

FLOW THROUGH POROUS MEDIA

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1. INTRODUCTION

The flow of a fluid through the complicated pore system geometry that is imposed by the boundaries of many solid particles broadly categorizes the hydraulic process called flow through porous media. This pore system geometry is often considered constant and not subject to change due to consolidation of the soil matrix or swelling of the individual particles themselves. This restriction will be imposed in the subsequent development, thus permitting us to concentrate on the hydraulic processes occurring within this constant pore system geometry. In most cases the civil engineer is concerned with the flow of liquids, hence only incompressible fluids will be considered.

Advances in our knowledge of flow through porous media are being reported in various technical journals (e.g. ASCE Technical Journals, Journal of Fluid Mechanics, Journal of Geophysical Research, Journal of Applied Physics, Journal of Hydraulic Research, Industrial and Engineering Chemistry, Chemical Engineering Science, Economic Geology, A.I.Ch.E. Journal, Chemical Engineering Progress, Proceedings of the Royal Society of London, etc.). Many professions (e.g. civil engineers, hydrologists, petroleum engineers, chemical engineers, soil scientists, nuclear engineers, etc.) are currently involved in research in the general area of flow through porous media. The specific areas of interest range from effects on a molecular scale (1) to considerations of entire ground water basins (2). Our theoretical advances result from an increased understanding of the physical process involved coupled with greater use of mathematics. High speed computational procedures have facilitated the obtaining of otherwise obscure solutions. The increased understanding of the physics of flow through porous media also results, at least in part, from more sophisticated research equipment. The work and advances in all disciplines has bearing on the broad topic of water resources.

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The objective of these notes will be to review briefly the latest thinking concerning the fundamentals of flow through porous media with the emphasis on new areas of interest.

2. BASIC EQUATIONS

2.1 Continuity Equation

The continuity equation or equation for the conservation of the mass of the bulk liquid takes the form

$$\frac{\partial \rho}{\partial t} + u \frac{\partial \rho}{\partial x} + v \frac{\partial \rho}{\partial y} + w \frac{\partial \rho}{\partial z} = 0 \quad (1)$$

where ρ = bulk density of liquid [M/L³] and u, v, w = seepage velocity components [L/T]. The seepage velocity is the average velocity of the liquid in the pores of a small but finite volume (macroscopic volume) which includes both pore spaces and solid particles.

2.2 Conservation of Dispersing Mass Equation

If a miscible contaminant is introduced into the flow of a liquid through porous media we can also write an equation for the conservation of the mass of the contaminant. (The term contaminant is used here in its broadest sense and could represent such things as sodium chloride, radioactive wastes, detergents, microorganisms, etc. In some cases heat could even be considered a contaminant.) For conservative contaminants (i.e. those which do not decay, are not absorbed, or do not engage in a chemical reaction) the conservation of dispersing mass takes the form (3)

$$\frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} + v \frac{\partial s}{\partial y} + w \frac{\partial s}{\partial z} = - \frac{\partial}{\partial x} (\overline{u \dot{s}}) - \frac{\partial}{\partial y} (\overline{v \dot{s}}) - \frac{\partial}{\partial z} (\overline{w \dot{s}}) \quad (2)$$

where s = average concentration of the mass of the contaminant in the macroscopic volume, $\dot{u}, \dot{v}, \dot{w}$ are the variations of the actual liquid velocity at a point from the seepage velocities, u, v, w , and \dot{s} is the variation of the actual concentration of the contaminant at a point from the average concentration, s , in the neighborhood of the point

(or in the macroscopic volume). On a microscopic level the actual velocity components, u_n, v_n, w_n are given by

$$\begin{aligned} u_n &= u + \overset{\circ}{u} \\ v_n &= v + \overset{\circ}{v} \\ w_n &= w + \overset{\circ}{w} \end{aligned} \quad (3)$$

and the actual concentration of the contaminant, s_n , is given by

$$s_n = s + \overset{\circ}{s} \quad (4)$$

The left-hand terms of equation (2) represent the local time rate of change of the average concentration of the contaminant in the macroscopic volume and the mass flux of the contaminant in and out of the macroscopic volume due to convection by the seepage velocities. The right-hand terms represent additional mass flux of the contaminant in and out of the macroscopic volume due to a process which will be called dispersion. The additional mass flux terms result from the macroscopic viewpoint taken in the derivation of equation (2). These dispersion terms account for the additional mass flux of the contaminant due to the complicated velocity pattern in the macroscopic volume and also, within a pore space, to the process of transverse molecular diffusion associated with the parabolic velocity distribution typical of laminar flow. Equation (2) is also valid for turbulent flow through porous media.

