comprise the most severe combination of inertial forces, and/or wind forces under which the tracking system can be expected to operate satisfactorily. The operational loading conditions used in design analyses depend to a large degree on what the antenna system is being devised to track. Some systems are designed to track objects which will remain fixed or which will move very slowly relative to points on the earth. With such "synchronous" systems, the magnitudes of the angular velocity and acceleration of the antenna while tracking are very small. Consequently, inertial forces due to movements of synchronous antenna systems during tracking operations are generally negligible. Several of the proposed satellite communication systems will involve the use of "slow-tracking" antennas.

Recent successes with one such system, known as SYNCOM, have been notable. SYNCOM is a satellite communication system being developed by the Federal Government. When perfected, it will have three satellite relays, each in an equatorial orbit, 22,300 miles above the earth's surface and revolving in the same direction as the earth rotates. If perfect orbits could be achieved, each relay would remain over a fixed point on the equator and would appear to remain motionless in the sky when viewed from points on the earth. Three satellite relays having orbits synchronized with the earth's rotation and spaced equidistant around the equator would be able to relay radio signals between any points on the earth with the exceptions of those in the remotest polar regions. However, it is unlikely that perfect orbits will be achieved. Furthermore, the orbits of earth satellites are warped in time by solar pressure and the gravitational field of the moon. As a consequence, the relays would drift slowly relative to the earth and orbital adjustments would have to be made from time to time if the system were to function properly for an extended period. The important point, insofar as antenna design is concerned, is that only a very low tracking rate would be required and, therefore, the inertial forces acting on the support structure and foundation would be small. The only significant transient loads acting on the support structure and foundation would be those due to wind.

There are many antennas which are devised to track objects that move rapidly relative to points on the earth. Such objects might be ballistic missiles or communication satellites having lower orbits than those in the above mentioned synchronous systems. High tracking rates are required to follow such objects, and the inertial force system resulting from the angular acceleration and velocity of the antenna dish has to be considered in the foundation design analysis. The magnitudes of the inertial forces, moments, and torques acting on the foundation are determined by the mass and geometry of the antenna system, and by the magnitudes of the angular acceleration and velocity of the antenna. Antennas with biaxial angular accelerations of up to 6 degrees per second per second and angular velocities of up to 10 degrees per second are not uncommon.

C. *Displacement Conditions*

The foundation angular displacements resulting from transient operational loads constitute only one source of aiming error in the overall tracking system, but nonetheless, are very important. Even an extremely small foundation rotation will result in a large linear tracking error when the object being tracked is hundreds, or even thousands of miles from the antenna. Foundation translation, on the other hand, is rarely, if ever, an important consideration.

Displacement design criteria vary widely. The author has designed two antenna foundations for which the maximum allowable foundation rotation about a horizontal axis under operational conditions was 1 second, which is equivalent to about 5 microradians. Maximum allowable foundation rotations of 10 to 20 microradians in either the azimuth or elevation planes under operational conditions are not uncommon in this type of design work. It follows that such criteria can only be met when subsurface soil and/or rock conditions are favorable, and when foundation stress changes are kept to a minimum.

D. *Natural Frequency Considerations*

If the top of an antenna tower were displaced horizontally and then released, the top of the structure would vibrate back-and-forth at an almost constant frequency, until the vibrational energy was dissipated by damping losses in the tower, the foundation, and the underlying soil or rock. This frequency of vibration, which is a function of the geometry, mass, and mechanical properties of the tower and foundation, is called the *natural frequency* of the structural system in the rocking mode. The system would also have natural frequencies for the other possible modes of vibration, i.e., the torsional, horizontal translational, and vertical translational modes.

Error correcting servo-systems are included as part of almost all tracking systems. The characteristics of the servo will depend on what the antenna is being designed to track, and on the tolerable aiming error. A servo is analogous to a system of masses, springs, and dashpots which has a cyclic force input. It follows, that a servo can become unstable in the same manner that a mass-spring-dashpot system will develop a resonant condition when the frequency of the cyclic force input corresponds to the natural frequency of the system. If the servo-system becomes unstable, it will not correct the antenna aiming error, but will cause the line of sight of the antenna to oscillate back-and-forth past the target.

In order to achieve a stable error correcting servo-system, the natural frequencies of the tower and foundation must meet certain natural frequency requirements. In cases where the characteristics of the servo-system are such that the structural natural frequencies are considered to be critical, minimum natural frequencies are specified in the foundation design criteria. Foundation design specifications requiring the natural frequency to exceed 4 or 5 cycles per second in each of the potential vibrational modes are common. Often, the natural frequency requirements of the foundation control the foundation design and possibly also the design of the superstructure, as will be explained later.

Elastic Characteristics of Foundation Soil and Rock

Both the natural frequency of a foundation, and the foundation displacement resulting from stress changes in the underlying soil or rock are functions of the compressibility of the underlying materials. Although neither rock nor soil are linearly elastic materials, linearly elastic behavior is approached when the stress changes in these materials are limited to small values. Generally, the assumption of elastic behavior is made when analyzing the vibrational characteristics of a foundation. Thus, the compressibility characteristics of the foundation soil and rock are defined in terms of three elastic coefficients which are inter-related; namely, Young's modulus (E), the shear modulus (G), and Poisson's ratio (ν). In the case of soil, these coefficients are functions of the magnitude of the stress changes imposed on the soil (Whitman, 1963). The methods commonly used to evaluate the coefficients often involve subjecting a specimen of the soil to stress changes which are smaller than those which will occur in the prototype situation. The ability of these methods to predict the behavior of the soil underlying the tower foundation is, therefore, one of the uncertainties in the foundation design.

Various testing methods have been used to evaluate the elastic coefficients of the foundation soils. Although a detailed discussion of each method is considered to be beyond the scope of this paper, the more prominent techniques will be described in brief in the paragraphs that follow.

A. *Laboratory Vibration Tests*

Laboratory vibration tests can be performed on specimens of soil to provide data from which E, G, and ν of the specimen can be estimated (Wilson and Dietrich, 1960). In these tests, vibrational energy is supplied to one end of a cylindrical specimen of soil, the ends of which are capped and the sides encased in a rubber membrane. A crystal pickup is attached to the other end and the vibrational energy transmitted through the specimen is measured. The frequency of the input energy is varied until the amplitude of the vibrational energy transmitted through the specimen reaches a peak, indicating a resonant condition.

It can be shown that, if compression waves are supplied to a cylindrical elastic specimen, Young's modulus can be estimated from the resonant frequency ($\bar{f}_c$), the length (L) of the specimen, and the density (ρ) of the material by means of the following relationship:

$$E = 16\bar{f}_c^2 L^2 \rho$$

If torsional vibrational energy is supplied to the specimen and the resonant frequency ($\bar{f}_t$) determined, the shear modulus of the material can be estimated with the following relationship:

$$G = 16\bar{f}_t^2 L^2 \rho$$

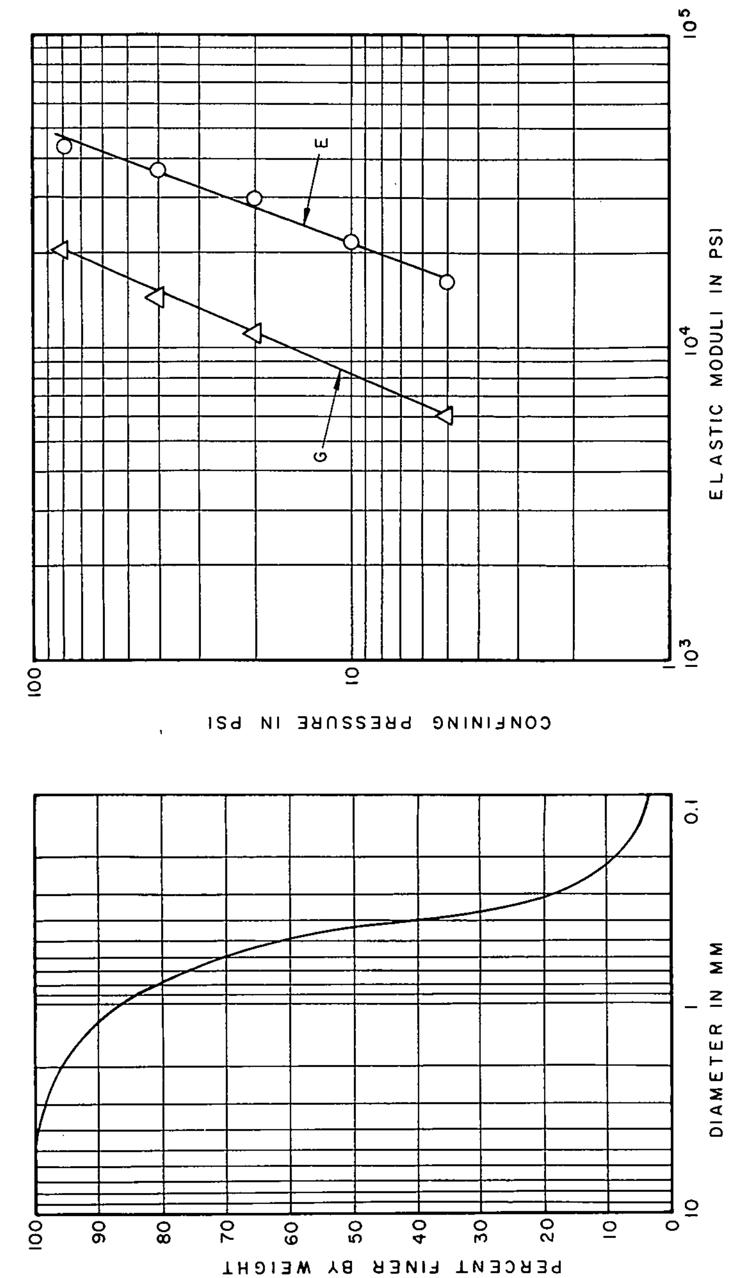
Once the shear modulus and Young's modulus have been determined, Poisson's ratio can be computed from the following expression:

$$\nu = \frac{E}{2G} - 1$$

In the case of cohesive soils, undisturbed samples are tested. Since it is usually not practicable to obtain undisturbed samples of non-cohesive soils, such as sand or gravel, these soils are tested in the reconstituted state. Considerable care must be taken to insure that the non-cohesive specimen being tested has a void ratio which is representative of that of the *in situ* soil.

The elastic coefficients of soils, particularly non-cohesive soils, are altered to a considerable degree by changes in the state of stress. The magnitudes of Young's modulus and the shear modulus of a deposit of non-cohesive soil increase with increasing depth. In order to get the effect of increasing depth when running laboratory vibration tests, the tests are performed at different levels of confining pressure, with the confining pressure being transmitted to the specimen through the rubber membrane and caps in the same manner as the chamber pressure is applied to a specimen in a conventional triaxial test. The results of a typical test series performed on a specimen of coarse to fine sand are presented in Fig. 2, along with the gradation of the material tested. It should be noted that the logarithm of the magnitude of both the shear modulus and Young's modulus varies approximately linearly with the logarithm of the magnitude of the confining pressure. This is generally true for non-cohesive soils. There is relatively little scatter in the data plotted in Fig. 2. This is often not the case with the results of laboratory vibration tests.

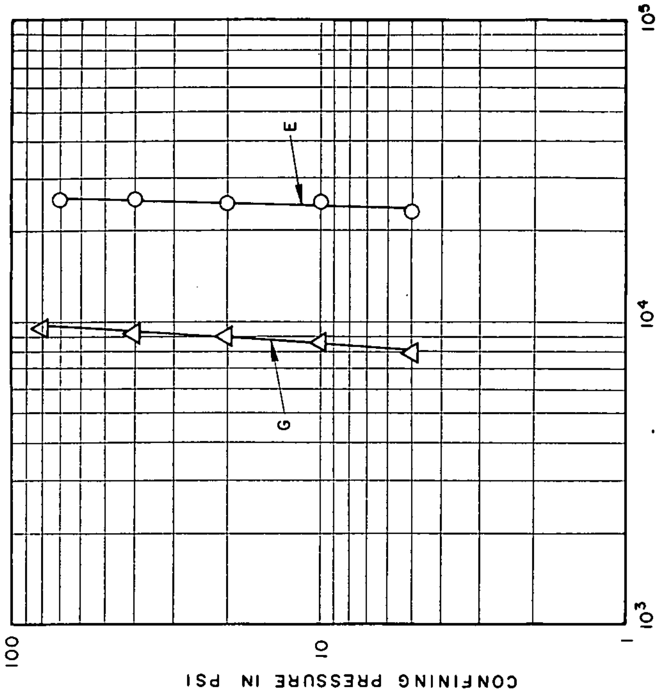
The results of a laboratory vibration test series performed on highly overconsolidated clay are presented in Fig. 3 along with the soil's Atterberg limits. It will be noted that both the shear modulus and Young's modulus were virtually unaffected by changes in the confining pressure over the range of confining pressure investigated. It is likely that the stability of these moduli would continue until the confining pressure exceeded the preconsolidation pressure of the clay. In areas underlain by such materials, relatively high values of E and G would be found, even at shallow depths. However, there is some question as to whether or not this is true when the overconsolidated clay is fissured. Laboratory vibration tests performed on specimens of stiff, fissured clay indicate that some of these soils have elastic moduli-confining pressure characteristics similar to non-cohesive soil,



VIBRATION TEST DATA FOR TYPICAL SAND

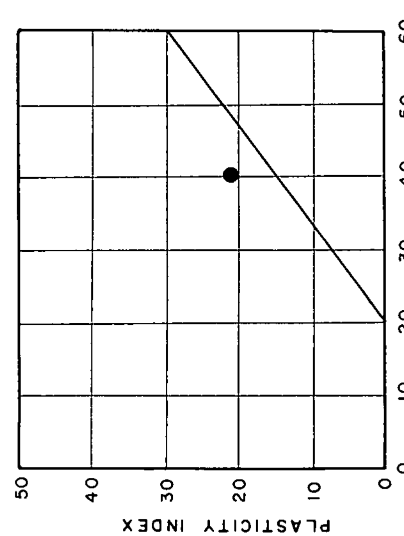
GRAIN SIZE DISTRIBUTION

FIGURE 2



VIBRATION TEST DATA FOR OVERCONSOLIDATED PARTIALLY SATURATED CLAY

Natural water content = 14.7%
Dry unit weight = 109 PCF
Degree of saturation = 74 %



PLASTICITY CHART

FIGURE 3

i.e., the moduli increase with increasing confining pressure. It is possible that this behavior is the result of the fissures opening up somewhat during sampling and preparation of the test specimen, and then being reclosed more and more as the confining pressure is increased.

The laboratory vibration test can be used to evaluate the elastic properties of sound rock. However, most deposits of rock are jointed to some degree. Joints reduce the overall elastic moduli of rock below the moduli of the sound fragments. Consequently, results of laboratory vibration tests performed on rock cores should be used with great discretion.

B. *Field Tests*

There are several field tests which can be used to estimate the shear modulus of *in situ* soil. These tests have certain inherent advantages over a laboratory test, since they are performed on essentially undisturbed soil, and the tests provide average values of the shear modulus between points in a soil mass. However, the field tests have some disadvantages which will be mentioned later.

1. *Seismic Tests*

The shear modulus of an elastic material can be computed if Poisson's ratio and the soil density are known, and if the velocity at which compression waves (v_c) propagate through the material can be determined (Jacobs, Russell and Wilson, 1959). The seismic test which will be described provides a means of determining the compression wave velocity.

Two borings are made several feet apart, with one boring extending to the bottom of the zone of interest and the other to the top of the zone. A small explosive charge is placed at the bottom of one of the borings, and a geophone in the bottom of the other. The charge is detonated and the time that it takes for the compression wave generated to propagate from the point of explosion through the soil to the geophone is measured. Since the distance between the two points is known, the compression wave velocity can be computed.

In order to evaluate the shear modulus from the compression wave velocity, the value of Poisson's ratio must be determined by some other means, possibly by laboratory vibration tests, or it must be assumed. Unfortunately, a small change in the value assumed for Poisson's ratio results in a relatively large change in the value computed for the shear modulus. Therefore, any computations based on assumed values of Poisson's ratio are of questionable validity. In addition, of the three elastic coefficients (E, G, and ν) which can be evaluated from the results of laboratory vibration tests, Poisson's ratio is subject to the most error, since it is equal to the difference between two numbers of relatively equal magnitude, the larger being the ratio of two experimentally determined quantities. It can be concluded, therefore, that values of shear modulus based solely on compression wave velocities are of questionable accuracy.

2. *Surface Vibration Tests*

Surface vibration tests have been developed in recent years as a means of appraising pavements, base courses, and subgrades (Heukelom and Foster, 1960). Techniques which have been developed can be used to evaluate the shear modulus of the upper portions of a soil mass. The test is based on the fact that the velocity of propagation of a surface wave in an elastic medium is related to the shear modulus of the medium.

A cyclic vertical load is applied to the surface of the soil by means of a variable-frequency vibrator. Waves are propagated away from the loaded area in all directions within and on the surface of the soil mass. It can be shown (Jones, 1958), that the velocity (v_s) of waves occurring at the surface of a homogeneous, semi-infinite, elastic medium is closely approximated by:

$$v_s \approx \sqrt{G/\rho} \qquad ;$$

where G and ρ are as defined previously. If the assumption is made that the soil mass is elastic and homogeneous, the shear modulus of the soil can be evaluated if the surface wave velocity and soil density are determined.

