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JOHN M. BIGGS
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# A BACKSIGHT 

## Presidential Address by Leslie J. Hooper

(Presented at Annual Meeting of Boston Society of Civil Engineers, March 23, 1966)
A young engineer fresh from school frequently feels that engineering has only started to develop at his graduation; that the boundaries of scientific and engineering knowledge only extend back for 100 years or so. Reference to an article of that vintage, however, shows that there already existed a very good foundation for that particular development and references are made to a previous century. This process cannot be continued indefinitely since, unfortunately, history of an engineering sort is usually fragmentary. There are no glorious military leaders nor victories to be remembered. The developments and improvements have been unspectacular. However, it might be interesting to take a glimpse into the past to get some idea of the basis of our present achievements.

If much that is referred to in the following discussion is of a hydraulic nature, it is a fact that water is a prime essential of life. It might be added that the author's mind at a very early date turned to water.

One engineer, when pressed to establish the ancient antecedents of his profession quoted Genesis $1: 3-31$ to support his claim. There it states that the heavens, the earth and all that is therein were created in a six-day working week. This, the engineer contended, was an engineering work of the first magnitude.

The development of New England depended in large part on water power. Many of our large towns are located at suitable sites for the development of power that was adequate in those days for the operation of grist mills, saw mills, and later cotton and woolen mills. Mr. Arthur Safford, a past president of this Society, wrote an excellent paper in 1922 on the development of the Francis turbine in New England. He carried this history back to approximately 1840 in a very detailed manner. Before that, as he pointed out, there existed tub wheels, flutter wheels, undershot wheels, overshot wheels and many other types that have since passed from the scene.

The most efficient of these early designs was the overshot wheel. It was brought to this country by the settlers from Europe. There is a record that a tidal mill was established in 1630 at Neponset. There was such a mill at Riverdale in Gloucester that was operating as a grist mill in 1910. The development of these overshot wheels is not easy to trace.

One excellent source is Mr. and Mrs. Herbert Hoover's translation of Agricola's "De Re Metallica." This is a description of mining techniques as of 1550 in Southern Germany. Here we find the overshot wheel frequently used for pumping and for hoisting. In the latter case, two overshot wheels were used side by side but operating in opposite directions so that the load could be lifted or dropped as necessary.

The only notice of an overshot wheel previous to this is by Friederich Klemm where he shows a cut of 16 wheels in series at Barbigal near Arles in France, with a date of approximately 250 A.D. From the cut, it is obvious that these were overshot wheels. Vitruvius, writing at the time of Augustus Caesar, speaks only of undershot wheels with flat blades, so far as can be determined from the translation. The undershot wheels were used from time immemorial in places where the current of the river was sufficiently rapid to drive the wheel of pots or similar devices.

It is well established that ancient civilizations started in arid areas. Apparently in such a place it was possible for a small number of people to kill off the wild animals and get the protection that they needed from other tribes and their surroundings. Naturally, in such a location they needed drinking water first for themselves and their flocks and later for irrigation. It is natural to suppose that methods of lifting water took a high priority in the development of the hydraulic art. The power required to lift water was largely supplied by animals and slaves until the end of the Roman Empire. For this reason, pumps probably have an older history that water wheels.

Our ubiquitous centrifugal pump is truly a recent development. Merriman indicated they were of very little utility before 1840. Ewbank indicated that Demour in France developed the first pump utilizing centrifugal action. This was a very crude device essentially to show only the fact that water could be lifted by such a means. Naturally the requirement of high speed of operation did not lend itself to the older forms of power available. Oddly enough, Hero's reaction turbine never seems to have been driven as a centrifugal pump.

Practically all of the older forms of lifting water depended on some manner of direct lifting. Agricola shows the chain of pots frequently used in deep mines and the Archimedean screw for low lifts. He also shows the piston force pump used occasionally. Inasmuch as the latter could only be placed at a limited distance above the water level, it was not largely used in mines where the water power of the time would be on the surface and the water to be pumped would be at a depth.

Vitruvius refers to the same two "engines" for lifting water in his Ten Books of Architecture. Inasmuch as he would be reporting on the commonly used methods of raising water at the time, this indicated that the two designs were commonly used at that time, one for high lifts and the other for low lifts.

The chain of pots is not specifically mentioned in Herodotus at the time of the Hanging Gardens of Babylon, but all references to that particular wonder of the ancient world speak of an engine located at the top of the gardens which lifted the water directly in a single lift. Since this lift was of the order of 300 feet, it is felt that a chain of pots was undoubtedly used in the Hanging Gardens for irrigation purposes. It is not known how much before that time ( 600 B.C.), that they might have been used.

The force pump consisting of two single acting pistons delivering into a common air chamber to eliminate pulsations of flow was mentioned by Agricola, and again it is mentioned by Ctesibius in Egypt at approximately 300 B.C. This particular development presents a very interesting puzzle. The design is essentially the same as we see in the 1850's as used in our fire engines. Yet it was shown completely developed in this form by Ctesibius as stated. The question as to how they would use a pump of this sort is puzzling.

One might think that the strength of available pipes might limit their use. Vitrified clay pipes were available for low pressure use and lead pipes for high pressure. Vitruvius and Frontinus give us the standards of lead pipe at approximately the time of Christ. It was quite customary to build such pipe with a wall thickness of $5 / 16^{\prime \prime}$ and interior diameters ranging from $1^{\prime \prime}$ up to $27^{\prime \prime}$. The larger size would only handle about 40 ft . head before bursting. However, the $1^{\prime \prime}$ pipe would carry something of the order of 760 ft . head which was quite adequate for most of the applications that might be considered in that age.

The force pump and the bellows are completely similar in construction and in operation. The primary requirement in this design is related to the valves which must be of simple construction but pressure tight. As to the construction of bellows, we find that Jeremiah refers to them. Further, Homer, in the Iliad describing the forge of Vulcan, indicated that a double action bellows with valves was being used at a very early date. Professor Smith at Massachusetts Institute of Technology, who is interested in the history of metallurgy, feels that all of the common metals were being reduced from their common ores possibly as early as 5000 B.C. He bases his supposition on the fact that when
the necessary heat can be raised in a fire by blowing, to reduce the ores of tin and copper, that an adequate technology exists to reduce many of the common ores or iron also. This particular problem probably lies with the prehistory of the Hittites who seem to have first made iron available. But in any case, our oldest description of the force pump comes from Ctesibius who was known to be a barber's son. It is interesting to contemplate in this case whether he was the developer of the force pump in one stroke of genius or whether he was reporting developments of an even earlier age by others.

The one type of lifting device that has not been mentioned is the wheel of pots which in Asia Minor is called the Persian wheel, in Egypt, the Egyptian wheel and in China the noria. The noria was probably developed to its highest degree by the Chinese. With them, it was constructed of bamboo throughout and with no iron whatsoever, which is an indication of its ancient origin. The axle and the pedestal were made of solid wood, this being the only concession to heavier loading. It consisted of a wheel with a number of pots tied to the circumference at an angle so that they would fill at the bottom of the travel when immersed in the river. They would not lose any substantial amount of water until they had reached the top of their travel. In some designs the pots were pivoted and were tipped by a suitable bar at the top of the travel so as to reduce spillage to a minimum. Ewbank, in describing this construction indicated that the Chinese excelled in construction of large wheels which were necessary to achieve the highest lifts in a single stage. In his second edition (1842) he indicated that the Chinese frequently built these wheels to a diameter of 90 feet which "was far in excess of the highest building in New York at the present time." Times have certainly changed rapidly in the last century with us.

Sometimes these norias were propelled by the treading of men or beasts but when the current was swift, a number of flat paddles would be mounted on the periphery of the wheel and the kinetic energy of the river would be used to lift the water to the required height. This was particularly true of the Persian wheels, found in the vicinity of Hamath, which apparently had a swift flowing stream adapted to such construction.

In China, at some ancient date, they also constructed windmills that operated in a horizontal plane instead of the vertical plane that we associate with the Dutch construction. These windmills were used to drive the equivalent of a screw or noria for lifting water. It is regret-
table that very little evidence is available in our American libraries to indicate the high stages of development of the hydraulic and engineering arts in China and, for that matter, in India. Civilization in both of these countries is old and there must be a wealth of information that is still awaiting revelation.

Turning to other civil engineering works, a hydraulic engineer naturally thinks of dams. Hathaway presented an interesting article on several ancient dams. He pointed out that so far as we know the most ancient dam of any size was constructed by the Egyptians about 18 miles south of Cairo and called the Sadd el-Kafara. This particular dam was probably constructed about 2800 B.C. It was 348 feet long at the top and had a crest height of 40 feet above the stream bed. The structure was made up of two separate rubble masonry dams each around 80 feet thick at the base with a space of 120 feet along the stream bed between them. This space was filled with random material from the stream bed and the adjacent hillsides. It appears that the dam was constructed to provide drinking water for the workmen at a nearby quarry. Although the Nile valley is subject to cloudbursts, no spillway was provided in the construction of this dam. Since there is no sign of sediment deposit in the reservoir area, it is felt that the dam was overtopped and failed soon after completion. In any case, there is no further evidence of any such dam being built in Egypt for more than 1000 years.

The most outstanding dam of antiquity was located approximately 40 miles from the ancient city of Marib in the kingdom of Yemen. This may well have been the kingdom over which the Queen of Sheba ruled around 950 B.C. as mentioned in the Old Testament. This dam, along with many other smaller dams, was the basis of the agricultural prosperity of this part of the country. This dam was constructed of large stone walls of huge stones very well fitted together. They also used headers to provide ties between the two layers of the stone. No mortar was used in filling the joints of the dam, but it was used as a covering on top of the dam to prevent damage from rain water. There is no evidence that the dam was provided with a spillway. However, because of the heavy type of construction, it is very possible that this dam would withstand overtopping in modest amounts and only in the case of a severe flood would it be breached. History indicates that while it was constructed approximately 1000 B.C. it had to be rebuilt in 449 A.D. In 450 A.D. it was breached again and a complete renovation was required. In 542 A.D. another breach of the dam occurred. Apparently the dam
was not repaired and the prosperity of the kingdom of Sheba or as it is more recently called, Saba, disappeared and the region reverted to desert. Naturally, it is referred to in the Encyclopedia of Islam as a punishment of God since the people turned away from Him.

Returning to Egyptian accomplishments, their greatest hydraulic work was the construction of Lake Moeris at approximately 2300 B.C. This reservoir was located in the province of Fayoum where there is a natural depression below the level of the Nile. It became a reservoir by being connected to the Nile by a canal through a low lying ridge of rock. This canal was constructed 300 feet wide and 10 miles long. The intake to this canal was at a point opposite the island of Nome where the Nile divided into two channels. Sir William Willcocks testifies that there is no indication that the Egyptians had any conception of control gates but rather used as a barrage of earth to fill the entrance of the canal when it was desired to close off the Nile from the lake. This was a work of considerable magnitude and expense and it is obvious from the records that have come down to us that even a Pharoah of Egypt was reluctant to undertake this work unless there was good reason for doing so. It is quite probable, according to Willcocks, that the use of Lake Moeris diminished and finally had to be abandoned when the channel on the western side of the island of Nome became silted up and therefore the lake was no longer effective in controlling the flood flows of the Nile.

The only other hydraulic work of note among the Egyptians was the construction of the canal from the Nile to the Red Sea. Herodotus tells us that this canal left the Nile near Memphis and was connected to the Red Sea at the point where the Suez Canal now joins the Red Sea. Herodotus states that he saw the remains of the Red Sea Canal during his own visit to Egypt (circa 450 B.C.). However, in our day, there is no trace of the existence of this canal, undoubtedly due to the fact that the desert quickly reclaims anything which is not continually maintained.

In Mesopotamia, however, canal building was an ancient art and they produced works of the first magnitude. The people of Mesopotamia said the original canals were constructed by the gods themselves which is only an indication that canal construction went back beyond the mists of oral tradition. An outstanding canal in Mesopotamia was the Nahrwan Canal, which diverted the waters of the Tigris near Opis, forcing that river to irrigate a large area eighteen to twenty-five miles
wide between the old bed of the Tigris and the Eastern hills. Another of the old and tremendous canals was the Pallocopas or Hindia, to use the older name. Both of these canals were of the order of 400 feet wide and 15 feet deep. Alexander sailed his seagoing fleet from the Persian Gulf into the Pallocopas after he had broken down some barrages to allow their passage. These canals went out of use and were lost after Tamerlane devastated the country of Mesopotamia so that there was no one to provide maintenance. Before that time, the country maintained agriculturally a population of over 12 million people, and at the present time, the population is about 5 million.

In the matter of aqueduct building, the Romans have left the most enduring monuments which are well known and well described in many references such as the Pont du Gard and the Claudian aqueduct; the former because of its beauty and size and the Claudian because of its length, which altogether was 62 miles. The reason for the building of aqueducts across the valleys was to shorten the path that the water had to follow to the city so as to make the water available at a higher elevation.

Frontinus indicates that the normal flow of the Claudian was about 11,700 GPM or 26 cfs . The total flow of all 9 aqueducts supplying Rome was estimated to be 58,000 GPM or 130 cfs . The water carrying cross-section of the Claudian was 3.3 ft . wide and 6.6 ft . high.

As compared to this, the first aqueduct of which any remains have been found was constructed by Sennacherib in 691 B.C. to bring drinking water from the mountains to his capitol city of Nineveh. At one spot he had to cross over a river valley and he built the Jehrwan Aqueduct for this purpose. The remains indicate that this aqueduct carried a channel 50 feet wide and 5 feet deep over a length of 920 feet across the valley. This was certainly a stupendous work as compared to the Roman aqueducts.

The Romans, Greeks and Persians are known to have done a great deal of tunnel work and mining. Before the advent of metal which could be hardened sufficiently to penetrate rock, there were only two methods of cutting into rock. In mining where the rock face was not to be preserved, it was customary to build a fire against the working face and then dash water or vinegar onto the heated rock in order to crack or spall the surface rock. Pliny states that vinegar was more effective in cracking the rock than water, and its use is indicated by Vitruvius as being the most desirable. With the poor ventilation that existed in mines
of those days, the use of fires and vinegar in the mines must have resulted in conditions that were hardly conducive to long life of miners.
${ }^{-}$When rock was being quarried, on the other hand, it was necessary to maintain a good working face and the Egyptians were among the foremost in this type of work. Their approach for cutting out a block of stone, a lintel or an obelisk for instance, consisted of digging a trench all around the block that was to be freed from the matrix. This was done by slaves pounding with balls of diorite, which was the hardest stone available at the time. This could smash or powder the limestone or granite that was being quarried. It was estimated by one archeologist, who has tried the process, that a large block of stone such as an obelisk, could be quarried in about eight or nine months by a suitable number of operators disposed all the way around the rock that was being freed.

Probably one of the outstanding examples of rock excavation that was performed by the Egyptians was the construction of Joseph's well near Cairo. In this case, the water bearing layer is 295 feet below grade. This was reached by two shafts with a small reservoir or basin excavated in the rock at the junction point between the two. The upper shaft was $24 \times 18$ feet in plan and 165 feet deep. Below the basin, the lower shaft was $15 \times 9$ feet in plan and continued another 130 feet deeper. Inasmuch as the water was lifted by two chains of pots, it was necessary to get oxen or donkeys down to the level of the basin to operate the lower chain of pots. Access was provided by a passageway $6^{\prime} 4^{\prime \prime}$ wide and $7^{\prime} 2^{\prime \prime}$ high, that was excavated in solid rock on the outside of the $24 \times 18$ foot shaft. This passageway was laid on a uniform grade all the way to the intermediate basin. In order to provide light to the passageway, openings were cut through the wall into the shaft itself. An examination of these openings shows that the original builders maintained a uniform thickness of wall of only 6 inches between the passage and the shaft. Although this well is ascribed to Joseph who lived approximately 1700 B.C., it could be more ancient than this due to the habit of people of that time ascribing memorable works to an outstanding public figure.

All through antiquity it is evident that the mining experts did not hesitate to sink wells, or construct tunnels as necessary. Dean Finch has indicated the example of the tunnel on the island of Samos, which is 3300 feet long, cut on a grade of fairly uniform slope, but also along a curved center line in plan. Due to the imperfect civil engineering techniques available in those days, it was customary to sink vertical shafts
to the location of the tunnel and then tunnel between these shafts. This allowed frequent checks of the alignment of the tunnel and kept the errors of alignment from getting too large. There are evidences in the ancient descriptions of tunnels that started from two ends that missed each other by very considerable amounts (Lanciani's "Ancient Rome") so that this precaution was deemed very necessary:

This method of tunnelling has been used for centuries in Islamic countries for intercepting water at a distance in the mountains and bringing it to the cities for a water supply. Butler, speaking of the water supply for the city of Teheran in 1933, comments that the entire water supply for the city at that time, which had a population of 275,000 was obtained from 36 tunnels which were from 8 to 16 miles in length and in places over 500 feet below the surface of the ground.

The space in such a paper is much too short to go into many areas of ancient accomplishments in engineering but there are several little vistas or anecdotes that are intriguing.

In the first place, our medical friends refer to a tradition of Hippocrates, but the most ancient authentic manuscript referring to his works that can be found is dated approximately 900 A.D. In engineering, we are most fortunate in having an engineering handbook of Vitruvius, a Roman architect-engineer, who wrote at the time of Caesar Augustus at approximately the time of Christ. In his Ten Books of Architecture, he speaks on city planning, the construction of walls, foundations, temples and buildings in general, the finding of water, the construction of engines of war and he even finds time to give advice or warning concerning the use of models. In regard to models, he remarks that "not all things are practicable on identical principles," or to the general effect that a model is a most unsafe thing to believe if you do not know the principles of its operation.

Similarly, we are fortunate in having the English translation of the report of "Frontinus on the Water Supply of Rome," when he became the water commissioner. As you all know, this work was essentially done by Clemens Herschel, who provided the translation into engineering English after others had provided the translation into French, German or English from the original Latin. Both of these experts have been quoted previously in this presentation.

It is fortunate indeed, for English speaking engineers that there are some competent people who can not only translate from the original language of a manuscript but also translate into engineering English of
our day. By way of example, Breasted gave a literal translation into English of the story of Creation according to the Babylonian epic. It was a description of a combat between gods which carried no particular implications to an American engineer. Sir William Willcocks was an English irrigation engineer who was brought up and spent his engineering life in Arabic speaking countries, understanding the idiom as well as the language. He translated this same epic into engineering English; he pointed out that in the combat, the god of the river was ensnared in a net which was created by the hero god Marduk. The magic net which was used to ensnare the river was the canal system of Mesopotamia, which made civilization in the lower reaches of the Tigris-Euphrates basin possible.

Similarly, in Exodus 15:25, Moses made the bitter waters of Marah sweet by laying a reed therein. Willcocks points out that this is an Oriental method of referring to the common manner of building low dams in water. A base of brush was covered with reeds and then overlaid with mud to make a watertight and safe dam. Moses, operating as an engineer, constructed a low dam to keep out the brackish backwater and allow the sweet water from the spring to be used for drinking purposes. So a miracle becomes an everyday engineering work and loses its glamor to the ordinary man.

Thus, it is also seen how necessary it is for an English speaking engineer to try to find a description of ancient works that have been translated not only into English but also engineering terms so that the comprehension of what was originally intended can be conveyed.

In review it is interesting to contemplate the position of the engineer-architect through the ages. In Egypt, inscriptions and monuments have been found that show that the engineer-architect was frequently of the nobility, highly educated, close to the king and greatly honored for his ability. However, in Mesopotamia, the works of creation were always ascribed to the gods. This may indicate that the priest-kings of that era may have also had a considerable engineering knowledge. No record exists that bears on this point. Later, in Persia and in Greece, we find that the engineer as such was a craftsman and almost always a slave. The Greeks greatly admired philosophy and the arts that did not soil the hands, and even Archimedes, who was a Greek of Syracuse, writing at approximately 300 B.C., disdained to mention any of the applications of the various works with which he is credited.

During the Middle Ages, our written history seldom records the names of engineers even though they were no longer slaves.

Thus, we see that today the lot of the engineer is gradually improving. He does receive more public recognition than was the case in the past. However, it has been the tradition of the engineer to serve, and it still continues. Few of us chose this profession with the idea of great worldly acclaim or gain. The most we can hope for is the recognition of our peers and the satisfaction of a work soundly conceived and well executed.

## References

1. Safford, Arthur T. and Hamilton, Edward Pierce. "The American Mixed Flow Turbine and Its Setting," trans. A.S.C.E. Vol. LXXXV 1922, pp. 1237-1356.
2. Hooyer, Herbert Clark and Lou Henry. "De Re Metallica," Dover Publications, 1950, New York City.
3. Rawlinson, George. "History of Herodotus" translation, edited by Manual Komroff 1939, Tudor Publishing Co., New York City.
4. Herschel, Clemens. "Frontinus and the Water Supply of the City of Rome, A.D. 97," Dana Estes and Co., 1899, Boston.
5. Ewbank, Thomas. "Hydraulic and Other Machines for Raising Water," D. Appleton and Co., 1842, New York City.
6. Morgan, Morris Hicky. Translation of Vitruvius, "The Ten Books on Architecture," 1926 Harvard University Press, Cambridge, Mass.
7. Winccocks, Sir William. "From the Garden of Eden to the Crossing of the Jordan," E. and F. N. Spon, 1920, London.
8. Singen, Dorothea Waley. Translation of Klemm, Friederich, "A History of Western Technology," Charles Schribner's Sons, 1959, New York City.
9. Fincte, G. K. "The Engineer Through the Ages," Civil Engineering, February 1957March 1958.
10. Hathaway, Gail. "Dams-Their Effect on Some Ancient Civilizations" Civil Engineering, January 1958, p. 58.
11. Butler, Millard A. "Irrigation in Persia by Kanats," Civil Engineering, February 1933, p. 69.
12. Gandolfo, J. H. "Tracing the Engineer in Early Egypt and Assyria," Civil Engineering, April 1931, p. 632.
13. Lane, E. W. "Ingenuity of the Ancient Chinese," Civil Engineering, October 1930, p. 17.

