

JOURNAL OF THE
BOSTON SOCIETY OF CIVIL
ENGINEERS

Volume 52

OCTOBER, 1965

Number 4

**RECENT DEVELOPMENTS IN SEDIMENT
TRANSPORT MECHANICS**

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(Presented as one of The John R. Freeman Lectures on Fundamental Hydraulic Processes
in Water Resources Engineering, Fall, 1963.)

1. INTRODUCTION

1.1 *Scope*

The field of sediment transport includes flow in alluvial channels and channel stability, coastal processes and sediment transport by waves, river morphology, sediment transport in closed conduits, and aeolian transport. It is not possible in the allotted space and time to summarize all of these topics in any meaningful way. Therefore, the scope of this treatment will be limited to topics related to flow in alluvial channels, including properties of sediments, initiation of sediment motion, channel roughness, and methods of determining rates of sediment transport. It is this area of sediment transport that is generally of greatest interest to hydraulic engineers.

1.2 *Features of Flow in Alluvial Channels*

In the last hundred years, the state of knowledge of flow in rigid-boundary conduits and open channels has improved until it is now possible to predict for these channels the depth and velocity of flow, velocity distribution, slope of the energy gradient, and other flow properties with accuracy that is usually adequate for engineering purposes. Such is not the case for flow in channels whose boundaries are composed of materials transported in significant quantities by the

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flow. The difficulty is that it is the flow itself that determines the channel geometry, both the gross dimensions of the channel (cross-section shape, slope, and alignment) and the roughness texture of the channel surface, by deforming the bed into ripples, dunes, sand bars, etc. In addition, the moving fluid is a mixture of a liquid and a solid, and does not behave in exactly the same way or exhibit the same properties as the same fluid without suspended sediment. Thus, the situation is significantly more complicated than the relatively simple case of flow of a homogeneous fluid in a rigid-boundary channel.

2. PROPERTIES OF SEDIMENTS

Just as it is necessary to know the properties of a fluid before its motion can be formulated, so also it is necessary to know the appropriate properties of transported sediment before sediment transport phenomena can be examined.

It is generally agreed that the grain size of the sediment particles and the fall velocity of individual particles in the transporting medium are the most important properties affecting sediment transportability. Actually, these two properties are not wholly independent; for a given particle size, shape, and weight and specified fluid properties, the fall velocity can be determined.

Measures of grain size that are in common use are:

- a. Sieve diameter: The size of the opening of a square mesh that will just allow the particle to pass.
- b. Sedimentation diameter: Diameter of a sphere with the same density and settling velocity in the same fluid at the same temperature.
- c. Fall diameter: The sedimentation diameter for water at 24°C.
- d. Nominal diameter: Diameter of a sphere of equal volume.

Natural sediments contain a range of particle sizes; hence it is necessary to speak of the frequency distribution of grain sizes. The grain diameters are usually measured by sieving the particles through a nest of calibrated, graduated sieves. The frequency distribution is obtained by plotting the logarithm of the grain size against the percent (by weight) of the sample smaller than any grain size, as shown in Fig. 1. The median diameter, D_g , is the diameter such that 50% of the sample has a smaller diameter, and the geometric standard

deviation is defined as $\frac{1}{2} \left(\frac{D_{50}}{D_{15.9}} + \frac{D_{84.1}}{D_{50}} \right)$, where $D_{15.9}$ is the di-