In the surface vibration test, a vibration pickup is moved along the surface away from the vibrator. The phase difference between the waves at the vibrator and the waves at the pickup is measured electronically. When the phase angle between waves at the two points equals 360 degrees, the distance between the vibrator and the pickup is equal to one wave length (λ). Since the frequency of the surface waves (f_s) is the same as that of the vibrator, and is therefore known, the wave velocity can then be determined by:

$$v_s = f_s\lambda$$

Therefore, the shear modulus can be approximated by:

$$G = f_s^2\lambda^2\rho$$

Often, the shear modulus of a natural soil deposit is a function of depth, in which case, the assumption of homogeneity is invalid. Empirical evidence indicates that the wave velocity determined in the surface vibration test is a measure of the shear modulus of the soil at a depth equal to half a wave length. If this is the case, the shear modulus can be evaluated for various depths by merely changing the frequency generated by the vibrator and determining the new surface wave length.

The depth to which the shear modulus can be evaluated by the above technique is limited by the maximum surface wave length that can be generated by the vibrator. The vibrators which are commonly used limit the depth of evaluation to about 10 to 15 feet.

3. *Mass Vibration Tests*

If the velocity of propagation of both the compression wave and shear wave are known over the distance between two points in an elastic medium, the average values of E, G, and ν between the two points can be determined. Mass vibration tests have been used successfully in recent years (Evison, 1956), to evaluate the elastic coefficients of rock formations by measuring the time intervals required for compression waves and shear waves to travel between two points in a formation. A vibrator is placed at some point in the rock formation, possibly in a tunnel or a shaft, and a geophone pickup placed at another point either in, or on a surface of the formation. The vibrator is then started and the starting time recorded electronically. The compression wave arrives at the geophone first and the arrival time recorded; the arrival time of the shear wave, which can easily be distinguished on the pickup trace, is also recorded. If the distance between the vibrator and the pickup point is known, the two wave velocities can be computed.

The author does not know if this method has ever been used successfully in evaluating elastic coefficients of soil deposits. However, the approach appears to offer considerable promise.

C. *Evaluation of Laboratory and Field Testing Methods*

Laboratory vibration testing techniques allow the evaluation of the three elastic coefficients of a specimen taken at a "point" in a soil mass which is to be subjected to small stress changes. A considerable number of specimens have to be tested before the variability of the elastic coefficients in the soil mass can be established. In addition, the specimens being tested will be disturbed to some degree, with the degree of disturbance varying between the slightly disturbed character of a good "undisturbed" sample of clay, to that of complete remolding in the case of a reconstituted specimen of non-cohesive soil. Little quantitative information is available at this time on the effect of sample disturbance on the magnitudes of the elastic coefficients of soil acted upon by small stress changes.

Despite the inherent short-comings of laboratory vibration testing methods for evaluating the elastic characteristics of foundation soils, the author feels that they provide the best single approach available at the present time, since both the shear modulus and Young's modulus can be estimated without having to assume a value for Poisson's ratio; and laboratory vibration tests can be performed on samples taken at relatively great depths, which may be an important consideration in some instances. In addition, soil samples recovered from borings made during the general site investigation can often be used to provide specimens for the vibration tests. The field testing methods presently available can provide valuable supplementary information, and their use should be considered when formulating the testing program. In addition, the field testing approach offers considerable promise, and hopefully, tests will be developed and made readily available which will provide the design engineer with an accurate picture of the compressibility characteristics of *in situ* soil to depths of at least 50 ft.

The best testing procedure is obviously the one which yields results that enable the designer to predict actual foundation performance. Until the performances of a considerable number of actual antenna foundations are studied and evaluated, the "best" testing technique to be used for a particular antenna foundation analysis will be debatable. Even in cases where test data show relatively little scatter, such as in Figs. 2 and 3, there is considerable uncertainty concerning how representative such data are of the actual *in situ* properties.

Theory Available for Design

The foundation design must satisfy angular displacement criteria and, in some cases, natural frequency criteria. Relationships have been developed, based on the theory of elasticity, which can be used to compute the angular displacement of a rigid footing resting on a homogeneous, isotropic, semi-infinite elastic material and which is acted upon by an overturning movement or torque. Other relationships have been developed to compute the natural frequencies of a mass resting on such a medium for the various vibrational modes, as well as the displacements which would develop if a resonant condition did occur. The ability of these relationships to predict the deflection and vibrational characteristics of real foundations will depend on how closely the actual foundation conditions agree with the idealized conditions assumed in deriving the relationships, and on how accurately the design engineer can estimate the elastic coefficients of the foundation soil. Despite the fact that soil formations never satisfy the idealized conditions completely, although the elastic condition is approximated when stress changes are very small, the theoretical relationships do provide some means of estimating the behavior of a proposed foundation. However, as with all foundation design evaluations, the engineer must exercise considerable judgement when using theoretical relationships to predict foundation behavior.

A. *Angular Deflections*

Transient angular deflections of a foundation can result from two types of loads; i.e., non-cyclic loads, such as those resulting from wind; and cyclic loads, such as the inertial forces which are generated when aiming errors are being corrected. The former can be treated as static loads when computing displacements. However, since cyclic loads result in a system of forced vibrations, the increase in the amplitude of vibration as resonance is approached should be considered when estimating displacements due to these loads. The displacement at resonance can be computed and considered as an upper limit.

Expressions relating the angular displacement of a circular or rectangular rigid footing to the magnitude of a static overturning moment acting on the footing, the dimensions of the footing, and the elastic properties of the foundation material have been published by Weissmann and White (1961) and are presented below.

Circular Footing:

$$\tan \theta_m = \frac{6\,(1 - \nu^2)\,M}{E\,d^3}$$

Rectangular Footing:

$$\tan \theta_m = \frac{16\,(1 - \nu^2)\,M;\ b >> a}{\pi E\,a^2 b} \qquad ;$$

where

θ_m = angular displacement of footing due to overturning moment
M = overturning moment
a = width of rectangular footing
b = length of rectangular footing
d = diameter of circular footing
E and ν are as defined previously

The latter expression is exact for cases where the length of the footing is much larger than the width. For the special case of a square footing, the following expression is applicable:

$$\tan \theta_m \approx \frac{16\,(1 - \nu^2)\,M}{\pi E\,c^3} \qquad ;$$

where c = the side dimension of the footing.

The above expression yields values for θ_m which are only approximately correct. However, the error is small.

Richart (1961) has published relationships for the angular displacement of a circular footing acted upon by a static torque about the axis of symmetry of the footing:

Rigid-base shear stress distribution:

$$\theta_t = \frac{3\,T}{16\,G\,r^3}$$

Linear shear stress distribution:

$$\theta_t = \frac{T}{\pi G\, r^3};$$

where

θ_t = angular displacement due to the torque
T = torque
r = radius of footing
G is as defined previously

B. *Foundation Vibrations*

Richart (1960 and 1961) has published theoretical relationships which can be used to compute the natural frequencies of an oscillator with a rigid circular base resting on an elastic foundation, and the maximum displacements which would occur if a resonant condition developed. Four modes of vibration were considered by Richart. Relationships for two of the modes (rocking, and torsional) are presented below:

Rocking Mode:

$$\bar{f}_r = \frac{a_0}{2\pi r}\sqrt{\frac{G}{\rho}}$$

$$\bar{\theta}_r = \frac{\overset{*}{A}\,\bar{M}}{Gr^3}$$

Torsional Mode:

$$\bar{f}_t = \frac{a_1}{2\pi r}\sqrt{\frac{G}{\rho}}$$

$$\bar{\theta}_t = \frac{\overset{*}{B}\,\bar{T}}{Gr^3};$$

where

$\overset{*}{A}$ = amplitude factor for the rocking mode
$\overset{*}{B}$ = amplitude factor for the torsional mode
$\bar{M}$ = amplitude of the cyclic moment
$\bar{T}$ = amplitude of the cyclic torque
a_0 = frequency factor for the rocking mode

a_1 = frequency factor for the torsional mode

$\bar{f}_r$ = natural frequency of the oscillator in the rocking mode

$\bar{f}_t$ = natural frequency of the oscillator in the torsional mode

$\bar{\theta}_r$ = maximum angular displacement in the rocking mode at resonance

$\bar{\theta}_t$ = maximum angular displacement in torsional mode at resonance

The parameters $\overset{*}{A}$, $\overset{*}{B}$, a_0, and a_1 are dimensionless, and are functions of the density and Poisson's ratio of the elastic foundation, the base dimensions and inertia of the oscillator, and the amplitude-frequency characteristics of the exciting moment or torque, as the case may be. Richart (1960 and 1961) has published these functions in the form of curves, examples of which are presented in Figs. 4 and 5, the latter for the torsional mode, the former for the rocking mode. The curves shown are based on the assumptions that the amplitude of the exciting moment or torque is not affected by changes in frequency, and that the oscillator has a rigid base.

Since a tower and foundation acted upon by a cyclic force system is effectively the same as an oscillator, the above relationships can be used to estimate the natural frequency of a foundation system, and the maximum displacement which would occur if a resonant condition developed.

Foundation Design

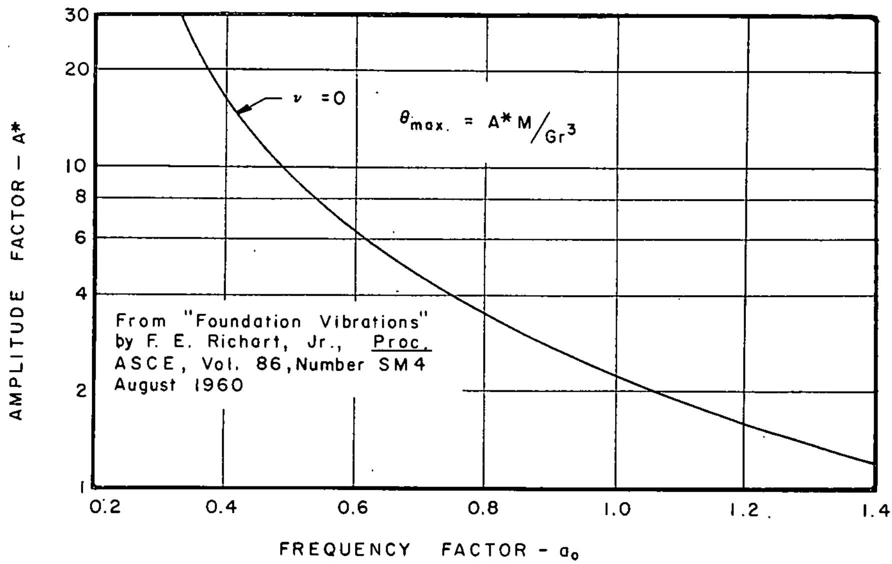
As explained earlier, some antenna foundations have displacement criteria but no natural frequency requirements, whereas others have both types of criteria. Each of these two basic types of foundation design problems will now be considered. The design criteria and subsoil conditions presented in the two illustrative design problems which follow are considered to be representative of those which an engineer is likely to encounter in antenna foundation design.

A. *Example 1: Natural Frequency Not Critical*

1. *Foundation Design Criteria*

The foundation is to support a steel tower having a regular octagonal cross-section with an inscribed diameter at the base of the tower of 31 ft. The antenna system will be expected to operate satisfactorily

VIBRATIONAL PARAMETERS IN ROCKING MODE FOR RIGID CIRCULAR FOOTING RESTING ON AN ELASTIC HALF-SPACE



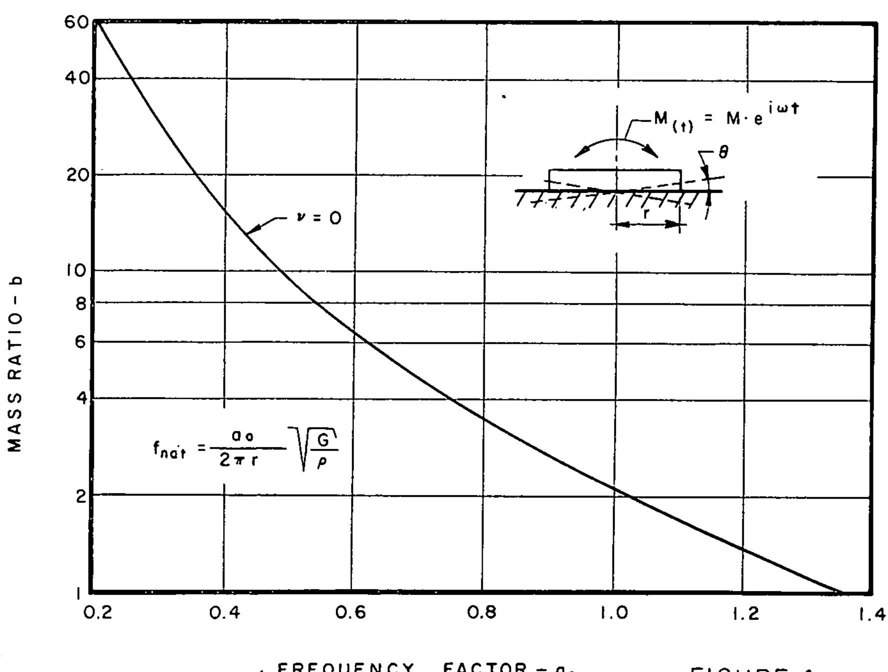


FIGURE 4

VIBRATIONAL PARAMETERS IN TORSION MODE FOR RIGID CIRCULAR FOOTING RESTING ON AN ELASTIC HALF-SPACE

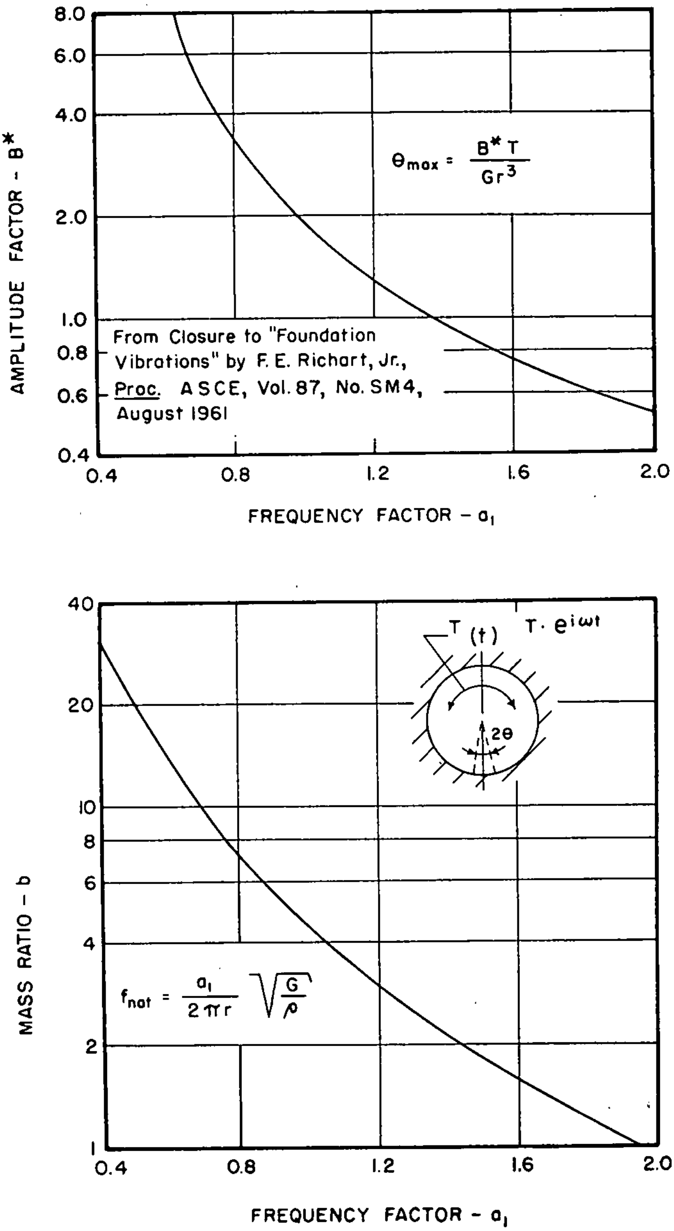


FIGURE 5

EXAMPLE 1: Slow-Tracking Antenna

Operational Design Loads:

$H = 24$ Kips
$M_o = 1,700$ Ft − Kips
$T = 295$ Ft − Kips
} Wind Forces

Inertia forces are negligible.

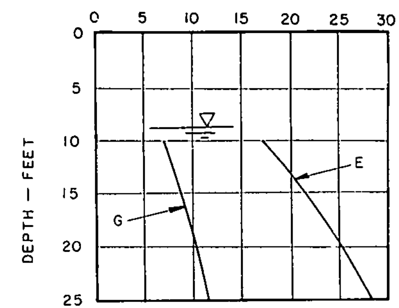
Operational Displacement Criteria:

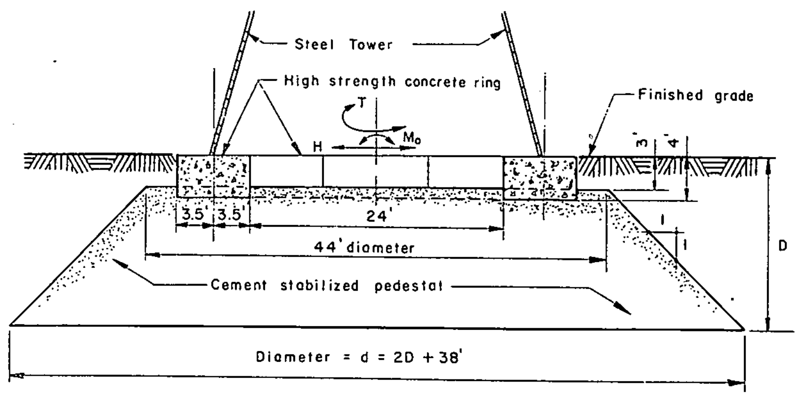
Angular displacements of the antenna about either the elevation axis or the azimuth axis due to strains in the subsoil should not exceed 0.001 degrees.