# GEOLOGY OF THE MALDEN TUNNEL, MASSACHUSETTS 

By Marland P. Billings* and David A. Raym**

The Malden Tunnel, trending approximately N. $8^{\circ}$ E. in the vicinity of Malden Square, is slightly less than a mile long and lies about 275 feet below mean low sea level. It is the southernmost section of the Spot Pond Brook Flood Control Project. The internal diameter of the concrete lining is $121 / 2$ feet.

The entire tunnel is in bedrock. The northern part is in various kinds of felsites belonging to the Lynn Volcanics, whereas the southern part is in the argillite of the Cambridge Formation. Dikes and, to a lesser extent, sills of mafic igneous rock (diabase and altered diabase) constitute a small percentage of the rock.

Most of the Cambridge Argillite here strikes northeast and dips southeast, as it is on the north limb of the Charles River syncline. But the northernmost 400 feet of the Cambridge Argillite dips northwest because it is overturned. The structure of the Lynn Volcanics is less clear, but available evidence indicates it is gently dipping.

The tunnel was of special interest because it crosses the Northern Boundary Fault of the Boston Basin, about which there has been considerable speculation for many decades. The Lynn Volcanics have been thrust southward over the Cambridge Argillite. In the tunnel the thrust fault dips $55^{\circ} \mathrm{N}$., but data from a bore hole indicates that below the tunnel the dip is nearly horizontal.

The various types of fractures (joints, faults, and shears), as well as dikes, are diversely oriented.

Fifty-two per cent of the tunnel was supported by structural steel, practically all of it in rocks directly north or south of the Northern Boundary Fault. Moreover, the flow of water into the tunnel was excessive. Both of these facts are due to the unusually large amount of fracturing of the rocks because of the fault.

## Introduction

## Location, Size, and Construction

The Malden Tunnel is part of the Spot Pond Brook Flood Control Project in Stoneham, Melrose, and Malden in eastern Massachusetts (Fig. 1). The building of this project is under the supervision of the Construction Division, Metropolitan District Commission. Section

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Fig. 1.-Location Map
Spot Pond Brook Flood Control Project consists of three parts: Part A, Malden Tunnel, completed; Part B, trenches and culverts, completed; Part C, trenches, culverts, and tunnels, in planning stage.

Dotted line between Malden Tunnel and City Tunnel Extension is presumed trace, at tunnel level, of a cross-bedded siltstone of use in correlating stratigraphy in two tunnels.

C, still in the planning stage, will probably consist of open trenches, culverts, and tunnels. Section B, already in operation, consists of open trenches and culverts. Section A is the Malden Tunnel, already in operation. The tunnel trends N. $8^{\circ} 17^{\prime} 52^{\prime \prime}$ E. This tunnel is 5265.7 feet long. The invert (bottom of the tunnel) ranges in altitude from minus 280 feet at the southern end to minus 270 feet at the northern end; altitudes are relative to Boston City Base, that is, mean low sea level. The land surface ranges from six feet to 57 feet above base. The internal diameter of the concrete lining is 12.5 feet. Two vertical shafts go from the surface to the tunnel level. The southerly shaft, known as Shaft A or Malden River Shaft, is on the Malden River 2400 feet southwest of Malden Square. The northerly shaft, known as Shaft B or Winter Street Shaft, is 3400 feet north-northwest of Malden Square.

The tunnel is entirely in bedrock. The total thickness of the cover, bedrock and unconsolidated material, ranges from 280 to 330 feet. Of this, 28.5 to 211.0 feet is unconsolidated material. The bedrock cover ranges in thickness from a minimuum of 75 feet to a maximum of 249 feet.

The contractor was the Coker Construction Company of Des Moines, Iowa. The work was done in 1957 and 1958. Shaft A was the construction shaft.

## Geological Investigations

Although several trips were made to the tunnel while it was being driven, detailed mapping, deferred until the walls were washed down, occupied four days in February, 1958. The mapping was done on a scale of one inch to 20 feet. The drafting of a folio on this scale, consisting of a map and section of the west wall, was completed in September, 1961. Copies of this folio have been deposited with the Metropolitan District Commission and the Department of Geological Sciences of Harvard University.

In the summer of 1956 the senior author, at the request of F . Lyle Tierney, then geologist for the Construction Division, Metropoli$\tan$ District Commission, examined the samples of bedrock obtained in the boring program. The logs made at that time have proved of value in the present study.

## Geological Setting

The most extensive account of the geology of the Boston area is by LaForge (1932). Recent papers based on data obtained from tunnels are by Rahm (1962) and Billings and Tierney (1964).

The rocks in the Boston area are of two kinds, the relatively young unconsolidated surficial deposits and the much older bedrock (LaForge, 1932, Pl. I and II).

The surficial deposits in the Boston area range in thickness from zero, where bedrock is exposed, to hundreds of feet. They consist largely of glacial deposits such as till, gravel, sand, silt, and clay, as well as peat, river deposits, and beach sands. These unconsolidated rocks are Quaternary and their age does not exceed a few tens of thousands of years.

The bedrock in the Boston area belongs to three groups: (1) "crystalline rocks," (2) Boston Bay group, and (3) dike rocks.

The "crystalline rocks," ranging in age from Precambian to Middle Paleozoic, consist of a complex series of metamorphic, granitic, and volcanic rocks. Further detailed description is here unnecessary, because none of this series, except the Lynn Volcanics, is involved in the Malden Tunnel.

The Boston Bay Group, of Upper Paleozoic age, has traditionally been divided into a lower unit, the Roxbury Conglomerate, and an upper unit, the Cambridge Argillite. However, a recent paper (Billings and Tierney, 1964, p. 148-149) has shown that the Roxbury and Cambridge are sedimentary facies of each other. Only the Cambridge Argillite is involved in the Malden Tunnel.

Mafic dike rocks, many of them diabase or altered diabase, cut the older rocks. Some are considered to be as young as Triassic.

For many decades it has been presumed that the Boston Basin is bounded on the north by a major fault separating the Cambridge Argillite on the south from the "crystalline rocks" to the north. Billings (1929) and LaForge (1932) show this fault as a thrust dipping $45^{\circ}$ north.

The major rock units shown in Fig. 8, which covers only part of the Boston Basin, are the Roxbury Conglomerate in the southwest corner, the Cambridge Argillite in the central portion, and the "crystalline rocks" in the northern part.

## Presentation of Data

The basic data are presented here in a geological map, a structure section, tables, and so-called point diagrams.

Fig. 2 is the geological map of the tunnel on a scale of 1 inch $=210$ feet. Since this map has been reduced 10 times from the original folio, much generalization has been necessary and only repre-

Fig. 2.-Geological Map of Malden Tunnel
Top of each strip corresponds to west wall of tunnel. Bottom of each strip lies 40 feet east of the west wall, that is, 27
feet east of the east wall. Elevations of invert refer to Boston City Base. Numbers below strips refer to stratigraphic units
in Table I.
sentative data are shown. Moreover, all the data on joints has been omitted to avoid crowding. Each strip represents 1200 linear feet, the top strip being the most southerly section of the tunnel, the bottom strip being the most northerly. The stations are given above each strip ( $12+00$, for example, means 1200 feet N. $8^{\circ} 17^{\prime} 52^{\prime \prime}$ E. of station $0+00$; Shaft A is at station $2+00$ ). The altitude of the invert of the tunnel is shown in the lower left-hand corner of each strip. Altitudes are referred to the Boston City Base, which is 5.65 feet below the U.S. Geological Survey base. The upper (west) side of each strip shows the geology exposed at breast level on the west side of the tunnel. On the scale employed, the east side of the tunnel would be only $0.06^{\prime \prime}$ below the top of the strip, as the tunnel was only 12.5 feet in diameter. To make a more readable map the base of each strip is shown 40 feet east of the west wall, that is about 27 feet east of the east wall.

Fig. 3 represents the geological structure along the line of the tunnel. It indicates the geology as it would appear on the west wall of a trench looking west, somewhat over 400 feet deep. Although considerable generalization and interpretation has been necessary, the section shows the thickness of the overburden (Quaternary), as well as the structure of the bedrock. Of course, the only "facts" are those observed in the tunnel, bore holes, and shafts. Shears and gouges, shown on the maps, have been omitted. Any upward or downward projection of these features would be largely conjectural. Moreover, an attempt to show them on an earlier version of Fig. 3 confused rather than clarified the diagram.

## Lithology

## General Statement

The superficial deposits along the line of the tunnel range in thickness from a minimum of 28.5 feet at bore hole 6 B to a maximum of 211.0 feet at bore holes 4 and 5 (Fig. 3). The samples of the surficial deposits obtained from the borings were not examined by the writers, but were studied by F. Lyle Tierney. Inasmuch as the present paper is concerned only with the bedrock geology, the surficial deposits are not considered further.

The bedrock geology along the tunnel line belongs to three units: the Lynn Volcanics in the northern half of the area, the Cambridge

 $50+00 \quad$ Shatt $B=55+00$ | $50+00$ | $\begin{array}{l}\text { Shatt } B= \\ 1\end{array}$ |
| :---: | :---: |
| B.H. 6 G |  |

Argillite in the southern half, and mafic igneous rocks (diabase and altered diabase) occurring as occasional dikes and sills throughout the tunnel. Fifty-five per cent of the tunnel was in the felsites of the Lynn Volcanics, 37 per cent was in the argillite of the Cambridge Formation, and 8 per cent was in the diabase in the dikes and sills.

## Lynn Volcanics

The Lynn Volcanics occupy all of the tunnel line north of station $24+56$ (the Northern Boundary Fault), except for a few mafic dikes. In this area the Lynn Volcanics are composed entirely of felsite. Felsite is a general term used for fine-grained igneous rocks rich in potash feldspar and sodic plagioclase, with or without quartz, and low in dark minerals. Quartz-rich varieties are called rhyolite, quartzpoor varieties are called trachyte. Along the tunnel line the felsites are light-green-gray, gray, pink to red, and, in a few instances, white. For descriptive purposes four varieties of felsite are shown in Figs. 2 and 3. Although transitions are common, and exact assignment is not always possible, these varieties and the approximate percentages are as follows: dense felsite ( $52 \%$ ), porphyritic felsite ( $10 \%$ ), felsitic tuff-breccia ( $38 \%$ ) and bedded felsitic tuff (trace).

The dense felsites are flinty, brittle rocks. The most common colors are light-green-gray, gray, and pink to red. Many tens of linear feet in the tunnel were of one color. But the change from one color to another was generally transitional in a zone not over a few feet wide. In places the dense felsite was mottled red and green-gray. Flow structure, consisting of alternating bands of somewhat different color a fraction of an inch thick, was noted in four places. The felsite is cut by numerous joints, as described below, but in many places it breaks along irregular hackly fractures between the joints.

Porphyritic felsite is the principal rock between stations $43+57$ and $46+80$. It is similar to the dense felsite, but contains phenocrysts of quartz and feldspar. Few of the former exceed 1 mm in diameter, but the latter range from 2 to 10 mm , in length.

The felsitic tuff-breccias are dense to fine-grained brittle gray, pink, or red pyroclastic rocks. The angular fragments, generally $1 / 4$ to 1 inch across, but in places as much as 3 inches, are set in a dense groundmass. The fragments, although in many cases differing slightly in color from the groundmass, are dense to fine-grained felsite. Where the fragments are the same color and texture as the groundmass, it
is difficult to distinguish felsitic tuff-breccia from dense felsite. Just like the dense felsites, many of the tuff-breccias break along hackly fractures between the joints.

Bedded felsitic tuff was exposed around station $42+64$. It is light-green-gray, well-bedded, 3 feet thick, strikes N. $25^{\circ}$ E. and dips $20^{\circ}$ NW. It is underlain by gray felsite and overlain by tuff-breccia.

The Lynn Volcanics here are composed of lava flows and pyroclastic rocks. Since the dip of the bedding along the tunnel line is very gentle, the known thickness would not exceed a few hundred feet (Fig. 3) here.

A description of the Lynn Volcanics as a whole is given by LaForge (1932, pp. 30-33). He gives no thickness, but it may well be several thousand feet. LaForge (1932, p. 29) correlates the Lynn Volcanics with the Newbury Volcanics, which are Upper Silurian or Lower Devonian (Toulmin, 1964, p. 17).

## Cambridge Argillite

The Cambridge Argillite occupies all of the tunnel, except for a few dikes and sills of mafic rocks, south of the Northern Boundary Fault at station $24+56$.

Along the tunnel line the formation consists entirely of argillite. These are gray well-indurated siltstones. The beds, generally conspicuous and ranging in thickness from paper thin to 2 inches, are due to slight differences in grain size or color, or both. The colors are all in shades of gray. A very striking rock, composing about half the formation, is cyclically layered, consisting of alternating lighter and darker gray beds, generally from 1 to 2 inches thick.

In other parts of the formation the siltstone is a more uniform gray, but slight differences in texture or color or both show beds $1 / 32$ to $1 / 16$ inch thick. In a few instances beds as much as 5 feet thick show no internal bedding. Between stations $10+21$ and $10+36$ the beds, $1 / 32$ to $1 / 4$ inch thick, pinch and swell. Between stations $4+96$ and $5+26$ beds $1 / 4$ inch thick show minute cross-bedding, indicating that the beds were derived from the south. As will be pointed out below, this unit is a value in correlating the strata in the Malden Tunnel with those in the City Tunnel Extension.

Petrographic and chemical studies (Rahm, 1962, p. 332) show that the argillites are composed chiefly of quartz, sericite, and chlorite, with small amounts of albite, and, locally, calcite and pyrite.

The rock is well jointed, but between joints the rock breaks with a hackly fracture.

A columnar section of the Cambridge Argillite exposed in the tunnel and Shaft A is given in Table I. Ten units have been established,

TABLE I
Stratigraphic Column, Cambridge Formation
Malden Tunnel

|  | Stations | Lithology | Unit thickness | Cumulative thickness |
| :---: | :---: | :---: | :---: | :---: |
|  | Shaft A | Argillite. Gray. Thin laminae. | 159.0 | 1210.1 |
|  | $\begin{aligned} & 2+00 \text { to } \\ & 4+96 \end{aligned}$ | Argillite. Gray. Laminae paper thin to $1 / 16^{\prime \prime}$ thick; but in many places bedding is obscure. Hackly fractures in places. | 89.1 | 1051.1 |
|  | $\begin{aligned} & 4+96 \text { to } \\ & 5+26 \end{aligned}$ | Argillite. Gray. Laminae paper thin to $1 / 32^{\prime \prime}$ thick. Some beds as much as $5^{\prime \prime}$ thick have obscure laminae. Minute cross-bedding, indicating source to south. | 19.2 | 962.0 |
|  | $\begin{aligned} & 5+26 \text { to } \\ & 6+21 \end{aligned}$ | Argillite. Gray. Bedding obscure. | 66.3 | 942.8 |
|  | $\begin{aligned} & 7+55 \text { to } \\ & 10+21 \end{aligned}$ | Dike <br> Argillite. Gray. Bedding generally conspicuous. Much of rock is cyclically layered, consisting of alternating lighter and darker gray layers $1 / 32^{\prime \prime}$ to $1 / 2^{\prime \prime}$ thick. Locally more uniform gray rock with laminae $1 / 32^{\prime \prime}$ to $1 / 16^{\prime \prime}$ thick. | 104.8 | 876.5 |
|  | $\begin{aligned} & 10+21 \text { to } \\ & 10+36 \end{aligned}$ | Argillite. Gray. Alternating lighter and darker gray bands $1 / 32^{\prime \prime}$ to $1 / 4^{\prime \prime}$ thick; beds pinch and swell somewhat. | 4.0 | 771.7 |
|  | $\begin{aligned} 10+36 \text { to } \\ 13+80 \end{aligned}$ | Argillite. Gray. Bedding generally conspicuous. Much of rock is thinly laminated in beds $1 / 16^{\prime \prime}$ to $1 / 32^{\prime \prime}$ thick. Some is layered, consisting of alternating lighter and darker gray beds $1 / 32^{\prime \prime}$ thick. Pyrite locally present. | 129.3 | 767.7 |
|  | $\begin{aligned} & 13+80 \text { to } \\ & 13+82 \end{aligned}$ | Argillite. Very light gray. | 1.5 | 638.4 |

TABLE I (continued)

| Stations | Lithology | Unit <br> thickness | Cumulative <br> thickness |
| :---: | :--- | :---: | :---: | :---: |
| 2. $13+82$ to |  |  |  |
| $15+68$ | Argillite. Gray. Rather uniform <br> gray, but in places laminae are <br> $1 / 32^{\prime \prime}$ thick. Hackly fracture. | 107.9 | 636.9 |
| $1.15+74$ to | Argillite. Gray. Bedding generally <br> conspicuous. Much of rock is cycli- <br> cally layered; alternating lighter <br> and darker gray beds $1 / 32^{\prime \prime}$ to $2^{\prime \prime}$ <br> thick. More uniformly gray rock <br> shows laminae $1 / 32^{\prime \prime}$ to $1 / 4^{\prime \prime}$ thick. | 529.0 | 529.0 |
|  | Locally splits into plates $2 / 5^{\prime \prime}$ to <br> $2^{\prime \prime}$ thick. |  |  |

but the formation is so relatively uniform throughout that the subdivisions have no great significance. In units 1,4 , and 6 cyclical layering is conspicuous. In units 2, 5, 7, 9, and 10 such cyclical layering is not conspicuous and in many cases bedding is obscure. In unit 5 the bedding pinches and swells, whereas unit 8 shows minute cross-bedding. Unit 3 was established primarily because it made a good break between differing units above and below it.

The total thickness of the Cambridge Argillite that was exposed in the Malden Tunnel was 1051 feet, but including the data from Shaft A the total is 1210 feet.

The thickness of the Cambridge Argillite that was exposed in the City Tunnel Extension was 6759 feet (Billings and Tierney, 1964, p. 111). How are the beds in the Malden Tunnel stratigraphically related to those in the City Tunnel Extension? Shaft A of the Malden Tunnel lies 3348 feet N. $50^{\circ}$ E. of Shaft 9A of the City Tunnel Extension (Fig. 1). Since the strata at both shafts strike northeast and dip southeast we may assume, as a first approximation, that the beds at the foot of the two shafts are approximately correlative. This would mean that the strata in the Malden Tunnel are stratigraphically below those in the City Tunnel Extension.

Several different methods may be used to establish a more precise correlation. Because they all lead to the same general conclusion, only one will be discussed here. This method assumes that the units characterized by minute cross-bedding may be correlated, that is, unit 8
of the MaldenTunnel may be correlated with unit 2 of the north limb of the Charles River syncline in the City Tunnel Extension (dotted line of Fig. 1). This means that the uppermost 353 feet of the Cambridge Formation in the Malden Tunnel correlates with the lowest 353 feet in the City Tunnel Extension. The lowermost 857 feet of the Cambridge Formation in the Malden Tunnel is stratigraphically below the lowest bed in the City Tunnel Exension. Thus the thickness of the Cambridge Argillite is at least $6759+857=7616$ feet. The equivalent of bed "zero," i.e., the lowest bed, of the City Tunnel Extension would be at station $7+90$ in the Malden Tunnel.

The age of the Cambridge Argillite has not been definitely determined. But the Boston Bay Group is younger than the Mattapan Volcanics, which may be correlated with the Lynn Volcanics. Hence the Boston Bay Group is younger than the Upper Silurian or Lower Devonian. Fossil trees (Rahm, 1962, p. 329) in the Roxbury Conglomerate indicate that the age may be Upper Devonian to Permian. Moreover, the Triassic Medford Diabase is younger than the Boston Bay Group.

## Mafic Igneous Rocks

Diabase, including altered diabase, occurs in dikes and sills. Seven sills and 16 dikes were mapped. Four hundred nineteen linear feet, or 8 per cent of the tunnel, was mafic igneous rock. More precisely, along a line at map level on the west side of the tunnel, mafic igneous rocks are the bedrock for 419 feet. Since the percentages obtained in a linear traverse are equivalent to the volume percentages, just as in petrographic studies, 8 per cent of the rock excavated from the tunnel must have been mafic igneous rock. However, inasmuch as many of these dikes and sills strike at an angle to the tunnel and dip at angles of less than 90 degrees, these mafic igneous rocks were found on one of the walls or in the roof over a linear distance considerably greater than 8 per cent of the tunnel.

The mafic igneous rocks are of two general types, diabase and altered diabase. The diabases are medium to coarse dark rocks with a typical ophitic texture and consist chiefly of plagioclase and augite, but contain biotite and secondary minerals such as hornblende, chlorite, and calcite. The diabases are found only in the dikes and not in the sills.

The altered diabases are light-gray, fine-to-medium-grained rocks
that have not yet been studied in thin section. All the sills and many of the dikes are altered diabase.

## Structural Geology

## General Statement

The Boston Basin is structurally a synclinorium composed of alternating anticlines and synclines plunging east. Major faults, some parallel and others diagonal or perpendicular to the major folds, are present (LaForge, 1932; Billings, 1929). The Malden Tunnel is of special interest to geologists because it crosses the Northern Boundary Fault, which both Billings (1929, p. 113) and LaForge (1932, p. 63) considered to be a thrust dipping north.

The ensuing paragraphs will be first concerned with the larger structural features-the Cambridge Argillite constituting the northern part of the Boston Basin, the Lynn Volcanics constituting the southern part of the "crystalline rocks," and the Northern Boundary Fault. Later paragraphs will discuss the minor structural features, such as the joints, shears, and dikes.