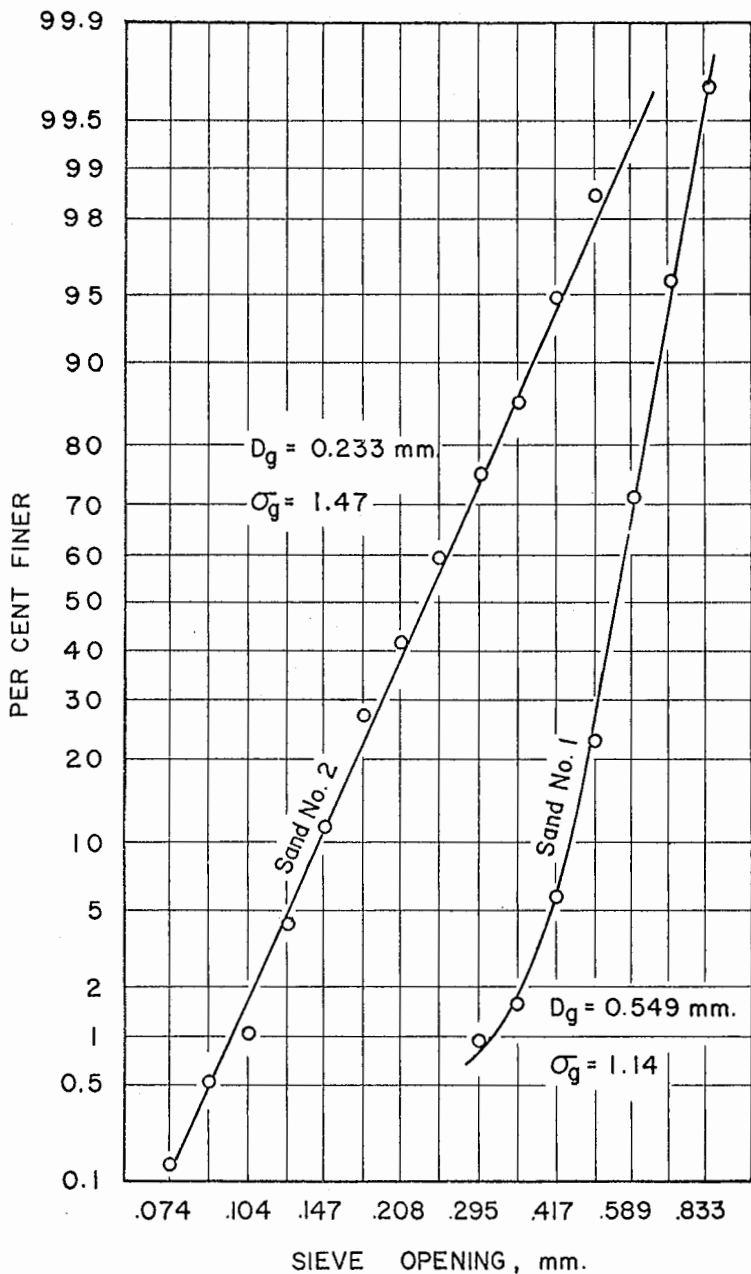


Fig. 1 Examples of cumulative distributions of sieve diameters plotted on log-probability paper.

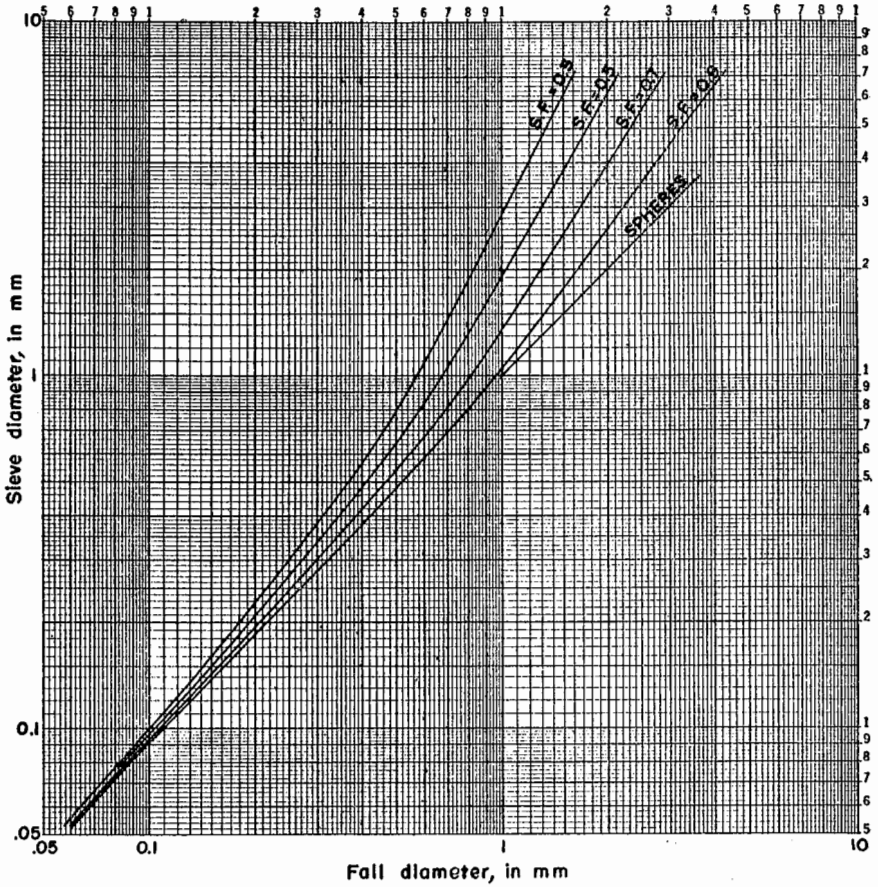


Fig. 2 Relation of sieve diameter to fall diameter for naturally worn quartz particles. (Ref. 1)

- Notes: 1. Fall diameter is sedimentation diameter for fall in water at 24°C only.
 2. Shape factor S.F. is defined in terms of the triaxial dimensions as:

$$S.F. = \frac{c}{\sqrt{ab}}$$

where

- a = longest axis
- b = intermediate axis
- c = shortest of the three mutually perpendicular axes of the particle.