Foundation Natural Frequency Criteria:

None specified for the foundation.

ELASTIC MODULI OF SUBSOIL BASED ON RESULTS OF LABORATORY VIBRATION TESTS





SUBSOIL

Medium compact, medium to fine quartz sand to a depth of more than 100 feet.

FIGURE 6

when acted upon by winds having velocities up to 45 miles per hour with gusts 25 percent higher. The weight of the superstructure is 350 kips. The operational design loads resulting from the wind are listed in Fig. 6. It should be noted that inertial forces are considered to be negligible. Angular displacements of the antenna about either the elevation axis or the azimuth axis, resulting from strains in the subsoil, should not exceed 0.001 degrees. The natural frequency of the foundation is not critical.

2. *Subsurface Conditions*

Borings indicate that a uniform deposit of medium compact, medium to fine quartz sand exists to a depth of over 100 ft., with the ground water level occurring at a depth of about 9 ft. Values for the elastic moduli of the sand, based on the results of laboratory vibration tests, are shown in Fig. 6 as functions of depth. Both moduli increase almost linearly with depth below the ground water level.

3. *Analysis and Design*

Preliminary studies indicated that a large diameter, cement-stabilized pedestal could be designed to satisfy the foundation displacement criteria. Such a foundation is illustrated in Fig. 6. The pedestal would consist of very lean concrete placed in thin layers and compacted in the same manner as compacted granular fill. The low cement content of the pedestal material would minimize displacements due to concrete shrinkage and, at the same time, would provide a pedestal having considerable rigidity. The high strength concrete ring shown in Fig. 6 would be provided to miminize displacement resulting from the relatively high stress changes which would develop immediately below the base of the tower. The base plate of the tower would be connected to the foundation by means of anchor bolts which would extend through the high strength ring into the cement-stabilized pedestal, thus assuring positive contact between the tower and the ring, and between the ring and the pedestal.

The design analysis consists of determining the minimum foundation depth needed to provide a pedestal satisfying the displacement criteria. The depth required is determined by a trial and error procedure. Since there is always considerable uncertainty regarding the elastic moduli of the foundation materials, particularly those of the soil, the author believes that a factor of safety (FS) of between 1.5 and 2.0 should be used on displacement requirements. A factor of safety of 1.5 will be used in this example.

Try a foundation depth of 16 ft.

$E = 22{,}200$ psi
$G = 9{,}200$ psi

a) Elevation Angular Displacement

$M = 1700 + 16 \times 24 = 2084$ ft. — kips
$d = 38 + 2 \times 16 = 70$ ft.

$$\theta_m = \frac{6\,M}{E\,d^3} \times FS = \frac{6 \times 2084 \times 1.5}{22.2 \times 144} \times \frac{360}{2\pi}$$
$$= 0.98 \times 10^{-3} \text{ degrees} < 1.0 \times 10^{-3} \text{ degrees}$$

b) Azimuth Angular Displacement

$$\theta_t = \frac{3T}{16\,Gr^3} \times FS = \frac{3 \times 295 \times 1.5}{16 \times 9.2(35)^3 \times 144} \times \frac{360}{2\pi}$$
$$= 0.1 \times 10^{-3} \text{ degrees} < 1.0 \times 10^{-3} \text{ degrees}$$

Therefore, *use* a foundation depth of 16 ft.

The above computations indicate that a foundation depth of 16 ft. would satisfy the displacement criteria. In an actual design problem, the displacements resulting from strains in the high strength concrete ring would have to be evaluated. Such displacements, however, are usually small and for the sake of simplicity, will not be considered in this example.

B. *Example 2: Natural Frequency Critical*

1. *Foundation Design Criteria*

The foundation is to support a tower having a base diameter of approximately 30 ft. Loads on the foundation under operational conditions result from both wind forces and cyclic inertial forces. The tower foundation under operational loading conditions must satisfy both displacement and natural frequency criteria. The operational design loads, displacement criteria, and natural frequency criteria are summarized in Fig. 7.

EXAMPLE 2: Fast-Tracking Antenna

Operational Design Loads:

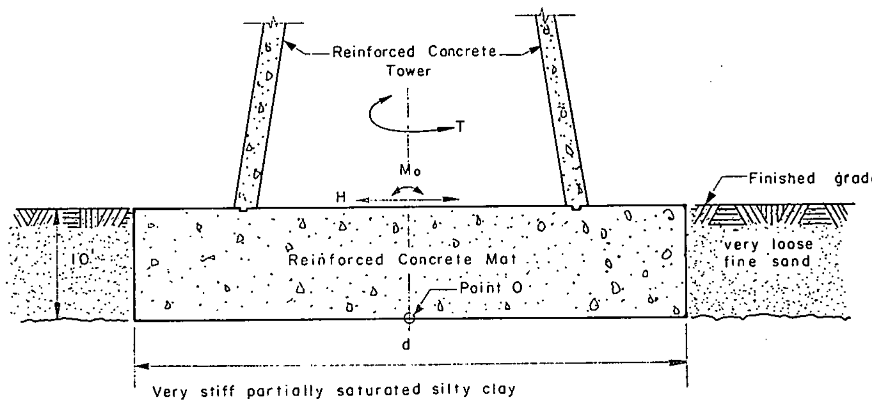
H:	H_w = 12 Kips (wind)
	H_i = 0 (inertial – cyclic)
M_o:	M_w = 1000 Ft. – Kips
	M_i = 200 Ft. – Kips
T:	T_w = 100 Ft. – Kips
	T_i = 200 Ft. – Kips

Operational Displacement Criteria:

Angular displacement of the antenna about either the elevation axis or the azimuth axis due to strains in the subsoil should not exceed 0.002 degrees.

Foundation Natural Frequency Criteria:

The foundation while supporting the mass of the superstructure shall have a natural frequency of 5 cycles per second or more in both the rocking and torsional modes.



γ = 115 lb/cu. ft.
E = 40,000 psi
G = 16,000 psi
ν = 0.25

FIGURE 7

2. *Subsurface Conditions*

The proposed antenna site is covered by a 10 ft. thick stratum of very loose fine sand, which is underlain to a great depth by a deposit of very stiff, partially saturated, silty clay. Laboratory vibration tests performed on the clay indicate that its shear modulus and Young's modulus are almost constant with depth and are respectively, 16,000 psi and 40,000 psi. Both of these moduli are quite high, but can be explained by the highly desiccated nature of the clay. The average unit weight of the clay is 115 lb./cu. ft.

3. *Analysis and Design*

Preliminary studies indicated that a reinforced concrete mat, 10 ft. thick and resting on the surface of the stiff silty clay, would offer the best possibility of satisfying the design criteria. The following information on the mass moment of inertia of the superstructure was supplied by the structural engineers:

Mass Moments of Inertia of Superstructure (I_0):

a) About horizontal axis through Point 0 (see Fig. 7)

Dish	16×10^6 slug-ft.^{2}
Turret and bridge	3×10^6
Concrete tower	35×10^6
Total	54×10^6 slug-ft.^{2}

b) About azimuth axis

Dish	0.60×10^6 slug-ft.^{2}
Turret and bridge	0.05×10^6
Concrete tower	9.00×10^6
Total	9.65×10^6 slug-ft.^{2}

Try a mat diameter (d) of 52 ft.

$$\text{Mass of mat} = \pi\,(52/2)^2 \times 10 \times \frac{150}{32.2} = 99{,}000 \text{ slugs}$$

Rocking Mode

a) Natural Frequency

$$I_0 = 54 \times 10^6 + 99 \times 10^3 \left[\frac{(26)^2}{4} + \frac{(10)^2}{12} + (5)^2\right]$$

$$= 74 \times 10^6 \text{ slug-ft.}^2 \text{ for entire system}$$

Mass ratio

$$= b = \frac{I_0}{\rho r^5} = \frac{74 \times 10^6 \times 32.2}{115\ (26)^5} = 1.73$$

Frequency factor (see Fig. 4)

$$= a_0 = 1.10 \text{ for } \nu = 0$$

$$\bar{f}_{r\,(\nu=0)} = \frac{a_0}{2\pi r_0}\sqrt{\frac{G}{\rho \times FS}}; \text{ use } FS = 1.5$$

$$= \frac{1.10}{2\pi \times 26}\sqrt{\frac{16 \times 10^3 \times 144 \times 32.2}{115 \times 1.5}} = 4.38 \text{ cps,}$$

but $\nu = 0.25$

$$\frac{\bar{f}_r\,(\nu = 0.25)}{\bar{f}_{r\,(\nu=0)}} \approx \sqrt{\frac{1-0}{1-0.25}} = 1.15$$

therefore, $\bar{f}_{r\,(\nu=0.25)} = 1.15 \times 4.38 = 5.04 \text{ cps} > 5 \text{ cps}$

b) Displacement

Due to Wind (θ_w)

$$\theta_w = \frac{6\,M_w(FS)}{E\,d^3} = \frac{6\,(1000 + 12 \times 10) \times 1.5}{40\,(52)^3 \times 144} \times \frac{360}{2\pi}$$

$$= 0.71 \times 10^{-3} \text{ degrees}$$

Due to inertial effects (assume resonance)

$$\text{Amplitude factor} = \overset{*}{A} = 1.9 \text{ (see Fig. 4)}$$

$$\bar{\theta}_r = \frac{\overset{*}{A} M_i\,(FS)}{G r^3} = \frac{1.9 \times 200 \times 1.5}{16 \times (26)^3 \times 144} \times \frac{360}{2\pi}$$

$$= 0.80 \times 10^{-3} \text{ degrees}$$

Total Displacement

$$= \bar{\theta} = (0.71 + 0.80) \times 10^{-3}$$

$$= 1.51 \times 10^{-3} \text{ degrees} < 2 \times 10^{-3} \text{ degrees}$$

Torsional Mode

a) Natural Frequency

$$I_0 = 9.65 \times 10^6 + \frac{99 \times 10^3}{2} \times (52/2)^2$$

$$= 43.05 \times 10^6 \text{ slug-ft.}^2 \text{ for entire system}$$

Mass ratio

$$= b = \frac{43.05 \times 10^6 \times 32.2}{115 \times (26)^5} = 1.01$$

Frequency factor, $a_1 = 1.97$ (see Fig. 5)

$$\bar{f}_t = \frac{1.97}{2\pi \times 26} \sqrt{\frac{16 \times 10^3 \times 144 \times 32.2}{115 \times 1.5}}$$

$$= 7.9 \text{ cps} > 5 \text{ cps}$$

b) Displacement

Due to wind

$$\theta_w = \frac{3\, T_w \times FS}{16G\, r^3} = \frac{3 \times 100 \times 1.5}{16 \times 16 \times (26)^3 \times 144} \times \frac{360}{2\pi}$$

$$= 0.04 \times 10^{-3} \text{ degrees}$$

Due to inertial effects (assume resonance)

Amplitude factor $= \overset{*}{B} = 0.54$ (see Fig. 5)

$$\bar{\theta}_t = \frac{\overset{*}{B} \times T_i\,(FS)}{G\, r^3} = \frac{0.54 \times 200 \times 1.5}{16\,(26)^3 \times 144} \times \frac{360}{2\pi}$$

$$= 0.23 \times 10^{-3} \text{ degrees}$$

Total Displacement

$$= \bar{\theta} = 0.27 \times 10^{-3} \text{ degrees} < 2 \times 10^{-3} \text{ degrees}$$

The above computations indicate that a reinforced concrete mat having a diameter of 52 ft. and resting directly on the stiff clay would satisfy the displacement and natural frequency criteria. It should be noted, however, that subsoil conditions at the site in Example 2 are extremely good since the clay is very stiff. Often, the engineer is asked to design a foundation for a tracking system located in an area underlain by soils much more compressible than those in the above design problem. If the natural frequency of the system in the rocking mode proves to be the critical factor, as it was in the example just cited, the designer may have to reduce the mass moment of inertia of the system about the base of the foundation to a bare minimum in order to satisfy the design criteria. One way of doing this is to design a foundation mat which is cellular. Unfortunately, this usually does not reduce the mass moment of inertia of the system to any marked degree because of the proximity of the cells to the base of the foundation. The most effective means of minimizing the mass moment of inertia of the tracking system about the base of the foundation is to use steel rather than reinforced concrete as the structural material for the tower. The reason for this is that steel is a much more rigid material than concrete. Consequently, a steel tower would have approximately 35 to 40 percent of the mass required by a reinforced concrete tower having the same stiffness. If a steel tower had been used in the previous design problem rather than one consisting of reinforced concrete, the computed natural frequency of the system in the rocking mode would have been approximately 16 percent higher, as shown in the following computations:

Moment of Inertia in Rocking Mode:

Dish	16×10^6 slug-ft.^{2}
Turret and bridge	3×10^6
Steel tower, 0.4×35	14×10^6
Superstructure	33×10^6 slug-ft.^{2}
+ Mat	20×10
Total	53×10 slug-ft.^{2}

$$b = \frac{53}{74} \times 1.73 = 1.24$$

$$a_0 = 1.27 \quad (\nu = 0)$$

$$\text{Therefore, } \bar{f}_r = \frac{1.27}{1.10} \times 5.04 = 5.82 \text{ cps}$$

The above points out the influence that foundation conditions may have on the selection of the material to be used for the antenna tower when natural frequency conditions have to be satisfied. Unfortunately, the selection of reinforced concrete rather than steel is often made before the foundation investigation has been completed. While it is true that a steel tower would generally cost more than one made of concrete, the use of steel may be necessary in order to satisfy the natural frequency requirements.

Design Uncertainties

The biggest uncertainties involved in the design of foundations for tracking systems are the mechanical properties of the subsoil.

The assumption of elastic behavior, which is usually made in the analysis, introduces an error. However, relatively little quantitative information is available at this time concerning the influence of inelastic damping of soil on the vibrational characteristics of prototype foundations. Even if linearly elastic behavior is approached, when stress changes in the foundation soil are limited to small magnitudes, the question regarding the ability of the available testing techniques to evaluate the *in situ* elastic coefficients of soil must be answered before the uncertainties of current design procedures can be reduced. Until these uncertainties are reduced, a substantial factor of "safety" will have to be included in this type of foundation design.

Frequently the selection of the site is made without prior consideration of existing soil conditions. In such cases, the engineer is often called upon to meet stringent design criteria at a site having less than ideal foundation conditions. Consequently, the engineer may have to work with a factor of safety approaching unity in the foundation design, not by choice or reason, but out of necessity. Such a situation is not desirable but is a current fact of life and will remain so until the people involved in site selection can be convinced of the important influence that subsurface conditions have on the overall performance of a tracking system.

Acknowledgements

This paper was inspired by experiences the author had during his association with the firm of Haley & Aldrich. He is indebted to Mr. James F. Haley and Dr. Harl P. Aldrich, Jr. for their cooperation and suggestions during the preparation of the paper.

The author also wishes to express his gratitude to his colleagues at M.I.T., particularly Professors T. William Lambe, and U. Luscher, and Mr. H. A. B. Rao, for their constructive criticism of the paper during its preparation.

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THE WANDERINGS OF THE CENTER OF GRAVITY

By Karl R. Kennison,* D.Sc., *Member*

The purpose of this paper is to reappraise and redefine the center of gravity. Although it will have little if any value to civil engineers in their design of structures, it should concern them and all others interested in accurate definition and in the correction of erroneous ideas. It is not easy to define the center of gravity. The mass of everything, of every portion or of every combination of things, attracts by a gravitational pull every other mass; and the location of its center of gravity relative to such other mass is at a point such that the amount and direction of that pull would be the same if its entire mass were actually concentrated at that point. The author suggests the above as a correct definition of the center of gravity and will use it herein although the c.g. is in some cases taken as the center of mass about which a mass can be balanced for free rotation. That might not always coincide with the c.g. as above defined. It is widely taken for granted by civil engineers, possibly because of instruction received in school and college, that every object, every thing, every part or every combination of things, has its fixed and definitely located center of gravity. Nothing could be farther from the truth.

Two of the authors' friends and colleagues, Thad Merriman and Tom Wiggin, both now deceased, authored a well known Civil Engineer's Handbook. They defined the center of gravity as a point that remains fixed "no matter how the body is turned about". The fact is that if a rod is horizontal its free balancing point is at its center, and it can be balanced on its center for free turning, but if turned to a vertical position its c.g. moves down a distance equal to its length^2 $\times$ about .000000012. Its c.g. is moved down only a very small amount and this may seem to be, and in some respects is, a matter of no importance. However a letter on this subject written to Mr. Wiggin shortly before his death was returned by his secretary to the author, and on the letter Tom had written: "An upsetting study by Kennison."

The upsetting started when the doings of Col. Glenn of orbit

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fame encouraged the author to do a little (?) figuring of his own, in the course of which he found that the spheroidal shape of the earth is out of round to an extent much more than can be attributed to centrifugal force alone, also that the earth's center of gravity wanders about over a space extending at least 20 and possibly as much as 28 miles. He concluded from this that we need to correct our thinking about, and to redefine, the center of gravity.