## Northern Part of the Boston Basin

The northern part of the Boston Basin was exposed between stations $2+00$ (Shaft A) and $24+56$ (Northern Boundary Fault). It is composed of the Cambridge Argillite which here strikes northeast and dips southeast except near the fault, where it is overturned and dips northwest (Fig. 3). Thus the southern part of the Malden Tunnel is on the northern limb of the Charles River syncline, the axis of which lies 4 miles to the south.

From station $2+00$ to $20+50$ the Cambridge Argillite dips southeast at angles ranging from $40^{\circ}$ to $80^{\circ}$ (Figs. 2 and 3). In general the dip increases toward the north, becoming vertical at station $20+$ 50. Between stations $20+50$ and $21+30$ the dip ranges from $85^{\circ} \mathrm{SE}$. to $75^{\circ}$ NW., the beds being overturned where the dip is northwest. North of station $21+30$ the dip ranges from $74^{\circ}$ to $89^{\circ}$ NW., all overturned. That is, the north limb of the Charles River syncline is overturned toward the south for a distance of 400 feet south of the Northern Boundary Fault.

Since all the strata in the Cambridge Argillite here "top", that is, get younger to the southeast, there is no synclinal axis in the tunnel.

Fig. 4.-Map and Structure Section of Area near Northern Boundary Fault at Tunnel Level Symbol for bedding and shears same as in Fig. 2. Short lines with square on one side represent joints. Triangles are tuff-
breccia; crosses are dense felsite.

Nevertheless, the variations in strike and dip show that broad open folds are present. The form of these folds can be analyzed most satisfactorily by a point diagram. Fig. 5A is such a diagram based on 97 readings of the bedding in the Cambridge Argillite. The plot is on the


Fig. 5.-Point Diagrams. Plotted on Lower Hemisphere of Equal Area Net
A. Poles of perpendiculars to bedding. Circles with dots: between stations $2+00$ and $10+00$. Open circles: between stations $10+00$ and $20+00$. Solid circles: between $20+00$ and $24+56$, not overturned. Crosses: between $20+00$ and $24+56$, overturned.
B. Poles of perpendiculars to shears in Lynn Volcanics north of Northern Boundary Fault. Those marked by cross contain clay gouge.
C. Poles of perpendiculars to joints in Lynn Volcanics north of Northern Boundary Fault.
D. Poles of perpendiculars to joints Cambridge Argillite south of Northern Boundary Fault.
lower hemisphere and the projection is an equal area net (Billings, 1954, pp. 108-115). The concentration of marks in the northwest quadrant is consistent with the northeast strikes and southeast dips; the concentration in the southeast quadrant represents the overturned beds dipping northwest. The dots in open circles represent strata between stations $2+00$ and $10+00$. They lie on an arc trending northnortheast; the arc signifies open folds or warps plunging $35^{\circ}$ in a direction $\mathrm{S} .74^{\circ} \mathrm{E}$. The open circles represent strata between stations $10+00$ and $20+00$. They lie on an arc trending north-northwest; the arc signifies open folds or warps plunging $40^{\circ}$ in a direction N . $75^{\circ} \mathrm{E}$. The solid circles and crosses are for strata between stations $20+$ 00 and $24+56$, and represent, respectively, normal and overturned strata. They lie on an arc that is essentially a straight line trending northwest; this signifies broad open folds or warps plunging nearly horizontally N. $40^{\circ}$ E.
Southern Part of Area of "Crystalline Rocks"
The structure of the Lynn Volcanics is difficult to deduce, inasmuch as bedding is rare. In the structural section (Fig. 3) the Lynn Volcanics are portrayed as consisting of two main units, an upper felsitic tuff-breccia and a lower dense felsite. The porphyritic felsite and felsite tuff are thin units.

The chief evidence favoring this interpretation is the following. Firstly, the bore holes in the Lynn Volcanics, except for their deepest parts, were all in felsitic tuff-breccia. The dense felsite was largely confined to the tunnel; its distribution is shown on Fig. 2. Secondly, bedding was observed in several places between stations $43+16$ and $36+57$; the average dip was $15^{\circ}$. Thirdly, in several places the tuffbreccia was observed to overlie the dense felsite. At station $42+58$, where the strike is $\mathrm{N} .25^{\circ} \mathrm{E}$. and the dip is $20^{\circ} \mathrm{NW}$., the following section was observed from bottom to top: (a) dense felsite; (b) three feet of well-bedded felsitic tuff; and (c) tuff-breccia. At station $37+$ 05 , where the strike is N. $45^{\circ} \mathrm{E}$. and the dip $5^{\circ} \mathrm{NW}$., the tuff-breccia overlies directly the dense felsite. The same relationship may be observed at station $36+73$, where the strike is $\mathrm{N} .45^{\circ} \mathrm{E}$. and the dip is $20^{\circ} \mathrm{SE}$.

## Northern Boundary Fault

The Northern Boundary Fault of the Boston Basin, that is, the contact between the Cambridge Argillite and the "crystalline rocks" to
the north, trends east-northeast (Fig. 1). For many decades this contact has been considered to be a fault. Although I. B. Crosby (1928) shows it as a normal fault, Billings (1929, p. 113) and LaForge (1932, p. 63) believed it to be a thrust fault.

Fig. 3 shows the Lynn Volcanics thrust southward over the Cambridge Argillite. Fig. 4 contains a map and section on a much larger scale than Figs. 2 and 3. The fault, exposed at station $24+56$ at map level on the west wall, strikes $\mathrm{N} .80^{\circ} \mathrm{W}$. and dips $55^{\circ} \mathrm{N}$. On the west wall the contact is knife-sharp and tight. In fact, a specimen right across the contact was collected. On the east wall the contact was an opening about an inch wide; clay gouge had presumably been present.

One might assume that the Northern Boundary Fault consistently dips $55^{\circ} \mathrm{N}$. But data from bore hole 6 indicates that the fault flattens out a short distance below the invert of the tunnel. The upper part of Fig. 4 is a map. Bore hole 6 is located at station $24+96$, L $20^{\prime}$, that is, it lies 20 feet N. $82^{\circ} \mathrm{W}$. of a station 40 feet N. $8^{\circ}$ E. of the trace of the fault in the tunnel. In BH 6 the contact of the Lynn Volcanics and the underlying Cambridge Argillite was encountered at an altitude of -278.5 feet, that is, 2.9 feet below the invert of the tunnel. The attitude of the contact could not be determined. The data from BH 6 are projected into the structural section in the lower part of Fig. 4. We would conclude that the Northern Boundary Fault flattens out just north of its exposure in the tunnel.

In projecting the data from BH 6 to the structure section in Fig. 4 it has been assumed that the contact does not dip away from or toward the tunnel. It cannot dip westward at any appreciable angle, otherwise it would come to tunnel level around station $24+96$. On the other hand, if it were to dip eastward it would be lower than shown in Fig. 4.

Consideration has been given to the possibility that BH 6 may not be vertical. Geologists are well aware of inclined and crooked bore holes. The surface at BH 6 is at an elevation of 41.9 feet. The LynnCambridge contact was reached at a distance of 320.4 feet. Assuming the bore hole to be vertical, it was calculated that the contact was at an altitude of -278.5 feet. A non-vertical bore hole could assume many different shapes. As one of many possibilities, let us assume that the bore hole was straight but inclined at an unknown angle. Let us further assume that the fault is a plane striking N. $80^{\circ} \mathrm{W}$., dipping $55^{\circ} \mathrm{N}$.
and located at map level at station $24+56$. Let us further assume that the inclined straight bore hole plunges in a vertical plane striking at right angles to the strike of the fault, that is, it plunges S. $10^{\circ} \mathrm{W}$. If such a bore hole crosses the fault at a distance of 320.4 feet from the surface, it would be inclined at an angle of $83^{\circ}$, that is, it would be $7^{\circ}$ from the vertical. Bore holes inclined at lower angles in a plane striking $\mathrm{S} .10^{\circ} \mathrm{W}$. would also cross the fault, but at lesser distances. But bore holes inclined at higher angles would not reach the postulated fault in the measured distance.

Some data are available to solve this problem. In the tunnel the dip of the bedding between stations $24+56$ and $20+50$ is relatively uniform, is overturned (with one exception out of 20) and averages $82^{\circ} \mathrm{NW}$. or $8^{\circ}$ from the vertical. In a core from a bore hole inclined $7^{\circ}$ in a direction $\mathrm{S} .10^{\circ} \mathrm{W}$. the bedding would make an angle of $15^{\circ}$ with the axis of the core. A specimen in the author's collection is a piece of core 5 inches long from a distance of 330 feet (altitude of -288 feet, assuming the hole is vertical) ; the bedding is inclined $5^{\circ}$ from the axis of the core. This agrees much more closely with what is expected from a. vertical boring than with a boring inclined at $8^{\circ}$.

An intriguing problem concerns the eastward extension of the Northern Boundary Fault. Geologists are interested in this subject. Moreover, this matter may be of importance in some future engineering project. Also, deep borings for other purposes may supply valuable data. As shown in Fig. 8, bedrock is not exposed in Winthrop, Revere, and the southern part of Lynn, but the North Metropolitan Relief Tunnel indicates that Winthop is underlain by Cambridge Argillite. Nahant belongs to the older rocks here classified as "crystalline rocks." It appears, therefore, that the Northern Boundary Fault swings east-southeast to go south of Nahant.

## Joints

The significance of joints in tunneling depends upon many variables, such as their orientation, spacing, and length. Vertical joints a few feet apart and crossing a tunnel at right angles are of no great concern. Flat joints a few inches apart in the roof would be of great concern.

In the Malden Tunnel the attitude of joint sets, that is, the attitude of a more or less parallel set of 3 to 10 joints, was measured in many places. In Fig. 5C each point represents a joint set in the

Lynn Volcanics. Gently dipping joints are represented by the dots in the center of the diagram. The strike of the other sets box the compass and the dip ranges from $30^{\circ}$ to vertical. It is apparent that the joints in the Lynn Volcanics are diversely oriented. Fig. 5D shows the orientation of the joints in the Cambridge Argillite. Many of these joints dip steeply and most strike in the sector ranging from northnortheast to north-northwest. Joints striking northwest and west-northwest are rare.

## Shears

The term shear as used in this report refers to planar features, generally a few inches to a few feet wide, in which the rock is platy, slickensided, or converted to clay gouge. A fault is a plane along which there has been visible offset of some older planar feature, such as bedding, a vein, or a dike. Many of the shears in the Malden Tunnel may be faults, but no displacement may be seen.

Forty shears were recorded in the Lynn Volcanics (Fig. 5B). Although they are diversely oriented, there is a slight tendency for two sets. One set has an average strike of northeast and an average dip of $45^{\circ}$ NW., but there is considerable spread. A second set has an average strike of $\mathrm{N} .80^{\circ} \mathrm{W}$. and dips steeply south.

## Veins and Gash Veins

Sixteen veins other than gash veins were recorded in the Lynn Volcanics. They range in thickness from $1 / 4^{\prime \prime}$ to $4^{\prime \prime}$, but most are an inch thick. The filling is mostly calcite, but some quartz is associated. The attitude of these veins is shown by crosses in Fig. 6A. Most strike east-west and dip steeply. One is horizontal and two strike north, dipping steeply west.

The gash veins are a rather unusual feature. They occur in a group of 6 to 10 veins arranged en echelon. The individual veins are generally $6^{\prime \prime}$ to $12^{\prime \prime}$ long, but range from $3^{\prime \prime}$ to $24^{\prime \prime}$. They rarely exceed $1 / 4^{\prime \prime}$ in thickness, but range from $1 / 16^{\prime \prime}$ to $1^{\prime \prime}$. Fig. 6A shows that the attitude of the gash veins is more systematic than the joints or shears. Although there is considerable spread, the average strike is N. $80^{\circ} \mathrm{W}$., the average $\operatorname{dip} 45^{\circ} \mathrm{S}$.

The en echelon zones always dip south at angles of $30^{\circ}$ to $70^{\circ}$. In some instances the zone dipped more steeply than the individual veins. This indicates that the hanging wall of the vein moved upward


Fig, 6.-Point Diagrams. Plotted on Lower Hemtsphere of Equal Area Net
A. Poles of perpendiculars to veins north of Northern Boundary Fault. Each dot represents 5 to 10 gash veins in a single set; each cross represents single veins.
B. Poles of perpendiculars to diabase and related dikes; numerals give thickness of dikes.
relative to the footwall, that is, a "reverse" movement. In other instances the zone dips less steeply than the individual veins. This indicates that the hanging wall moved down relative to the footwall, that is, a "normal" movement.

## Summary of the Attitude of Joints, Shears, and Veins

It is clear that these planar features are diversely oriented. The lack of a uniform or systematic pattern in these rocks is of significance in planning future projects in bedrock. Moreover, any attempt to deduce the orientation of the principal stress axes when the rocks were being deformed is fraught with difficulties.

## Dikes and Sills

In a previous page it has been stated that seven sills and 16 dikes of mafic igneous rock, diabase or altered diabase, were recorded. By definition a sill is parallel to the bedding, a dike is crosscutting. Since bedding is rare in the Lynn Volcanics, all the mafic igneous rocks in it are classified as dikes.

The seven sills range in thickness from 3 inches to 14 feet, and average 4.4 feet. They are confined to the Cambridge Argillite, that is, the area south of the Northern Boundary Fault.

The five dikes north of the fault range in thickness from one foot to 14 feet, averaging 4 feet. The dikes south of the fault range in thickness from 1-1/2 feet to 134 feet, and average 20 feet. Excluding the dike that is 134 feet thick, the average thickness is 7 feet.

Fig. 6B shows that the dikes dip steeply. Although there is considerable spread in the strike, a general northeasterly strike is more common than a northwesterly strike.

## Engineering Aspects of the Geology

Two subjects deserve special consideration: the amount of support necessary and the amount of water encountered.

In the planning stages it was realized that the tunnel would cross the Northern Boundary Fault. It was anticipated that a wide wet shear zone might be present. This did not materialize, but the fault was accountable for other problems.

Structural steel was used for support continuously between stations $6+85$ and $33+17$, a distance of 2632 feet, and between stations $44+40$ and $45+45$, a distance of 105 feet. Thus $52 \%$ of the tunnel was supported by structural steel. About 35 roof bolts were used. Twenty-two bolts were used between stations $4+05$ and $4+31$, nine were used between stations $5+20$ and $5+41$, and the rest were used elsewhere.

The tunnel was unusually wet. Table II gives the average number of gallons of water pumped daily per month. The engineers indicate that this is an exceptionally large amount for a tunnel of this size and length. Detailed data on places where water came into the tunnel are unavailable, but the writers' notes record that unusually large amounts came into the roof at stations $26+93$ and $27+14,137$ and 158 feet, respectively, north of the fault.

Fig. 7 shows the distribution by 500 foot intervals of the various fractures recorded in the Lynn Volcanics north of the Northern Boundary Fault. Although the data are somewhat subjective, they appear to be significant. Structural steel was used for support where the gouge zones are present, where the shear zones exceed 5 per 500 foot interval, and where the total number of recordings exceeded 16 per 500 foot interval. The number of joint sets does not show such a definite relationship, but a satisfactory method of recording joints within a reasonable time has not been devised. Moreover, no effort was made to record the density of the joints.

Table II
Average Number of Gallons of Water Pumped Daily

| Month | Gallons per Day |
| :--- | ---: |
| May, 1957 | 45,602 |
| June | 195,994 |
| July | 220,500 |
| August | 537,000 |
| September | $1,115,000$ |
| October | 997,000 |
| November | 982,000 |
| December | $1,200,000$ |
| January, 1958 | 948,500 |
| February | $1,462,000$ |
| March | $1,353,000$ |
| April | $1,107,000$ |
| May | 994,000 |
| June | 414,000 |
| July | 389,000 |

Unfortunately, satisfactory data were not recorded in the Cambridge Argillite. In this section of the tunnel the writers concentrated on the attitude of the bedding and neglected the details of the shears and joints.

The use of structural steel continuously for 861 feet north of the fault and 1771 feet south of the fault, as well as the large amount of water, indicates that the rocks were fractured more than usual. The Cambridge Argillite required little support in the Main Drainage Tunnel (Rahm, 1962) or in the City Tunnel Extension (Billings and Tierney, 1964). Similarly, the Mattapan Volcanics (correlated with the Lynn Volcanics) in the West Roxbury Tunnel, now under construction, required little support. It is apparent that the rocks on either side of the Northern Boundary Fault were greatly strained and consequently fractured more than elsewhere.

General geological experience shows that conditions elsewhere along the Northern Boundary Fault may be different. All the deformation may occur in a single wide shear zone or even this may be absent.

## Acknowledgments

We wish to express our appreciation to Frederick W. Gow, Chief Engineer of the Construction Division of the Metropolitan District Commission, and to Martin W. Cosgrove, Deputy Chief Engineer, for


Fig. 7.-Density of Fractures in Lynn Volcanics North of Northern Boundary Fault


Fig. 8.-Northern Boundary Fault Shows Presumed Eastward Extension


Fig. 9.-Conway Mucking Machine. Looking Southerly from Heading at Station $34+35$. Rock Is Tuff-Breccia of Lynn Volcanics


Fig. 10.--Looking Southerly Toward Long Section of Structural Steed, Northern end of Which Is at Station $33+17$. Rock Is Tuff-Beccia of Lynn Volcanics


Fig. 11.-Looking Southerly from Station $7+65$ Nlar South End of Long Section of Structural Sterl. Rock Is Argillite of Cambridge Formation in Immediate Foreground; Rock Is Diabase Further South
the opportunity to study the geology. Mr. F. Lyle Tierney, geologist of the Construction Division at the time of the mapping, expedited the the logistics. Mr. Douglas S. Maclain furnished us with data on the support and amount of water pumped.

## References

Billings, M. P. Structural geology of the eastern part of the Boston Basin, Am. Jour. Sci., 1929, 5th series, 48, p. 97-137.
Billings, M.P. Structural Geology, New York, 1954, 514 pp.
Blllings, M. P., and Tierney, F. L. 1964, Geology of the City Tunnel Extension, Greater Boston, Massachusetts: Jour. Boston Soc. Civ. Eng., 51, p. 111-154.
Crosby, I. B. Boston Through the Ages, Boston, 1928, 166 pp.
LaForge, L. Geology of the Boston area, Massachusetts: U.S. Geol. Survey, 1932, Bull. 839, 108 pp.
Rahm, D. A. Geology of the Main Drainage Tunnel, Boston, Massachusetts: Jour. Boston Soc. Civil Engineers, 1962, 49, p. 319-368.
Toulmin, P., 3rd. Bedrock geology of the Salem Quadrangle and vicinity, Massachusetts: Bull. 1163-A, U.S. Geol. Survey, 1964.

# COASTAL PROCESSES AND BEACH EROSION 

By Joseph M. Caldwell*<br>(Presented as one of the John R. Freeman Lectures on Fundamental Hydraulic Processes in Water Resources Engineering, Fall, 1963.)

1. First, let it be recognized that none of the coastal processes relating to beach erosion are understood with the exactitude of some other hydraulic phenomena. The gross processes are known and fairly well understood; however, the desirable quantitative understanding has not yet been established, though continuing research is directed to this end. Nevertheless, the understanding which is available is applied to actual engineering problems around our coasts, with increasingly encouraging results. The fundamentals of these coastal processes and their application to selected engineering problems are given in this paper.

## Nature of the Coast

2. Most of our ocean coasts-and indeed most of the coasts of the world-are composed of unconsolidated, granular deposits in the form of sand or gravel. This material has reached the shore face through fluvial transport, glacial transport, or by deterioration and fragmentation of the adjacent uplands. By and large, we find that we work with the materials which have been laid down on the coast over the centuries. In the United States, a few rivers, such as the Mississippi and some of the West Coast rivers and streams, still contribute material in quantities which are significant in the local shore processes. However, in most of our shore problems, we are working with the sand that has already reached the shore and we can depend little if at all on new contributions.
3. As an example, our Atlantic beaches from Long Island to Key West receive practically no new shore building material from the uplands. Thus, what is lost in any manner from the shore face is essentially a permanent loss unless replacement is effected by works of man.

## Wave Action

4. The dominant forces in coastal waters are the wind-generated waves from the sea. The internal currents and turbulence in the waves

[^1]stir up and transport the sand in varying degrees depending on the type of wave and the type of bed material. Most of this activity takes place in the zone between the 50 -foot contour and the limit of wave uprush on the shore; this may be defined as the active zone. Now in what direction is the sand moved by the waves? Onshore or offshore? Or along the coast? Let us first consider the question of the wave action itself.
5. Wave trains generated by the wind are complex in that they contain high waves, low waves, long waves, and short waves. The actual wave pattern, or spectrum, depends on the wind duration, the woind velocity, and the fetch-the fetch being the overwater distance which is under the influence of a wind blowing in a single direction. The spectrum of waves at the end of the fetch can be predicted with fair accuracy. (1) A study of this wave spectrum will show that certain wave heights and wave periods tend to dominate the spectrum; these dominant waves are frequently referred to as the significant waves of the spectrum. The wave length in deep water is related to the wave period by the formula:
$$
\mathrm{L}=\frac{\mathrm{gT}^{2}}{2 \pi}
$$
where $L=$ wave length in feet
\[

$$
\begin{aligned}
& \mathrm{T}=\text { wave period in seconds } \\
& \mathrm{g}=\text { gravity }(32.2 \mathrm{ft} / \mathrm{sec} / \mathrm{sec})
\end{aligned}
$$
\]

Thus, knowing the significant wave period, the significant wave length can be computed. This length can then be compared to the significant wave height to give us a height/length ( $\mathrm{H} / \mathrm{L}$ ) ratio or wave steepness ratio. This ratio will be shown to be very important in determining the effect of the wave on the beach.
6. It can be shown (2) that generally the wave steepness ratios of the significant waves in a wave generating area (i.e., within the fetch area) lie in the range of 0.014 to 0.040 . Once the wave train leaves the generating area, the short period waves dissipate their energy more rapidly than the long period waves. Thus there is a gradual shift in the significant wave period and length to the longperiod end of the spectrum. This shift is accompanied by a decrease in significant height due to a gradual dissipation of energy throughout the
spectrum. The net result is a shift in the $\mathrm{H} / \mathrm{L}$ ratio of wave steepness ratio to smaller and smaller values. An example of this shift in steepness is shown in the following tabulation.