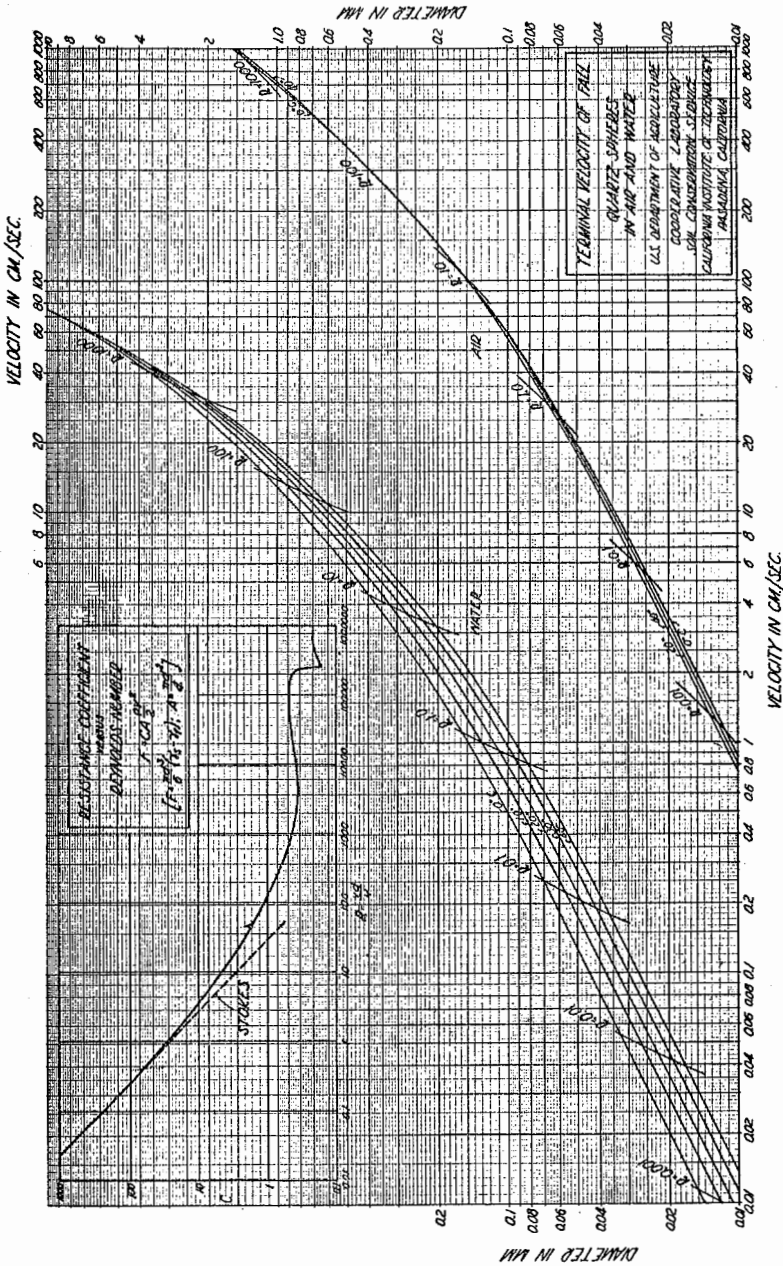


Fig. 3 Terminal velocity of fall for quartz spheres in water and air.

ameter such that 15.9% of the material is finer, and D_{50} and $D_{84.1}$ have similar meanings. It is noteworthy that the size distributions of natural sediments generally plot as straight lines on log-probability paper.

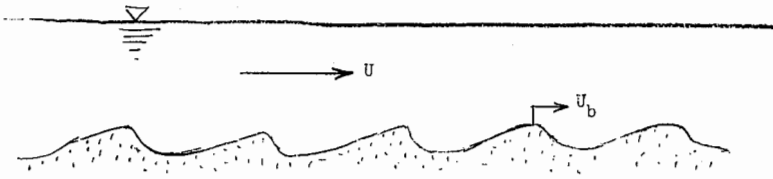
The fall diameters (sedimentation diameter in water at 24°C) can be obtained from the sieve diameter from Fig. 2 (1), which is based on a large body of experimental fall velocity and sieve diameter data. In this figure, the shape factor is c/\sqrt{ab} , where a, b and c are respectively the longest, intermediate, and shortest of three mutually perpendicular axes of the particle. For most natural sand grains, the shape factor is about 0.7. With the fall diameters or sedimentation diameters known, the fall velocities can be obtained from Fig. 3.

3. THE FLUID-BED INTERFACE

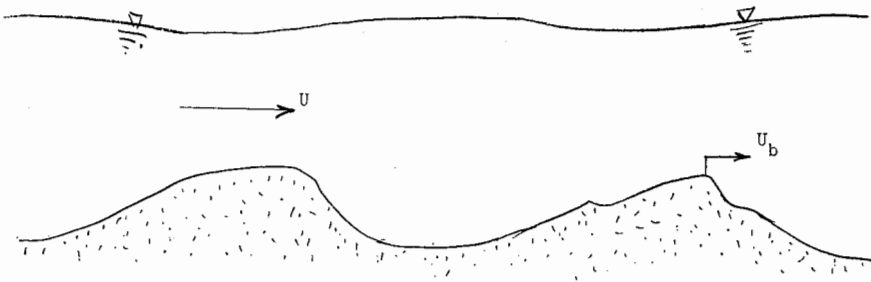
The behavior of the interface between the fluid and the sediment bed is of interest because this is the region from which the transported sediment originates, and the configuration of the interface plays a large role in determining the roughness of the channel. In this section discussions of the different types of bed roughness and the conditions for the initiation of sedimentation are presented.

3.1 *Forms of Bed Roughness in Alluvial Channels*

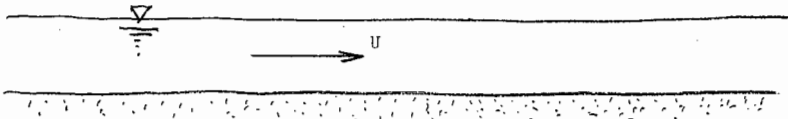
The interface between a moving fluid and an erodible bed is inherently unstable over a wide range of flow conditions, and an initially flat bed will spontaneously develop surface irregularities. For a constant depth of flow, the sequence of bed features with increasing velocity is ripples, dunes, flat bed, and antidunes. These are illustrated schematically in Fig. 4. The primary distinction between ripples and dunes is size, dunes being longer and higher than ripples by a factor of five or more. These two types of features can exist simultaneously, ripples being superimposed on the upstream slopes of the dunes. The flat bed configuration generally occurs at Froude number between about 0.5 and 0.8; however, for some sand sizes and flow depths the flat bed does not occur and instead, features that are not of any one well defined type occur between dunes and antidunes. Antidunes generally occur at Froude numbers greater than about 0.8. At Froude numbers greater than about 1.0, the stationary surface waves that form above and in phase with antidunes become so high that they are



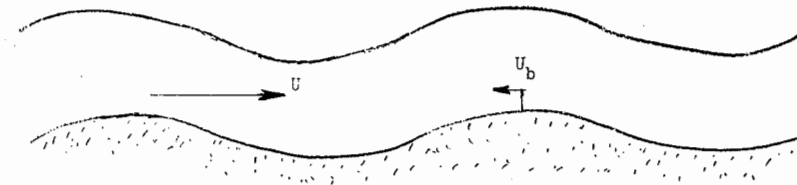
a) Ripples. Bed features move downstream, and water surface is nearly flat.



b) Dunes. The dunes move downstream, and the water surface is slightly wavy.



c) Flat bed.



d) Antidunes. Antidunes move upstream, and the water surface is wavy with an amplitude greater than the antidune amplitude.