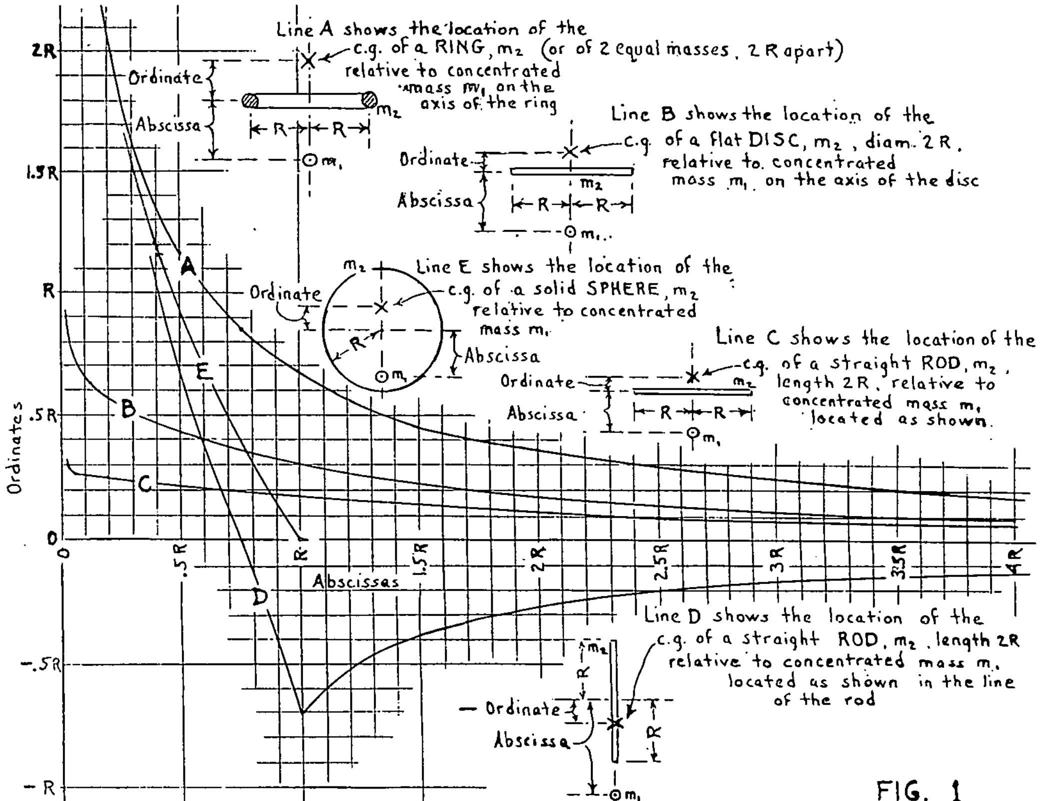
There is nothing really new about this to the up-to-date physicist, and everything herein is well covered in his latest texts, of at least the last three or four years. However, these are very technical and very seldom if ever found on engineers' bookshelves.

Just as there is no such thing as sound unless there is an ear to hear it, so an object has no such thing as a gravitational pull unless some other object exists to create the pull. We are forced to conclude that the c.g. of any mass must be defined in relation to some other mass, and its location must be determined and defined relative to that other mass. The reason this seems to be a strange idea and an unnecessary refinement is that in our experience gravity appears to be always in the same direction, vertical, and if we think about it at all we think of it as aimed at the center of the earth. In other words the "other mass" in our case is the earth and it is so overwhelmingly tremendous that its effect overshadows and for most practical purposes obliterates every other consideration.

The selected graphs shown in Fig. 1 and Fig. 2 should clarify our thinking on the subject. They show the wanderings of the center of gravity and should make it clear why the location of the c.g. of every object, every mass, depends on the shape of that mass and its position relative to the other mass which is, which must be, involved.

The well known fundamental equation for F, the force of the gravitational pull between two masses m_1 and m_2, the distance between their centers of gravity being r, is $F = km_1 m_2 \div r^2$. The constant $k = 3.436 \div 10^8$ using ft.-lb.-sec. units; and about $6.54 \div 10^5$ in cm.-gram (weight)-sec. units. We shall use only ft.-lb.-sec. units. We shall let m_2 be the mass of any object the c.g. of which we wish to locate relative to the mass m_1, and for convenience we shall in every case let m_1 be an object so small that its shape is immaterial because it and its c.g. are concentrated in a single pin-pointed location.

Certain objects selected for illustration in Fig. 1 and Fig. 2 show how the location of the c.g. of any mass m_2 depends on its shape and its



Line A shows the location of the c.g. of a RING, m2 (or of 2 equal masses, 2R apart) relative to concentrated mass m1 on the axis of the ring
Line B shows the location of the c.g. of a flat DISC, m2, diam. 2R, relative to concentrated mass m1 on the axis of the disc
Line E shows the location of the c.g. of a solid SPHERE, m2 relative to concentrated mass m1
Line C shows the location of the c.g. of a straight ROD, m2, length 2R, relative to concentrated mass m1 located as shown.
Line D shows the location of the c.g. of a straight ROD, m2, length 2R relative to concentrated mass m1 located as shown in the line of the rod
Ordinate
Abscissa
Ordinates
Abscissas
A
B
C
D
E
2R
1.5R
R
.5R
0
-.5R
-R
.5R
1.5R
2R
2.5R
3R
3.5R
4R

FIG. 1

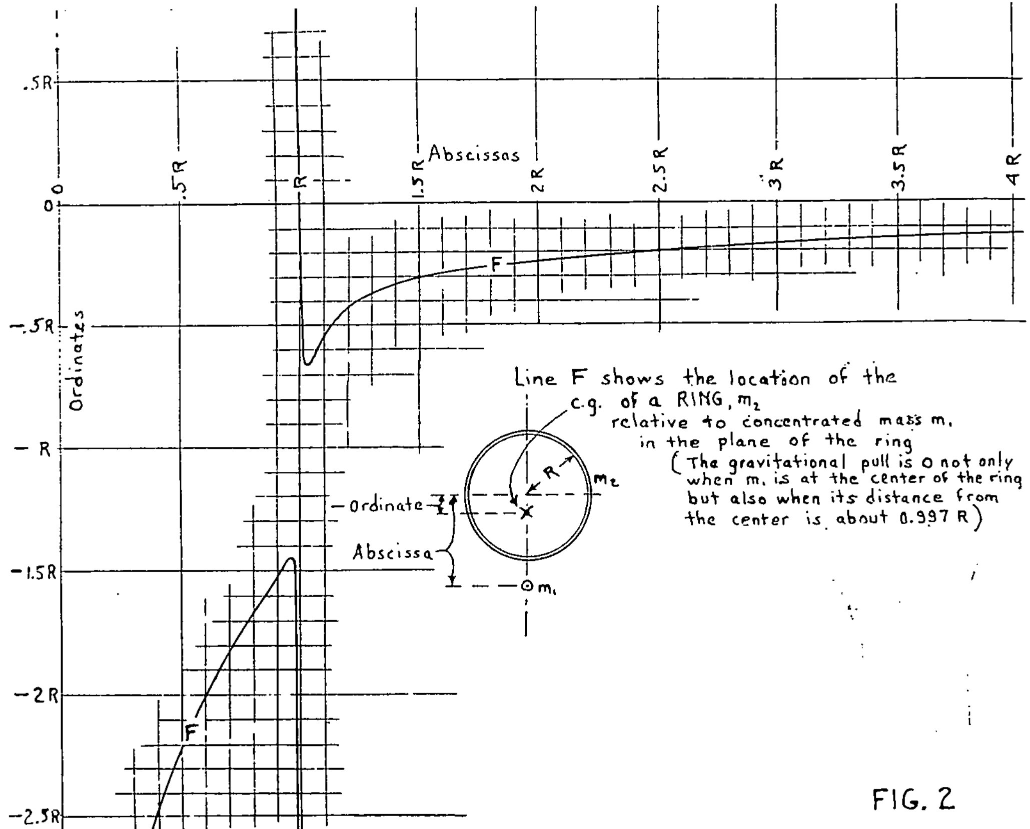

.5R
0
-.5R
− R
−1.5R
−2R
−2.5R
Ordinates
0
.5R
R
1.5R
Abscissas
2R
2.5R
3R
3.5R
4R
F
F
Ordinate
Abscissa
R
m_2
m_1
Line F shows the location of the c.g. of a RING, m_2 relative to concentrated mass m_1 in the plane of the ring
(The gravitational pull is 0 not only when m_1 is at the center of the ring but also when its distance from the center is about 0.997 R)

FIG. 2

position relative to the necessary other mass m_1. Some of these objects, especially circular rings and discs, furnish data useful in solving the problem of locating the wandering c.g. of our spheroidal earth. Each graph plots the relation between the abscissa, the known distance of m_1 from m_2's center, and the ordinate, the desired distance of m_2's c.g. from its center. Line A traces the wanderings of the c.g. of m_2 when it is a doughnut shaped ring or hoop of radius R, relative to m_1 located anywhere on the axis of the ring; and incidentally this same graph applies if m_2 is, instead of a ring, a combination of two equal masses separated by a distance of 2R. Line B traces the wanderings of the c.g. of m_2 when it is a flat disc of radius R, relative to m_1 located anywhere on the disc's axis. Line C does the same for a straight rod with m_1 located anywhere on a line perpendicular to the rod at its center. Line D does the same for a straight rod with m_1 located anywhere on the line of the rod itself.

Line E does the same for a solid sphere with m_1 located anywhere. This Line E is unique and illustrates what we understand was discovered by Newton, namely that the c.g. of a perfect sphere is always at the center of the sphere as far as the gravitational pull on any object on the surface of the sphere, or outside it, is concerned. This fact is one that is too easily taken for granted. Actually it is by no means axiomatic. To the author it is astonishing and, until demonstrated, unbelievable. For example if you stand at the north pole why should not the northern hemisphere which is right under your feet attract you with much greater force than the more remote southern hemisphere, especially since distance is so important because as shown in the above fundamental equation the pull of m_2 on m_1 is inversely proportional not just to the distance between them but to the square of that distance. The answer is that although every portion of the northern hemisphere pulls much more heavily than its southern counterpart it has a relatively small vertical or radial component and it is only this small vertical component of the pull that counts, and contributes to the resultant pull of the earth as a whole.

Incidentally the author can't help wondering how the time he has spent on this entire matter would compare with the time similarly spent by Newton. For two reasons Newton probably spent more. First, he is reported in some circles as dallying with his thoughts while lying on the grass under his apple trees. Second, although the logarithmic studies and discoveries of Briggs anteceded him he may not have had the benefit of the author's 7-place logarithmic tables, which have been very useful since the precision of the trusty slide rule was not entirely adequate.

On account of the above noted Newtonian fact, the Line E for a solid sphere in Fig. 1 shows only the situation for locations of m_1 beneath the surface, in the interior of the sphere, since for abscissas greater than R the ordinate is always 0.

Note that all these graphs, Lines A, B, C, D and E, show that the wanderings of the c.g. finally land at infinity—with the gravitational pull reduced to 0, where m_1 can truly be called weightless. As ordinarily used this term is a misnomer. Although Glenn in his capsule was apparently weightless, civil engineers, as well as mechanical engineers, know that his weight was what kept him in orbit and that if he were really weightless he would have shot off in a straight line for parts unknown. Note that if he used his braking jet to stop his capsule he would in his drop to earth be pulled down by his ever increasing weight until he reached the surface. Then, as Line E shows us, if we could imagine for our illustrative purpose that he dropped into an open well of unlimited depth, his weight would stop increasing and immediately start to decrease until at the center of the earth he would be in fact weightless, with no misnomer about it.

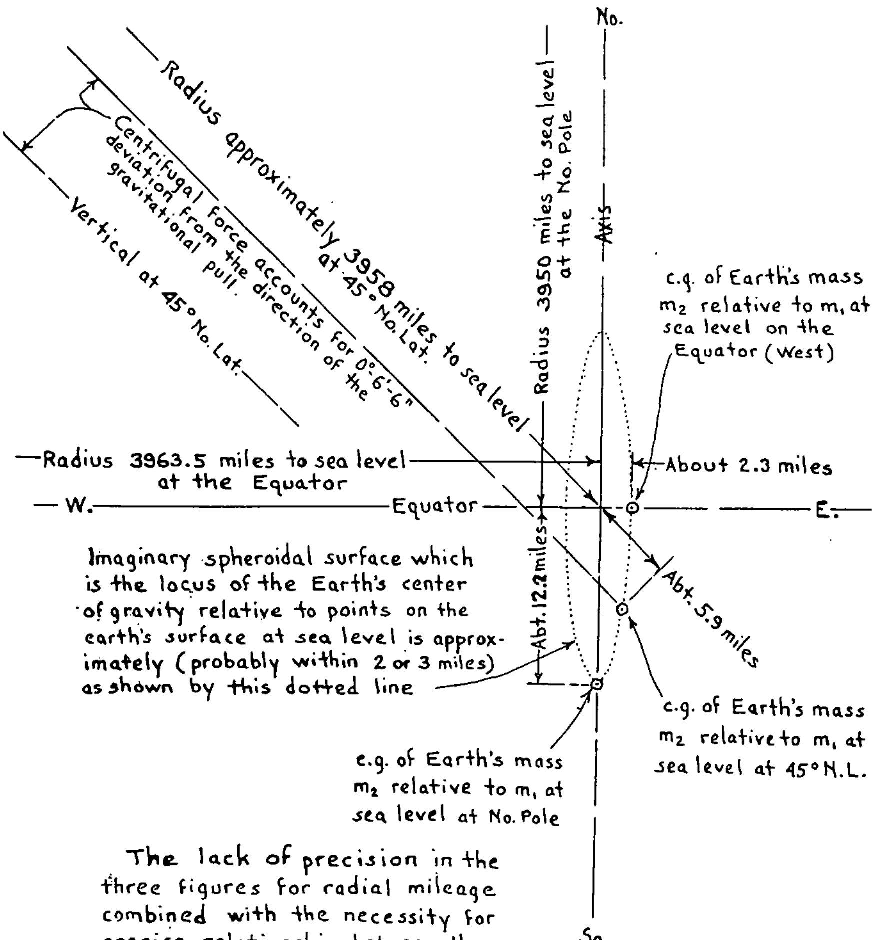
On Fig. 2 Line F traces the wanderings of the c.g. of m_2 when it is a ring or hoop of radius R similar to the ring of Line A except that it traces its position relative to m_1 located anywhere in the plane of the ring itself instead of on its axis. This situation is peculiar and very interesting because here, quite unlike the case of the solid sphere, we find two locations where m_1 is weightless. For example, consider m_1 approaching the ring attracted by the ring's gravitational pull. When m_1 has practically reached the ring the c.g. of the ring, which also has been approaching, stops its approach and wanders back to the center and beyond to infinity, leaving m_1 weightless as soon as it has entered the ring an unbelievable distance of only about 3/10 of 1% of the radius R. Then as the location of m_1 is pushed more toward the center of the ring the sprightly c.g. of the ring, having jumped to infinity in the opposite direction, approaches but never gets nearer than about 1 1/2 R to the ring. It then wanders further away, and again reaches infinity when m_1 is pushed to the center of the ring and for the second time becomes weightless, this time in unstable equilibrium.

It should be clear that the above descriptions, definitions, remarks about weightlessness, etc., involve situations which we ourselves cannot experience because of the tremendous and overwhelming mass of the nearby earth. However, using the fundamentals thus described we can now proceed to examine the wanderings of the c.g. of the earth itself, taken as mass m_2, in relation to us or to any other mass m_1 anywhere on its surface at sea level. To this end we could for example, since we know that a perfect sphere's c.g. stays put, consider the earth as a combination of two masses, one a sphere with a diameter equal to its polar diameter of 7900 miles and the huge mass of over 4000×10^{20}, and the other a ring or multiplicity of rings constituting the equatorial bulge which protrudes 13½ miles.

The bulge is not only in this 13½ mile increase but it involves also a considerable increase or radial bulging of interior masses in the equatorial plane and in all other planes normal to the earth's axis. The resulting c.g., relative to m_1 at the north pole, of all such masses, over and above those that might constitute a perfect sphere with uniform increase of density with depth, must be in the neighborhood of 2500 miles south of the equator, i.e. at a distance from m_1 which exceeds the earth's radius by that amount. Lack of data as to how the bulging affects specific gravity throughout the earth's interior makes it impractical for the author to combine sphere and bulge and thus compute with anything that could be called precision the location of the earth's c.g.

The earth's average specific gravity computed from various data is about 5.535, a figure which makes m_2, the earth's mass, about 4108×10^{20}. Where such assumption is pertinent we have assumed that the material in the bulge near the surface weighs about 250 lbs. per cu. ft. However it would take a considerable variation in this assumption to move the c.g. more than a fraction of a mile, or thus materially affect the results.

We can approach satisfactory precision by involving the published values of g, very familiar to civil engineers as the acceleration due to gravity. These values attest that they have been precisely measured at all latitudes. The result is presented in Fig. 3 which shows the location of the c.g. of the earth, mass m_2, relative to any and all locations of m_1 at sea level. It apparently wanders on the surface of an imaginary spheroid represented by the dotted line. For example, its location relative to m_1 at the north pole is at a distance about 12.2 miles greater than the polar radius of 3950 miles, and relative to m_1



The lack of precision in the three figures for radial mileage combined with the necessity for precise relationship between them is illustrated by the fact that an uncertainty of only 1/10 of 1% in the earth's specific gravity, or in the value of the fundamental constant K, would change the radial increments 2⅓, 5.9, and 12.2 to 4.1, 7.7, and 14.

FIG. 3

at the equator it is at a distance about 2.3 miles greater than the equatorial radius of 3963.5 miles, and at 45 degrees latitude at a distance approximately 5.9 miles greater than the radial distance. For reasons above stated and as stated on the chart, these individual distances are not to be taken as anything like precise. However taken together they are correctly related to each other, as will be shown in the following comparison.