Generating Conditions

| Wind velocity | 40 knots |
| :--- | ---: |
| Wind duration | 12 hours |
| Fetch | 200 miles |


|  | Significant <br> Wave Period | Significant <br> Wave Length | Significant <br> Wave Height | Steepness <br> H/L |
| ---: | :---: | :---: | :---: | :---: |
| At end of fetch | 11.0 sec. | 620 ft | 19.3 ft. | 0.031 |
| 100 miles later | 13.0 | 865 | 11.6 | 0.013 |
| 500 miles later | 14.6 | 1090 | 8.7 | 0.008 |
| 2,000 miles later | 15.8 | 1280 | 4.4 | 0.003 |

From this tabulation, it can be seen that storms in the open ocean may generate wave trains which will eventually reach and affect shore hundreds of miles away. Waves from local storms reach the local shore as tempestuous storm waves with significant wave periods usually from 5 to 12 seconds and with heights up to twenty feet or more: waves from distant ocean storms generally reach the shore as long, low swells with significant periods ranging from about 10 to 20 seconds. The transition or decay, from storm waves to swells is, of course gradual. Methods exist for computing the decay of the wave train after it leaves the generating area. (3)

## Onshore-Offshore Movement

7. As stated previously, the waves advancing shoreward finally reach a depth where the internal wave currents begin to disturb and move the bottom particles. The action is particularly pronounced at the point where the wave breaks; here in the surf zone large quantities of sand are thrown into semi-suspension and quantities are also moved on the bottom somewhat as bed-load. Numerous laboratory experiments supported by a number of field studies have established that the short, tempestuous storm waves tend to drag material from the beach face and deposit it in deep water. Conversely, the long swells tend to push the offshore material back onto the beach.
8. Thus we find on most beaches a shuttling of the sand back and
forth between the inshore and offshore zone. A series of local storms, or a local storm of long duration, can erode the beach and dune area sufficiently to lay open the back shore area to considerable destruction. For instance, the beach width on the south shore of Long Island frequently erodes 50 to 100 or more feet during the winter storms only to rebuild itself during the subsequent summer season.

## Effect of Wave Angularity

9. Wave trains seldom approach the shore at right angles. Even though refraction does tend to swing the crests into parallelism with the shore contours, the residual angle of the waves with the beach results in the generation of an alongshore current called the littoral current. This current is noticeable even seaward of the surf zone, but is most pronounced in and shoreward of the surf zone. Recent studies (4) indicate the incident wave and littoral current relationship for small approach angles for a laboratory beach on a $1: 10$ slope and for a natural beach to be:

$$
\mathrm{V}=\mathrm{gm} \mathbf{T} \sin 2 \alpha
$$

where $\mathrm{V}=$ longshore current velocity, m is beach slope, T is wave period, and $\alpha$ is angle between wave crest at breaking and shoreline. Though having some theoretical basis, the relationship is largely empirical. Under this relationship, the maximum velocity would be

$$
V_{\max }=\mathrm{gm} T \text { when } \alpha=45^{\circ}
$$

Refraction effects would, however, seldom, if ever, permit a breaker angle of $45^{\circ}$, values between zero and about $20^{\circ}$ being much more common. Littoral current velocities estimated by the above relationship are given in the following tabulation although laboratory data indicate that a correction must be made to the equation for V at the larger angles, the correction acting to reduce the predicted V's for the larger angles. Though these estimated velocities are based on a relationship derived largely from laboratory tests, the estimated velocities appear to be in the right order in light of known field conditions. The absence of the wave height $H$, from this relation is due to the fact that an increase in H causes the wave to break farther offshore; this in turn increases the total littoral current volume but apparently has only a secondary effect on its velocity.

Estimated Littoral Drift Velocities
( $\mathrm{ft} / \mathrm{sec}$ )

| m | . 01 |  |  | . 025 |  |  | . 04 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\alpha^{\text {T }}$ | - 5 | 10 | 15 | 5 | 10 | 15 | 5 | 10 | 15 |
| $5^{\circ}$ | . 28 | . 56 | . 84 | . 70 | 1.40 | 2.10 | 1.12 | 2.24 | 3.36 |
| $10^{\circ}$ | . 55 | 1.10 | 1.65 | 1.38 | 2.75 | 4.13 | 2.20 | 4.40 | 6.61 |
| $15^{\circ}$ | . 81 | 1.61 | 2.42 | 2.01 | 4.02 | 6.04 | 3.22 | 6.44 | 9.66 |
| $20^{\circ}$ | 1.03 | 2.07 | 3.11 | 2.58 | 5.18 | 7.77 | 4.13 | 8.28 | 12.43 |

where:
$\mathrm{m}=$ beach slope
$\mathrm{T}=$ wave period in sec.
$\alpha=$ angle of breaker with shore
10. Thus we find that the incoming waves usually generate an alongshore, or littoral current. The velocity of this current is in many cases too low to, of itself, move the bed materials; however, it always acts to move with itself any material which has been placed into even temporary suspension by wave action. This combination of the stirring action of the waves and the littoral current acts to transport sizeable quantities of sand along the shore face. Measurements on coasts subjected to the constant pounding of the trade winds have indicated littoral drift rates of in excess of five million cubic yards per year, though most coastal areas exhibit less than one million cubic yards per year of littoral drift.
11. Numerous laboratory tests coupled with a few field observations show an empirical relationship (5) between incident wave energy and alongshore sand transport. Though admittedly needing additional study and refinement, the relationship is useful. The relationship is expressed as

$$
\mathrm{Q}=\mathrm{KE}_{\mathrm{i}}^{0.8}=210 \mathrm{E}_{\mathrm{i}}^{0.8}
$$

where $\mathrm{Q}_{\mathrm{i}}=$ rate of alongshore sand transport in cubic yard per day
$\mathrm{E}_{\mathrm{i}}=$ alongshore component of incident wave energy in millions of foot-pounds per foot of beach per day
$\mathrm{K}=$ factor of proportionability (tentatively 210 ).

The value of $\mathrm{E}_{\mathrm{i}}$ is in turn defined as:

$$
\mathrm{E}_{\mathrm{i}}=\mathrm{E}_{\mathrm{p}} \sin \theta \cos \theta
$$

where $E_{p}=$ potential energy of incident wave train.
$\theta=$ angle of wave crest with shore at point where wave height and length are measured.

The potential energy, $\mathrm{E}_{\mathrm{p}}$, is of course that portion of the total wave energy which moves shoreward as opposed to the kinetic energy, $\mathrm{E}_{\mathrm{k}}$, which does not move shoreward. The value of $\mathrm{E}_{\mathrm{p}}$ is expressed by:

$$
\mathrm{E}_{\mathrm{p}}=0.64 \mathrm{WTh}_{\mathrm{s}}{ }^{2} \mathrm{n}\left(\tanh 2 \pi \mathrm{~d}_{\mathrm{s}} / \mathrm{L}_{\mathrm{s}}\right) \mathrm{t}
$$

where $\mathrm{E}_{\mathrm{p}}=$ summation of forward-moving (potential) energy, $\mathrm{ft} / \mathrm{lbs}$.
$T=$ wave period, sec.
$\mathrm{h}_{\mathrm{s}}=$ wave height (ft.) at point of observation
$d_{s}=$ depth (ft.) of water at point of observation
$L_{\mathrm{s}}=$ wave length (ft.) of water at point of observation
$\mathrm{n}=$ fraction of total energy transmitted forward with wave form
$t=$ time interval (seconds) over which energy is summarized
$\mathrm{W}=$ specific weight of liquid in pound per cubic foot ( 64.0 for sea water).

Of course, if the wave measurements are made in deep water ( $\mathrm{d}_{\mathrm{s}}>$ $1 / 2 \mathrm{~L}_{\mathrm{s}}$ ) the energy expression can be simplified to

$$
\mathrm{E}_{\mathrm{p}}=0.64 \mathrm{WTh}_{\mathrm{s}}{ }^{2}
$$

12. From the above, it can be seen that, given the wave height, length, and direction, it is possible to predict the value of the alongshore component of wave energy, $\mathrm{E}_{\mathrm{i}}$. From this value it then is possible to estimate the rate of littoral drift. Or, as a further possibility, given the applicable weather maps, it is possible to predict the incident wave pattern and from this the alongshore drift. This latter computation was made for four points on the New Jersey shore for which a set of wave predictions (or wave "hindcasts") covering three years had been made. by the Beach Erosion Board for other purposes. The rate of drift at

the four selected points is known fairly accurately from field measurements made by the Corps of Engineers. A comparision of the various points established by the above work is shown on Fig. 1. Though showing a promising degree of consistency, the plotted points indicated that additional refinement is still needed.
13. Another approach to the question of littoral transport is through measurements of the material thrown into suspension on the shore face coupled with the velocity of the littoral current. Measurements of sand in suspension in and near the surf zone were made in 1950-51 at Mission Bay, California, by the Beach Erosion Board. (6) A pump-type sampler was used to take over 170 samples under a wide range of wave conditions and indicated concentrations ranging from about 0.10 to over 4.5 parts per thousand by weight with most samples being in the 0.15 to 0.70 ppt . range. The samples were taken between the 12 -foot contour and the shore. The samples, when combined, indicated the amount of sand in suspension per foot of shore to about as follows: (6)

| Range of wave height | Sand in suspension in cubic <br> yard per foot of shore |
| :---: | :---: |
| $1-2 \mathrm{ft}$. | 140 |
| $2-3 \mathrm{ft}$ | 260 |
| $3-4 \mathrm{ft}$ | 475 |
| $4-6 \mathrm{ft}$. | 450 |

14. Combining these suspensions according to the relative incidence of the various wave conditions at Mission Bay it is possible to construct a hypothetical table of the rate of littoral drift movement due to suspended material being carried along by the littoral current. (6) This table assumes no reversals in drift which is seldom the actual

| Littoral Current |  | Rate of Drift <br> $\mathbf{f t} / \mathrm{min}$ <br> $\mathrm{ft} / \mathrm{sec}$ <br> $\mathrm{cu} \mathrm{yd} / \mathrm{yr}$ <br> 2 |
| :---: | :---: | ---: |
| 6 | 0.03 | 16,000 |
| 15 | 0.10 | 50,000 |
| 30 | 0.25 | 125,000 |
| 45 | 0.50 | 250,000 |
| 60 | 0.75 | 375,000 |
| 90 | 1.00 | 500,000 |
| 120 | 1.50 | 750,000 |
| 180 | 2.00 | $1,000,000$ |
| 360 | 3.00 | $1,500,000$ |
|  | 6.00 | $3,000,000$ |

case; so in a sense it represents the total drift rather than the net drift. The littoral current velocities given above are in the ranges normally encountered along the shore. Also, the indicated rates of littoral drift are of the same order as those observed along our shores. This correlation indicates that while movement of sand along the bottom as bed load is probable, movement as suspended load is a major if not the predominant manner of movement.
15. From the above, it can be seen that the incident wave energy has a certain capacity for stirring up the sand and for generating a littoral current. These two actions result in a transport of material along the shore face as littoral drift. Varying wave directions may reverse the drift direction; however, most areas have a dominant drift direction and the net movement in that direction is referred to as the net drift. The net drift is of course the difference between the gross drift upcoast and downcoast.
16. A shore sector can be in equilibrium only if the net amount of material leaving the sector at one end is balanced by an equal net amount entering the sector at the other end. The presence of inlets, estuaries, bays, and other irregularities in the shore alignment coupled with changing patterns of wave energy from one shore point to another seldom permits the equilibrium balance to be maintained in a shore sector. The result will be either accretion or erosion of the sector. Unfortunately, the factors producing erosion dominate in most areas. Let us now examine the ocean shore of New Jersey as a field laboratory of shore processes.

## Littoral Processes in New Jersey

17. The ocean coastline of New Jersey extends for some 120 miles, being oriented roughly in a north-south direction (see Fig. 2). It is a sandy shore broken by ten major inlets and somewhat sheltered at the north by the eastward extension of Long Island. Except for three small headland sectors-one at Cape May, one near Bayhead, and one near Long Branch-the entire coast is a series of barrier islands separated from the mainland by bays, lagoons, and tidal marshes in various states of deterioration. The tide range is about 5 feet and the shore is subject to extra-tropical storms (northeasters) and to hurricanes. The entire coast comprises a large and important summer resort area.
18. Wave Energy. Wave hindcasts (7) were made by the Beach


Fig. 2

Erosion Board for points off Sandy Hook and off Cape May using three years of North Atlantic weather maps. The analysis showed that the wave energy pattern would be essentially the same at the two points, except for the sheltering action of Long Island. In other words, the wave energy approaching from the southeast quadrant is essentially equal over the entire coast: that from the northeast quadrant is more prominent at Cape May than at Sandy Hook due to the greater shelter which Long Island provides the Sandy Hook area. Further, the wave energy patterns indicate that the wave energy from the north is greater than that from the south; thus a net littoral drift to the south can be expected where the effect of Long Island is too great.
19. Measured Drift Rates. In connection with shore erosion problems, the Corps of Engineers has determined the rates of littoral drift at two points along the New Jersey coast, one at Cold Springs Inlet and the other at Sandy Hook. Sandy Hook accumulates essentially all the sand moved northward to it from the New Jersey beaches to the south. The drift movement south off Sandy Hook is, to all purposes, zero; so the rate of sand accumulation on Sandy Hook represents both the gross northbound drift and the net drift at this point. This rate is determined as 500,000 cubic yards per year; the determination was made from a series of accurate surveys made at various times of the past 100 years.
20. Cold Springs Inlet is maintained artificially to provide direct access from the Atlantic into Cape May Harbor. The inlet is flanked by two jetties of considerable length constructed in 1910. The north jetty has provided an impoundment area which has entrapped the littoral drift moving down from the north. The average rate of impoundment, as determined from successive surveys is 200,000 cubic yards per year. This rate is the net rate as waves from both the northeast and southeast quadrants reach the impounded area and the impounded material represents the difference in the resulting movements.
21. As stated previously, the wave energy from the south is essentially uniform over the New Jersey coast. Thus the northbound drift rate of 500,000 cubic yards/year at Sandy Hook can also be taken to approximate the northbound drift at Cold Springs Inlet. Combining the above two figures we get for Cold Springs Inlet:

Northbound drift $=500,000$ cubic yards/year
Net drift (south) $=200,000$ cubic yards $/$ year
Southbound drift $=700,000$ cubic yards/year

From this figure, it can be seen that the total sand in transit along the beach face in the Cape May area is $(500,000+700,000)=$ $1,200,000$ cubic yards per year. Thus, any unprotected inlet in this area is potentially subjected to shoaling from some $1,200,000$ cubic yards/year of sand being shuttled back and forth along the shore face.
22. North Jersey. Starting from the nodal point at Dover Township, 35 miles south of Sandy Hook, we find the net rate of littoral drift increasing from essentially zero at Dover Township to 500,000 cubic yards per year northbound at Sandy Hook. Obviously, then, the 500,000 cubic yards furnished to Sandy Hook must come from the 30 miles or so of shore between Dover Township and Sandy Hook. There is no river-borne sand added to the shore face over these 30 miles. Also, surveys dating back about 100 years show no evidence of sand being supplied to the shore by offshore deposits over these 30 miles. Thus, the conclusion is that the material to supply 500,000 cubic yards per year to Sandy Hook must come from beach face itself. How does this check with the facts?
23. Comparisons of surveys made in 1838 and 1953, a 115-year span, show that the 30 miles of shore south of the base of Sandy Hook retreated shoreward some 500 feet (or about 5 feet per year) during this period. During the first sixty years or so the rate of accretion on Sandy Hook was 525,000 cubic yards/year. During the last fifty years, the rate has been 475,000 cubic yards/year. This $12 \%$ reduction is thought to be due to the construction of multitudinous groins and jetties in the 30 miles of eroding shore. It is interesting to note that the wholesale attempt to halt erosion by groin construction apparently reduced the rate of erosion by only $12 \%$; thus, in this area at least, groins were found to be ineffective as beach building or beach stabilizing devices.
24. Studying in more detail the 50 -year period, 1885-1935, the average rate of accretion on Sandy Hook was 493,000 cubic yards per year. The erosion from the thirty miles of shore south of Sandy Hook for this 50 -year period averaged 723,000 cubic yards/year. This excess of erosion over accretion is due to the fact that only about $2 / 3$ of the eroded material is sand, the remaining $1 / 3$ being silt and clay which, once eroded, are lost into the sea and play no part in the beach building processes at Sandy Hook. Thus, we find the entire 30 miles of shore under erosion in order to supply the net littoral drift demanded by the wave action impinging on this section of the shore.
25. Practical Solutions. The erosion described above, about 5
feet per year, has been very damaging to the economy of the thirty miles of shore. Attempts to control the erosion by groins and seawalls have been largely unsuccessful and the continuing erosion gradually undermined and destroyed the shore structures. The replenishment of the shore face by adding sand to the littoral drift stream presents itself as a desirable solution. Let us examine this possibility.
26. The north end of Barnegat Bay extends some 4 or 5 miles north of Dover Township, and the Bay bottom contains large deposits of sand suitable for replenishing the eroding shore face. The question then arises as to the efficacy of pumping 500,000 cubic yards of sand per year from Barnegat Bay onto the beach near Mantoloking just north of Dover Township. To visualize whether or not this would provide a solution to the North Jersey erosion problem, it is advisable to construct the following table:

TABLE
Erosion and Littoral Drift, Cubic Yards/Year

|  | $\begin{aligned} & \text { Mantoloking } \\ & \text { to } \\ & \text { Manasquan } \end{aligned}$ | $\begin{gathered} \text { Manasquan } \\ \text { to } \\ \text { Asbury Park } \end{gathered}$ | Asbury Park to base of Sandy Hook | Base Sandy Hook to tip of Sandy Hook |
| :---: | :---: | :---: | :---: | :---: |
| Net litt. drift entering sector | 0 | 74,000 (n) | 319,000 ( n ) | 493,000 (n) |
| Net litt. drift leaving sector | 74,000 (n) | 319,000 (n) | 493,000 (n) | - |
| Litt. drift deficit in sector | 74,000 | 245,000 | 174,000 | - |
| Average shore erosion (incl. silt \& clay) | 109,000 | 364,000 | 256,000 | 493,000 (acc.) |
| Sand erosion | 74,000 | 245,000 | 174,000 | 493,000 (acc.) |

From this table, it is seen that the net littoral forces are capable of moving only 74,000 cubic yards/year out of the first sector into the second sector, whereas the second sector supplies 310,000 cubic yards/year to the third sector. Thus the second sector needs an additional supply of 245,000 cubic yards/year of sand to overcome the present erosion in the second sector. However, the littoral forces in the first sector are already moving their full capacity of sand into the second sector. Thus, the only potential capability in the use of Barnegat

Bay sand is to supply the 74,000 cubic yards/year presently being lost in the first sector. If we place an excess of sand in the first sector, it will only serve to widen the beach in this sector without benefiting the second sector. Thus, sand placed to aid the second sector will have to be placed on the beach in the second sector. This relationship between sectors holds true for successive sectors to the north and shows the impossibility of assisting adjacent sectors by over-supply to the preceding sectors.
27. This same relationship explains why it is inevitable that the northerly shore of New Jersey will erode under natural conditions, for each successive sector shows increasing net littoral forces and net littoral drift. A beach erosion control study of this section of the shore was made in 1954; this study recommends that restoration of the shore be accomplished by rebuilding the beaches with sand and then maintaining them by periodic replenishment at intervals along the shore face. The cost of the initial restoration was estimated to be about $\$ 22,000,000$ with a yearly maintenance cost thereafter of about $\$ 1,700,000$. This plan was considered satisfactory and acceptable by the State, but funds have to date been appropriated to initiate only about one-third of the restoration project.
28. South Jersey. From the nodal area at Dover Township, the New Jersey shore extends southerly some 85 miles to its terminus at Cape May. The same reasoning as developed for North Jersey brings us to a similar conclusion to the effect that the South Jersey shore must show a net loss of 200,000 cubic yards per year between Dover Township and Cape May; this is indicated by the net drift of zero at Dover Township and 200,000 cubic yards/year to the south of Cold Springs Inlet. Without further examination, it might be proposed that the 85 miles of South Jersey shores could be stabilized by adding 200,000 cubic yards/year along the 85 miles. But let us now examine the actual shore processes.
29. A comparison of the North and South Jersey shores will show one great difference. The North shore is pierced by only two relatively small inlets which play only a minor part in the shore processes. The South shore, in contrast, is pierced by eight large inlets each of which has a great potential for interrupting the normal littoral drift movement along the shore. To determine if there is in fact such an effect, an erosion table for this 85 miles is drawn up as follows:

TABLE
Accretion and Erosion, Southe Jersey
cu. yd./year

|  | Gross drift | Net drift | Net erosion |
| :--- | :---: | :---: | :---: |
| Long Beach Island | $1,050,000$ | 50,000 | 520,000 |
| Pullen Island | $1,070,000$ | 75,000 | 270,000 |
| Brigantine Island | $1,080,000$ | 75,000 | 200,000 |
| Absecon Island | $1,100,000$ | 100,000 | 110,000 |
| Peck Beach | $1,125,000$ | 150,000 | 320,000 |
| Ludlam Island | $1,150,000$ | 175,000 | 300,000 |
| Seven Mile Beach | $1,175,000$ | 200,000 | 40,000 |
| $\quad$ Totals | $7,750,000$ | - | $\underline{1,760,000}$ |
| Two Mile Beach |  |  | 200,000 (accretion) |

A study of this table shows the average annual loss by erosion of the shore face to be $1,760,000$ cubic yards. Surveys show that this material is not deposited offshore or in the outer bars at the inlets, but rather that it is swept into the inlets on flood tide where a significant portion remains permanently. In other words, the flood tide aided by the ocean waves carries more material into the interior bays and lagoons than the ebb tide, unaided by the waves, can return to the shore face.
30. A review of the above table shows that the gross drift shuttles approximately 1.1 million cubic yards of material back and forth at each of the seven inlets north of Cold Springs Inlet; thus a total of 7.7 million cubic yards per year is brought within the influence of the seven inlets. The permanent entrapment of the inlets is in the order of $23 \%$ or $1,760,000$ cubic yards per year, or an average of about 250,000 cubic yards per year per inlet. From these figures, it can be seen that the net drift is grossly inadequate to compensate for the losses into the inlets; therefore general erosion of the shore face on a continuing basis can be expected unless steps are taken to bring the losses under control.
31. A study of the southerly 75 miles of shore will show that much of the beach has retreated by 500 feet or more during the past 100 years, though there has been a noticeable tendency for accretion at the south shoulders of the inlet. This erosion has of course presented a severe problem to the recreation communities along the shore.
32. Temporary solutions to this problem involve periodically
supplying new sand to the eroding shores and the use of groins and seawalls. The long-term solution will probably involve bringing the inlets under control by jetties of the proper length and spacing and the provision of sand by-passing arrangements to move the net drift past the jettied inlets; the elements of a long-term solution are already being constructed in phases at Absecon Inlet at Atlantic City. The proper control of the inlets will not only aid in the shore stabilization but will also reduce the cost of maintenance of the navigation channels in the inlets.