Figure 4. Major Forms of Bed Roughness in Alluvial Channels.

unstable and break in the upstream direction, resembling the breaking of ocean waves as they approach a shore. The accompanying agitation levels the bed. Antidunes then reform and grow until the accompanying waves break, the whole process being repeated with a period of one to several minutes.

The mechanism responsible for the formation of these different types of bed features is not well understood. Various theories have been put forth to explain their occurrence, but none of the theories is without some major flaw, and the problem cannot be considered as wholly solved. Furthermore, there does not exist at the present time any reliable method for predicting what type of bed configuration will accompany a given flow, or what the length and height of the bed features will be. The lack of an adequate understanding and formulation of the mechanics of dunes and antidunes is without question one of the greatest impediments to further progress in fluvial hydraulics.

3.2 *Initiation of Sediment Motion*

In the design of stable channels, it is frequently necessary to know the upper limit of velocity or bed shear stress below which the sediment particles on the bed will not move. This information is also required as a parameter in several sediment transport formulas.

For the case of a flat bed, Shields (2) deduced from physical and dimensional arguments based on the forces involved that incipient motion is primarily dependent on the shear stress acting on the bed, and that the dimensionless quantities involved are

$$\frac{\tau_c}{\rho_f g (S_s - 1) D}$$

and

$$\frac{\sqrt{\tau_c / \rho_f} D}{\nu} = \frac{U_* D}{\nu}$$

where

τ_c = bed shear stress for incipient motion

ρ_f = fluid density

g = gravitational constant

S_s = specific gravity of sediment

D = grain diameter

ν = kinematic viscosity of fluid

$U_* = \sqrt{\tau_c / \rho_f}$ = shear velocity at incipient motion.

From experimental data he obtained the relation shown in Fig. 5. For sufficiently large values of $U \cdot D/\nu$ (greater than about 10),

$$\frac{\tau_c}{\rho_f g (S_s - 1) D} = \text{constant} = 0.06.$$

For quartz ($S_s = 2.65$) particles in water ($\rho_f g = 62.4 \text{ lb/ft}^3$),

$$\tau_c = 0.020 D$$

if τ_c is in lbs. per sq. ft. and D is in mm.

Shields curve (Fig. 5) is applicable only to a flat bed. Actually a flat bed at low velocities is seldom encountered in earth canals because if the flow has even been great enough to move the bed particles, ripples or dunes will have been generated and will remain after the velocity has decreased. The problem of ascertaining the critical tractive force or velocity for incipient motion on a dune-covered or rippled bed is far more complicated than for a flat bed because the problem involves the complex geometry and flow conditions at the bed. It has

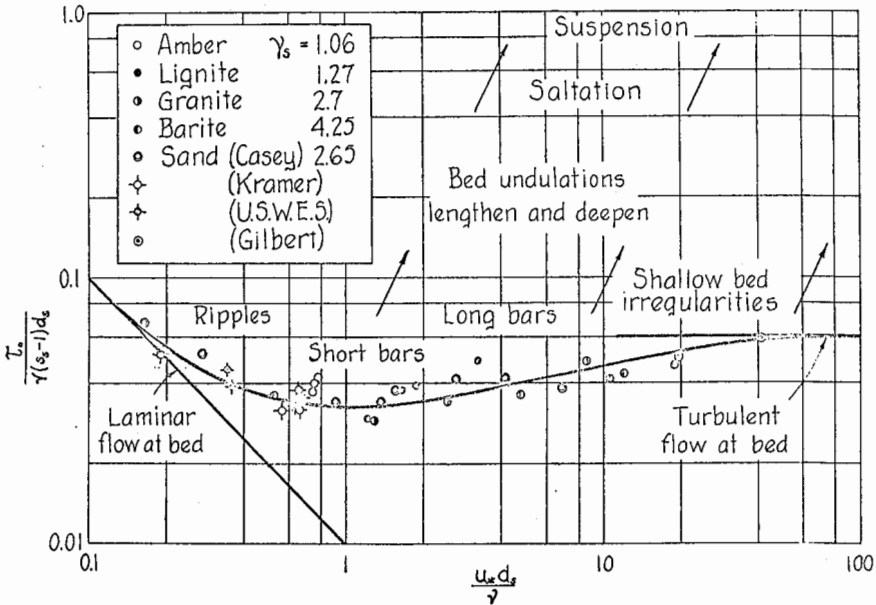


Fig. 5 Shields' curve giving the conditions for incipient sediment motion on a flat bed. (Ref. 2)

been found experimentally (3) that on a rippled or dune-covered bed, incipient motion occurs at a lower velocity and slightly higher bed shear stress, τ_c , than on a flat bed.