It will suffice to deal with the situation at only the three locations indicated in the following lines. The figures at the left are for the equator, at the right for the poles, and those in the center for the half-way points at a latitude of 45 degrees. For convenience let m_1 in every case be an object which would weigh one pound at sea level at 45 degrees latitude. Since g at that latitude is known to be 32.174 ft. per sec^2 the mass of m_1 is .031081; and the fundamental equation, $F = km_1 m_2 \div r^2$, can be simplified and for this study becomes $F =$ about $43{,}880 \times 10^{10} \div r^2$.

The three values of r, from Fig. 3, are about

3963.5	3958	3950
2.3	5.9	12.2
3965.8 miles	3963.9 miles	3962.2 miles
or	or	or
209.395×10^5 ft.	209.295×10^5 ft.	209.195×10^5 ft.

Hence F is equal to

1.00077 lb.	1.00173 lb.	1.0026 lb.

This is the true gravitational pull. However to find the apparent force which is customarily, not to say erroneously, referred to as gravity, we must make a correction by subtracting the effect of the centrifugal force due to the rotation of m_1 about the earth's axis once in 23 hours 56 minutes and 4 seconds.

The velocity of m_1 is

1526 ft. per sec.	1080 ft. per sec.	0

at a radial distance from the earth's axis of

3963.5 miles	2805 miles	0

which makes the centrifugal force about

.00346 lb.	.00246 lb.	0

and its radial component, which is what offsets the gravitational pull, is

.00346	.00173 lb.	0

Hence the effective gravitational pull is the difference

1.00077	1.00173	
.00346	.00173	
0.9973 lb.	1.0000 lb.	1.0026 lb.

and g in ft. per sec.^{2} is this force divided by the mass .031081, or

32.088	32.174	32.258

Precise agreement with the published measured values confirms the correctness of the above results and the data herein presented that produced them. Agreement in the case of any single one of the three figures is not significant, because of the possible lack of precision in determining the earth's mass. It is the simultaneous agreement in the case of all three that is significant.

It is hardly necessary to add that the wanderings of the c.g. of the earth, as shown in Fig. 3, apply as clearly stated only to gravity as it affects us here on the earth's surface. In relation to remote distances such as are important in space studies, there is very little difference, and no important difference, between the earth's center of gravity and its center.

COMPUTERS IN SURVEYING AND ENGINEERING PROBLEMS

By Joseph F. Willard,* *Member*

(Presented in the Lecture Series of the Surveying and Mapping Section, B.S.C.E., April 16, 1963.)

Before describing some of the problems which may be readily solved by present-day computer methods, let us look at a question which frequently arises when the subject of electronic computation is brought up for discussion. The question is—why should we use the computer for this particular type of work? I suppose the same question was asked about logarithms when they were first introduced, and I know that the gradual transition to the use of desk calculators brought out the same question. Yet in each case, the newer method gradually replaced, or rather, complemented, the previous ways. The answer seems to be the ease of problem solution by the newer method, for repetitive problems, especially.

I just mentioned that the newer method in each case has complemented the previous ways, rather than completely replaced them. This can be shown by the fact that many simple problems are solved today by long-hand methods; logarithms are sometimes used where access to mechanical assistance is not immediately available, and so on. This points out another fact of which we must all be constantly aware, and that is that proper judgement must be continually exercised to avoid using any method of computation which is actually too advanced for the problem at hand. There are times when the proper combination of all of the tools at our disposal becomes essential, and in the use of computers this is very important. Short, simple problems generally are not economical if a computer approach is looked upon as the "only way." Also, I would recommend that a beginner in the profession be *required* to do a certain amount of computation by the older, established methods, so that he may apply the experience thus gained, in the event he suddenly finds himself removed from the newer techniques. Also, by understanding the conventional methods, he will be

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better able to communicate with the computer, and may, we hope, be able to make suggestions for improvement.

Some Basic Computer Facts

The computer, regardless of the particular make or model, is only a machine. It is only capable of doing those things which we tell it to do, whether it be simple arithmetic or complicated design procedures. Every step of each problem must be entered into the machine, before it is capable of performing anything. The organization of the problem into a logical sequence of mathematical operations is referred to as a *program*. This may be considered to be similar to a very detailed set of instructions which one might give to a subordinate who is being exposed to a particular problem for the first time.

The major difference between these two conditions, however, is that the person is able to think for himself, whereas the machine cannot. A slight change in the instructions, or a difference in problem type, may be readily perceived by the individual, but the computer *must* do each problem the same way every time. Moreover, the whole series of instructions must be repeated to the computer, every time the problem is to be solved.

A computer program, therefore, must give the details of a particular problem in a straightforward, logical manner, and should include every step required to reach the solution. Many times, we find ourselves doing certain parts of a problem mentally, because we have become so well acquainted with a particular problem that this is possible. When preparing a computer program, however, we cannot do this, since the machine must have every instruction in detail. Therefore, a logical diagram of the problem is generally prepared, to assist in the preparation of the program.

It should be pointed out here that the actual program for the computer may be written by a person who is not a specialist in the problem to be solved. The logic of the problem, however, is generally best prepared by the person who knows the details of the work at hand; in our case, an engineer or surveyor.

Computer as Engineering Aid

Just as the transit, level, and rod are considered as engineering or surveying equipment, so should the computer be considered. Knowledge of the use of the standard surveying instruments does not make one a surveyor or engineer, but their use is often necessary for the proper execution of a particular job. The same is true, to a certain extent, of this new piece of equipment, the computer. A machine operator could not easily learn to be a surveyor or engineer, but an engineer, with basic instructions, may learn to communicate with this new tool.

Electronic computers have been used in the solution of engineering problems for perhaps ten years. During this period, a wide range of problems have been successfully applied to this new method of solution, but our present discussion will be limited to those problems related closely to surveying.

One of the first problems which was presented in this area was the computation of taping notes. Since the solution of this problem is relatively simple, and quite repetitive in nature, it lends itself readily to computer methods. Basically, a single measurement is considered, where it is not possible to hold the tape horizontally, but rather an inclined measurement is made in the field, and the difference in elevations is taken between the ends of the tape. The usual approximation, $H^2/2S$, is often applied to get the correction to horizontal. However, in the computer, it is as easy to solve the Pythagorean Theorem, to get the horizontal distance exactly, so we use $D = (S^2—H^2)^{0.5}$ to get the horizontal distance.

Many surveyors use tables for this reduction to horizontal, rather than the approximation, but if a large number of calculations are involved, the machine method is more rapid, and possibly more accurate, since fatigue is no longer a factor.

Correction for temperature must also be considered, if any great length is to be measured at other than standard temperature. Again, tables may be used, but in the computer, multiplication by the standard correction for the material used in the tape is easier than the use of tables. Also, the tape used in the field generally is not exactly true in length at standard temperature; however, if the correction for the tape is supplied to the computer, this correction is easily made.

If the slope distance, difference in elevation, temperature, and standard tape correction are provided to the computer, a book of about eighty pages of notes may be reduced to horizontal, and sums between major measuring stations made, in about half an hour, using today's medium-sized computer. If conditions demand it, a much more rapid solution may be made on a high speed computer.

Before proceeding further with descriptions of other problem types, I would like to say a few words about error detection. All modern computers have built-in error detection devices to prevent calculations of wrong answers in the event of mechanical or electronic failure of the equipment. These vary with the different machines used, but generally are used to check against such items as incorrect data transmission, invalid operation codes due to damage to the program instructions, and the like.

Errors in the data submitted to the computer for solution, however, becomes an entirely different problem. Even with care in data preparation an occasional value which is incorrect might be presented to the computer. This type of error may be further divided into two classes, those which may be detected by the program, and those which, although incorrect, are still within a reasonable range for the problem at hand, and are nearly impossible to detect at compute time. Because of the latter class of possible error, care must be taken in the submission of data, and some human judgement is often required in the interpretation of the final results.

The first class of error, however, may be detected by the program in the computer; a few examples may illustrate this type of error. Suppose, in the taping problem which we have been discussing, a temperature of well over one hundred degrees is entered as part of the data. This can be automatically rejected by the program as invalid, and no answer will result from the data presented. However, if the correct temperature is, for example, 84 degrees, and the value is presented as 32 degrees in error, the computer program will consider the temperature as valid, and proceed with a solution.

Many times, certain totals may be used for checking, also. In the case of taping, for example, an approximate check on the total distance is frequently made in the field, before the taping party moves along to the next location. If this value is entered with the data, it may be used as a check on the accuracy of the individual measurements upon which the final answer is to be based.

The reduction of field notes for vertical measurements is another problem which may be programmed for easy solution by a computer. This problem generally is run to prepare a record of ground elevations for later use in earthwork calculations or for establishing profiles, but ordinary bench levels may also be run in a similar manner.

Generally, a bench of known or assumed elevation must first be provided, as a reference for the calculations. Ordinary leveling calculations are then performed by the computer, until a series of values are detected which represent the readings of points on the ground. These values usually are coded in a certain manner so that the computer program may readily detect their presence. In the case of cross section notes for earthwork, the base line station is given for identification, followed by a series of offsets left or right, with the corresponding rod readings. Using the last computed height of instrument, the computer proceeds to calculate ground elevations corresponding to the rod readings.

Again, the detection of errors becomes an important part of the problem. Obviously, incorrect readings may be recognized, causing error messages to be generated, and halting the computation, if necessary. In the establishment of successive instrument heights, the checks on bench marks previously established provide an easy means of detecting errors. Since in many cases the level run does not meet the known elevation of the bench exactly, a tolerance factor may be provided to the computer, in keeping with the particular job being run. If the elevations are within the tolerance limit, computation proceeds; if not, the problem may be stopped for analysis. Since the solution of problems of this type may often be separated into logical groups of calculations, such as when a new bench is used for a section of notes, current practice often provides for the computer to print out an error message if an error is detected, and then to by-pass all following data until a valid starting point is reached; computation then proceeds as usual, giving answers to as much of the problem as is possible under the particular conditions presented.

Quantity Calculations

Perhaps one of the first computer applications in civil engineering was the computation of earthwork quantities. A wide variety of computer programs have appeared in the past few years, for different types of machines and various earthwork problems. Since there are different purposes for using this type of calculation, we find several concepts being employed, depending on the particular application.

One very simple earthwork program, designed for approximate volumes, or comparisons of general locations, uses a very simple design template, of perhaps five points, to define the proposed work. This is combined, station by station, with the information available on the existing terrain, which might have been obtained either from ground surveys or by photogrammetry. The computer determines the cross section area in cut and in fill, and then the quantities of material between each station. This type of program, using a relatively simple template, runs very quickly on the computer, but is only intended for approximate values.

For more advanced design purposes, programs with a more extensive template definition have been developed. These programs generally allow a relatively large number of break points to be presented to describe the design in detail. Various back slopes are also provided, and in most programs of this type, the slope to be applied at any point is determined by the computer, on the basis of design criteria written into the program. Changes in the template to be applied may be made at any time during the run. One method employs the storing of various templates in the computer's memory; this method, however, is not really practical on a small- or medium-sized computer, if the template definitions are involved. A method employed in the Digital Terrain Model series of programs, prepared by the Civil Engineering Department at the Massachusetts Institute of Technology, employs a series of links extending across the section. Any one of these links may be varied at any time, thus automatically correcting the entire section for any special conditions.

Another problem which must be considered in earthwork computations for design is the superelevation of the pavement in banked curves. This may be presented to the computer as a series of changes in the template during the transition and extending through the banked section. However, since the rules being employed for banking and transition may generally be formulated mathematically, we often find the computer program being called upon to apply the particular rules to the standard section. All that is required in this case is a signal to the program indicating where superelevation is to be applied, and the parameters necessary to generate the proper amount of banking.

Although other methods might be employed, most current earthwork programs, once the terrain information and design template have been established for a particular section, proceed in the following manner. The necessary slopes are formed digitally, and intercepts with the existing ground are determined. Also, crossover points, where the section changes from cut to fill, are computed. The computer then establishes vertical lines at every point where either the terrain or the template changes slope. The resulting figure, represented in the machine in digital form, may be quickly evaluated for areas, by successive application of the rule for the area of a trapezoid, $A = \frac{a+b}{2} \times d$, where a and b are the end heights and d is the distance between the verticals. For the two ends of the section, and the crossover points, either a or b becomes zero, and the formula reduces to that for a triangle. Whether the area is cut or fill is determined by proper use of signs on the vertical measurements. The resulting areas are summed in separate tables for cut and for fill, and the volumes are subsequently determined, generally by the average end area method.

The mass haul diagram may also be computed during this process, and the ordinates presented as part of the computer output, along with such information as the slope stake points, the cut and fill areas, the excavation and embankment, and the net excavation and embankment. Corrections to the quantities for shrinkage or swell, or both, may be applied if desired, and many programs also provide for corrections for unsuitable material and other special conditions.

If classified materials are encountered, successive applications of the computation process, working between the lines of demarcation of the various layers, may be employed to generate the desired results.

The same general concepts are also employed for determining quantities of as-built projects, for final pay purposes. Here, however, the final measurements are combined with the definition of the original ground, to make the necessary computations.

Geometrics and Traverse Calculations

Another topic of interest to civil engineers and surveyors is the computation of various geometric problems, involving both horizontal and vertical geometry. One fairly large segment of the former involves the solution of traverses, for problems in location or land surveying. These problems may be analyzed carefully and found to be composed of a series of types of computation, depending upon which values are known and which are to be determined. With these general types in mind, a series of commands may be given to a computer for the solution of specific problems. Some typical computer instructions would include the ability to store one or more coordinates of known points for control, and the routine to determine coordinates of other points if distances and directions are known. Other desired solutions involve the determination of distance, direction, or both between known positions or points previously computed.

In many problems involving horizontal measurements, an adjustment of a series of courses becomes necessary to meet previously established positions. These adjustments can be made in a variety of ways; the standard methods have been programmed for solution on electronic computers.

Various computer approaches have been tried for problems of this class. One method uses individual programs for each major type of solution. Another system uses several fairly large routines, combined in one program, for the solution of all the major traverse types encountered in practice, and computing missing parts of any set of data, following the general rules for geometry, such as intersecting two straight lines, intersecting two arcs, or intersecting a line with an arc. A third approach to the problem involves a detailed analysis of each problem, and allows the engineer or surveyor to combine a fairly complete series of commands adaptable to individual problems. One of these systems was developed by Professsor C. L. Miller, at M.I.T., and is called COGO (Coordinate Geometry). Working with a table of stored coordinates, and a series of geometric commands, the user may develop the solution to his problem in a logical manner, much in the way he would approach the same problem by conventional methods.

After at least one starting position is defined by coordinates, other points may be located by distance and direction. The direction from one point to another may be defined, generally, in a number of ways: by bearing, azimuth, deflection angle, and so forth. All of these methods are available to the computer user.

Various intersection problems are also possible, as pointed out previously. However, in certain cases, such as intersecting arcs, care must be taken to arrive at the correct answer, since mathematically there are generally two solutions to such a problem. The computer must be given additional information to determine which solution is desired, since it cannot otherwise make a judgement, as we would, to disregard the unwanted values.

Adjustment of traverses, either open type for route surveys or closed type for property surveys, may be accomplished with ease by computer methods. Generally, the same information is needed by the computer as is required for solution by other methods. Coordinates of the beginning and ending points, and distances and directions (unadjusted), together with the method of solution desired are entered into the computer. Results include closure error, adjusted coordinates, and adjusted distances and directions. The methods of adjustment available usually include the compass rule, the transit rule, and the method of least squares.

The practical use of the COGO-type approach has been shown by continued use of the system in various places, including our own use for about one and a half years. During this period certain improvements have been made in the form of additional routines and also in the ways of applying the existing system as experience is gained by the users. Because of the flexibility of the system additions are easily made and do not require re-writing the entire program.

Solution of various problems in vertical geometry have also been programmed for computer operation. Basically, these programs allow the solution of the standard vertical parabolic curve for transition between lines of different rates of grade. Various other features, however, give a great deal of flexibility to the answers possible from this type of program.

Map Projections

The last topic which I will discuss involves the computations required for conversion of surveying measurements to those used for representation of the earth's surface in mapping. Various projections of the earth's sphere are used to represent the surface on a flat map; since our own state of Massachusetts has its major dimension in an east-and-west direction, the Lambert Conformal Conic Projection has been adopted, and now is programmed to the electronic computer.

The various equations for the projection are coded to computer language, and stored along with tables of the necessary corrections for the projection. For any given problem the measured coordinates of the points are given and the computer proceeds to compute the necessary adjustments and apply them for the solution of the problem. The resulting coordinates, and the grid distances and azimuths between adjacent points, are brought out as results.

Due partly to the very large distances employed in the calculations, such as the radius of the earth in feet at the location being computed, and the small angles, a high degree of precision must be carried in all the computations, especially the trigonometric functions which are used. This, however, is no serious problem for the computer.

I hope that these few remarks have summed up the current use of computers in this field, and perhaps have created more interest in further advancing the use of this valuable engineering and surveying instrument.

OPTICAL INSTRUMENTS FOR MODERN SURVEYING

By Albert M. Weinbrecht*

(Presented as two of the lectures in the Lecture Series of the Surveying and Mapping Section, B.S.C.E., May 14 and 21, 1963.)

I plan to discuss Optical Theodolites, Automatic Levels, Optical Alidades and special optical instruments during these lectures.

Optical Theodolites

The history of theodolites can be divided into two periods, with the first ending about 1910 and the second bringing it up to date. The reason for this division is two-fold. To begin with, during the first period manufacturing techniques were pretty much unchanged; the instruments were built more or less individually, by highly skilled craftsmen who used their knowledge and experience rather than detailed drawings and specifications. The highly precise machinery we use today did not exist, and the optical science devoted its main interest to microscopes, binoculars, telescopes, and military instruments before it got around to surveying instruments. As an example of how individualistic the manufacture of surveying instruments was, the Kern catalog of 1897 lists no less than thirty-two different leveling instruments, ten alidades, forty-seven direction and repeating theodolites, and nine universal instruments. This was possible as long as the demand for surveying instruments was relatively small, and skilled labor cheap.