## Summary

33. From the material presented above, it can be seen that the beach face is an active zone with a constant shuttling of the sands back and forth along the shore. Permanent beach losses occur along most of our shores due to: (a) an excess of littoral drift leaving a given sector; (b) material lost inland by the tidal action of inlets; and (c) material pulled offshore into deep water by local storms. The first two losses are permanent losses; the third loss may or may not be permanent depending on local conditions. The continuing erosion of a beach narrows the beach to where it is extremely vulnerable to severe erosion and wave overtopping during severe storms. The proper solutions to our many shore erosion problems involve basically a quantitative evaluation of the overall processes causing the erosion followed by the development of a plan to either prevent or to compensate for these losses. Much research still needs to be done to enable proper qualitative evaluation to be made in most coastal areas.

## References

1. "Shore Protection Planning and Design," U. S. Army Beach Erosion Board, Technical Report No. 4 (1961), Chapter 1.
2. Ibid., Chapter 1.
3. Ibid., Chapter 1.
4. Galvin, C. J. "Experimental and Theoretical Study of Longshore Currents on a Plane Beach," doctoral thesis, Department of Geology and Geophysics, MIT, 1963 (unpublished).
5. Caldwell, J. M. "Wave Action and Sand Movement near Anaheim Bay, California," U. S. Army Beach Erosion Board, Technical Memorandum No. 68 (1956).
6. Watts, G. M. "Development and Field Tests of a Sampler for Suspended Sediment in Wave Action," U. S. Army Beach Erosion Board, Technical Memorandum No. 34 (1953).
7. Saville, T., Jr. "North Atlantic Coast Wave Statistics Hindcast by Bretschneider (rev)—Sverdrup-Munk Method," U. S. Army Beach Erosion Board, Technical Memorandum No. 55 (1954).

# WAVES AND TIDES IN COASTAL PROCESSES 

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## Introduction

It is naturally impossible to compress the broad developments of water-wave theory and of the related laboratory and field observations on wave phenomena into a manageable presentation in a few pages. The task of this presentation on coastal phenomena has therefore been conceived as one to give a first basis of the elements of wave theory in order to establish certain definitions as a means of communication and then to concentrate on some of the tidal phenomena of primary engineering interest in recent years. It is in the latter phase that certain contributions to knowledge have originated from the author and some of his colleagues, thus providing the personal relation to the material, which hopefully will enhance this presentation.

## A. Principles of Wave Motion-Auxiliary Notes

## 1. Properties of Single Wave Trains

As a first approximation water-wave motion is described by the "small amplitude wave theory." This is the classical approach extremely useful in many engineering problems, but failing in others, such as the breaking of waves, which must be described by more refined "finite amplitude" approaches. The basic assumptions of the "small amplitude" theory are those inherent generally in irrotational motion and in addition, that all motions are small enough so that terms containing the squares of velocities may be neglected in the equations of motion. The available equations are therefore for the unsteady motion considered, with density constant throughout:

$$
\begin{equation*}
-\frac{\partial \phi}{\partial \mathrm{t}}+\frac{\mathrm{p}}{\rho}+\mathrm{gz}=0 \tag{1}
\end{equation*}
$$

derived from Newton's law, and:

[^2]\[

$$
\begin{equation*}
\frac{d u}{\partial x}+\frac{\partial w}{\partial z}=0=-\left(\frac{\partial^{2} \phi}{\partial x^{2}}+\frac{\partial^{2} \phi}{\partial z^{2}}\right) \tag{2}
\end{equation*}
$$

\]

derived from continuity.
The introduction of the velocity potential $\phi$ has reduced the number of variables by one, since $u=-\frac{\partial \phi}{\partial x}$ and $w=-\frac{\partial \phi}{\partial z}$ by definition.

With appropriate boundary conditions these equations can be solved:

$$
\begin{aligned}
& \text { at the surface, where } \frac{p}{\rho}=0: \eta=\frac{1}{\mathrm{~g}}\left(\frac{\partial \phi}{\partial \mathrm{t}}\right) \mathrm{t}=\eta \\
& \text { at the bottom, where } \mathrm{z}=-\mathrm{h}: \mathrm{w}=-\frac{\partial \phi}{\partial z}=0
\end{aligned}
$$

The periodic solutions with the notations of Fig. 1a are:

$$
\begin{align*}
& \text { for the surface: } \eta=a \sin (k x-\sigma t)  \tag{3}\\
& \text { for the pressure at any depth: } \frac{p}{\gamma}=\eta \frac{\cosh k(h+z)}{\cosh k h}-z  \tag{4}\\
& \text { for the velocity } u: u=\frac{a g k}{\sigma} \frac{\cosh k(h+z)}{\cosh k h} \sin (k x-\sigma t)  \tag{5}\\
& \qquad w=\frac{a g k}{\sigma} \frac{\sinh k(h+z)}{\cosh k h} \cos (k x-\sigma t) \tag{6}
\end{align*}
$$

wherein:

$$
\begin{aligned}
& \mathrm{k}=\frac{2 \pi}{\mathrm{~L}} \text { the "wave number" } \\
& \sigma=\frac{2 \pi}{\mathrm{~T}} \text { the "wave frequency" }
\end{aligned}
$$

hence:

$$
\begin{aligned}
\frac{\sigma}{\mathrm{k}}=\frac{\mathrm{L}}{\mathrm{~T}}= & \mathrm{C} \text { the "wave velocity" as the ratio of wave } \\
& \text { length } \mathrm{L} \text { to wave period } \mathrm{T} .
\end{aligned}
$$

The most important implications of these particular solutions may be summarized as follows:


Fig. 1a.-Smali Amplitude Wave Notations


Fig. 1b.-Standing Wave in Channel or Length Less Than One Quarter Wave Length
1.1 The wave profile: $\eta=\mathrm{a} \sin (\mathrm{ks}-\sigma \mathrm{t})$ represents a periodic variation of the surface at any given station $x$ between the limits +a and -a for time intervals equal to the wave period. The same amplitude $\eta$ is present at the same instant of time at all stations ( $\mathrm{x}+\mathrm{nL}$ ) wherein $\mathrm{n}=1,2,3, \ldots \ldots$.
1.2 If we move along with the wave speed, i.e., we are attaching ourselves to a point so that $\eta=$ const., then ( $k x-\sigma t$ ) $=$ constant.

Hence:

$$
\mathrm{x}=\frac{\sigma}{\mathrm{k}} \mathrm{t}+\text { const. }
$$

$$
\frac{\mathrm{dx}}{\mathrm{dt}}=\mathrm{C}=\frac{\sigma}{\mathrm{k}}=\frac{\mathrm{L}}{\mathrm{~T}}
$$

The wave velocity is in the positive x direction and the profile represents a "progressive wave" moving to the right. Consequently, a progressive wave moving in the opposite direction is obtained simply by reversing the sign of $\sigma t$, i.e.,

$$
\eta=\mathrm{a} \sin (\mathrm{kx}+\sigma \mathrm{t})
$$

1.3 A general expression for the wave velocity $C$ is obtained from the equations by observing that the vertical velocity component w at the surface $(z=0)$ is approximately equal to $\partial \eta / \partial t$. This results in:

$$
\begin{equation*}
C=\sqrt{g h}\left(\frac{\tanh k h}{k h}\right)^{1 / 2} \tag{7}
\end{equation*}
$$

or

$$
\begin{equation*}
\mathrm{C}=\sqrt{\frac{\mathrm{gL}}{2 \pi}}\left(\tanh \frac{2 \pi \mathrm{~h}}{\mathrm{~L}}\right)^{1 / 2} \tag{8}
\end{equation*}
$$

For depths $h$ in excess of one-half of wave length $L\left(\frac{h}{L}>1 / 2\right)$ the hyperbolic tangent is close to unity and the velocity of "deep water waves" is obtained:

$$
\begin{equation*}
C *=\sqrt{\frac{g \mathrm{~L}}{2 \pi}} \tag{9}
\end{equation*}
$$

For small depths $h$ of the order of $5 \%$ of wave length $L$ the hyperbolic tangent assumes the value of k h and the velocity of "shallow water waves" results:

$$
\begin{equation*}
\mathrm{C}_{\mathrm{o}}=\sqrt{\mathrm{gh}} \tag{10}
\end{equation*}
$$

1.4 The wave length generally can be derived from equation (8) when the wave period is known, since $\mathrm{C}=\frac{\mathrm{L}}{\mathrm{T}}$

$$
\begin{equation*}
\mathrm{L}=\frac{\mathrm{g} \mathrm{~T}^{2}}{2 \pi} \tanh \frac{2 \pi \mathrm{~h}}{\mathrm{~L}} \tag{11}
\end{equation*}
$$

For given depths, however, this equation must be solved by trial or reference to wave tables.

$$
\begin{align*}
& \text { For "deep water" conditions : } \mathrm{L}^{*}=\frac{\mathrm{gT}^{2}}{2 \pi}=5.12 \mathrm{~T}^{2}  \tag{12}\\
& \text { and for "shallow water" }: \mathrm{L}_{o}=\mathrm{T} \sqrt{\mathrm{gh}} \tag{13}
\end{align*}
$$

1.5 The paths of individual fluid particles as a wave passes may be derived by integration with time of equations (5) and (6). The total orbit of a particle is described generally by an ellipse of horizontal axis 2 A and of vertical axis 2 B , which are given by:

$$
\begin{align*}
& A=a \frac{\cosh k(h+z)}{\sinh k h}  \tag{14}\\
& B=a \frac{\sinh k(h+z)}{\sinh k h} \tag{15}
\end{align*}
$$

It is of interest to note that for deep water waves (see definition under 1.3) with good approximation:

$$
\begin{equation*}
\mathrm{A}=\mathrm{B}=\mathrm{a} \mathrm{e}^{\mathrm{k} z} \tag{16}
\end{equation*}
$$

which indicates circular orbits of exponentially decreasing amplitude.
For shallow water waves, however:

$$
\begin{align*}
& A=a \frac{1}{k h}=a \frac{L}{2 \pi h}  \tag{17a}\\
& B=a \frac{k(h+z)}{k h}=a\left(1+\frac{z}{h}\right) \tag{17b}
\end{align*}
$$

The maximum horizontal "excursion" of a water particle regardless of its vertical location is therefore in shallow water:

$$
\begin{equation*}
2 \mathrm{~A}=\frac{1}{\pi}\left(\frac{\mathrm{a}}{\mathrm{~h}}\right) \mathrm{L} \tag{18}
\end{equation*}
$$

1.6 The horizontal velocity $u$ is seen to be "in phase" with amplitude $\eta$, velocities are positive for positive values of $\eta$ and vice versa.

For shallow water waves equation (5) reduces to:

$$
u=\frac{a g k}{\sigma} \sin (k x-\sigma t)
$$

with equation (10)

$$
\begin{equation*}
\mathrm{u}=\frac{\mathrm{a}}{\mathrm{~h}} \mathrm{C}_{\mathrm{o}} \sin (\mathrm{kx}-\sigma \mathrm{t}) \tag{19}
\end{equation*}
$$

Maximum velocities are therefore quickly computed from:

$$
\mathrm{u}_{\mathrm{o}}=\frac{\mathrm{a}}{\mathrm{~h}} \mathrm{C}_{\mathrm{o}}
$$

1.7 The total energy of a wave can be obtained from the potential and kinetic energies integrated over the depth and over a wave length utilizing equations (3) to (6). The result, expressed as average energy per unit surface area, is:

$$
\begin{equation*}
\mathrm{E}=\mathrm{PE}+\mathrm{KE}=\gamma \frac{\mathrm{a}^{2}}{4}+\gamma \frac{\mathrm{a}^{2}}{4}=\gamma \frac{\mathrm{a}^{2}}{2} \tag{20}
\end{equation*}
$$

1.8 Through developments beyond the scope of this review it can be shown that energy is propagated at a velocity less than the wave or phase velocity C given by equation (8). This so-called "group velocity" $\mathrm{C}_{\mathrm{G}}$ has a minimum value of $1 / 2 \mathrm{C}$ in deep water and approaches C for shallow water waves. It is expressed as a function of depth and wave number:

$$
\begin{equation*}
C_{G}=C \frac{1}{2}\left(1+\frac{2 \mathrm{kh}}{\sinh 2 \mathrm{kh}}\right) \tag{21}
\end{equation*}
$$

## 2. Superposition of Waves

Waves given by the harmonic expressions in the preceding section can be superimposed and complex waves may be built up by this process. As a corollary, complex systems of waves observed in nature may be analyzed by determining the harmonic components, differing in frequency, amplitude and phase angles. This superposition applies to amplitudes, pressures and velocities, however to wave energies only under special conditions.
2.1 Waves traveling in the same direction usually at different velocities will give rise to amplification and to interference, as waves move through each other. A few simple systems may be presented for illustration:
a. Two waves of amplitudes $\mathrm{a}_{1}$ and $\mathrm{a}_{2}$ of the same frequency with a difference in phase $\delta$ will be given by:

$$
\begin{equation*}
\eta_{\text {tot. }}=\mathrm{a}_{1} \sin (\mathrm{kx}-\sigma \mathrm{t})+\mathrm{a}_{2} \sin (\mathrm{kx}-\sigma \mathrm{t}+\delta) \tag{22}
\end{equation*}
$$

if $\delta=0$ : wave will be amplified and

$$
\eta_{\text {tot. }}=\left(\mathbf{a}_{1}+\mathbf{a}_{2}\right) \sin (\mathrm{kx}-\sigma \mathrm{t})
$$

if $\delta=180^{\circ}$ : wave will be decreased by interference and $\eta_{\text {tot. }}=\left(\mathrm{a}_{1}-\mathrm{a}_{2}\right) \sin (\mathrm{kx}-\sigma \mathrm{t})$
if $\delta=90^{\circ}$ : wave will be intermediate and $\eta_{\text {tot. }}=a_{1} \sin (k x-\sigma t)+a_{2} \cos (k x-\sigma t)$
b. Two waves traveling in the same direction, but having different amplitudes $\mathrm{a}_{1}$ and $\mathrm{a}_{2}$ and different frequencies $\sigma_{1}$ and $\sigma_{2}$, thus

$$
\eta_{\text {tot. }}=a_{1} \sin \left(k_{1} x-\sigma_{1} t+\delta_{1}\right)+a_{2} \sin \left(k_{2} x-\sigma_{2} t+\delta_{2}\right)
$$

The resulting wave is not harmonic, but periodic since it is readily seen that after a certain time interval $\mathrm{T}_{\mathrm{b}}$ the same relative constellation of the waves must recur. This time is given by:

$$
\begin{equation*}
\mathrm{T}_{\mathrm{b}}=\frac{2 \pi}{\sigma_{2}-\sigma_{1}}=\frac{\mathrm{T}_{2} \mathrm{~T}_{1}}{\mathrm{~T}_{1}-\mathrm{T}_{2}} \tag{23}
\end{equation*}
$$

An envelope drawn of the maximum values of $\eta_{\text {tot. }}$ shows no zero values for ( $\eta_{\text {toot. }}$ ) max and the phenomenon is known as an "incomplete beat." However, this so-called "beat effect" becomes very pronounced when we let the amplitudes $a_{1}$ and $a_{2}$ become equal and the frequency difference ( $\sigma_{2}-\sigma_{1}$ ) become very small. In this case equation (22) can be reduced to:

$$
\begin{equation*}
\eta_{\mathrm{tot} .}=2 \mathrm{a} \cos \left(\frac{\sigma_{2}-\sigma_{1}}{2}\right) \mathrm{t} \sin \left(\frac{\sigma_{2}+\sigma_{1}}{2}\right) \mathrm{t} \tag{24}
\end{equation*}
$$

It is seen that maximum values of $\eta_{\text {tot. }}$ vary from 0 to 2 a as the cosine term slowly changes with the small differential frequency, while the sine term passes through many cycles with the basic frequency $\frac{\sigma_{2}+\sigma_{1}}{2}$. Values of $\eta_{(t o t) \text { max. }}=0$ represent the nodic points of the beat.
2.2 One of the most important aspects of shore line structures is their property of wave reflection to various degrees. Such reflection also occurs at steep changes of the bottom elevation or by channel contraction. In the simplest case we may assume here for illustration that a wave of amplitude $a_{1}$, is partly reflected by a vertical barrier,
thus that the reflected wave has an amplitude $\mathrm{a}_{2}$. The two waves may then be given by:

$$
\begin{equation*}
\eta_{\text {tot. }}=\mathrm{a}_{1} \sin (\mathrm{kx}-\sigma \mathrm{t})+\mathrm{a}_{2} \sin (\mathrm{kx}+\sigma \mathrm{t}) \tag{25}
\end{equation*}
$$

adding and subtracting $\mathrm{a}_{1} \sin (\mathrm{kx}+\sigma \mathrm{t})$ :

$$
\begin{equation*}
\eta_{\text {tot. }}=2 a_{1} \sin k x \cos \sigma t-\left(a_{1}-a_{2}\right) \sin (k x+\sigma t) \tag{26}
\end{equation*}
$$

representing a progressive wave of amplitude ( $\mathrm{a}_{1}-\mathrm{a}_{2}$ ) being reflected plus a standing wave of amplitude $2 \mathrm{a}_{1}$. Equation (25) can also be resolved into the alternate form:

$$
\begin{equation*}
\eta_{\text {tot. }}=\left(\mathrm{a}_{1}+\mathrm{a}_{2}\right) \sin \mathrm{kx} \cos \sigma \mathrm{t}-\left(\mathrm{a}_{1}-\mathrm{a}_{2}\right) \cos \mathrm{kx} \sin \sigma \mathrm{t} \tag{27}
\end{equation*}
$$

This latter form represents 2 standing waves, however, with nodal points displaced by $\mathrm{kx}=\frac{\pi}{2}$. Therefore the envelope of the combined system must pass through ( $a_{1}+a_{2}$ ) maxima and ( $a_{1}-a_{2}$ ) minima, which remain fixed with respect to $x$. When $\mathrm{a}_{1}=\mathrm{a}_{2}$ a single standing wave results from either equation (26) or (27) and the envelope shows maximum amplitudes 2 a at the anti-nodes and zero amplitudes at the nodal points, characteristic of the true standing waves (see Fig. 1b).

It can readily be shown that horizontal velocity components, $u$, reach maximum values under the nodes and are zero at the antinodes, while vertical velocities, w , are reversed in this respect.

It is clear that $u$ and $w$ retain finite values if $\mathrm{a}_{1}$ is not equal to $\mathrm{a}_{2}$.
2.3 Reflection phenomena in the general case become rather complex and are not completely resolved theoretically. Hence experimental evidence must be depended upon in many cases. For a barrier partly reflecting an incoming wave $\mathrm{a}_{1}$, however, the preceding may be used to define a reflection coefficient $\mathrm{K}_{\mathrm{r}}$ from the wave envelope as indicated by equations (25) to (27). Considering no energy loss at the reflection point and requiring that the sum of incoming wave amplitude $\mathrm{a}_{1}=\mathrm{a}_{1}$ and of reflected wave amplitude $\mathrm{a}_{2}=\mathrm{a}_{\mathrm{r}}$ be equal to the transmitted wave amplitude $a_{t}$ the reflection coefficient becomes:

$$
\begin{equation*}
\mathrm{K}_{\mathrm{r}}=\frac{\mathrm{a}_{2}}{\mathrm{a}_{1}}=\frac{\mathrm{a}_{\mathrm{r}}}{\mathrm{a}_{\mathrm{i}}} \tag{28}
\end{equation*}
$$

and the transmission coefficient is defined as:

$$
\begin{equation*}
\mathrm{K}_{\mathrm{t}}=\frac{\mathrm{a}_{\mathrm{t}}}{\mathrm{a}_{\mathrm{i}}}=1+\mathrm{K}_{\mathrm{r}} \tag{29}
\end{equation*}
$$

When $\mathrm{a}_{2}=\mathrm{a}_{1}$ obviously $\mathrm{K}_{\mathrm{r}}=1$, when $\mathrm{a}_{2}<\mathrm{a}_{1}$ these amplitudes may be determined from the fixed amplitude envelope giving ( $a_{1}+a_{2}$ ) and $\left(a_{1}-a_{2}\right)$.