4. ROUGHNESS OF ALLUVIAL CHANNELS

The friction factor for flow in rigid boundary conduits can be predicted quite accurately because the roughness of the boundary is constant for a given material and can be determined once and for all time. This happy situation does not exist for the case of erodible-bed channels in which the bed configuration, and hence the roughness, varies radically, as discussed in Section 3.1 and illustrated in Fig. 4. In fluvial hydraulics the problem that arises in predicting channel roughness is that the channel roughness depends strongly on the bed configuration which in turn depends on the depth and velocity of flow (and also the properties of the bed material and fluid). However, the depth and velocity of flow are a function of the channel roughness. Thus there is no fixed starting point from which to proceed to calculate the rate of energy dissipation for the stream. An additional difficulty is that the relation between the flow conditions and the bed configuration, and the relation between the bed geometry and channel roughness have not yet been adequately formulated.

An example of the variability of alluvial channel roughness is shown in Fig. 6 (4), which shows the variations of the bed friction factor,¹ f_b , slope, s , and bed shear velocity, U^*_b , with depth and velocity. These data were obtained in laboratory experiments with a constant discharge of 0.50 cfs per foot. It is seen that for these experiments the bed friction factor is the most variable quantity involved, varying by a factor of almost six, while the slope varies by a factor of less than five, and the depth and velocity by a factor of about 2.5. Furthermore, it is seen that for a given slope (e.g., $s = 0.0021$) and discharge, it is possible to have two different depth-velocity combinations; a small depth, high velocity and flat bed, or a large depth, low velocity and dune bed. Many natural alluvial channels also have this non-uniqueness of depth and velocity as a function of slope and discharge (5,6).

The question that thus arises is: How does a stream with a given discharge and slope, in the range of values where the non-uniqueness occurs, know which depth and velocity to assume? It is observed in

¹ The subscript "b" indicates that the quantity has been corrected for the effects of the smooth, vertical walls of the laboratory flume.

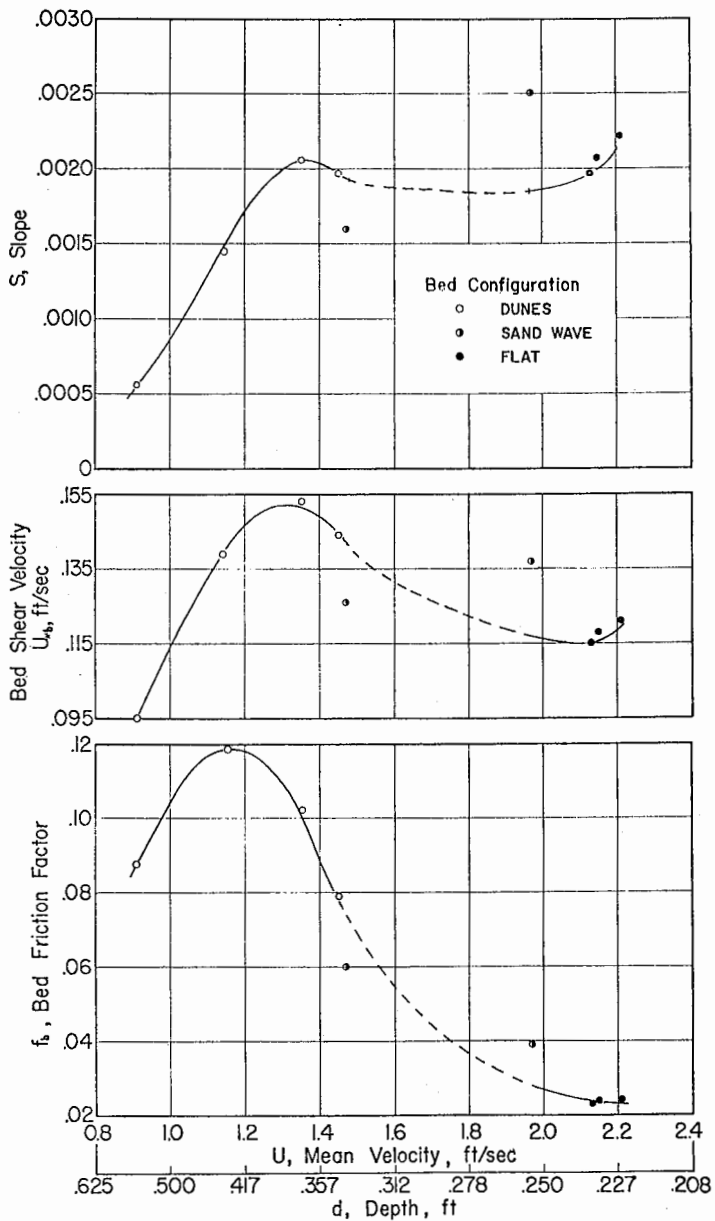


Fig. 6 Variation of slope, bed shear velocity, and bed friction factor with mean velocity and depth for constant discharge (0.50 cfs/ft) experiments in a laboratory flume. (Ref. 4)

both laboratory experiments and in natural streams that the deciding factor is the sediment transport rate; the flow with the smaller depth and higher velocity has a greater capacity to transport sediment. Therefore, when a stream has a relatively large sediment load imposed on it, the bed will be flat, and the velocity will be relatively high.

The above discussion indicates that the roughness of alluvial channels and thus the friction factor for flow in such channels is intimately connected to the form of the bed configuration. As was discussed in Section 3, the problem of predicting the bed form is by no means solved, and therefore, neither is the problem of predicting the roughness of alluvial channels. Several methods have been proposed for predicting the friction factor for flow in alluvial channels. A good summary of available methods has been given by Simons and Richardson (7). Most of these utilize the regime concept of flow in alluvial channels, which attempts to predict the properties of flow in alluvial channels from data on many other channels that have performed satisfactorily.