For the beginning of the second period, I selected the end of the year 1910, primarily because at that time a man began making himself heard who was to start a complete revolution in instrument design and in manufacturing techniques. This man was Dr. Heinrich Wild. Fortunately, he lived at a time when the optical and instrument industry was just getting ready to become industrialized and to take advantage of the new methods, materials and machinery which were gradually being introduced in other fields.

Let me give you a thumbnail sketch of Dr. Wild's life. He was

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born in a mountain valley in Switzerland in 1877, and when still in his teens he had a keen interest in surveying. At the age of fifteen, he was apprenticed to a surveyor who was in charge of flood control of the treacherous stream in Wild's native valley. Wild bought a small transit out of his pocket money and made extensive surveys on his own, as a hobby.

Later on he studied surveying at the Winterthur Engineering School. In 1899, he was employed by the Swiss Ordnance Department, and in 1900, he joined the Swiss Topographic Survey. Very soon he was promoted to Engineer First Class and was put in charge of the verification of all forest surveys. His keen interest in surveying made him realize that the instruments then in use were lacking in many respects. Some of his findings were made the hard way. For instance, in 1902, he had to triangulate from the peak of the 10,400 foot Dent du Midi. His party left early in the morning, hoping to complete the job by noon, but instead of being able to start working, he had to spend close to three hours adjusting the spindle of his theodolite. Then came a thunderstorm with a considerable snow fall, which forced the party to stow the instrument under some rocks and to retreat hurriedly to the valley. Not until several days later was it possible to resume and complete the triangulation.

The leveling instruments were no better. Again, Wild found out through practical experience. He had to run a high order level traverse between two cities in Western Switzerland, about nineteen miles apart. He was assigned a German instrument which was the newest thing then available. He tells that it took him about a month to complete the job, during which time he had to tighten tripod wing nuts 300 times, read the rod with a poor telescope 2,400 times, and carry the heavy instrument 600 times from one station to the other. Being a practical man, he did not just complain and let it go at that.

Between 1905 and 1907, Wild was on a Federal Commission which studied the feasibility of introducing artillery rangefinders into the Swiss Army. He had made an invention in this field and had sold it to Zeiss in Jena, Germany. This contact with the then largest manufacturer of optical instruments proved most beneficial. He was able to convince Zeiss of the need for better surveying instruments, and so he was employed in 1908 and given the enviable assignment of organizing a new division for this purpose. He had the assistance of astronomers, mathematicians, specialists in optics, and of people who knew how to mount optical elements in other, mostly military, instruments, so that they would not come loose.

The years Henry Wild spent with Zeiss were extremely productive. During that time he made numerous inventions which today are in general use, although few people know he was their originator—for instance, the anallatic (internal focusing) telescope, the coincidence level, the cylindrical vertical axis, and the wide frame or "European type" tripod, just to name a few.

World War I stopped the manufacture of surveying instruments in favor of military equipment, and only at the end of the war was Zeiss able to produce a theodolite based on Dr. Wild's design.

Right after World War I, the situation in Germany was not favorable for an ambitious inventor, so Wild left Zeiss and returned to his native Switzerland, where, with some financial backing, the company was established which still bears his name. He designed a new line of instruments, beginning wtih the T-2 which many of you know, and the small company took on the uphill fight against the gigantic Zeiss firm in Germany. In 1930, at the meeting of the International Society of Photogrammetry in Zurich, the Swiss Federal Institute of Technology awarded Heinrich Wild an honorary doctor's degree of technical sciences, in recognition of his outstanding contribution to the design of geodetic instruments. In the early thirties, Dr. Wild left the firm he had helped to create, and for a few years he worked independently on further developments of geodetic and photogrammetric instruments, selling some of his patents to his former company. In 1935, he became associated with Kern & Co., Ltd., which obtained exclusive manufacturing rights for all his new designs. To produce the new line developed by Dr. Wild, Kern had to re-tool completely, and shortly before World War II, production was rolling.

Dr. Wild passed away after a heart attack late in 1951, at the age of 74, leaving a rich heritage of ideas, designs, and high principles to his son, Heinrich Wild, Jr., who is now Technical Director of Kern & Co., Ltd.

The instruments of the first period were very bulky and heavy. Their controls and leveling screws were exposed to dust and dirt. They had provisions for a lot of adjustments because their construction was unstable. The telescopes had poor optics and were not dust- and moisture-proof because they had external focusing. To read the theodolites, it was necessary to walk around the instrument and endanger the stability of the set-up, and to squint through magnifiers or microscopes placed at opposed points of the circle. This situation had existed for a good century, with some improvements here and there, interesting but without far-reaching consequences.

If we are to study the progress made in surveying instrument design since the 19th Century, we might as well be guided by the fundamental goals which Dr. Wild set for himself on the strength of his tribulations in the field, and then we can examine how far we have gone towards reaching them. Dr. Wild endeavored to make instruments:

(a) More reliable, so that they would require as few adjustments as possible over a long period of time and be ready for use immediately whenever needed.
(b) More portable, that is, lighter in weight, more compact, and packaged for easy and safe transportation even in very rough terrain.
(c) More convenient to operate, so that a man would not have to be a mathematician to use them efficiently.
(d) More versatile by designing accessories for various purposes.
(e) Easier to repair by standardization of component parts. This requires switching to mass production techniques where strict tolerances are prescribed, rather than leaving it to the individual workman to obtain perfect fit.

All these goals are governed by one most important prerequisite: any instrument which is to be truly useful must be fundamentally balanced. By this I mean that every element of which it is composed must have a quality and precision correlated to the rest of the elements and to the specific requirements which the whole instrument has to meet. For instance, the finest circles do not make a good theodolite out of an instrument which has a poor vertical axis or an inadequate telescope. Therefore, balance, or you might call it harmony of design, is of the utmost importance.

I would like to subdivide the advancements into three categories:

(A) Mechanical improvements
(B) Optical improvements
(C) Improvements in tripod design

In order that we may follow the designer's thoughts, let us recapitulate some basic facts which, I am sure, are known to all of you. Let us start on the ground and work our way up:

(1) The tripod is nothing else but a platform on which the surveyor sets up his instrument. It should be very stable, because accurate measurements are to be made from its top. Hence, no instrument is better than the tripod on which it rests. This ought to induce some instrumentmen to treat their tripods with greater care. To permit efficient work, the desired stability must be obtained quickly, with a minimum of effort. It should be possible to set up this "platform" over any station point, even in rough terrain, with little effort and waste of time.

(2) The leveling base has only one function. It is an adjustable link between the top of the tripod and the instrument proper which permits us to "plumb" the vertical axis. The conventional solution consists of three or four vertical screws. It is necessary that they afford the same stability as the tripod.

(3) The vertical axis of a surveying instrument has two functions:
 (a) It has to carry the weight of the rotatable upper part of the instrument with as little friction as possible, so that it will not "bind" and so that we obtain immediate reaction without drag or backlash when we turn the tangent screw.
 (b) It must position the upper part of the instrument so that the standards rotate in a horizontal plane with a precision commensurate with the overall precision of the instrument.

(4) A surveyor wants to be able to read the circles of his theodolite or transit easily and quickly, and he wants consistently accurate and reliable results. He does not like to walk around his instrument and endanger the stability of his set-up. The best circles are of little use if they cannot be read properly.

(5) A surveyor also wants to be able to use his level without having to trample around his tripod to check the bubble. He prefers to perform all operations from his position at the eyepiece end of the instrument and to have a positive check immediately before and after he reads the rod.

(6) The most precisely calibrated reading system is useless if we cannot sight our target with equal accuracy. Therefore, optical quality (brightness and definition and commensurate magnification) of the telescope is of paramount importance.

(7) When we go out into the field—which will be under all climatic conditions—we want to use our instrument. Therefore, its construction must be inherently stable. We prefer an instrument which does not have to be adjusted to an instrument on which there is an adjusting screw for every component part. In other words, the instrument should be ready for immediate use any time.

Before going into details, I might add that many of the improvements I shall refer to became possible only due to great technological progress, and by changing, strange as it may sound, to quantity production methods. From innumerable hand-made types, the industry switched to fewer models, the parts of which could be made to some extent with consistently precise machine tools, and with strict quality control at the end of each manufacturing and assembling stage.

Now, then, let us look at some of the improvements which were made, following the "credo" of Dr. Wild and of those who were inspired by him.

Mechanical Improvements in Instrument Design

Let us start at the bottom, with the leveling base, our adjustable connection between instrument and tripod. Right here, European and American concepts clash. The Europeans say that three points determine a plane and that, therefore, a three-point support is the only geometrically correct solution. Of course, this is true. American surveyors who at times in the old days had to work hundreds of miles from the nearest instrument shop, without the fast means of transportation we have nowadays, contended quite correctly that if you have worn screws on a transit with a three-screw base, you are in trouble because you will have serious errors with an instrument not firmly connected to its tripod. In the pioneer days, it may have been a costly, time-consuming, and often almost impossible proposition to replace worn leveling screws. A four-screw base had the advantage that the slack due to wear could be taken up by tightening one pair of opposite screws against the other pair, although there was always the danger that over-tightening could cause stresses and, therefore, errors of a different nature. Nowadays there are better screws on the market, and worn ones can be replaced quickly, so that there seems to be little or no justification for four-screw leveling bases. Nobody would think of putting four legs on a tripod for "greater stability."

Two fairly recent improvements eliminate even the chance of looseness due to wear. On theodolites, the three vertical leveling screws were replaced by horizontal knobs, provided with an eccentric shoulder or cam on the side facing the instrument. From each corner of the triangular base plate of the instrument, a finger reaches into one of the knobs. By turning a knob, the corresponding side of the instrument is raised or lowered, depending upon the direction of the turn. Since the cam is held firmly against the finger, by a double guide, by the weight of the instrument, and in some cases by a spring, even whatever little wear we might get will not cause any looseness. The range of tilt thus obtained is smaller than with vertical screws, but this is desirable, because a big range means a greater amount of wear. The preliminary leveling is, therefore, accomplished by means of a new type of tripod head which is not affected by wear and needs no close fit to achieve stability. I shall say more about this when talking about tripods.

Let us proceed to the vertical axis, the stability of which is very critical on theodolites, slightly less so on leveling instruments.

In the old days, European instruments had tapered spindles. Even today, such spindles have not disappeared by any means. This longevity is primarily due to the fact that they can be manufactured to a good fit relatively easily and with simple means. On older instruments, not only the vertical axis, but other elements were tapered, like micrometer arms, circle seats, etc.

A tapered axis can be fitted and re-fitted on a good lathe, while a cylindrical axis either does or does not fit. A tapered spindle, however, has serious disadvantages. If the tapered bushing is to carry the weight of the rotatable upper part of the instrument without additional support, a considerable surface pressure results within the bearing. For instance, if the sides of the bushing are inclined by 4° from the vertical, the surface pressure amounts to about fourteen times the weight of the alidade, and the friction is increased proportionally. This friction cannot be tolerated because the spindle will not move freely enough and will sooner or later corrode. Therefore, the center must be fitted in such a manner that its upper shoulder takes up some of the weight, and the bushing carries only enough weight to give the center and the upper part of the instrument sufficient stability when rotated. This immediately brings out the weak point of a tapered spindle: If the taper carries too little weight, the axis wobbles; if it carries too much weight, it binds. Therefore, the more precise and heavier European theodolites used to have devices to adjust the fit of the spindle.

This sounds simpler than it is, because a great deal depends on the roundness of spindle and bushing, the metals used, the surface finish, the cleanness and the type of lubricant. Dr. Wild found that out on a mountain peak. But even if a spindle is properly fitted and adjusted, it still actually floats in an oil bath, and while it rotates, it goes through a slight wobble which cannot be eliminated.

In 1908, Dr. Wild tried to develop a vertical axis which would not need adjustment. He introduced the cylindrical axis, first for levels and later for theodolites. To build such axes, it is necessary to have high quality steel which does not warp and which can be hardened, and also good grinding machines. On the cylindrical axis, the full weight is carried by the shoulder of the bushing or by the end of the center, either directly or by a ball bearing. If the upper part is well balanced and the axis is vertical, the bushing carries no weight at all. Experience has shown that such cylindrical axes can perform satisfactorily for many years without maintenance or adjustment. But the center still floats on an oil film, and with proper clearance for the oil film Dr. Wild found this had a limit of precision of about 3 seconds.

To further increase the precision of the vertical axis, Dr. Wild introduced a new system in his latest designs. This axis consists primarily of two rings or disks, each with a precisely ground plane surface which, together with meticulously matched steel balls, form a precision ball bearing. This bearing carries the weight of the upper portion of the instrument and at the same time positions it in a plane. To center the bearing, only a plug with a relatively small diameter is needed. Such a plug causes little friction and it need not be ground to extreme precision if the circle readings are taken simultaneously at diametrically opposed points. At the same time, this type of axis makes an instrument base wide and squat, and, therefore, very stable.

The precision of this axis depends upon the flatness of the two ring surfaces and upon the uniformity of the diameter of the balls. Since the balls deviate from each other by no more than ± 0.2 micron (about 8 millionths of an inch), and since the bearing runs on numerous balls, they do not cause any inaccuracies worth considering. Furthermore, the weight of the instrument suffices to compress the balls and the steel races enough to equalize the small differences in the ball diameters.

In this manner it is possible to build a vertical axis which has a running accuracy of ± 0.5 second for use in a very compact one-second theodolite. This precision can be further increased by increasing the diameter and thickness of the ball races and by using more balls. This axis design has the further advantage that it is relatively easy to manufacture and that, with proper designing, it can be checked again with a test glass (optical flat) after it has been installed. I feel it should be pointed out here that although the ball race provides great accuracy, like all precise equipment it is subject to damage of the races when dropped on its base in an upright position.

Going further up on the instrument, the next mechanical improvement is the enclosing of all slow motion screws, so that they can be permanently lubricated and are not exposed to dirt which acts as an abrasive and may cause stiffness, drag, backlash, and uneven motion. The entire instrument is now practically dust- and moisture-proof. This became possible because of some of the optical improvements which I shall mention in a short while. On early instruments, the leveling screws were also exposed, which added to the normal wear.

Better clamps were designed, which require no force at all and lock the instrument axes without drag and without wearing them down. There are some clamps which simply release a spring, so that the axis is held under a certain pressure provided by the manufacturer rather than by a heavy-handed operator.

The trunnion or horizontal axis of the theodolites has changed little over the last forty to fifty years. It was always the endeavor of the designers to equip the trunnion axis with two cylindrical bearing surfaces with as small a diameter as possible and as far apart as possible. Naturally, the trunnions should both have the same thickness. Most horizontal axis bearings are wye shaped, so that the axis is clearly determined by four points, two on each side. The major task of a manufacturer is to achieve perfect roundness of the trunnions.

On modern theodolites, this task has been made a little more difficult because the optical path of the reading system sometimes goes through the horizontal axis and requires a larger diameter of the trunnions. This is compensated by the greater precision of today's grinding machines. The influence of the oil film on the horizontal axis is much smaller than on the vertical axis, because the horizontal axis is under constant pressure.

Optical Improvements

Originally, the only optical part of a surveying instrument was the telescope, and in some instances a built-in magnifier or reading microscope to ease the strain of reading a vernier.

Since the first surveying instruments were purely mechanical (open peepsights), the telescope was just a convenient means to get a more accurate sight at a given target or rod. Therefore, manufacturers did not think in terms of optics, but approached their designs from a purely mechanical standpoint. An instrument was a mechanical device with an optical sighting aid. This concept has changed radically. Nowadays, a theodolite has primarily an optical system—which includes the circles—around which the mechanical designer has to build an axis system and an outer shell, always aiming for the "balance of design" I mentioned earlier. This change is primarily due to the invention by Dr. Wild of the internal optical reading system which results in greater accuracy and much increased operator comfort.

But let us first have a look at what happened to the long, bulky old telescopes with external focusing. They were not even dust-proof, since focusing was accomplished by sliding the objective or the eyepiece backward and forward. While the telescopes were long and heavy, they had only a small aperture and, hence, a dim image. The optics were poorly corrected for the various aberrations, making accurate pointing difficult even in broad daylight. The crosshairs were made of spider web. During the last century, the Kern factory had regular "spider hunts" along the banks of the nearby Aar River, combined with an outing. Around the turn of the century, the method of scribing lines on glass was first used, and later on the process of etching became general practice. Photographic application of a metallic deposit is sometimes used today, but still poses some problems. The etching method is still the most precise and the most durable one, especially where instruments are used in the tropics, with fungus problems.

In 1909, Dr. Wild invented the anallatic telescope with a negative focusing lens moving inside the barrel, making it possible to produce shorter, waterproof telescopes which have an addition constant of zero.

As optical science made progress, so did the quality of telescopes improve. Nowadays we have refracting telescopes corrected for astigmatism, coma, spherical, and almost all the chromatic aberrations. A new telescope designed by Dr. Wild, consisting of a combination of lenses and concave spherical mirrors, eliminates even the last traces of chromatic aberration, the so-called secondary spectrum. It is used on the Kern DKM3 first-order and astronomical theodolites and on some missile tracking instruments.

The coating of lenses made it possible to increase the light-gathering power of telescopes by as much as 50 per cent, so that better work can be done at night, at dawn, and at dusk, when atmospheric conditions are most favorable, and underground.