## 3. Transformation of Wave Properties

Inherent in the fundamental properties of waves discussed under sections 1 and 2 are the tools by which changes in the character of wave systems may be analyzed. Implicit in equation (8) for the wave velocity, for example, is the fact that long waves will "outrun". shorter waves. From a storm-generated random wave system over the ocean after a long distance the long waves will become segregated and with little damping arrive first near the shores. With the effect of depth becoming pronounced these waves will tend to assume smaller angles relative to the shore line, their lengths will decrease and their amplitudes will increase, as they transform from deep water to shallow water waves. This refraction process can be followed by methods derived from the equations with various refinements considering the simultaneous damping by internal and especially boundary resistance. When waves encounter barriers or are of limited extent transverse to their direction of travel, energy is dispersed laterally or diffracted. For further discussion of wave-decay, diffraction, refraction and reflection the extensive literature must be consulted. In the following discussion more will be said about these problems in relation to their bearing on beach processes. For the present, therefore, the discussion will be centered more on the properties of very long waves in shallow water with particular emphasis on tidal waves.
3.1 Transformation without change of energy (2). Assuming initially no energy loss and no reflection, the rate of energy transmission is readily obtained from the product of equations (20) and (21). Considering in addition the possible variation in width $\mathbf{B}$ of the confining boundaries between station (0) and station (x) along the direction of travel:

$$
B_{0} E_{0} C_{G_{0}}=B_{x} E_{x} C_{G_{x}}
$$

or

$$
\begin{equation*}
\mathrm{B}_{0} \mathrm{a}_{0}{ }^{2} \mathrm{C}_{\mathrm{G}_{0}}=\mathrm{B}_{\mathrm{x}} \mathrm{a}_{\mathrm{x}}{ }^{2} \mathrm{C}_{\mathrm{G}_{\mathrm{x}}} \tag{30}
\end{equation*}
$$

For shallow water waves the wave velocity $\mathrm{C}=\sqrt{\mathrm{gh}}$ will be equal to the group velocity Ca. Hence, equation (30) may be reduced to the so-called Green's Law:

$$
\begin{equation*}
\frac{a_{x}}{a_{0}}=\left(\frac{B_{0}}{B_{x}}\right)^{1 / 2}\left(\frac{h_{0}}{h_{x}}\right)^{1 / 4} \tag{31}
\end{equation*}
$$

which holds strictly, however, only if the reach of the gradual transition is larger than the wave length. It is seen that amplitudes increase with decreasing depth and hence decreasing wave length, while decreasing width affects amplitudes even stronger. As high amplitudes are attained the "small amplitude" theory must give way to the more appropriate, but highly complex "finite amplitude" theory. Eventually the breaking of waves presents a special problem, as yet inadequately approached. The effects of bottom resistance by friction and porosity also become more important with the "steepening" of the wave. In the following the effect of damping will be considered for very long, shallow water waves.
3.2 Transformation with energy dissipation for shallow water waves (2). Severe difficulties are encountered with the introduction of boundary friction into the general wave equations. Small amplitudes are assumed, and the resistance term must be linearized as first utilized by the prominent Dutch physicist H. A. Lorentz in planning the closure schemes of the Zuider Zee. The solution can then be stated for a channel of uniform depth with a maximum reference amplitude $a_{0}$ (for $\mathrm{x}=0$ ) as a harmonic function:

$$
\begin{equation*}
\eta=\mathrm{a}_{0} \mathrm{e}^{-\mu \mathrm{x}} \cos (\sigma \mathrm{t}-\mathrm{k} \mathrm{x}) \tag{32}
\end{equation*}
$$

$\mu$ represents a damping constant and the wave number k also is dependent on $\mu$. Maximum amplitudes along x decrease exponentially. $\mu$ and k are related by:

$$
\begin{align*}
& \tan 2 \alpha=\frac{\mathrm{f}}{3 \pi} \frac{\mathrm{u}_{\mathrm{o}}}{\sigma \mathrm{~h}}  \tag{33a}\\
& \tan \alpha=\frac{\mu}{\mathrm{k}}  \tag{33b}\\
& \mathbf{k}_{0}=\frac{\sigma}{\sqrt{\mathrm{gh}}}=\mathrm{k}\left[1-\left(\frac{\mu}{\mathrm{k}}\right)^{2}\right]^{1 / 2} \tag{33c}
\end{align*}
$$

hence all unknowns $\alpha, \mu$ and k are defined by the quantities:

$$
\begin{aligned}
\mathrm{u}_{o} & =\text { maximum velocity at the channel entrance } \\
\mathrm{f} & =\text { Darcy-Weisbach coefficient of resistance } \\
\mathbf{h} & =\text { channel depth } \\
\sigma & =\text { tidal frequency }=\frac{2 \pi}{\mathrm{~T}} \\
\mathrm{C}_{o} & =\sqrt{\mathrm{gh}}=\text { wave velocity without friction }
\end{aligned}
$$

The instantaneous velocity for any section is obtained for a given distance x from the entrance by:

$$
\begin{equation*}
\mathrm{u}=\frac{\mathrm{a}_{0}}{\mathrm{~h}} \mathrm{C}_{0} \mathrm{e}^{-\mu \mathrm{x}} \frac{\mathrm{k}_{0}}{\sqrt{\mu^{2}+\mathrm{k}^{2}}} \cos (\sigma \mathrm{t}-\mathrm{kx}+\alpha) \tag{34}
\end{equation*}
$$

The maximum value of $u$ therefore also decreases with $x$ as does the maximum value of $\eta$ for any given $x$. The occurrence of the maximum velocity is displaced in time with respect to maximum tidal amplitude by a time angle $\alpha$ dependent on frictional effects. The wave velocity $C$ is also reduced over the value $C_{o}=\sqrt{g h}$ since:

$$
\begin{equation*}
\frac{C}{C_{0}}=\frac{k_{0}}{k}=\frac{L}{L_{0}}=\frac{1}{\sqrt{1+\left(\frac{\mu}{k_{0}}\right)^{2}}} \tag{35}
\end{equation*}
$$

For example, this ratio is for the: Delaware Estuary . 94
Cape Cod Canal .91
WES Tidal Flume . 78 to .80
It should finally be noted that in this approach valid for tidal wave phenomena the flow at any instant is assumed as a shear flow extending over the entire depth as in uniform flow in open channels. For shorter wave lengths certain boundary layer solutions for unsteady motion have been obtained, which should be referred to in reference (1).

## B. Tidal Wave Characteristics in Estuaries

4.1 With the elements of wave theory developed toward the end of the last section various tidal wave problems may now be approached
analytically. For the general case, the basic differential equations of motion must be used directly since the cross sections may vary widely along the estuary channel. For this condition two methods of numerical analysis are available: numerical integration by finite differences (greatly aided now by computers) and solutions by the method of characteristics (3) (4) (5). The difficulty for both methods is primarily with the proper adjustment for friction effects and with results limited to tidal elevations and velocities. Extensive observations are therefore needed to check out the existing tides and then to proceed from there to predict the changes anticipated from engineering measures modifying the channel geometry or flow characteristics.
4.2 Whenever possible therefore, solutions by harmonic analysis are preferred which give general solutions for the entire estuary in terms of the basic wave components developed in equations (32) to (35) by superposition. Such solutions exist now for the Thames, Bay of Fundy (6) and the Delaware (7) estuaries. The method will be illustrated by application to the simple case of a rectangular tidal channel at the Waterways Experiment Station (8). The basic assumptions are:
a. A reference tide at the estuary mouth and complete reflection at the upstream end.
b. a (amplitude) $<\mathrm{h}$ (depth) $<\mathrm{B}$ (width) $<l$ (length of estuary).
c. Tidal wave length $L$ is of the order of the length $l$ of the estuary or larger.
d. Salinity variations and fresh-water flow give small velocities as compared to the tide.

Some estuaries show little variation in cross sections, others may be approximated by an exponential decrease in section to the tidal limit, where reflection takes place due to a dam or rapidly decreasing depth and width. In the WES flume complete reflection may be assumed as also proved acceptable for the Bay of Fundy and the Delaware. Since the damping constant $\mu$ must be the same for the incoming tidal wave and the reflected wave the equations of the combined system for a channel of constant depth and width are:

$$
\begin{equation*}
\eta=\eta_{1}+\eta_{2}=a_{o}\left[e^{-\mu x} \cos (\sigma t-k x)+e^{\mu x} \cos (\sigma t+k x)\right] \tag{36}
\end{equation*}
$$

$u=u_{1}+u_{2}=\frac{a_{0} C_{0}}{h} \frac{k_{0}}{\sqrt{\mu^{2}+\mathrm{k}^{2}}}\left[e^{-\mu x} \cos (\sigma t-k x+\alpha)\right.$

$$
\begin{equation*}
\left.-\mathrm{e}^{\mu \mathrm{x}} \cos (\sigma \mathrm{t}+\mathrm{kx}+\alpha)\right] \tag{37}
\end{equation*}
$$

The constants can be evaluated from maximum tidal stage observations only as a function of $x$.
4.3 The results can be introduced into the energy balance at the estuary mouth, expressing the difference between the energy of the incoming and the reflected wave as the energy dissipated in the channel of length $l$. The average rate of dissipation per unit mass in the estuary is given by:

$$
\begin{equation*}
\mathrm{G}=\frac{\mathrm{gC}_{0} \mathrm{a}_{0}{ }^{2}}{\mathrm{~h} l} \frac{\mathrm{k}_{0}}{\sqrt{\mu^{2}+\mathrm{k}_{0}^{2}}} \sinh 2 \mu l \tag{38}
\end{equation*}
$$

while the local rate of dissipation $G_{x}$ may be expressed by the ratio:

$$
\begin{equation*}
\frac{\mathrm{G}_{s}}{\mathrm{G}}=\frac{1-\frac{\sinh 2 \mu \mathrm{x}}{\sinh 2 \mu l}}{1-\frac{\mathrm{x}}{l}} \tag{39}
\end{equation*}
$$

The amplitude $\mathrm{a}_{0}$ represents here one half of the maximum tidal amplitude at the reflecting end of the estuary as observed. It may be replaced by the maximum tidal amplitude at the ocean end of the estuary $\eta_{1 \mathrm{H}}$ from:

$$
\begin{equation*}
\eta_{l \mathrm{H}}=\mathrm{a}_{\mathrm{o}} \sqrt{2(\cos 2 \mathrm{k} l+\cosh 2 \mu l)} \tag{40}
\end{equation*}
$$

It is seen that in addition to the geometric properties h and $l$ only the existing stages need to be introduced from observations to define the damping terms, whereupon velocities and dissipation may be computed. This is the advantage of the harmonic analysis over the other methods, in which representative values of velocities and friction factors must be estimated. Further developments of this method for estuary sections exponentially decreasing from the mouth have been applied to the Delaware.
4.4 Extensive experimental results as well as field data confirm the value of the harmonic analysis. The tidal wave behavior in a rectangular channel was studied with this approach in order to check the
possible effects of salinity and of freshwater flow on the tidal velocities and stages. It was shown (8) (9) that the superposition of an incoming and reflected wave with proper damping coefficients as stated by equations (36) and (37) matched the experimental results with sufficient accuracy to warrant computation of tidal energy dissipation by equation (38). The influence of freshwater flow and of salinity remained of small consequence, however, it produced a slight rise in the mean level of the surface with the increase in these quantities toward the landward end of the channel. Damping factors $\mu$ were increased as expected from equation (33a) as the maximum computed tidal velocity $u_{o}$ increased with amplitude a a . Fig. 2 illustrates the entire set of results. The maximum tidal amplitudes at any station x are plotted here in terms of the maximum amplitude at the reflecting end of the channel (landward) versus the local time of high water measured again from the time of high water at the reflecting end. For any test (with constant amplitude of ocean tide) the points are close to a line of constant $\phi=2 \pi\left(\frac{\mu}{\mathrm{k}}\right)$. This also illustrates the method of determining $\frac{\mu}{\mathrm{k}}$ from the tidal observations. The primary value of the harmonic method lies in the possibility to predict changes in energy dissipation with tidal changes and will become evident from the following discussion on salinity intrusion.


Fig. 2.-Determination of Damping Coefficient $\mu$ and of Wave Number $k$ from Tmal Elevations (8)

## C. Salinity Intrusion in Estuaries

5.1 The general problem of salinity intrusion in tidal estuaries is one of extreme importance with respect to sedimentation and pollution. Two extreme cases may be illustrated by Fig. 3. The upper sketch shows the type referred to as "fully stratified" exhibiting a wedge of salt water, which remains unmixed with the fresh water flowing over it out to sea. The interface or boundary between the two layers is stable even though it may move slowly to and fro with the tides. In this country the Mississippi mouth contains such a salinity wedge. Its shape can be completely described and the length of intrusion can also be computed. The longitudinal pressure forces are indicated at the ends and give rise to a moment resulting in internal circulation within the wedge. Note, however, that the salt water flow $Q_{s}$ must be compensating in downstream and upstream direction for a stationary wedge so that no net saltwater flow exists.

The lower sketch of Fig. 3 by contrast illustrates the extreme of salinity conditions at the opposite end of the spectrum. Severe tidal currents have generated sufficient turbulence to overcome the stabi-


WELL MIXED ESTUARY


Fig. 3.-Schematic Representation of Salinity Intrustons in Estuaries (10)
lizing effects of the density difference and to produce a state of mixing between the salt and the fresh water which may be termed "well mixed," i.e., the vertical salinity differences are small. The sketch represents a state which is the result of looking at the time average of the current and salinity pattern rather than at the instantaneous patterns. The salinities are given as average salinities plotted vertically from the bottom and the average salinity varies from the maximum at the entrance section to zero at the toe of the intrusion. The static forces at the ends still produce upstream bottom velocities and a general circulation. Due to the mixing there is no "interface" as before, but a line of zero velocity (temporal mean) separates zones of adverse end of seaward flow. While it was shown in the preceding section that this temporal mean flow engendered by density differences has no effect on the much more intense tidal currents, the density underflow is of extreme importance for sedimentation. It retains all material settling into the bottom zones in the estuary, if it is too heavy to be lifted into the upper seaward flow. The presence of large amounts of silt in all estuaries is evidence of this mechanism and its detailed exploration is thus of extreme importance to the planning of engineering works in estuaries.
5.2 The two-dimensional aspects of this internal circulation may be illustrated by presenting the results of an experiment carried out in the WES tidal flume as analyzed at MIT (10). With an established tidal condition and freshwater flow, instantaneous velocity and salinity measurements were carried out at various stations from the ocean end at Sta. 5 ( 5 ft . from end) to Sta. 240 ( 240 feet from end). The velocity measurements were then averaged over several tidal cycles and plotted in Fig. 4 in terms of the mean freshwater velocity $\mathrm{U}_{\mathrm{f}}$ against depth. The existence of strong upstream currents is apparent through Sta. 120 over a sizable portion of the section. Similarly, mean salinities are given in Fig. 5 in terms of the ocean salinity $\mathrm{S}_{\circ}$ versus depth exhibiting vertically an increase with depth and longitudinally decreasing values from Sta. 5 to Sta. 160. Large vertical gradients exist over the central portion of the intrusion length. These large gradients of salinity and hence density are responsible for the form of the distribution of horizontal velocities shown again in different form in Fig. 6. Values derived from Fig. 4 for various constant elevations in the stream are plotted versus distance and show maximum upstream components near the bottom between Sta. 40 and 80 and correspondingly strong downstream


Fig. 4.-Ttme-Average Horizontal Velocity Distribution (10)


Fig. 5.-Time-Average Salinity/Basin Salinity (10)
velocities near the surface. This pattern of mean velocity distribution is related to the corresponding mean salinity profiles of Fig. 7, in which the strong longitudinal gradients of salinity between Sta. 40 and 80 are evident.


Fig. 6.-Mean Velocities along Channel at Various Depths (10)
Similar distributions, particularly of the typical velocity profiles of Fig. 4, have been observed in practice and have been presented by Simmons (11) as so-called "flow predominance curves." The correlation in many estuaries with shoaling patterns is obvious and has led to the planning of new techniques in coping with sedimentation in estuaries.
5.3 One-dimensional solution of salinity intrusion. Theoretical approaches on a two-dimensional basis to predict the behavior of salinity and internal velocities from the governing parameters of channel geometry, tidal wave motion, and freshwater flow are faced with as yet unresolved difficulties. The only assumption which leads to an approximate solution is the reduction of the general diffusion equation to a one-dimensional form, which eliminates the gradients of salinity in the vertical direction and considers uniform velocities over the vertical only. Salinity and velocity vary only with time and distance. The equation therefore reduces to the form:

$$
\begin{equation*}
\frac{\partial s}{\partial t}+\left(u_{x, t}-U_{f}\right) \frac{\partial s}{\partial x}=-\frac{\partial}{\partial x}\left(\overline{u^{\prime} s^{\prime}}\right)=\frac{\partial}{\partial x}\left(D_{x}^{\prime} \frac{\partial s}{\partial x}\right) \tag{41}
\end{equation*}
$$

It is noted that the tidal velocity $\mathrm{u}_{\mathrm{x}, \mathrm{t}}$ averages out to zero over a tidal cycle and the average salinity s will not change with time for constant $\mathrm{U}_{\mathrm{f}}$ and constant tidal conditions. Therefore, averaging the equation with respect to time, the corresponding terms become zero and the equation reduces to:


Fig. 7.-Time-Average Salinity versus Distance from Ocean (10)

$$
\begin{equation*}
-\mathrm{U}_{\mathrm{f}} \frac{\partial \stackrel{\rightharpoonup}{s}}{\partial \mathrm{x}}=\frac{\partial}{\partial \mathrm{x}}\left(\mathrm{D}_{\mathrm{x}}^{\prime} \frac{\partial \bar{s}}{\partial \mathrm{x}}\right) \tag{42}
\end{equation*}
$$

This equation states that changes in the mean salinity due to $\mathrm{U}_{\mathrm{f}}$ are prevented by the diffusion upstream of salinity due to the diffusion coefficient $\mathrm{D}_{\mathrm{x}}{ }^{\prime}$ which embodies all turbulent and convective diffusion processes. The equation has been integrated (8) and put into the final
form describing the mean salinity distribution within the intrusion length at low tide:

$$
\begin{equation*}
-\ln \left(\frac{\overline{\mathrm{s}}}{\mathrm{~S}_{0}}\right)=\frac{\mathrm{U}_{\mathrm{f}}}{2 \mathrm{BD}_{0}^{\prime}}\left(\mathrm{x}_{l}+\mathrm{B}\right)^{2} \tag{43}
\end{equation*}
$$

This equation is based on the assumption that the diffusion coefficient $\mathrm{D}_{\mathrm{x}}{ }^{\prime}$ is inversely proportional to distance from the ocean end, where it reaches a maximum value of $\mathrm{D}_{0}^{\prime}$. B is a constant to be derived from the observation of minimum values of $\bar{s}$ at low tide. Both constants $\mathrm{D}_{0}{ }^{\prime}$ and B are thus subject to definition from a few observations. It has also been shown that complete correlations can be obtained by relating $D_{0}{ }^{\prime}$ to tidal energy dissipation $G$ and the mixing energy rate $J$ expended on the freshwater flow through the estuary. The expression for J is:

$$
\begin{equation*}
\mathrm{J}=\frac{\Delta \gamma}{\gamma} \mathrm{gh} \frac{\mathrm{U}_{\mathrm{f}}}{l} \tag{44}
\end{equation*}
$$

Since $G$ is given by equation (38) an empirical law has been established from all experimental results (see Figs. 8 and 9)

$$
\begin{equation*}
\frac{D_{0}^{\prime}}{\mathrm{G}^{1 / 3}}=108\left(\frac{\mathrm{G}}{\mathrm{~J}}\right)^{-1 / 2} \tag{45}
\end{equation*}
$$

Hence the salinity distribution at low tide is completely described in terms of estuary geometry, tidal characteristics and freshwater flow. The salinity distribution for all tidal times is readily obtained by translating the low tide salinities with the tidal velocities obtained from the tidal analysis. For these solutions reference (8) is again to be consulted.
5.4 Effects of salinity intrusion on sedimentation. The effect of salinity in estuaries is twofold. On the one hand the internal dynamics are such as to produce a stagnation zone for the temporal mean velocities near the bottom thus retaining the sediments in the bottom zone near the end of the salinity intrusion. On the other hand the salinity promotes the flocculating of fine clay and silt and the much higher settling velocities of the flocs causes the fine suspensions coming from upland sources to accumulate in the bottom zones. The estuarine environment in addition provides fine suspended load due to entrainment by turbulence due to wave action on tidal flats and shallow portions of


Fig. 8.-Diffuston Coefricient versus Rate of Tidal Energy Dissipation (8)
the estuary as well as from human and industrial sources. The conclusions from the internal dynamics of estuarine flow patterns are therefore:

1. Sediments reaching the bottom zones in estuaries will be transported upstream and not downstream.
2. Shoaling takes place primarily near the stagnation zones of the internal flow.
3. The higher the degree of stratification, i.e., the higher the salinity gradients both vertically and horizontally, the more intense is the shoaling.
General rules with regard to all engineering measures modifying the existing conditions are derived as follows:
a. The major portion of sediments in an estuary, if entrained by the periodic tidal velocities will be transported to the limits of the salinity intrusion. All measures leading to increasing depths and tidal range cause higher entraining velocities and thus promote localized shoaling.