If we consider the case of unidirectional flow parallel to the x -axis the convective terms containing v and w vanish. But it is clear that $\overset{\circ}{u}$, $\overset{\circ}{v}$ and $\overset{\circ}{w}$ still exist, hence the additional mass flux terms must be retained. Assuming that the additional mass flux terms can be related to the gradient of the average concentration by a Fickian-type diffusion equation, we have

$$\begin{aligned} \overline{\overset{\circ}{u} s} &= -D_1 \frac{\partial s}{\partial x} \\ \overline{\overset{\circ}{v} s} &= -D_2 \frac{\partial s}{\partial y} \\ \overline{\overset{\circ}{w} s} &= -D_3 \frac{\partial s}{\partial z} \end{aligned} \quad (5)$$

where D_1 is called the longitudinal dispersion coefficient since the flow is in the x -direction and D_2 and D_3 are called lateral or transverse dispersion coefficients. The dispersion coefficients are kinematic quantities having the units of L^2/T . Substituting (5) into (2) gives

$$\frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} = \frac{\partial}{\partial x} \left(D_1 \frac{\partial s}{\partial x} \right) + \frac{\partial}{\partial y} \left(D_2 \frac{\partial s}{\partial y} \right) + \frac{\partial}{\partial z} \left(D_3 \frac{\partial s}{\partial z} \right) \quad (6)$$

Equation (6) is sometimes called the convective-dispersion equation. A discussion of the dispersion coefficient will be found in section 5.

2.3 Equation of Motion

For laminar flow through porous media the Navier-Stokes equations are sufficient within the pore system. But in order to obtain a useful equation of motion for the complicated flow pattern we must resort to a consideration of the macroscopic volume similar to the treatment of dispersion. Assuming that the inertia terms in the Navier-Stokes equations are vanishingly small for the extremely slow motions usually associated with flow through porous media and integrating the remaining normalized terms through the pore system of the macroscopic volume leads to the following equations for the superficial velocity components:

$$\begin{aligned} U &= -C_x \frac{d^2 g}{v} \frac{\partial h}{\partial x} \\ V &= -C_y \frac{d^2 g}{v} \frac{\partial h}{\partial y} \\ W &= -C_z \frac{d^2 g}{v} \frac{\partial h}{\partial z} \end{aligned} \quad (7)$$

The coefficients C_x , C_y , C_z are dependent only upon the geometry of the pore system. For an isotropic medium $C_x = C_y = C_z = C$. The superficial velocity components (U , V , W) represent a fictitious average liquid velocity that would exist in the macroscopic volume if no particles were present. The seepage velocities and superficial velocities are related by the expressions

$$\begin{aligned}U &= \theta u \\V &= \theta v \\W &= \theta w\end{aligned}\tag{8}$$

where θ is the porosity. $d[L]$ is the normalizing length and is equal to the average grain size (e.g. the 50 per cent grain size determined from a standard gravimetric sieve analysis). g is the gravitational acceleration $[L/T^2]$ and $\nu[L^2/T]$ is the kinematic viscosity of the bulk liquid. $h[L]$ is the piezometric head and is given by

$$h = \frac{p}{\gamma} + y\tag{9}$$

where $p =$ liquid pressure $[F/L^2]$ and $\gamma [F/L^3]$ is the specific weight of the bulk liquid. Gravity has been assumed here to act in the negative y -direction.

Equations (7) can be written as

$$\begin{aligned}U &= -\frac{k_x g}{\nu} \frac{\partial h}{\partial x} \\V &= -\frac{k_y g}{\nu} \frac{\partial h}{\partial y} \\W &= -\frac{k_z g}{\nu} \frac{\partial h}{\partial z}\end{aligned}\tag{10}$$

where $k_i (k_i = C_i d^2, i = x, y, z)$ is called the intrinsic permeability and is dependent upon only the geometry of the pore system. It has the units of length squared. The Darcy permeability, K , is given by $K_i = \frac{k_i g}{\nu}$ and has the units of velocity. The Darcy permeability is

seen to be dependent upon the pore system geometry and the liquid that is flowing.