On leveling instruments, the operator had to walk over to the side of his instrument to check the bubble. Not only did this involve the hazard of disturbing the stability of the tripod, but the fact that the bubble was not always viewed at the same angle caused quite serious parallax errors. Sometimes folding mirrors were used to view the bubble, which helped some. As early as 1909, Dr. Wild invented the coincidence level, which the operator views from the eyepiece end of the instrument—sometimes even through the telescope itself. The coincidence principle, by increasing the precision of the bubble centering, greatly increases the effective sensitivity of a level vial. Therefore, coincidence levels are also used for indexing the vertical circle of theodolites.

The metal circles of theodolites were replaced by fully enclosed glass circles. Glass can be graduated much more finely and precisely than metal and can be viewed under much higher magnification. Lines on metal always appear jagged when viewed under high magnification. Glass circles are read through an optical train ending in an eyepiece located conveniently for the operator. Since these optical parts, circles, prisms, and lenses are located inside the instrument, there is very little risk of breakage, and the instrument can be sealed against dust and rain. The necessary light to illuminate the circles and micrometer is furnished by one or two reflectors in daylight, and by small electric light fixtures in the dark. A further advantage of glass circles is that, since they can be graduated so precisely, their diameter can be cut down to less than half of metal circles of comparable precision, which makes the entire instrument extremely compact.

For precise instruments, even the fine graduation on glass is not enough. It is not feasible to divide a small circle into single seconds, or 1,296,000 intervals. The optical micrometer invented by Dr. Wild permits breaking down larger circle intervals into small fractional units very easily. An optical flat is introduced into the path of light between the circle and the observer's eye. By rotating the optical flat, we deflect the light and obtain an apparent rotation of the circle. The thickness and the range of motion of the optical flat can be so chosen that the apparent motion of the circle image is exactly one interval. A separate small circle or scale is connected with the optical flat. If we turn the optical flat until an index coincides with the nearest circle interval, we feed the fractional reading into the micrometer scale and can now read the full interval on the circle and add to it what we read on the micrometer scale.

This principle was first used on leveling instruments where the estimation of a fraction of a rod interval was replaced by actual measurement on a scale or drum attached to the micrometer. Here the rotation of the optical flat about a horizontal axis results in an upward or downward motion of the rod image. The flat is rotated until the horizontal cross-hair appears to be on the nearest full rod interval which is read. To this reading is added the reading of the micrometer drum, which is the amount by which the line of sight was moved down, from the zero drum position, parallel to the telescope axis.

One of the faults of old theodolites, which Dr. Wild tried to eliminate after working on triangulation jobs in the high mountains, was the need to walk around the instrument to read the two or more verniers. Especially on precise instruments, opposed verniers had to be read to eliminate circle eccentricity errors. To overcome this cumbersome procedure, Wild devised a system to bring the two opposed circle readings to one common reading microscope equipped with a drum micrometer. He filed his patent in January, 1907. This was before he had introduced glass circles, and that particular system was not used commercially as far as I know. But as soon as glass circles were introduced, Dr. Wild applied the same principle. In fact, all the theodolites designed by him have this feature. He was a very outspoken antagonist of repetition theodolites, especially the ones with an internal, optical reading system which permits the reading of only one circle point at a time. As you undoubtedly know, repetition transits or theodolites are not as precise as direction instruments. There is no repetition mechanism in existence which does not cause small amounts of circle drag when operated.

Therefore, practically no repetition instruments are built which give a least direct reading of better than 20 seconds. Most of them are basically one-minute instruments. This is due partly to circle eccentricity errors which may be reduced in magnitude by repetition. However, repetition causes circle drag errors. Dr. Wild contended very outspokenly that it was nonsense to use a repetition instrument if more precise and more reliable results could be obtained by using direction instruments with automatic elimination of all eccentricity errors.

Today, all theodolites with glass circles reading to one second or better, have this system, because if only one side of the circle were read, the eccentricity errors might at times exceed the least direct reading of the instrument and make the reliability of the measurements questionable.

In his later designs, Dr. Wild used two concentric sets of graduations in the reading systems, with the result that if a direction is measured in the direct and the reversed telescope positions, two completely different pairs of graduations are used for the readings. The two diameters of these pairs of circle graduations are located 180° apart on the curve of systematic graduation error. Analyzing this curve of graduation error according to Fourier, it is found that for measurements in both telescope positions all members of the series with an odd frequency are eliminated automatically. To go into the vast field of circle evaluation is beyond the scope of this paper, so let me go on to other advancements in the optical field. One quite simple addition to theodolites and transits which has proven a boon to surveyors is a small, right-angle telescope sighting through the hollow center of the vertical axis. It is known as the optical plummet and replaces the plumb bob. Its great advantage is that the line of sight does not start swinging when the wind blows. The optical plummet is particularly practical when it is built into the alidade or a transit; its adjustment can be checked by turning the alidade 180 degrees.

Improvements in Tripod Design

If you look at old instruments, you will find that their tripods consisted, to express it crudely, of three heavy sticks hinged with bolts and wing nuts to a very narrow plate at the top. In spite of their substantial weight, these tripods were not stable at all. The wing nuts had to be tightened at every set-up, which was a waste of time and did not always give the same stability to each leg.

In 1909, Dr. Wild designed the wide-frame tripod with fixed legs, and in 1911 with extension legs. This tripod, which has a flat plate at the top, is still used practically unchanged, by a number of manufacturers. It offers greater stability, and the fairly large hole in its plate permits the instrument to be shiften horizontally by a greater amount than was previously possible. The tripod hinges can be adjusted to the proper snugness and need only periodic re-tightening.

In 1935, it was again Dr. Wild who came up with a new development, the tilting plate tripod, which has an upper plate mounted in the head, suspended in gimbals. This head plate can be leveled by means of a set of recessed cross levels, and locked in position. It has two advantages:

(a) Instruments with optical plummets can be shifted laterally without requiring major re-leveling, which makes the optical plummet considerably easier and faster to use.

(b) Long vertical leveling screws can be abandoned in favor of the more stable cam system which has a more limited range.

The hinges on these tripods are tapered, and they are tightened by means of a turnbuckle which pulls them against their seats. This causes them to wear evenly. The earlier designs were tightened by pressing the leg hinge bolts between two plates, so that eventually they wore down to an oval shape and to a poor fit.

The next major advancement, developed by Mr. R. F. Haller, chief designer of Kern, was a tripod head with a double ball-and-socket point which operates smoothly, and by means of which greater stability could be obtained than had been achieved by others in earlier attempts. This ball-and-socket tripod head permits rapid preliminary leveling and makes it possible to set up the tripod with ease in the roughest terrain because of the wide range of tilt of the head. This principle was used for three types of tripods, the simplest of which is the one used for leveling instruments. It was also used for the planetable tripod, combined with a built-in planetable head, a clamp, and a tangent screw.

The most sensational use of the ball-and-socket head was made in the centering tripod for theodolites, where the head was combined with a centering rod and bullseye level. This is really a semi-automatic tripod which, as it is set up, does three things simultaneously: it centers the instrument over the station point with an accuracy of .02 to .04 inch; it levels the top of the tripod head; and it gives the height of the trunnion axis above the station point.

A bushing in the center of the tripod head fits a plug in the base of the theodolites, traverse targets, and subtense bars, as well as other accessories so closely that autocentering—an interchange of heads with an accuracy of 0.002 inch and without disturbing the centering over the station point—is possible simply by flicking a bayonet lever. An article on Autocentering was published in the June, 1960, issue of *Surveying and Mapping*, and reprints can be obtained from my company. The centering tripod can be set up over a point with almost incredible speed in any kind of terrain, with an accuracy which closely approaches that which can be achieved with the optical plummet.

The centering rod can be unscrewed, and the centering checked by means of the optical plummet. This can also be done if a wall has to be straddled where the rod would be in the way. Finally, I am happy to say that Kern pioneered the use of metal legs on tripods which proved very successful, especially under humid conditions where wooden legs swell up, warp and rot, and in mines where the high reflectivity of the aluminum alloy makes them visible at great distances. The expansion coefficient of the alloy used did not prove harmful. On the contrary, wooden tripods also go through a rotational movement or torque when exposed to prolonged sun irradiation. While wood, due to its grain, tends to build up a stress and then "jump," metal with its uniform composition goes through a smooth motion which can be determined and taken into consideration.

A NOTE ON THE REGRESSION BETWEEN GAMMA VARIATES

BY MYRON B. FIERING,* *Member*

INTRODUCTION

In connection with investigations of the statistical structure of hydrologic records, gamma functions play a significant role. A variety of hydrologic variables are characterized by skewed distributions, whence log-normal and gamma density functions are logical candidates for describing their distributions. Typically, extreme values originate from skewed populations (Matalas, 1959) but recent studies (Thomas and Fiering, 1963) consider as well the skewness in mean annual and mean monthly flows. The United States Geological Survey subjects a large portion of its gaging station data to logarithmic transforms in order to render it more nearly normal and hence more amenable to standard analyses.

This paper develops the first phase of the theory required to construct regression functions between two variables whose joint distribution is bivariate gamma. Apart from the specific hydrologic applications cited above, the results comprise a useful start in the systematic analysis of regressions between non-normal joint distributions.

THE JOINT DISTRIBUTIONS

W. F. Kibble (1938) derives a bivariate gamma density by considering first a bivariate normal density in two standardized variates X and Y (with correlation R)

$$f(X,Y) = \frac{1}{2\pi\sqrt{1-R^2}} \exp\left[\frac{-1}{2(1-R)^2}(X^2 - 2RXY + Y^2)\right] \tag{1}$$

Let $x = X^2/2$, $y = Y^2/2$, and ρ be the correlation between x and y; then

$$g(x,y) = \frac{(xy)^{-1/2}}{4\pi\sqrt{1-\rho^2}} \exp\left[\frac{-1}{1-\rho^2}(x - 2\rho\sqrt{xy} + y)\right] \tag{2}$$

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or, taking account of the fact that there are four pairs (X,Y) corresponding to each pair (x,y), and correcting an apparent omission in Kibble's derivation,

$$\phi(x,y) = \frac{(xy)^{-1/2}}{2\pi\sqrt{1-\rho^2}} \{\exp[-(x - 2\rho\sqrt{xy} + y)/(1-\rho^2)] + \exp[-(x + 2\rho\sqrt{xy} + y)/(1-\rho^2)]\} \quad (3)$$

Equation 3 contains two exponents which are referred to here as the first and second exponents, respectively. The first exponent arises from positive combinations of X and Y, and thus, apart from a scale constant, is the more useful term in hydrologic studies because variate values (e.g., rainfall, streamflow, barometric pressure) are positive. However, for completeness, two regression solutions are developed; one pertains to the first exponent only and the second includes both terms of the joint distribution $\phi(x,y)$.

We integrate to verify that the marginal distributions are gamma. Consider only the first exponent, or the function

$$\phi'(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2}\sqrt{x}} \int_0^\infty \exp\left[-\frac{x - 2\rho\sqrt{xy} + y}{1-\rho^2}\right] / \sqrt{y}\, dy$$

Let $u = \sqrt{2}(\sqrt{y} - \rho\sqrt{x})/\sqrt{1-\rho^2}$

whence $dy = \sqrt{2y}\sqrt{1-\rho^2}\, du$

and

$$u^2/2 = \frac{y - 2\rho\sqrt{xy} + x}{1-\rho^2} - x$$

Thus, by substitution

$$\int \phi' dy = \frac{e^{-x}}{\sqrt{2x}\,\pi} \int_0^\infty \exp[-u^2/2]\, du = \frac{e^{-x}}{2\sqrt{\pi x}} \quad (4)$$

For the second exponent, $\phi''(x,y)$, we make the transform $v = \sqrt{2}[\sqrt{y} + \rho\sqrt{x}]/\sqrt{1/3\rho^2}$

whence

$$\int \phi'' dy = \frac{\sqrt{1+3\rho^2}}{\pi\sqrt{2x}\sqrt{1-\rho^2}} e^{-x} \int \exp\left\{\left(\frac{-v^2}{2}\right)\left(\frac{1+3\rho^2}{1-\rho^2}\right)\right\} dv$$

Now if we make the further transform $w = v\left[\frac{1+3\rho^2}{1-\rho^2}\right]^{1/2}$, the second integration is easily made

$$\int \phi'' dy = \frac{e^{-x}}{\pi\sqrt{2x}}\int_0^\infty \exp\left[-w^2/2\right] dw = \frac{e^{-x}}{2\sqrt{\pi x}} \tag{5}$$

Adding Equations 4 and 5, the marginal distribution of x is

$$h(x) = \frac{e^{-x}}{\sqrt{\pi x}} \tag{6}$$

so that h(x) is a gamma distribution with parameter 1/2. Similarly the marginal distribution of y is $e^{-y}/\sqrt{\pi y}$, also a gamma distribution.

To deal with the first exponent only—that is, to consider only positive variate values—it is necessary to multiply the distribution by 2 to obtain a valid distribution function (whose integral is unity). Thus the corrected density is

$$\phi'(x,y) = \frac{(xy)^{-1/2}}{\pi\sqrt{1-\rho^2}}\left[\exp\left(-(x-2\rho\sqrt{xy}+y)/(1-\rho^2)\right)\right] \tag{7}$$

where the prime indicates that only one exponent is included.

The Conditional Distribution and Regression Functions

The conditional distribution $\theta(y|x)$ is given by the ratio

$$\theta(y|x) = \frac{\phi(x,y)}{h(x)} = \theta'(y|x) + \theta''(y|x) \tag{8}$$

$$= \frac{(xy)^{-1/2}}{2\pi\sqrt{1-\rho^2}}\left\{\exp\left[-\frac{(x-2\rho\sqrt{xy}+y)}{1-\rho^2}\right] + \exp\left[-\frac{(x+2\rho\sqrt{xy}+y)}{1-\rho^2}\right]\right\} \div e^{-x}/\sqrt{\pi x} \tag{9}$$

Integrating the first exponent

$$\int \theta'(y|x)dy = \frac{1}{\sqrt{2\pi}}\exp\left[-u^2/2\right] du = 1/2$$

and, by symmetry, the second exponent integrates to 1/2; the sum of the integrands, $\theta(y|x)$, is indeed a valid frequency function.

The regression of y on x is given by the expectation $E[y|x]$, or by the equivalent integral $\int y\theta(y|x)dy$. Performing the division in Equation 9, the expected value of the first exponent is

$$\int_0^\infty \frac{\sqrt{y}}{2\sqrt{\pi}\sqrt{1-\rho^2}} \exp\left[\frac{-(y-2\rho\sqrt{xy}+\rho^2 x)}{1-\rho^2}\right] dy.$$

Letting $u = \sqrt{2}\,[\sqrt{y} - \rho\sqrt{x}]/\sqrt{1-\rho^2}$ the expectation becomes

$$\frac{1}{\sqrt{2\pi}}\int_0^\infty \left\{\frac{u^2(1-\rho^2)}{2} + \rho^2 x + \rho u\sqrt{2x(1-\rho^2)}\right\} \exp\left[-u^2/2\right] du$$

which expands to

$$\frac{1-\rho^2}{2\sqrt{2\pi}}\int u^2 \exp\left[-u^2/2\right] du + \frac{\rho^2 x}{\sqrt{2\pi}}\int \exp\left[-u^2/2\right] du$$

$$+ \frac{\rho\sqrt{1-\rho^2}\sqrt{x}}{\sqrt{\pi}}\int u \exp\left[-u^2/2\right] du$$

The first two terms integrate directly to the sum $\left[\frac{1-\rho^2}{4} + \frac{\rho^2 x}{2}\right]$; the last term can be integrated by writing $z = u^2/2$ from which

$$\int_0^\infty u \exp\left[-u^2/2\right] du = \int_0^\infty e^{-z}\, dz = 1$$

so that the expectation of the first exponent $E[y|x]$ is

$$E[y|x] = \frac{(1-\rho^2)}{4} + \frac{\rho^2 x}{2} + \rho\sqrt{1-\rho^2}\sqrt{x/\pi} \qquad (10)$$

If only the first exponent is used, it is necessary to multiply Equation 10 by 2 so the resulting non-linear regression is

$$E'[y|x] = \frac{1-\rho^2}{2} + \rho^2 x + 2\rho\sqrt{1-\rho^2}\sqrt{x/\pi} \qquad (11)$$

where the prime denotes that all variates are positive. If the full regression function is desired, the second exponent is expanded in a similar fashion, and its expectation is

$$\frac{1-\rho^2}{4}+\frac{\rho^2 x}{2}-\rho\sqrt{1-\rho^2}\sqrt{x/\pi}$$

so that the complete regression is the linear function

$$E[y|x]=\frac{1-\rho^2}{2}+\rho^2 x \tag{12}$$

The non-linear terms cancel in this case.