Fig. 9.-Correlation of Apparent Diffusion Coefficient $D_{o}{ }^{\prime}$ witil Stratification Number (8)
b. Engineering measures increasing normal freshwater flow by regulation or reducing tidal action by constriction lead to increased stratification and thus to increasing shoaling tendencies.
c. Dredging of navigation channels in estuaries should be accompanied by permanent removal of all dredge spoils from the estuaries. Dumping within the estuary confines back into the tidal streams is practically always useless. The criterion for dredging costs is not to be based on cost per cubic yard of dredging but on minimum annual costs of channel maintenance. In the case of the Delaware, complete removal of dredge spoils from the channel, while increasing the cost per cubic yard greatly, nevertheless produced a deeper channel year round for less annual cost of maintenance.
The definition of the hydraulic characteristics of the shoaling materials encountered in estuaries is a problem largely unresolved. Colloidal clays and fine sands of less than one quarter millimeter diameter are generally encountered as shoaling materials. The former constitute by far the predominant portion in all U. S. estuaries with
the exception of the Columbia River mouth. Colloidal clays will flocculate upon contact with saline water and due to the mean currents discussed before accumulate rapidly in shoals. These shoals become voluminous due to the small densities of only 1050 to 1200 grams per liter which only slowly consolidate due to the periodic disturbances by larger tides and large changes in fresh water flow. In general, the denser colloidal mixtures of water and clay-flocs have non-Newtonian characteristics as fluids, i.e., their initial displacement requires a finite shear and only for hydraulic shear forces in excess of this critical value will the mixtures behave as fluids. Needless to say the analysis of the complex interaction of sediment shoals and of the complicated flow regimes in estuaries is still only in the beginning stages and has led to extensive studies in the laboratory and in the field. The publications of the Tidal Hydraulics Committee of the Corps of Engineers (U.S. Waterways Experiment Station, Vicksburg, Mississippi) may be consulted with respect to progress made in this area, particularly with respect to planning of engineering works in San Francisco Bay.

It is hoped that this brief summary has at least produced an awareness of the complicated mechanics of estuary flow and sedimentation systems. Engineering measures interfering in this natural environment must be planned with full consideration of the organic interaction of tidal regime, freshwater flow, salinity and sediment characteristics.

## References

1. Eagleson, P. S. and Dean, R. G. "Small Amplitude Wave Theory," Chapter A of "Estuary and Coastline Hydrodynamics," Hydrodynamics Laboratory, Department of Civil Engineering, MIT, 1963.
2. Ippen, A. T. and Harleman, D. R. F. "Tidal Dynamics in Estuaries," Chapter J of "Estuary and Coastline Hydrodynamics," Hydrodynamics Laboratory, Department of Civil Engineering, MIT, 1963.
3. Proudman, J. "Dynamical Oceanography," 1952, Methuen and Co., Ltd., London.
4. Dronkers, J. J. and Schoenferd, J. C. "Tidal Computations in Shallow Water," Journal of the Hydraulics Division, ASCE, Separate No. 714, 1955.
5. Pillsbury, G. B. "Tidal Hydraulics," Revised edition 1956, Sales Agent, Superintendent of Documents, Waterways Experiment Station, Vicksburg, Miss.
6. Ippen, A. T. and Harleman, D. R. F. "Investigation of Influence of Proposed International Tidal Power Project on Tides in the Bay of Fundy," 1958 Report to New England Division, U.S. Corps of Engineers, Boston, Mass.
7. Goulis, D. "Damped Tides and Energy Dissipation in Exponential Estuaries with Particular Reference to the Delaware," 1962, S.M. Thesis, MIT, Department of Civil Engineering.
8. Ippen, A. T. and Harleman, D. R. F. "Analytical Studies of Salinity Intrusion in Estuaries and Canals," 1961, Committee on Tidal Hydraulics, Corps of Engineers, Technical Bulletin No. 5.
9. Ippen, A. T. "Salinity Intrusion in Tidal Estuaries," Chapter M of "Estuary and Coastline Hydrodynamics," Hydrodynamics Laboratory, Department of Civil Engineering, MIT, 1963.
10. Ippen, A. T. and Harleman, D. R. F. "Two-Dimensional Aspects of Salinity Intrusion in Estuaries," 1964, Committee on Tidal Hydraulics, Corps of Engineers, Vicksburg, Miss.
11. Simmons, H. B. "Some Effects of Upland Discharge on Estuarine Hydraulics," Proc. ASCE, Separate No. 742, 1955.


## WACHUSETT INTAKE AND POWER PLANT

By Chester J. Ginder*

## General Description

The Wachusett Intake and Power Plant is a facility of the Boston Metropolitan Water Supply System located on the easterly shore of the Wachusett Reservoir in Clinton about 1800 feet south of the main dam built about 1900 to form the reservoir. Its purpose is to provide a controlled means of introducing water from the reservoir to the recently completed Wachusett-Marlborough Tunnel which is the first section of the pressure aqueduct built to carry water from the sources of supply to Norumbega Reservoir in Weston and to the District distribution lines. The Wachusett-Marlborough Tunnel is a deep rock tunnel 14 feet in diameter extending from Clinton to Marlborough a distance of 8 miles where it connects with section 2 of the pressure aqueduct. Fig. 1 shows the location of Wachusett Reservoir with respect to the system as a whole.

The intake structure contains the usual features of control and operation such as coarse and fine screens, stop shutters, sluice gates and control valves. In addition the structure will house two turbines and two generators which will furnish plant power and power for sale to the New England Electric System on a contract basis. A second function of the power generation equipment will be to use up excess hydraulic head which exists with Wachusett Reservoir at the spillway elevation of 395 on Boston City Base. The pressure aqueduct discharges into Norumbega Reservoir at elevation 274.5. The head available is more than required for the flow requirements and somewhat in excess of the design pressure allowed for certain sections of the pressure aqueduct and certain valve installations along the line. Bypass lines are provided for use when the turbines may be shut down or for use when the demand exceeds the capacity of the turbines. These bypass lines will be equipped with head dissipating valves of the Howell-Bunger type. Tail water for the turbines and the Howell-Bunger valves will vary from elevation 315 for low discharge conditions to elevation 338 for maximum

[^3]

Fig. 1.-Entire System.
flow. Maximum flow in the remote future is considered as being 600 mgd and the intake structure is designed on this basis. However, the maximum capacity of the present single pressure aqueduct from Shaft 4 to Norumbega under permissible maximum gradients is limited to 312 mgd . A second pressure aqueduct will be needed between these points to meet future demand.

The safe yield of present sources is estimated to be 330 mgd . The safe yield under the proposed future use of the Millers River is estimated at 519 mgd .

## Construction Sequence

The entire project has involved construction under three stages. Stage I is the Wachusett-Marlborough Tunnel previously mentioned. Stage II is the site preparation for the intake structure and Stage III the intake and power plant.

Stage II, the site preparation, was completed under a previous contract at a cost of about $\$ 1,200,000$. The work of this contract included the construction of a sheet pile cofferdam composed of six sand-filled cells 60 feet in diameter and five connecting cells having a maximum depth to rock of about 65 feet. The cell cofferdam and end dikes permitted unwatering an area of the reservoir about 350 feet square in which the intake structure will be located.

Arrangements were made to maintain Wachusett Reservoir at elevation 380 during the construction period in order to reduce the depth of the cofferdam cells. Emergency spillways through the cofferdam we provided at elevation 380 with stop logs to allow controlled flooding of the site if a rise in the reservoir level could not be prevented. This stage was completed without the necessity of using these spillways. The extent of this work is indicated in Fig. 2. Rock excavation for the structure amounted to about $50,000 \mathrm{cu}$. yds. with side slopes on the land side of the excavation about 95 feet deep from elevation 393 at the top to elevation 298 at the bottom of the foundation. The rock consisted mostly of Worcester Phylite of rather poor quality and was subject to weathering. Since the side slope called for was 8 on 1, it was necessary to protect the slopes with wire mesh and some mortar covering as a safety measure. Figs. 3, 4 and 5 show work involved in the cofferdam and site preparation contract, Stage II.

Elaborate provisions were made for grouting under the cofferdam and the building foundation to prevent leakage into the area but very little grouting was actually done as the leakage was negligible.


Fig. 2.-Existing Conditions at Site.


Fig. 3.-Cofferdam and Excavation.


Fig. 4.-Cofferdam and Excavation.
This contract also included a 12 foot diameter connecting tunnel from the bottom of the excavation to Shaft A of the Wachusett-Marlborough Tunnel, a distance of 340 feet, and the installation in this section of tunnel of a steel bulkhead to protect the Wachusett-Marlborough Tunnel from being flooded and the Wachusett Reservoir from being drained should the cofferdam fail.

Sanitary protection of the reservoir required all drainage from the shore areas occupied by the contractor and pumpage from the excavation to be chlorinated and filtered in a settling basin adjacent to the site.

Stage III construction comprises the concrete substructure with intake channels to carry water from the reservoir through water wheels and through bypass lines into the Wachusett-Marlborough Tunnel. It also includes the construction of the superstructure which provides control centers and service areas. Figs. 6, 7 and 8 show the general design of the structure and the location of equipment. The water wheels, governors, and generators were purchased by the Commission for installation by the general contractor.


Fig. 5.-Cofferdam and Excavation.


[^4]
Fig. 7.-Power Plant Elev. 343.0

Fig. 8.-Power Plant Cross Sections.

Downstream from the by-pass lines and the water wheels a stilling basin is provided wherein the tailwater elevations will vary with changes in the discharge through the structure. Under minimum discharge conditions the draft tube seal is provided by a low weir at the entrance to the connecting tunnel to Shaft A. Under minimum discharge conditions the connecting tunnel will be flowing less than full depth and a free fall will occur at Shaft A. Since this condition will exist at times for many years, a cast iron lining has been placed in Shaft A where turbulence would tend to erode the concrete lining of the Shaft. Fig. 9 shows a portion of the cast iron lining which will protect the


Fig. 9.-Cast Iron Lining.
throat at the tunnel and shaft intersection and twenty feet of the shaft below the throat.

The substructure provides for a generator room at elevation 357 which is above the expected high tailwater elevation but below the reservoir spillway elevation, which is at elevation 395. The facing consists of granite panels having a thermal finish and polished granite trim combined with glass panels with stainless steel trim.

Landscaping of the grounds adjacent to the intake structure is
provided with the expectation that an administration building will later be built nearby.

## Design

The intake structure is located on the shore adjacent to Shaft A of the tunnel. In this structure there are six intake channels, two for the water wheels and four for the bypass lines. These channels are grooved for stop shutters and for screens. The channels extend full height from elevation 335 to elevation 405 with 4 foot by 6 foot sluice gates at elevations 340 and 360 in each channel. Additional sluice gates 8 feet square are provided to control flow to the water wheels. The screens are to be made in sections about $5^{\prime}-6^{\prime \prime}$ high and $4^{\prime}-3^{\prime \prime}$ wide with stainless steel screening. For one set the screening is to be $14 \mathrm{ga} .1 / 2$ inch mesh, and for the other set the screening is to be $10 \mathrm{ga}$.1 inch mesh. Lifting devices operated from a crane in the screen room at elevation 405 will be used for the installation and removal of the screens and shutters. The sluice gates will be motor operated on gate stands in the screen room.

Four bypass lines are provided of which two are to be completed under the present construction contract and two at a later date. The completed lines will each include a 54 inch flow tube and a 36 inch Howell-Bunger type of discharge valve set at elevation 332. The Howell-Bunger valves will operate under free discharge conditions with a maximum head of 68 feet for low tailwater conditions and will discharge submerged under high tailwater conditions. The maximum capacity of each bypass line will be about 240 mgd .

Two waterwheels with Woodward Company governors are being furnished by the James Leffel Company. These turbines will be vertical shaft Kaplan type with adjustable blades having a rated output of 2240 horsepower under an effective head of 80 feet at a speed of 450 revolutions per minute. The wheels will be set at elevation 325. The spiral cases will be steel plate and these will be embedded in the concrete substructure of the building. The two turbines will be capable of discharging about 380 mgd total to supply the district demand. Generally speaking, the operation of the turbines and the bypass lines together or separately will be such as to match as nearly as possible the water demand by the District in order to keep wastage of water to a minimum.

The generators are being furnished by Electric Machinery Com-
pany. Each is to be the vertical shaft, waterwheel driven type having a rated capacity of 200 kva . The exciters will be direct connected. The generator room at elevation 357 is approximately 44 feet by 100 feet. It is open to the roof of the superstructure at elevation 437.23 and is served by a 10 ton traveling crane located in the superstructure. Flow through the bypass lines and through the turbines will be measured and recorded on indicating and totalizing charts.

The stilling basin downstream of the generator room performs the functions of sealing the draft tubes, dissipating the remaining discharge head and establishing a free water surface gradient for the tunnel under high flow conditions.

Halfway along the tunnel at Shaft B an overflow weir will be built with the crest at elevation 325 as indicated on Fig. 10. This overflow, which is to be built at the top of the former tunnel construction shaft, will serve as an overflow relief for the system should some operation failure cause the gradient to exceed the safe limit. Under planned operating procedures no overflows at Shaft B are expected.

At the downstream end of the tunnel at Shaft C, which is about eight miles from the intake structure, a connection to the previously constructed pressure aqueduct will be made, as shown on Fig. 11. At this point two 72 inch butterfly valves in the main aqueduct line will permit control of the flow through the tunnel. Normally, however, the control of flow through the tunnel will be done at the intake structure by varying the discharge through the water wheels and the bypass lines.

Part of the construction at Shaft C includes a 66 inch by-pass line controlled by a 48 inch butterfly valve. This valve will be motorized and actuated by signals from Norumbega Reservoir in Weston, and will be called upon to prevent an uncontrolled rise in Norumbega Reservoir by helping to match the flow through the pressure aqueduct with the water demand of the District. This bypass line will be equipped with pressure relief valves sensitive to the effects of surges in the aqueduct line. Flows through this bypass line will be lost to the high service lines of the system, but through Sudbury Reservoir and Weston Reservoir will be available to the low service lines. Normal operation calls for discharges through the bypass line at Shaft C to be kept to a minimum.

Presently the pressure aqueduct begins at Shaft C, taking water from the adjacent forebay at elevation 278.5. This gradient will not

Fig. 10.-Spillway at Shaft B.
/
Fig. 11.-Connections at Shaft C.
permit Norumbega Reservoir to be maintained at elevation 274.5, where it should be to satisfy the requirements of the District. Eventually a second pressure aqueduct will be required between Shaft C and Norumbega Reservoir. In the meantime, the gradient at Shaft C will be raised from elevation 278.5 to about elevation 310 or 312 , which will greatly increase the flow to Norumbega Reservoir and relieve the difficulty caused by a low gradient at Norumbega.

The work required to make the pressure aqueduct operative includes the installation of a control system based on maintaining Norumbega Reservoir within close limits. Compared with Wachusett Reservoir, which is backed up by Quabbin Reservoir, Norumbega Reservoir is very small. The top foot of Norumbega has a capacity of only 13.6 million gallons and the spillway capacity is so small that overflows at Norumbega must be prevented. Therefore, the controls are set up to maintain Norumbega Reservoir within the top foot. Signals from Norumbega Reservoir will automatically adjust the discharge of the turbines or bypass valves at the intake structure to meet the demand on Norumbega Reservoir and will tend to keep the elevation at elevation 273.9. Such signals will also control the bypass valves at Shaft C. A high water alarm in the stilling basin will call for shutting off flow through the intake should the signals from Norumbega fail to properly regulate the flow.

A contract not yet awarded for miscellaneous electrical equipment and control equipment will include flow and pressure transmitters and receivers, power transformer, substation, main control board, motor control centers and turbine control board. The control system will be fully automatic except for the manual operation required to add or remove one turbine from the system.

When water is being released through the bypass valves, the valves will be automatically controlled in a manner similar to the turbines by the float control at Norumbega. The automatic control on the bypass valves may be switched over to manual control at any time and then be operated by push-button control by the station operator.

The construction contract was awarded in November, 1964, to Wes-Julian Company on the basis of a bid price of $\$ 5,239,557.00$.

The design was done by the Construction Division of the Metropolitan District Commission, under the supervision of Mr. Frederick W. Gow, Chief Engineer. Chas. T. Main, Inc. acted as consultant and prepared the detailed drawings and specifications.

## THE VIRTUAL DIFFERENTIAL SETTLEMENT METHOD

By Z. Getzler, Ing. M.Sc.*

The paper defines the different concepts of "basic," "natural" and "damped" differential settlement and the difficulties involved in their exact determination as well as in evaluation of their influence on the structure.

Complete statical computation of a structure, due to the analytically determined differential settlements (for all points) being, in most cases, impracticable, a method is proposed in which a virtual differential settlement at all supporting points is determined separately, on the basis of a simplified statical scheme and extreme soil conditions. In framed structures, based on separate footings to which the proposed method is confined, several typical situations are considered with their corresponding coefficients: end support and internal support in a two- and multi-bay frame, respectively. Soil properties are represented, instead of by an average value of the modulus of elasticity (or modulus of subgrade reaction), by its maximum value and maximum-minimum ratio:

$$
\frac{\mathrm{E}_{\mathrm{s}}^{\prime} \max }{\mathrm{E}_{\mathrm{s} \text { min }}^{\prime}} \text { or } \frac{\mathrm{K}_{\max }}{\mathrm{K}_{\min }}
$$

permitting allowance for extreme soil conditions.
A formula is derived for a simple case of a two equal span beam with one of its supports settling more than the others under virtual extreme soil conditions. This formula for the resulting differential settlement damped by the rigidity of the structure, can be adapted for more complicated statical schemes and other modes of presentation of soil-elasticity properties. Coefficients of statical conditions for typical cases are tabulated and a numerical example is added to illustrate the procedure.

## Introduction

The stability of foundations is conditioned by two main factors:
a) safety against soil failure,
b) safety against structural failure or damage due to soil deformations.

The first factor comprises soil stability, and its analysis entails examination of the limit equilibrium of the latter. The second factor comprises the behavior of the structure and the soil, both separate

[^5]and joint. The soil may be far below its limit shear stress level, but the deformations of its surface and those of the structure due to these soil movements may result in structural failure or damage.

The final allowable load on the foundation lies within the allowable bearing stress limits in the soil and results in settlements below the danger level. Settlement of the whole structure entails usually no danger as long as it is uniform for all its parts. Connections with other structures (or installations such as pipes) may be damaged, but the structure itself is ordinarily not affected. The same applies for nonuniform but still linear settlement, resulting in tilting of the structure in one piece. This kind of settlement, normally confined to rigid structures, involves an aesthetic aspect, but it can be assumed that a tilt of less than $1 / 250$ can be disregarded (Skempton \& MacDonald '56 (1), Bjerrum '61 (2)).

The main structural damage involved in foundation settlement is due to unequal and non-linear settlement of different parts, i.e., the non-linear differential settlement, which forms the subject of the present paper.

## Definitions

The factors determining differential settlements can be grouped in three main categories (Terzaghi, '35 (3)):

1. Basic (or theoretical) differential settlement is that part of the differential settlement which is due to shape of the stress distribution soil, assuming perfectly homogeneous subgrade and completely flexible superstructure. This part of the settlement can usually be predicted quantitatively if the load conditions and the elastic properties of the soil are more or less simple and known. This definition may be extended to include the influence of the size of the foundation, the effectiveness and character of the load, and all other factors capable of analytical expression based on theoretical or empirical data.
2. Natural differential settlement is that which includes, in addition to the basic one, deviations due to inhomogeneity* of the soil, unexpected moisture fluctuations, unexpected load distribution or concentration, and other factors not yielding themselves to analytical treatment. Under this definition the structure is again assumed infinitely flexible.

[^6]3. The final differential settlement of the structure is a resultant of the natural settlement and the restraining (damping) effect of the rigidity of the structure, namely the damped differential settlement.

An infinitely-flexible structure settles as a direct result of natural settlement of the soil; the latter undergoes complete damping in the case of an infinitely rigid structure, resulting in uniform settlement and in transfer of the loads from the deeper-settled parts of the foundation to their less affected counterparts. Most superstructures which have limited rigidity partially damp the natural settlement and settle due to this damped settlement line. According to the magnitude of this resulting differential settlement the structure either remains undamaged or undergoes varying degrees of damage.

## Prediction of Basic Settlement

The settlement of foundation in elastic homogeneous soils can be computed theoretically, and so can the basic differential settlement. For example, the maximum basic differential settlement between center and circumference of a circular flexible foundation plate is 0.36 of the maximum basic settlement at the center, while for a square plate it is 0.50 of the maximum (Timoshenko '51 (4)). In more complicated cases these values may be found from the stress distribution and elastic properties of the soil at different depths and under different parts of the structure. The consolidation theory can be utilized in such cases for regular clay layers. In other soils subgrade moduli or statistical coefficients may be used. The Terzaghi-Peck formulas (1948) (5) permitting extrapolation for foundations of different sizes from the settlement of a 1 sq . ft . test plate are in widespread use. In literature and practice also other methods are to be found which may be of help in analytical determination of settlements and differential settlements on the basis of more or less specified soil properties and load conditions.

## Natural Differential Settlement

The theoretical settlement profile obtained by one of the above mentioned methods usually fails to provide a true picture for a variety of reasons. For one thing, even soils defined as fairly homogeneous may differ in their properties within limits of about $\pm 50 \%$, and these differences may sometimes have more influence on foundation behavior than the absolute mean (Terzaghi '35 (3)). (Hence the need for data supplied by soil-mechanics laboratories to provide the foundation
engineer with a complete picture of the results, instead of confining reports to average values.) Other factors distorting the theoretical settlement distribution are time, the real load distribution, settlements during construction, nearby works, etc. In addition, there is difficulty in isolating the natural settlement, which can only be measured after the building has been completed, when in most cases it is no longer "natural" but affected by the rigidity of the structure. (The only exception is a completely elastic element, such as the flexible bottom of a reservoir.)