Equations (7) and (10) are valid for laminar flow through porous media when the inertia terms in the Navier-Stokes equations are negligible. This regime of flow is referred to as the linear laminar regime because the velocities are linearly related to the piezometric gradient. The parameter which generally describes the relative magnitude of the inertial forces to viscous forces is the Reynolds number. For porous media flow this can be written using the seepage velocity

and the average particle size as the characteristic length, $Re = \frac{ud}{\nu}$. As the Reynolds number approaches unity the inertia forces approach the same order of magnitude as the viscous forces and hence can no longer be neglected in the equation of motion. Although this type of flow may still be laminar, equations (7) and (10) are no longer valid. This regime of laminar flow which is characterized by a non-linear relationship between the seepage velocity and the piezometric gradient is called the non-linear laminar regime. At some higher Reynolds number ($Re \approx 60$) (4) inception of turbulence occurs and as the velocity increases the flow will become fully turbulent.

For an isotropic porous medium $k_x = k_y = k_z = k$ and from equations (10) we see that the superficial velocity components are derivable from

a potential function, $\phi = -\frac{kg}{\nu} h$, (i.e. $U = \frac{\partial\phi}{\partial x}$, $V = \frac{\partial\phi}{\partial y}$

$W = \frac{\partial\phi}{\partial z}$) The substitution of ϕ into the condition for an incom-

pressible fluid ($\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0$) yields Laplace's equation

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0 \quad (11)$$

Two-dimensional problems and problems with radial symmetry have been the focus of most attention. For these two-dimensional problems we may also define a stream function, ψ , such that

$$U = \frac{\partial\psi}{\partial y}$$

$$V = -\frac{\partial\psi}{\partial x} \quad (12)$$

The existence of a velocity potential implies irrotationality. If the stream function is substituted into the condition for irrotational flow for two-dimensional flow ($\frac{\partial V}{\partial x} = \frac{\partial U}{\partial y}$) we see that the stream function also satisfies Laplace's equation. The properties of these two

mutual orthogonal families of curves ($\psi = \text{constant}$ and $\phi = \text{constant}$) are utilized in the construction of flow nets.

3. BOUNDARY CONDITIONS

If we have a bounded region within which Laplace's equation is valid everywhere and if the conditions on the boundaries of the region are specified, then a unique solution exists for ϕ and ψ . Commonly encountered boundary conditions will be reviewed briefly (5).

3.1 Impermeable Boundary

Along an impermeable boundary such as line ED in Fig. 1, $\frac{\partial \phi}{\partial N} = 0$ and $\psi = \text{constant}$.

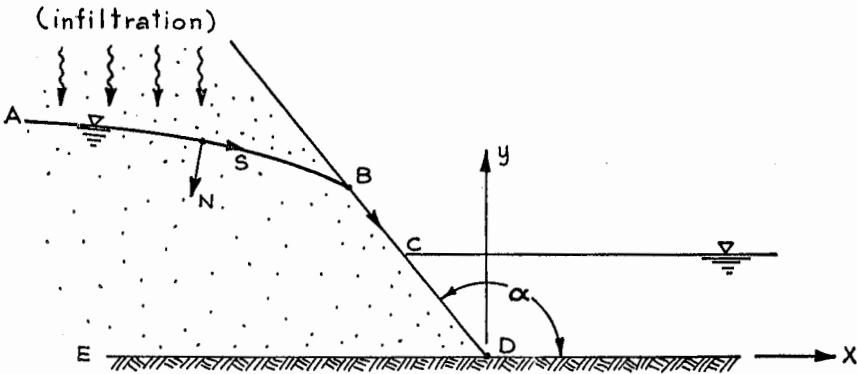


FIG. 1 DIAGRAM ILLUSTRATING BOUNDARY CONDITIONS

3.2 Seepage Surface

A surface of seepage (line BC in Fig. 1) is a boundary of the medium itself and also the flow region along which the flow is exposed to atmospheric conditions. If the atmospheric pressure is taken as zero, then along this surface $\phi + Ky = 0$. If α is the angle which the surface of seepage makes with the x-axis, then taking the derivative of $\phi + Ky = 0$ with respect to S, the direction of flow, gives

$$\frac{\partial \phi}{\partial x} \cos \alpha + \frac{\partial \phi}{\partial y} \sin \alpha = -K \sin \alpha$$

where $\cos \alpha$ and $\sin \alpha$ have been substituted for $\frac{dx}{dS}$ and $\frac{dy}{dS}$ respec-

tively. For vertical surfaces of seepage, $\alpha = \pi/2$, and $\frac{\partial\phi}{\partial y} = -K$, (i.e. the velocity is vertical and is equal to $-K$).