The Standard Error of Estimate

We require to find the variance $E[y^2|x]-[E[y|x]]^2$. Multiplying Equation 9 by y^2, the first exponent gives

$$E'[y^2|x]=\frac{1}{\sqrt{2\pi}\sqrt{1-\rho^2}}\int_0^\infty y^{3/2} \exp\left\{\frac{-(y-2\rho\sqrt{xy}+\rho^2 x)}{1-\rho^2}\right\}dy \tag{13}$$

If we make the usual transform $u=\sqrt{2}[\sqrt{y}-\rho\sqrt{x}]/\sqrt{1-\rho^2}$, an explicit expression for $y^{3/2}$ can be found

$$y^{3/2}=u^3\sqrt{\left(\frac{1-\rho^2}{2}\right)^3}+\frac{3}{2}u^2\sqrt{x}\,\rho(1-\rho^2)+\frac{3}{\sqrt{2}}ux\rho^2\sqrt{1-\rho^2}+\rho^3x^{3/2} \tag{14}$$

so that substitution into Equation 13 gives

$$\begin{aligned}E'[y^2|x]=&\frac{1}{\sqrt{2\pi}}\sqrt{\left(\frac{1-\rho^2}{2}\right)^3}\int_0^\infty u^3\exp[-u^2/2]\,du\\&+\frac{3}{2\sqrt{2\pi}}\rho\sqrt{1-\rho^2}\sqrt{x}\int_0^\infty u^2\exp[-u^2/2]\,du\\&+\frac{3}{2\sqrt{\pi}}x\rho^2\sqrt{1-\rho^2}\int_0^\infty u\exp[-u^2/2]\,du\\&+\frac{1}{\sqrt{2\pi}}\rho^3x^{3/2}\int_0^\infty \exp[-u^2/2]\,du\end{aligned} \tag{15}$$

The second and fourth terms integrate immediately, and the first and third terms are readily evaluated after making the transform $z = u^2/2$; the entire integral is

$$\sqrt{\frac{2}{\pi}}\sqrt{\left(\frac{1-\rho^2}{2}\right)^3} + \frac{3}{4}\rho\sqrt{x}(1-\rho^2) + \frac{3}{2\sqrt{\pi}}\rho^2 x\sqrt{1-\rho^2} + \frac{1}{2}\rho^2 x^{3/2}$$

The second exponent is evaluated similarly; the necessary transforms are again $v = \sqrt{2}\,[\sqrt{y} + \rho\sqrt{x}]/\sqrt{1+3\rho^2}$ and $w = v\sqrt{1+3\rho^2}/\sqrt{1-\rho^2}$. Finally, the sum of both integrals yields

$$E[y^2|x] = \sqrt{\frac{(1-\rho^2)^3}{\pi}} + \frac{3\rho^2 x\sqrt{1-\rho^2}}{\sqrt{\pi}} \tag{16}$$

from which we subtract $[E[y|x]]^2$ giving the variance

$$\mathrm{Var}(y|x) = \frac{1}{\sqrt{\pi}}\left[\sqrt{(1-\rho^2)^3} + 3\rho^2 x\sqrt{1-\rho^2}\right] - \left[\frac{(1-\rho^2)^2}{4} + \rho^2 x(1-\rho^2) + \rho^4 x^2\right] \tag{17}$$

if both exponents are used and

$$\mathrm{Var}'(y|x) = 2\left\{\sqrt{\frac{(1-\rho^2)^3}{\pi}} + \frac{3}{4}\rho\sqrt{x}\,(1-\rho^2) + \frac{3}{2\sqrt{\pi}}\rho^2 x\sqrt{1-\rho^2} + \frac{1}{2}\rho^3 x^{3/2}\right. - \left\{\frac{(1-\rho^3)}{4} + \frac{\rho^2 x}{2} + \rho\sqrt{1-\rho^2}\sqrt{\frac{x}{\pi}}\right\}^2 \tag{18}$$

if only the first exponent is used. In either case the regression function is not homoscedastic.

The Correlation Coefficient

Matalas (1964) derives the expectation

$$E[xy] = \frac{1+2R^2}{4} \tag{19}$$

Since the gamma variates x and y have mean and variance 1/2, the product-moment correlation coefficient is

$$\rho = \frac{E[xy] - E[x]E[y]}{\sigma_x \sigma_y}$$

so that $\rho = R^2$, where R is the correlation between X and Y. Thus $\rho \geqslant 0$ whether X and Y are positively or negatively correlated.

Conclusions

1. The marginal distributions $e^{-x}/\sqrt{\pi x}$ and $e^{-y}/\sqrt{\pi y}$ are special gamma distributions which decrease monotonically from $x,y = 0$ to $x,y \rightarrow \infty$. Consequently the results derived in this note are limited because they apply only to gamma distributions with parameter 1/2. In order that the parent gamma functions shall be unimodal the parameters must exceed unity. Nonetheless, these results provide the basis for further work. It should be emphasized that certain hydrologic records seem to exhibit non-linear regressions of the general sort predicted by Equation 10; moreover, these regressions are generally not homoscedastic. Thus it is to be expected that the general bivariate gamma regression function will be neither linear nor homoscedastic.

2. Another restriction of this model is that $\rho \geqslant 0$, which limits the spectrum of possible applications because many data exhibit negative correlation.

3. Despite these drawbacks, the integrations result in estimating equations which are markedly different from the traditional bivariate normal case. Hopefully the lack of generality in these results can be reduced as further work is completed.

Acknowledgement

The author wishes to thank Dr. Nicholas C. Matalas of the United States Geological Survey for reviewing the derivations; this work was sponsored, in part, by a grant from the United States Public Health Service, and the author is appreciative of this support.

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KARL TERZAGHI

KARL TERZAGHI
1883-1963

Karl Terzaghi, founder of soil mechanics and eminent civil engineer, died at his home in Winchester on October 25, 1963. An Austrian by birth, he had lived in Greater Boston during a major part of his professional career. This period embraced the years in which soil mechanics attained a secure place among the engineering sciences and his own professional capacities received world-wide recognition.

He was born on October 2, 1883, in Prague, where his father, an officer in the Austro-Hungarian army, was stationed. Upon his retirement, Colonel Terzaghi moved his family to Graz, where Karl grew to manhood. In Graz and in a military school in Hungary, he received the conventional education of an officer's son. It was supplemented, however, by his own eager indulgence in such extra-curricular activities as the study of the geography of exotic lands and the invention of a system of trigonometry long before he had encountered this subject in his textbooks. His education was frequently punctuated by the perpetration of imaginative pranks. One of the last of these came close to causing his expulsion from the Technical University of Graz. Owing largely to the intercession of one of his professors, he was allowed to continue his studies, and in 1904 he received his engineering degree. Two years later, he returned to the University for an additional year of study, chiefly in geology.

After leaving his *alma mater,* Terzaghi worked as a civil engineer, mainly as superintendent of construction, in various parts of the old Austrian Empire and in czarist Russia. Many of his jobs involved earthwork and foundation engineering, at that time necessarily performed on the basis of local custom and empirical rules lacking general validity. With each new job, he recognized more clearly the need to replace the old rules-of-thumb by a rational approach to the problems of earthwork and foundation engineering. In the search for such an approach, his rebellious and creative nature found a purpose which was to occupy his thoughts and consume his vast energies until the day of his death.

Terzaghi began the search in earnest after obtaining the doctorate from the Technical University in Graz in January 1912. He went first to the United States because he hoped to obtain insight into fundamental principles by means of a systematic study of the relation between foundation conditions and performance of a large number of major structures, then recently completed or still under construction in this country. Although he had been able to collect a wealth of interesting observational data, he was disappointed and discouraged when he returned to Austria at the end of 1913, knowing that he had failed to obtain the answers he sought.

Military service in World War I interrupted his efforts to salvage useful conclusions from his notes. He was not free to renew his attack upon the problems of soils engineering until 1916, when he was appointed professor at the Imperial School of Engineering in Constantinople (later Istanbul). By this time, Terzaghi was convinced of the inadequacy of case studies unsupported by theoretical insight into the behavior of soils under conditions imposed by civil engineering operations. Hence he embarked upon a program of experimental and theoretical research designed to provide the required fundamental knowledge. Upon conclusion of the armistice in 1918, he was, like other subjects of the Central Powers, dismissed from the Imperial School of Engineering. Shortly thereafter he joined the faculty of American Robert College in Istanbul. There, in spite of a continual struggle with poverty and a heavy burden of lectures, he returned enthusiastically to his research program. Terzaghi described this period of hardship and discovery in a letter written on February 15, 1920, to his former teacher, Professor F. Wittenbauer. "At the beginning of March [1919]" he wrote, "I listed on a single sheet of paper everything that we needed to know about the physical properties of clay in order to be in a position to treat the fundamentals of earthwork engineering on a scientific basis. My requirements seemed excessive even to myself, and I doubted that I would live to see all the questions answered. In April, I started to build primitive apparatus . . . and by the end of the month the first experiments were running. After six weeks of 12-hour days in which I experienced uninterrupted failures, my luck changed, and since that time I have not carried out a single experiment that did not provide the anticipated enlightenment. Today, in mid-February, I can say that I have solved all the problems of last March . . . . It is really a remarkable dispensation of the fates that my old dream of a scientific basis of engineering geology has been fulfilled in a period of such general disaster, and in such an incredibly short time. I believe that the cares, misfortunes, and insecurity of my situation have multiplied my powers many times over." Five years later, the results of his work were presented to the engineering profession in a book, *Erdbaumechanik auf bodenphysikalischer Grundlage,* published in Vienna in 1925.

In the latter part of the period of intense creative activity in Istanbul, Terzaghi enjoyed a small consulting practice in which the results of his research bore their first practical fruits. However, the large-scale applications that were to demonstrate the revolutionary character of his findings still lay ahead, and he was frequently beset by doubts whether he would live to see the full fruition of the new engineering science of soil mechanics. The long-awaited opportunity to extend his professional horizon came in 1925 in the form of an invitation to lecture at Massachusetts Institute of Technology during the academic year 1925-26. This appointment opened a new period in his own life and a new chapter in the history of soil mechanics. For the next four years, he was a member of the MIT staff. During these years he continued his fundamental research and also served as consultant on a large number of important projects on which he was able to demonstrate the immediate practical value of his discoveries. His research as well as his consulting activities gave rise to a steady stream of highly original and important papers, several of them published in the JOURNAL of the Boston Society of Civil Engineers. When he left MIT in 1929 to accept a professorship at the Technical University of Vienna, soil mechanics was not only established as an academic subject; it had begun to revolutionize earthwork and foundation engineering.

Terzaghi returned to Cambridge after the incorporation of Austria into the German Reich in 1938. He accepted a part-time lectureship at Harvard, and in 1946 the University appointed him Professor of the Practice of Civil Engineering. After his retirement as *emeritus* in 1956, he continued to lecture on engineering geology for several years.

During his years at Harvard, and particularly after the close of World War II, his consulting practice, which he limited to exceptionally difficult projects, spread to all parts of the globe, and continued undiminished until the end of 1960, when failing health confined him to his home in Winchester. There, during the last three years of his life, Terzaghi wrote six important papers, a large number of illuminating discussions, and several reports on projects with which he had been intimately and actively connected during the preceding years.

Of the many honors that were bestowed upon him during his long and active career, the first was the Clemens Herschel Prize awarded by the Boston Society of Civil Engineers in 1926. He received the same prize for the second time in 1942, the Desmond FitzGerald medal in 1953, and he was made an honorary member of the Society in 1952. The New England Award of The Engineering Societies of New England was conferred upon him in 1955. He was a member of the Institution of Civil Engineers (London), of the American Academy of Arts and Sciences, and the Wiener Akademie der Wissenschaften. He was an honorary member of the American Society of Civil Engineers and many other professional associations. He was elected honorary president of the International Society for Soil Mechanics and Foundation Engineering after having served as its president for twenty-one years. A complete list of his honorary degrees and awards follows.

Honorary Degrees

1949 Dr. sc., h.c., Trinity College, Dublin (Ireland)
1950 Dr. h.c., Istanbul Teknik Universitesi (Turkey)
1951 Dr. h.c., Universidad Nacional Autonóma de Mexico (Mexico)
1953 Dr. tech., h.c., Eidgenössische Polytechnische Hochschule, Zürich (Switzerland)
1954 Dr. eng., h.c., Lehigh University, Bethlehem, Pa.
1958 Dr. Ing, h.c., Technische Universität, West Berlin (Germany)
1960 Dr. tech., h.c., Norges Tekniske Hogskole, Trondheim (Norway)
1962 Dr. tech., h.c., Technische Hochschule, Graz (Austria)
1963 Dr. sc., h.c., The Ohio State University, Columbus, Ohio

Medals and Awards

1926 Clemens Herschel Prize, Boston Society of Civil Engineers
1930 Norman Medal, American Society of Civil Engineers
1939 James Forrest Lecturer, Institution of Civil Engineers, London
1942 Norman Medal, American Society of Civil Engineers
1942 Clemens Herschel Prize, Boston Society of Civil Engineers
1946 Norman Medal, American Society of Civil Engineers
1946 Frank P. Brown Medal, Franklin Institute, Philadelphia, Pa.
1948 Thomas Fitch Rowland Prize, American Society of Civil Engineers

1948 Goldene Ehrenmünze, Österreichischer Ingenieur and Architekten Verein
1953 Desmond FitzGerald Medal, Boston Society of Civil Engineers
1955 Norman Medal, American Society of Civil Engineers
1955 The New England Award, The Engineering Societies of New England
1960 "The Karl Terzaghi Award" established by the American Society of Civil Engineers
1960 James Alfred Ewing Medal, Institution of Civil Engineers, London
1962 The Moles Award, The Moles, New York

RUTH D. TERZAGHI,* *Member*

* Research Fellow, Harvard University.

OF GENERAL INTEREST

PROCEEDINGS OF THE SOCIETY

MINUTES OF MEETING

Boston Society of Civil Engineers

MAY 13, 1964.—A Joint Meeting of the Boston Society of Civil Engineers with the Structural Section was held this evening in the Society Rooms, 47 Winter Street, Boston, Mass., and was called to order by President William A. Henderson at 7:00 P.M.

President Henderson stated that the Minutes of the previous meeting April 15, 1964 would be published in a forthcoming issue of the Journal and that the reading of those Minutes would be waived unless there was objection.

President Henderson announced the deaths of the following members:-

James F. Brittain, who was elected a member February 15, 1928, who died April 11, 1964.

J. Henry Leon, who was elected a member January 19, 1955, who died May 2, 1964.

The Secretary announced the names of applicants for membership in the Society and that the following had been elected to membership May 11, 1964:—

Grade of Member—Charles E. O'Bannon, Terry S. Hoyt, Nalinikant K. Parekh

Grade of Junior—Mark W. Tenney

President Henderson stated that this was a Joint Meeting with the Structural Section and turned the meeting over to Max D. Sorota, Chairman of that Section to conduct any necessary business at this time.

Chairman Sorota introduced the speaker of the evening, Mr. William J. LeMessurier, W. J. LeMessurier & Associates Inc., who gave a most interesting illustrated talk on "Structural Design of Tall Buildings."

A discussion period followed the talk.

Eighty-five members and guests attended the meeting.

The meeting adjourned at 8:50 P.M.

CHARLES O. BAIRD, JR., *Secretary*

STRUCTURAL SECTION

APRIL 8, 1964.—A regular meeting of the Structural Section was held in the Society Rooms and was called to order by the Chairman, Max D. Sorota, at 7:00 P.M.

The Chairman announced the program for the balance of the year and reminded those present of the remaining lectures in the series on prestressed concrete.

The Chairman introduced the speaker of the evening, Dr. James L. Sherard of the firm of Woodward, Clyde, Sherard & Associates of New York, who spoke on "Earth Dams—Design Procedures to Minimize Construction Problems and Claims by Contractors."

The speaker described various problems encountered in the construction of earth dams, and recommended that more extensive study of construction problems be made during design, and that the results of this study be made available to the contractor in the form of a written report.

After an extensive question and answer period, the meeting was adjourned at 8:15 P.M.

Attendance was 69.

ROBERT L. FULLER, *Clerk*

SANITARY SECTION

APRIL 29, 1964.—The annual outing of the Sanitary Section was held at Deer Island in Boston Harbor. From 3:30 to 5:30 P.M. guided tours of the M.D.C. Sewage Treatment Plant, under construction, were conducted. By the time the last group was formed, the total registration at the resident engineer's office reached 103. At about 6:00 P.M. Chairman Francis T. Bergin invited the members and guests to sit down to a fine roast beef dinner in the dining hall of the Suffolk County House of Correction. Following the dinner Mr. Bergin introduced Mr. Edgar L. Shepard, Master, SCHC, who presented welcoming remarks. The Chairman then introduced Mr. Martin F. Cosgrove, Deputy Chief Engineer, Construction Division, Metropolitan District Commission, who gave an illustrated progress report on the construction of the plant. Blasting problems during caisson construction, concrete placement and temperature control in the 27 ft. thick mat at the caisson bottom (8000 c.y. in an 80 hr. continuous operation), and the mammoth radial dual fuel engines were some of the interesting features discussed in his presentation. The after-dinner talk was enjoyed by 90 (pleasantly stuffed) members and guests.

ROBERT L. MESERVE, *Clerk*

ADDITIONS

Members

William J. Carey, 108 Jersey Street, Boston 15, Mass.
Paul C. Danforth, 9 Glidden Street, Beverly, Mass.
Alvin S. Goodman, 132 Sewall Avenue, Brookline, Mass.
Terry S. Hoyt, 4636 E 9th Avenue, Denver, Colorado
Arthur H. Mallon, 26 Mystic Valley Pky., Winchester, Mass.
Nalin Perekh, 64 Littleton Road, Ayer, Mass.
Olaf Tabur, Tahanto Trail, Harvard, Mass.
Donald M. Thornquist, 21 Greenleaf Road, Natick, Mass.

Junior

Conrad C. Fagone, 12 Browning Road, Arlington, Mass.

Student

Brian S. Gaylord, 10 Beals Street, Brookline, Mass.
Thomas J. Quinn, Jr., 26 Kensington Street, Newtonville, Mass.
Paul Taurasi, 3rd, 4 Taurasi Road, Hingham, Mass.
Peter Thompson, 409 Houston Hall, Tufts Univ., Medford, Mass.

DEATHS

James F. Brittain, April 11, 1964
J. Henry Leon, May 2, 1964

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