The method utilizing analysis to some extent that has gained the widest acceptance is that of Einstein and Barbarossa (8). These investigators assumed that the force exerted by the fluid on the bed can be divided into two additive parts: one due to the grain roughness (i.e., the "skin friction" exerted on the bed), and one resulting from the form drag on the ripples and dunes. They further divided the hydraulic radius into two parts

$$R = R' + R''$$

where R is the hydraulic radius of the section, R' is the hydraulic radius of the equivalent area A' for which the flow energy is dissipated by skin friction, and R'' is the hydraulic radius for the equivalent area A'' for which the flow energy is dissipated by form drag on the bed irregularities. The effect of grain roughness can be computed from either the Manning-Strickler equation,

$$\frac{U}{\sqrt{g R' s}} = 7.66 \left(\frac{R'}{D_{65}} \right)^{1/6}$$

or the logarithmic equation

$$\frac{U}{\sqrt{g R' s}} = 5.75 \log_{10} \left(12.2 \frac{R'}{D_{65}} \right)$$

where U is the mean velocity, D_{65} is the sand size such that 65% of the bed material is finer, and s is the channel slope. The effect of bed irregularities is obtained from a graphical relation, obtained from data on natural streams, between the dimensionless quantities

$$\frac{U}{\sqrt{g R'' s}}$$

and

$$\psi' = 1.68 \frac{D_{85}}{R'S}$$

Einstein and Barbarossa's empirical relation between these quantities is shown in Fig. 7. Using the Einstein-Barbarossa procedure to obtain the depth of flow for a given discharge in a channel of known slope,

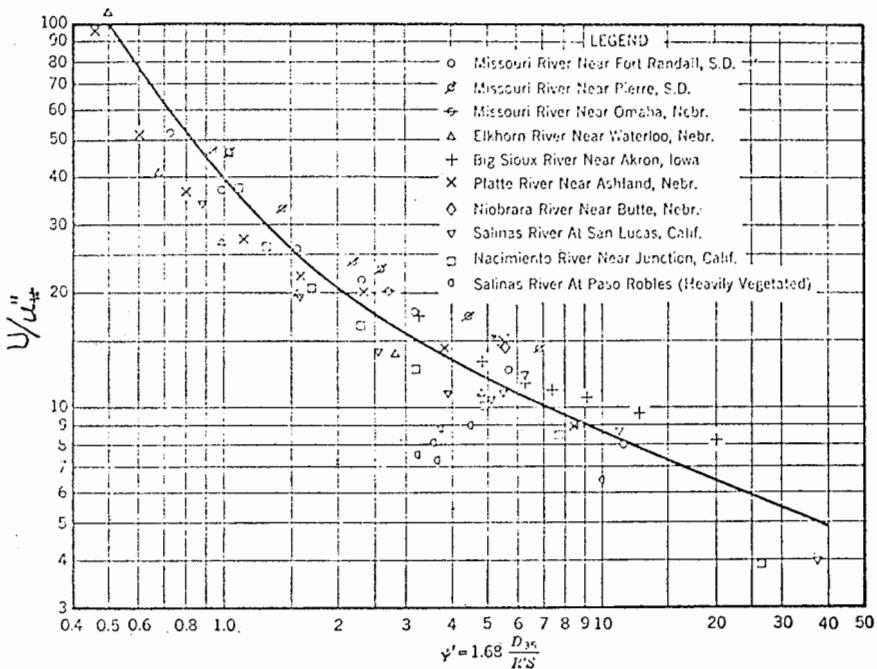


Fig. 7 Bar resistance curve for the Einstein-Barbarossa method of predicting alluvial-channel roughness. (Ref. 8)

geometry and bed material involves straightforward, but rather tedious, trial and error procedure that is described by Einstein and Barbarossa (8). Thus, the Einstein-Barbarossa procedure can be used to develop a stage-discharge relation for an alluvial stream. The parameters involved have been replotted by Vanoni and Brooks (9) in a way that eliminates the need for trial and error solution.

The agreement reported between measured stage-discharge relations and those predicted by the Einstein-Barbarossa method ranges from poor (10) to very good (11). With the possible exception of the regime methods, this is probably the best available method for predicting stage-discharge relations for alluvial streams. Its obvious deficiency is that it does not take account of the effects, discussed in Section 3, of the sediment discharge rate on the bed configuration and channel roughness.

5. SEDIMENT DISCHARGE

If the problem of predicting the roughness of an alluvial channel appears to be a dense forest, predicting the sediment transport rate of an alluvial stream can only be described as a jungle. With the single exception of the suspended local equation, presented below, the present state of knowledge of the mechanisms involved in the entrainment, transportation, and deposition of sediment particles is quite unsatisfactory.

5.1 *Definitions*

The total sediment discharge of a stream is the average quantity of sediment passing a section of the stream per unit time. It is convenient to distinguish two different types of load, as follows:

Suspended load: That part of the total load that is transported at some distance above the bed, supported by the vertical components of the fluid turbulence, and carried along at approximately the forward velocity of fluid.

Bed load: The fraction of the total load that moves along or very close to the bed by rolling, sliding, and saltating.

Note that there is continuous interchange between the bed load and suspended load, and in the region near the bed it may not be possible to say whether the transported material is bed load or suspended load.

Another classification is on the basis of the relative particle size, as follows:

Bed material load: The part of the total load that is composed of particles found in appreciable quantities in the bed.

Wash load: Finer particles, which are not found in appreciable quantities in the bed, and which have a nearly uniform concentration over the depth of the channel.