3.3 Phreatic Surface

The phreatic surface, water table, or free surface (line AB in Fig. 1) is a boundary of the flow region which is still contained in the porous medium but considered exposed to atmospheric conditions. The basic difference between the surface of seepage and the phreatic line is that generally the surface of seepage is known and its geometry can be stated, but the position of the phreatic line and its shape are not known a priori. The condition $\phi + Ky = 0$ is also valid along the phreatic line. If there is no infiltration or evaporation, the phreatic line may be considered a streamline (i.e., $\psi = \text{constant}$). If water is added or lost at a constant rate along the phreatic line, then $\psi = ex + \text{constant}$ along the phreatic line where e is the volume of water added or lost per unit length along the phreatic line per unit time.

3.4 Reservoir Boundaries

When boundaries of the flow region are reservoirs of water (line CD in Fig. 1), the condition $\phi = \text{constant}$ pertains.

3.5 Dispersion Problems

Boundary conditions for the contaminant must also be established for dispersion problems. Generally we can state that no mass flux of the contaminant occurs across impermeable boundaries or the phreatic surface (i.e., $\frac{\partial s}{\partial N} = 0$). The distribution of the concentration of the contaminant is sometimes known and constant along other boundaries. If it is not constant, its time variation must be known. One important exception to the condition of no mass flux across the phreatic line would be the addition of a contaminant to the flow system through infiltration due to rainfall (e.g., radioactive fallout).

4. METHODS OF SOLUTION

Many groundwater flow problems do not require consideration of the dispersion of some contaminant. In some problems where two liquids of different density are involved, approximate solutions may be obtained on the assumption that the liquids are immiscible (i.e. no

mixing at the interface between the two liquids). The method of solution to these problems where dispersion is absent requires only the determination of ϕ and ψ . An excellent treatment of many such problems is found in the recently translated text by Polubarinova-Kochina (6). For the problems where the distribution of some contaminant is desired, $s(x,y)$ must also be determined. The methods of solution can be generally classified as (a) analytical, (b) numerical, (c) graphical, (d) models and analogs and (e) approximations.

4.1 Analytical Solutions

Exact analytical solutions for ϕ and ψ usually require simple geometries to the flow boundaries. The most common techniques for exact solutions are the use of conformal mapping including the hodograph plane and separation of variables. Transforms, such as the Fourier transform, are useful for infinite or semi-infinite regions. Superposition of separate solutions is also valid since Laplace's equation is linear in ϕ and ψ . The treatment of the conformal mapping technique is beyond the scope of these notes and the reader is referred to (5), (6) and (7).

4.2 Numerical Solutions

Numerical solutions are feasible using finite difference representation for the differential equations and high speed computational procedures. The boundary conditions must be specified.

Laplace's equation for the stream function, ψ , becomes in finite difference representation

$$\psi_0 = \frac{1}{4} (\psi_1 + \psi_2 + \psi_3 + \psi_4) \quad (13)$$

where ψ_i ($i = 1, 2, 3, 4$) represent the value of ψ at the four lattice points surrounding the interior node point with a value of ψ_0 . The precision of the numerical procedure depends on the fineness of the lattice grid. Special difference equations similar to equation (13) must be written for nodes having unequal arms to the surrounding lattice points.

Consider the flow under a sheetpile (see Fig. 2). The flow picture is symmetrical about the sheetpile. Line BC is a streamline and may be given the value of $\psi = 0$. Line ED is a streamline and may arbitrarily be given the value of $\psi = 1$ for ease in computation. Line AE is a continuation of ED and also has the value of $\psi = 1$. It happens that the location of AE does not significantly affect the flow picture near the

sheetpile as long as it is reasonably far from the sheetpile. Lines AB and CD are lines of constant potential and hence the streamlines must be perpendicular to them at intersection. This requirement induces symmetry of the ψ values about these potential lines and permits writing the difference equation at the nodes along AB and CD in terms of interior lattice points only. For the problem shown in Fig. 2 there are 36 nodes and hence 36 difference equations. Electronic digital computers are capable of solving this set of 36 simultaneous algebraic

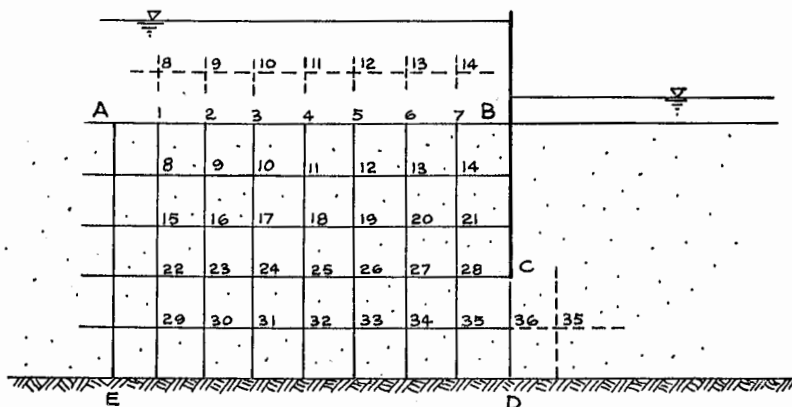


FIG. 2 FLOW UNDER A SHEETPILE WITH LATTICE GRID SUPERIMPOSED FOR NUMERICAL SOLUTION

equations in seconds. Once the values of ψ at the node points are known, the streamlines may be sketched in. The accuracy of the procedure can be increased by superimposing a finer grid over all or only a portion of the flow.

4.3 Flow Nets

Sketching of flow nets is still a very useful tool. Theoretically there is only one correct net for a given problem (e.g. the one shown in Fig. 2). The experience of the sketcher determines how closely this exact net is approached.

4.4 Models and Analogs

Models and analogs are particularly useful in research. The sand model and Hele Shaw model are widely used (8), (9). The electric analogy and membrane analogy are other types of models (10).

4.5 Approximations

The most widely used approximation is the Dupuit-Forcheimer approximation. It is especially useful when there is a free surface (i.e., unconfined flow). The Dupuit-Forcheimer approximation permits reformulation of the problem so as to eliminate the free surface boundary which is not known a priori. The basic assumption used in the formulation of the Dupuit-Forcheimer equation is that the variation of the piezometric head with y is negligible. This assumption requires that vertical velocity components also be negligible which implies small gradients of piezometric head. The final equation, including infiltration due to rainfall ($e > 0$) or evaporation ($e < 0$) is

$$\frac{\partial^2(h^2)}{\partial x^2} + \frac{\partial^2(h^2)}{\partial y^2} + \frac{\partial^2(h^2)}{\partial z^2} + e = \frac{2\theta}{K} \frac{\partial h}{\partial t} \quad (14)$$

Provisions for leakage through the base of the aquifer can also be included. This non-linear partial differential equation can be linearized by introduction of a new variable that represents small deviations of the piezometric head from some average value, \bar{h} , typical to the problem under consideration. This linearized equation takes the form

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} + \frac{e}{2\bar{h}} = \frac{\theta}{K\bar{h}} \frac{\partial h}{\partial t} \quad (15)$$

There is no need to resort to equation (15) for steady state problems since equation (14) is linear in h^2 in those cases.

5. DISPERSION COEFFICIENTS

For the simple case of unidirectional flow parallel to the x -axis through an isotropic homogeneous porous medium, we need only consider two dispersion coefficients (11), D_1 and D_2 . Equation (6) becomes for this case

$$\begin{aligned} \frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} = \frac{\partial}{\partial x} \left(D_1 \frac{\partial s}{\partial x} \right) \\ + \frac{\partial}{\partial y} \left(D_2 \frac{\partial s}{\partial y} \right) + \frac{\partial}{\partial z} \left(D_2 \frac{\partial s}{\partial z} \right) \end{aligned} \quad (16)$$

We see that even for this simple case we must retain D_1 and D_2 inside the derivative. We intuitively suspect that D_1 and D_2 are dependent

upon the geometry of the pore system which is constant in this case since the medium was specified isotropic and homogeneous. The other important variable that may still vary is the seepage velocity, u . From experiments (12), it is found that the dispersion coefficients are dependent upon the absolute magnitude of the seepage velocity. This dependency can be written in equation form as (13), (14)

$$\begin{aligned}\frac{D_1}{v} &= a_1 \text{IR}_d^{n_1} \\ \frac{D_2}{v} &= a_2 \text{IR}_d^{n_2}\end{aligned}\tag{17}$$

where a_1 and a_2 are coefficients that are primarily dependent upon particle shape and possibly size distribution. For nearly uniform media, experiments have shown that a_1 and a_2 are independent of particle size. IR_d is called a grain size Reynolds number and is given by

$$\text{IR}_d = \frac{|u|d}{v}\tag{18}$$

The exponents n_1 and n_2 are primarily dependent upon particle size distribution. Experimental evidence confirms that n_1 and n_2 are independent of particle shape and size.