Finally, there is a distinction based on the limitations of the measuring method employed.² Due to the physical size of samples used for field measurements, it is impossible to measure the bed load, or the suspended load near the bed, without seriously disturbing the bed and hence the transport rate. The load that is sampled by the measuring device used is called the measured load. The difference between the total load and the measured load is called the unmeasured load.

5.2 *Suspended Load*

The suspended load per unit width, q_s , of a stream is

$$q_s = \int_b^a c(y) u(y) dy \quad (1)$$

where

b = small distance above the bed, taken as the dividing line between the bed load and suspended load

d = stream depth

$c(y)$ = sediment concentration at a distance y above the bed

$u(y)$ = horizontal velocity at a distance y above the bed.

The distribution of sediment concentration is given by the theoretically derived equation

$$\frac{c}{c_a} = \left(\frac{a-y}{y} \frac{a}{d-a} \right)^z \quad (2)$$

where

a = distance above the bed at which the sediment concentration is c_a

$$z = \frac{w}{kU_*}$$

w = particle fall velocity

k = Kármán's universal constant

$$U_* = \sqrt{gRS}$$

² A good summary of the methods and instruments used for field measurements of sediment has been prepared by the Federal Inter-Agency Committee on Water Resources (12).

Kármán's constant, k , varies from about 0.2 for heavily sediment-laden flow to 0.4 for clear fluid. Equation (2) is known as the suspended load equation. It was first developed by A. T. Ippen, at the suggestion of Theodor von Kármán, and was first published by Rouse (13). It is based on the equation for continuity of sediment motion, and utilizes the hypothesis that the rate of turbulent mixing of the sediment is proportional to the rate of turbulent momentum exchange within the fluid. Experiments by Vanoni (14) and others have demonstrated that equation (2) predicts the sediment distribution quite accurately. It has also been verified in field measurements by Nordin and Dempster (11), and many others.

The quantity $u(y)$ appearing in equation (1) can be obtained from the Kármán-Prandtl velocity distribution

$$u = u_{\max} + \frac{U_*}{k} \ln y/d \quad (3)$$

where u_{\max} is the maximum velocity in the section (at $y = d$). Equations (2) and (3) may now be substituted into equation (1) for q_s . The resulting integral is not readily evaluated, but tables (15) and a nomograph (16) have been prepared for its evaluation.

5.3 Bed Load

Because no entirely satisfactory formula or method for prediction of bed load discharge has been obtained, there are many formulas and methods in the literature, and new ones are still appearing. In general, there are two different types of equations:

- a) Those of the form³

$$q_b = f[(\tau_o - \tau_c)^n]$$

where

q_b = bed load discharge per unit width

τ_o = shear stress acting on bed

τ_c = critical shear stress for incipient motion

n = number that is dependent of flow parameters and sediment properties.

Duboy's formula and the Meyer-Peter formula are of this type.

- b) Equations based on detailed analyses of the fluid motion (es-

³ f [] is to be read "function of."

pecially the fluid turbulence) and the sediment motion induced by the fluid motion. Examples of these types of analyses are the Einstein bed load function, and the Lane and Kalinske entrainment function.

It is not possible in the allotted space to present, even in summary form, the available bed load formulas. For a presentation and discussion of the different formulas, the reader is referred to the excellent book by Leliavsky (17), the chapter of Rouse's *Engineering Hydraulics* written by Brown (18), and the lecture notes prepared by Vanoni, Brooks, and Kennedy (10).

5.4 Total Sediment Load

The total load is the sum of the bed local and suspended load. Actually, many of the formulas that go by the name "bed load formula" are total load formulas for a limited range of conditions. The difficulty encountered in evaluating the suspended load discharge from equation (1) arises in obtaining c_a in equation (2). Einstein (15) has resolved this by evaluating c_a a distance of two grain diameters above the bed. He presents a rather involved method for obtaining c_a at this level. The Einstein method (15), coupled with the Einstein-Barbarossa procedure for predicting channel roughness, constitutes the only available analytic method for predicting the stage-discharge and sediment transport relations for an alluvial stream. The original Einstein procedure has been simplified by Colby and Hubbell (22).

5.5. Summary and Recommendations

It is not possible to recommend strongly any of the available transport formulas or methods since they all give errors of up to 100% (10). Only the distribution of suspended sediment can be reliably predicted, from equation (2), but this formula requires knowledge of the concentration c_a at some reference level.

The following recommendations for computing sediment discharge rates were put forth by Vanoni, Brooks, and Kennedy (10):

"1. Values of sediment discharge calculated by formulas are to be considered as estimates only since the errors involved may be 100 percent or more.

2. When possible, it is desirable to use more than one formula to calculate sediment discharge. By comparing the results from several formulas one gets a rough idea of their reliability.

3. Calculated sediment discharge rating curves which have slopes on logarithmic graphs which are in the neighborhood of unity are too flat. These curves should not be used, especially in cases where one is interested in the difference between or the ratio of sediment discharges for different flows in the same channel. In such cases the use of sediment discharge rating curves with slopes of between 2 and 3 on logarithmic graphs should give more reliable results."

6. REGIME METHODS

The preceding sections have discussed analytic and semi-analytic methods for determining the various quantities needed to analyze or design alluvial channels. With the possible exception of the suspended load equation, the various methods were observed to be, in most cases, only marginally adequate, and have provided the practicing engineer with few useful tools. Another approach to the problem is to put analysis aside and study the interrelation of the various quantities in existing channels that have been found to perform satisfactorily. The relations between the variables are presented either in graphs, or in equations (usually not dimensionally homogeneous) derived from the graphical relations.