The ratio of the longitudinal and lateral dispersion coefficients should be of the form

$$\frac{D_1}{D_2} = a \text{IR}_d^n\tag{19}$$

where $a = a_1/a_2$ and $n = n_1 - n_2$. In accordance with the above discussion, a and n are functions only of the media characteristics. For nearly uniform porous media composed of spheres (13), (14), $a_1 = 0.66$, $a_2 = 0.037$, $n_1 = 1.2$ and $n_2 = 0.7$. For nearly uniform angular particles $a_1 = 0.90$ and $n_1 = 1.2$. Preliminary evidence indicates that n_1 may approach unity for highly non-uniform media.

The longitudinal dispersion coefficient has been correlated with the intrinsic permeability (13). This correlation is expressed as

$$\frac{D_1}{v} = b \text{IR}_k^{n_1}\tag{20}$$

where b is a dimensionless coefficient dependent upon only the particle shape and size distribution. The exponent n_1 is primarily a function of

size distribution. For nearly uniform media composed of spheres, $b = 54$ and $n_1 = 1.2$. For nearly uniform media composed of angular particles $b = 88$ and $n_1 = 1.2$. R_k is called the permeability Reynolds number and is formed using the square root of the intrinsic permeability as the characteristic length. The correlation expressed in equation (20) provides a method of estimating the dispersion parameter from a knowledge of the more easily measured permeability.

6. DISPERSION WITH ADSORPTION

Some phenomena involving the transport of a contaminant through porous media are subject to dispersion and adsorption. The adsorption process involves the transfer of the contaminant between the liquid and solid phases (i.e., the contaminant may be absorbed from the liquid by the solid particles, or the liquid may be acquiring the contaminant from the solid phase). To account for this adsorption process, an additional term must be included in the conservation of dispersing mass equation (6). This becomes (15)

$$\begin{aligned} \frac{\partial s}{\partial t} + \left(\frac{1-\theta}{\theta} \right) \frac{\partial F}{\partial t} + u \frac{\partial s}{\partial x} = \frac{\partial}{\partial x} \left(D_1 \frac{\partial s}{\partial x} \right) \\ + \frac{\partial}{\partial y} \left(D_2 \frac{\partial s}{\partial y} \right) + \frac{\partial}{\partial z} \left(D_3 \frac{\partial s}{\partial z} \right) \end{aligned} \quad (21)$$

where F is the concentration of the contaminant in the solid phase. The relationship between F and s depends on the exact mechanism of interphase transfer. One such relationship is

$$F = C_1 s + C_2 \quad (22)$$

which gives

$$\frac{\partial F}{\partial t} = C_1 \frac{\partial s}{\partial t} \quad (23)$$

Equation (23) states that the rate of change of the concentration in the solid phase is proportional to the rate of change of the concentration in the liquid phase. This implies that the adsorption process takes place instantaneously when changes in the local concentration, s , of the contaminant occur. This is often referred to as the equilibrium case.

Assuming the above relationship between F and s , equation (21) becomes

$$\varepsilon \frac{\partial s}{\partial t} + u \frac{\partial s}{\partial x} = \frac{\partial}{\partial x} \left(D_1 \frac{\partial s}{\partial x} \right) + \frac{\partial}{\partial y} \left(D_2 \frac{\partial s}{\partial y} \right) + \frac{\partial}{\partial z} \left(D_3 \frac{\partial s}{\partial z} \right) \quad (24)$$

where

$$\varepsilon = 1 + \frac{C_1(1 - \theta)}{\theta}.$$

An expression such as equation (21) which includes the adsorption-desorption term would be applicable to problems such as reclamation of salt encrusted soils by fresh water flushing (15) and removal of some contaminant (e.g., detergents) by passage of the contaminated solution through a porous bed of surface active particles (16).

7. EXPANDED MEDIA

The flow of a liquid around the grains of a porous medium exerts a seepage force on the particles. For sufficiently high vertical seepage velocities the individual particles will be supported by these upward forces. This condition is referred to as an expanded or fluidized bed. It has applications in many chemical processes, hydraulic gradation of sands, back-washing of filter beds, sediment entrainment, etc.

The analysis of the total resistance to flow by an expanded bed is complicated because of the interference of one particle on the flow pattern of another. This interference produces unsteady effects resulting in violent random movements of the particles within the constant limits of the expanded bed.

Consider the upward flow of water through a column containing a porous medium. At some critical seepage velocity the drag force on a typical particle will equal the submerged weight of the particle and a state of incipient expansion of the bed results. A further increase in the seepage velocity will cause the particles to expand, thus reducing the seepage velocity (due to the increase in porosity) and a quasi-steady state obtains wherein the limits of the expanded bed remain constant but the individual particles are subject to violent random fluctuations within the expanded bed limits. Statistically the average drag force on each particle must equal the submerged weight of the particle in this quasi-steady condition. Utilizing the drag equation and conservation of

energy principle, an expression for the superficial velocity can be derived for the expanded bed,

$$V = \sqrt{\frac{4}{3(1-\theta)} \frac{gd}{C_D} \frac{\partial h}{\partial y}} \tag{25}$$

where C_D is the drag coefficient appropriate for this condition.

Experimental results (17) on uniform spheres are plotted in Fig. 3 as C_D versus IR , where $IR = \frac{Vd}{\nu}$.

IR_i is the Reynolds number which defines the state of incipient expansion. IR_s is the extrapolation of the experimental curve for the expanded porous bed to intersection with the well-known curve for one sphere. Significantly, IR_s agrees with the Reynolds number that would be given using the settling velocity of a single sphere. The Reynolds numbers

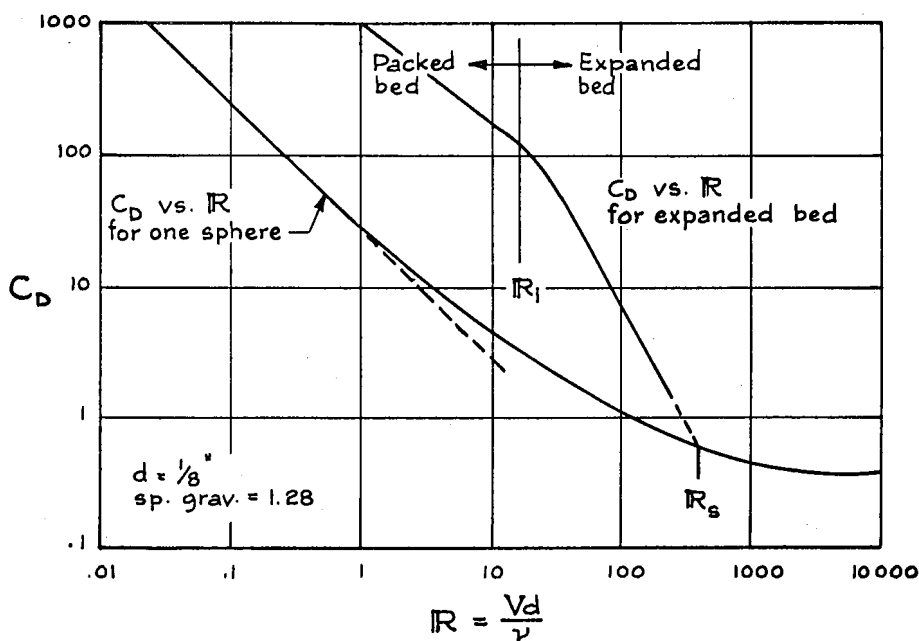


FIG.3 C_D VS. IR FOR A SINGLE SPHERE AND A PACKED AND EXPANDED BED OF SPHERES

between IR_i and IR_s give the velocity at which the whole bed would settle were the flow to be abruptly stopped.

8. PRESENT STUDIES

Present studies in the area of water resources include salt water intrusion and dispersion in coastal aquifers, mechanics of flow in the vicinity of recharge wells, spread of contaminants from recharge wells, clogging phenomenon, seepage from surface reservoirs, underground disposal of radio-active wastes, adsorption associated with leaching of salt encrusted soils, and removal of detergents by flow through activated carbon.

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