Three relations are usually presented:

- a. A flow formula which gives the required slope.
- b. A relation between channel depth, discharge, and bed material size.
- c. A relation between channel width and the flow parameters and bed material properties.

The regime method had its origin in India among the British engineers who were responsible for building and operating the extensive canal systems that distributed irrigation water. Originally, the method was quite unreliable, but as more data became incorporated into the relations and the formulas were refined, the reliability improved until now the regime method is widely used with considerable success. It is a valuable approach for the engineer who must design alluvial channels.

Probably the best (because it is based on the largest body of accurate data) set of regime relations for canals has been developed by Simons and Albertson (20). An extensive treatment of the regime of natural streams has been prepared by Leopold and Maddock (21). Blench's book (22) summarizes the philosophy and history of the regime method, and also presents a set of regime relations.

REFERENCES

1. U.S. Interagency Committee on Water Resources, Subcommittee on Sedimentation, "Some Fundamentals of Particle Size Analysis," Report No. 12, 1957.
2. SHIELDS, A., "Application of Similarity Principles and Turbulence to Bed-Load Movement," Mitteilungen der Preussischen Versuchsanstalt für Wasserbau und Schiffbau, Berlin, Heft 26, 1936. (Translated by W. P. Ott and J. C. von Uchelen, Soil Conservation Service Cooperative Laboratory, Calif. Inst. of Tech., Pasadena.)
3. KENNEDY, J. F. AND RAICHELN, F., "Critical Tractive Force on a Dune Bed," Tech. Memo 63-1, W. M. Keck Laboratory of Hydraulics and Water Resources, Calif. Inst. of Tech., Pasadena, 1963.
4. KENNEDY, J. F., "Further Laboratory Studies of the Roughness and Suspended Load of Alluvial Streams," W. M. Keck Laboratory of Hydraulics and Water Resources, Report KH-R-3, Pasadena, 1961.
5. DAWDY, D. R., "Depth-Discharge Relations of Alluvial Streams—Discontinuous Rating Curves," U.S. Geological Survey, Water Supply Paper No. 1498-b, 1961.
6. COLBY, B. R., "Discontinuous Rating Curves for Pigeon Roost and Cuffowa Creeks in Northern Mississippi," U.S. Dept. of Agriculture, Agricultural Research Service, Publ. No. ARS41-36, 1960.
7. Progress Report of the Task Force on Friction Factors in Alluvial Channels, Proc. ASCE, Jour. of Hydraulics Division, Vol. 89, No. HY2, pp. 97-143, March 1963.
8. EINSTEIN, H. A., AND BARBAROSSA, N. L., "River Channel Roughness," Trans. ASCE, Vol. 117, pp. 1121-1132, 1952.
9. VANONI, V. A. AND BROOKS, N. H., "Laboratory Experiments on the Roughness and Suspended Load of Alluvial Streams," Report E-68, Sedimentation Laboratory, Cal. Inst. of Tech., Pasadena, Calif. Dec. 1957.
10. VANONI, V. A., BROOKS, N. H. AND KENNEDY, J. F., "Lecture Notes on Sediment Transportation and Channel Stability," Report No. KH-R-1, W. M. Keck Laboratory of Hydraulics and Water Resources, Calif. Inst. of Tech., Jan. 1961.
11. NORDIN, C. F., JR., AND DEMPSTER, G. R., JR., "Vertical Distribution of Velocity and Sediment, Middle Rio Grande, New Mexico," U.S. Geological Survey, Prof. Paper 462-B, 1963.
12. "Federal Inter-Agency Sedimentation Instruments and Reports," Subcommittee on Sedimentation, U.S. Federal Inter-Agency Committee on Water Resources, Report AA, U.S. Govt. Printing Office, May 1959.
13. ROUSE, H., "Modern Conceptions of the Mechanics of Fluid Turbulence," Trans. ASCE, Vol. 102, p. 463, 1937.
14. VANONI, V. A., "Transportation of Suspended Sediment by Water," Trans. ASCE, Vol. 111, pp. 67-133, 1946.
15. EINSTEIN, H. A., "The Bed-Load Function for Sediment Transportation in Open Channel Flows," U.S. Dept. Agric. Tech. Bull. No. 1026, Sept. 1950.
16. BROOKS, N. H., "Calculation of Suspended Load Discharge from Velocity and Concentration Parameters," Proc. Federal Inter-Agency Sedimentation Conference, Jackson, Miss., Jan. 1963 (proc. in press).
17. LELIAVSKY, S., "An Introduction to Fluvial Hydraulics," Constable, London, 1955.
18. ROUSE, H. (Ed.), "Engineering Hydraulics," Wiley, New York, 1950. (See Chapter XII, "Sediment Transportation" by Carl B. Brown.)
19. COLBY, B. R. AND HUBBELL, D. W., "Simplified Methods for Computing Total Sediment Discharge with the Modified Einstein Procedure," U.S. Geological Survey Water Supply Paper, No. 1593, 1961.

20. SIMONS, D. B. AND ALBERTSON, M. L., "Uniform Water Conveyence Channels in Alluvial Material," Proc. ASCE, J. of Hyd. Div., Vol. 86, No. HY5, pp. 33-71, May 1960.
21. LEOPOLD, L. B. AND MADDOCK, T. JR., "The Hydraulic Geometry of Stream Channels and some Physiographic Implications," U.S. Geological Survey Prof. Paper 252, 1953.
22. BLENCH, T., "Regime Behavior of Canals and Rivers," Butterworths Scientific Publications, London, 1957.