## Damped Differential Settlement

A rigid structure resists deformations due to soil movement and transfers loads and pressures from the more to the less settling points, thus reducing the potential differential settlement to a restrained or damped one. This damping of the soil movement produces a new load distribution in the structure and new loading conditions on the soil. This interaction of the two factors, controlled by their respective elastic and geometric properties, lends itself to theoretical analysis (Meyerhof '47 (6), Chamecki '56 (7), Yokoo \& Yamagata '56 (8)), insofar as these properties are known; but the accuracy of this analysis is limited by that of the available data. The data and settlement values supplied by the soil engineer are averages based on tests with varying degrees of scatter and on theories whose relation to reality is mostly qualitative rather than quantitative. On the other hand, statical analysis of the structures is normally based on simplified assumptions. Bearing in mind that in most computations all secondary elements (such as floors and partitions) are disregarded, discrepancies of up to $\pm 50 \%$ between calculated and actual values should be expected (Skempton \& MacDonald (1)). Another source of discrepancies lies in inaccurate assumptions with regard to the elasticity and rigidity of members and joints.

## The Proposed Method

In these circumstances, more suitable tools are required for estimating the expected differential settlements and the degree of danger to the structure entailed by them. In the author's opinion, the method outlined below permits prediction of the behavior of the structure with an accuracy commensurate with that of other tools used in foundation design. The proposed method (confined at present to framed structures on separate footings) consists in determining virtual differential settle-
ments at all supporting points of the building. Under "virtual differential settlement" is understood the settlement which corresponds to the worst alternative of soil properties scatter. Under this simple approach accuracy is automatically reduced to a very rough level. Recourse to more accurate tools will not help, since if the settlement under footings undergoes changes within the limits determined by the scatter, any specific assumption with regard to the subgrade modulus will still refer to a single alternative, certainly not the worst. Chamecki's and other methods are based on such assumption in actual disregard of the unlimited number of other (not less real) possible alternatives.

Although for the exact solution an infinity of situations should be considered covering the whole range of subgrade moduli, the proposed method is confined to their extrema, thus reducing the number of situations to minimum.

For this purpose the soil properties should be represented, instead of by an average value of E and K , by their maximum value and maximum-minimum ratio:

$$
\begin{equation*}
x=\frac{\mathrm{E}_{\mathrm{s}, \max }^{\prime}}{\mathrm{E}_{\mathrm{s}^{\prime} \min }^{\prime}} \tag{1}
\end{equation*}
$$

or

$$
\begin{equation*}
x=\cdot \frac{\mathrm{K}_{\max }}{\mathrm{K}_{\min }} \tag{2}
\end{equation*}
$$

Now, with the degree of accuracy reduced as noted above, a simplified and time saving statical scheme suffices to find the damping influence of the structure.

Two typical schemes will be considered, each for two positions of the settling support: a two-span and a multi-span structure, and settling of the end or interior support.

Another simplification refers to the possibility of checking each support separately. This is effected by means of the fact that all footings are designed for the same contact pressure or settlement value. Thus, with a single proportion for all supports, the adjoining column load values become superfluous leaving only the footing dimensions in the formulas.

In what follows the damping process is illustrated for a two-span beam on three footings. First, the basic differential settlements ( $\Delta \rho^{\circ}$ ) and the damped differential settlements $\left(\Delta_{\rho}\right)$ are calculated on the
assumption of fixed subgrade moduli under all footings. In that case the differential settlement may be (theoretically) eliminated altogether by adjusting footing sizes to yield equal settlements. Substituting the maximum-minimum ratio ( $\kappa$ ) for E (in clays) or for K (in sands) the virtual damped differential settlements are obtained.

It is also assumed that the dimensions of the footings and the character of the subgrade allow neglect of the influence of the load of one footing on the settlement of others.*

Assuming the footings in the first stage as non-settling, we have:

$$
\begin{gather*}
\mathrm{R}_{\mathrm{A}}{ }^{o}=\mathrm{P}_{\mathrm{A}}+\frac{\mathrm{g} l}{2}-\frac{\mathrm{M}_{\mathrm{B}}^{(-)}}{l}=\mathrm{R}_{\mathrm{c}^{\mathrm{o}}}  \tag{3}\\
\mathrm{R}_{\mathrm{B}}{ }^{\mathrm{o}}=\mathrm{P}_{\mathrm{B}}+\mathrm{g} l+\frac{2 \cdot \mathrm{M}_{\mathrm{B}}^{(-)}}{l} \tag{4}
\end{gather*}
$$

where:
$\mathrm{R}_{\mathrm{A}}{ }^{0} \mathrm{R}_{\mathrm{B}}{ }^{0} \mathrm{R}_{\mathrm{C}}{ }^{\mathrm{o}}$ - are loads on footings $\mathrm{A}, \mathrm{B} \& \mathrm{C}$, respectively, before settlement,
$P_{A} P_{B} P_{c}$ - loads independent of the differential settlement, g - load on beam per unit length,
$\mathrm{M}_{\mathrm{B}}{ }^{(-)}$— moment at support B before settlement, $l$-span.
Basic subgrade settlement will induce settlements $\rho_{\mathbf{A}}{ }^{\circ}, \rho_{\mathbf{B}}{ }^{\circ}, \rho_{0}{ }^{\circ}$ in the three supports. Assuming $\rho_{\mathrm{B}}{ }^{\circ}>\rho_{\mathrm{A}}{ }^{0}=\rho_{\mathrm{C}}{ }^{\circ}$, the basic differential settlement would be:

$$
\begin{equation*}
\Delta \rho_{A B^{0}}{ }^{0}=\rho_{B^{0}}{ }^{0}-\rho_{\mathrm{A}}{ }^{0} \tag{5}
\end{equation*}
$$

The damped differential settlement, resulting exclusively from the basic settlement and structural rigidity will be:

$$
\Delta \rho_{\mathrm{AB}}=\rho_{\mathrm{B}}-\rho_{\mathrm{A}}=\left(\rho_{\mathrm{B}}{ }^{0}-\Delta \rho_{\mathrm{B}}\right)-\left(\rho_{\mathrm{A}^{0}}+\Delta \rho_{\mathrm{A}}\right)=\Delta \rho_{\mathrm{A}}{ }^{0}-\left(\Delta \rho_{\mathrm{B}}+\Delta \rho_{\mathrm{A}}\right)
$$

where $\rho_{A}$ and $\rho_{B}$ are the final settlements of footing $A$ and $B$, and $\rho_{\mathrm{C}}=\rho_{\mathrm{A}} ; \Delta_{\rho_{\mathrm{A}}}$ and $\Delta_{\rho_{\mathrm{B}}}$ the changes in footing settlement due to the influence of the rigidity of the structure, as defined later.

This differential settlement reduces the $\mathrm{M}_{\mathrm{B}}{ }^{(-)}$at the middle support by:

[^7]\[

$$
\begin{equation*}
\Delta \mathrm{M}=\frac{3 \mathrm{E}_{\mathrm{C}} \mathrm{~J}_{\mathrm{c}}}{l^{2}} \Delta_{\rho_{\Delta \mathrm{B}}} \tag{7}
\end{equation*}
$$

\]

( $\mathrm{E}_{\mathrm{C}}$ and $\mathrm{J}_{\mathrm{C}}$ being the modulus of elasticity and moment of inertia of the beam), with the following effect on the reactions $\left(\Delta R=\frac{\Delta M}{l}\right)$ :

$$
\begin{align*}
\mathrm{R}_{\mathrm{A}} & =\mathrm{R}_{\mathrm{A}}{ }^{0}+\frac{3 \mathrm{E}_{\mathrm{C}} \mathrm{~J}_{\mathrm{C}}}{l^{3}} \Delta \rho_{\mathrm{AB}}  \tag{8}\\
\mathrm{R}_{\mathrm{B}} & =\mathrm{R}_{\mathrm{A}^{0}}-2 \frac{3 \mathrm{E}_{\mathrm{C}} \mathrm{~J}_{\mathrm{C}}}{l^{3}} \Delta \rho_{\mathrm{AB}} \tag{9}
\end{align*}
$$

The effect of the change in the reactions on the settlement ( $\Delta_{\rho}$ ) will be determined here with the aid of the subgrade modulus $K$, calculated from the modulus of elasticity or computed directly on the basis of laboratory (or field) tests, or taken from tables, once the soil has been identified. Although the simplest, it is not the best and the only method available; as pointed out earlier, however, allowance should be made for accuracy considerations before more exact or more refined methods are resorted to.

So, in general,

$$
\begin{equation*}
\rho=\frac{\sigma}{\mathrm{K}} \tag{10}
\end{equation*}
$$

( $\rho-$ settlement, $\sigma$ - stress, $\mathrm{K}-$ subgrade modulus).
The final differential settlement will be from Eq. (6) and (10):

$$
\begin{align*}
\Delta \rho_{\mathrm{AB}}=\rho_{\mathrm{B}}-\rho_{\mathrm{A}}=\left(\rho_{\mathrm{B}}{ }^{0}-\frac{\Delta \sigma_{\mathrm{B}}}{\mathrm{~K}_{\mathrm{B}}}\right) & -\left(\rho_{\mathrm{A}_{\mathrm{A}}}{ }^{0}+\frac{\Delta \sigma_{\mathrm{A}}}{\mathrm{~K}_{\mathrm{A}}}\right) \\
& =\Delta \rho_{\mathrm{AB}^{\circ}}{ }^{0}-\left(\frac{\Delta \sigma_{\mathrm{B}}}{\mathrm{~K}_{\mathrm{B}}}+\frac{\Delta \sigma_{\mathrm{A}}}{\mathrm{~K}_{\mathrm{A}}}\right) \tag{11}
\end{align*}
$$

but as: $\quad \Delta \sigma_{A}=\frac{R_{A}-R_{A}{ }^{0}}{a^{2}} \quad \Delta \sigma_{B}=\frac{R_{B}-R_{B^{0}}}{b^{2}}$
where:
$\Delta \sigma$ - change in soil stress due to differential settlement
$\mathrm{a}, \mathrm{b}$ - size of square footings, so, combining Eq. (11) and (12) with (8) and (9):

$$
\begin{equation*}
\Delta_{\rho_{\mathrm{AB}}}=\Delta \rho_{\mathrm{AB}}{ }^{0}-\left(\frac{6 \cdot \mathrm{E}_{\mathrm{C}} \mathrm{~J}_{\mathrm{C}}}{\mathrm{~K}_{\mathrm{B}} l^{3} \mathrm{~b}^{2}}+\frac{3 \mathrm{E}_{\mathrm{C}} \mathrm{~J}_{\mathrm{C}}}{\mathrm{~K}_{\mathrm{A}} l^{3} \mathrm{a}^{2}}\right) \Delta_{\rho_{\mathrm{AB}}} \tag{13}
\end{equation*}
$$

the following formula is obtained:

$$
\begin{equation*}
\Delta \rho_{\mathrm{AB}}=\frac{\Delta \rho_{\mathrm{AB}^{0}}}{1+\frac{3 \mathrm{E}_{\mathrm{C}} \cdot \mathrm{~J}_{\mathrm{C}}}{l^{3}} \cdot \frac{2 \mathrm{~K}_{\mathrm{A}} \mathrm{a}^{2}+\mathrm{K}_{\mathrm{B}} \mathrm{~b}^{2}}{\mathrm{~K}_{\mathrm{A}} \cdot \mathrm{~K}_{\mathrm{B}} \cdot \mathrm{a}^{2} \mathbf{b}^{2}}} \tag{14}
\end{equation*}
$$

which permits calculation of the damped differential settlement from the basic one.

With the basic differential settlement $\Delta_{p_{\mathbf{A}} \mathbf{B}^{0}}$ (5) computed by the same method (subgrade coefficient), the complete relationship can be expressed in a single formula (putting $\rho_{\mathrm{A}}{ }^{\circ}=\mathrm{R}_{\mathrm{A}}{ }^{\circ} / \mathrm{K}_{\mathrm{A}} \cdot \mathrm{a}^{2}$ and $\left.\rho_{\mathrm{B}}{ }^{0}=\mathrm{R}_{\mathrm{B}}{ }^{0} / \mathrm{K}_{\mathrm{B}} \cdot b^{2}\right):$

$$
\begin{equation*}
\Delta_{\rho_{\mathrm{AB}}}=\frac{\mathrm{R}_{\mathrm{B}}{ }^{0} \mathrm{~K}_{\mathrm{A}} \mathrm{a}^{2}-\mathrm{R}_{A^{0}} \mathrm{~K}_{\mathrm{B}} \mathrm{~b}^{2}}{\mathrm{~K}_{\mathrm{A}} \cdot \mathrm{~K}_{\mathrm{B}} \mathrm{a}^{2} \mathrm{~b}^{2}+\frac{3 \mathrm{E}_{\mathrm{C}} \mathrm{~J}_{\mathrm{C}}}{l^{3}}\left(2 \mathrm{~K}_{\mathrm{A}} \mathrm{a}^{2}+\mathrm{K}_{\mathrm{B}} \mathrm{~b}^{2}\right)} \tag{15}
\end{equation*}
$$

which allows the calculation of the damped (basic only!) differential settlement directly.

## Direct Determination of the Damped Differential Settlement in Clayey (Elastic) Soils

Assuming elastic behavior of the clay, a simple expression may be used for the subgrade modulus in this soil:

$$
\begin{equation*}
\mathrm{K}_{\mathrm{i}}=\alpha \frac{\mathrm{E}_{\mathrm{s}}^{\prime}}{\mathrm{B}_{\mathrm{i}}} \quad\left(\mathrm{E}_{\mathrm{s}}^{\prime}=\frac{\mathrm{E}_{\mathrm{s}}}{1-\mu^{2}}\right) \tag{16}
\end{equation*}
$$

where $\alpha$ is a shape factor (Terzaghi '43(9); Skempton '51(10)); $B_{i}$ is width of the footing. Then:

$$
\mathrm{K}_{\mathrm{A}}=\alpha \frac{\mathrm{E}_{\mathrm{s}}^{\prime}}{\mathrm{a}} \quad \mathrm{~K}_{\mathrm{B}}=\alpha \frac{\mathrm{E}_{\mathrm{s}}^{\prime}}{\mathrm{b}}
$$

where from, for equal $\alpha$ and $\mathbf{E}_{\mathbf{s}}^{\prime}$ :

$$
\begin{equation*}
\mathrm{K}_{\mathrm{A}} / \mathrm{K}_{\mathrm{B}}=\mathrm{b} / \mathrm{a} \quad \text { and } \quad \rho_{\mathrm{A}} / \rho_{\mathrm{B}}=\sigma_{\mathrm{A}} \cdot \mathrm{a} / \sigma_{\mathrm{B}} \cdot \mathrm{~b} \tag{17}
\end{equation*}
$$

Substituting this in the $\Delta_{\rho_{A B}}$ formula (15), and assuming a homogeneous soil ( $\mathrm{E}_{\mathrm{s}}^{\prime}=$ const.), we obtain:

$$
\begin{equation*}
\Delta_{\rho_{\mathrm{AB}}}=\frac{\mathrm{R}_{\mathrm{B}^{0} \mathrm{a}}-\mathrm{R}_{\mathrm{A}^{0} \mathrm{~b}}}{\alpha \mathrm{E}_{\mathrm{s}^{\prime}} \mathrm{ab}+\frac{3 \mathrm{E}_{\mathrm{C}} \mathrm{~J}_{\mathrm{o}}}{l^{3}}(2 \mathrm{a}+\mathrm{b})} \tag{18}
\end{equation*}
$$

Using the well-known rule (Terzaghi-Peck '48 (5)) for adjusting the design pressure of footings so as to yield the same settlements, one can (theoretically, of course) eliminate differential settlements altogether.

For $\rho_{\mathrm{B}}=\rho_{\mathrm{A}}$ we have, from Eq. (17):
so

$$
\begin{gather*}
\frac{\sigma_{B}}{\sigma_{A}}=\frac{a}{b} \text { and } \frac{a}{b}=\frac{R_{A}{ }^{0}}{{R_{B}}^{o}}  \tag{19}\\
\mathrm{R}_{B^{0}} \mathrm{a}-\mathrm{A}_{\mathrm{A}}{ }^{0} \mathrm{~b}=0 \text {, i.e., } \Delta \rho_{\mathrm{AB}} \text { vanishes. } \tag{20}
\end{gather*}
$$

## Maximum-Minimum Ratio

However, as already mentioned, even homogeneous soils show deviations from their average properties; these may be expressed for clayey soils by the maximum-minimum ratio of the modulus of elasticity:

$$
\begin{equation*}
\frac{\mathrm{E}_{\mathrm{s}}{ }^{\prime} \max }{\mathrm{E}_{\mathrm{s}^{\prime} \min }}=\% \tag{21}
\end{equation*}
$$

Now, one footing may be supported by soil having $\mathrm{E}^{\prime}$ min and its neighbors by soil with $\mathrm{E}_{\text {max }}^{\prime}$; this extreme case is involved when the maximum natural virtual differential set̂tlement for the discussed structure is to be taken into consideration. Assuming:

$$
\begin{equation*}
\mathrm{E}_{\mathrm{A}}{ }^{\prime}=\mathrm{E}_{0}^{\prime}{ }^{\prime}=\mathrm{E}_{\mathrm{s}}^{\prime}{ }^{\prime} \max \quad \mathrm{E}_{\mathrm{B}}{ }^{\prime}=\mathrm{E}_{\mathrm{s}^{\prime} \min }^{\prime} \tag{22}
\end{equation*}
$$

we have:

$$
\begin{equation*}
\frac{\mathrm{E}_{A^{\prime}}}{\mathrm{E}_{\mathrm{B}^{\prime}}}=x \quad \mathrm{E}_{\mathrm{A}^{\prime}}=\mathrm{E}_{\mathrm{B}^{\prime}} x \tag{23}
\end{equation*}
$$

the virtual damped differential settlement is then (by (15) and (17)):

$$
\begin{equation*}
\Delta_{\rho_{A B}}=\frac{\mathrm{R}_{\mathrm{B}^{0}} \alpha \mathrm{E}_{A^{\prime}} \mathrm{a}-\mathrm{R}_{A^{0}} \alpha \mathrm{E}_{\mathrm{B}} \mathrm{~b}}{\alpha^{2} \mathrm{E}_{\mathrm{A}}^{\prime} \mathrm{E}_{\mathrm{B}^{\prime}} \mathrm{ab}+\frac{3 \mathrm{E}_{\mathrm{C}} \mathrm{~J}_{\mathrm{C}}}{l^{3}}\left(2 \alpha \mathrm{E}_{A^{\prime}} \mathrm{a}+\alpha \mathrm{E}_{\mathrm{B}^{\prime}} \mathrm{b}\right)} \tag{24}
\end{equation*}
$$

or

$$
\begin{equation*}
\Delta_{\rho_{A B}}=\frac{\mathrm{R}_{B^{0}} \alpha \mathrm{a}-\mathrm{R}_{A^{0}} \mathrm{~b}}{\alpha \mathrm{E}_{\max }^{\prime} \mathrm{ab}+\frac{3 \mathrm{E}_{\mathrm{C}} \mathrm{~J}_{\mathrm{C}}}{l^{3}}(2 \times \mathrm{a}+\mathrm{b})} \tag{25}
\end{equation*}
$$

or, for $\sigma_{\mathrm{A}}$ and $\sigma_{\mathrm{B}}$ adjusted according to (19) with the basic differential settlement eliminated:

$$
\begin{equation*}
\Delta \rho_{\mathrm{AB}}=\frac{\mathrm{R}_{\mathrm{B}^{o}}(x-1)}{\alpha \mathrm{E}_{\max }^{\prime} \mathrm{b}+\frac{3 \mathrm{E}_{\mathrm{C}} \mathrm{~J}_{\mathrm{C}}}{l^{3}}\left(2 x+\frac{\mathrm{b}}{\mathrm{a}}\right)} \tag{26}
\end{equation*}
$$

whence $\Delta \rho_{\text {AB }}$ can be estimated separately for each footing, with the effect of adjacent footings expressed by the size ratio $\frac{b}{a}$, and span $l$ only.

## Direct Damped Differential Settlement in Sand

For sand, using the empirical formula for settlement estimation as proposed by Terzaghi \& Peck (5), we have:

$$
\begin{equation*}
\rho_{1}=\rho_{o}\left(\frac{2 B_{i}}{B_{i}+1}\right)^{2} \quad\left(B_{i} \text { footing width in feet }\right) \tag{27}
\end{equation*}
$$

Hence,

$$
\begin{equation*}
\mathrm{K}_{\mathrm{i}}=\frac{\sigma}{\rho_{\mathrm{i}}}=\frac{\sigma}{\rho_{o}}\left(\frac{\mathrm{~B}_{\mathrm{i}}+1}{2 \mathrm{~B}_{\mathrm{i}}}\right)^{2}=\mathrm{K}_{o}\left(\frac{\mathrm{~B}_{1}+1}{2 \mathrm{~B}_{\mathrm{i}}}\right)^{2}=\mathrm{K}_{o} \eta_{\mathrm{i}} \tag{28}
\end{equation*}
$$

$\mathrm{K}_{\mathbf{i}}$ and $\rho_{1}$ being respectively the modulus of subgrade reaction and settlement of a footing of width $B_{i}$ (in feet). $K_{o}$ and $\rho_{o}$ are, respectively, the modulus of subgrade reaction and the settlement of a $1 \mathrm{sq} . \mathrm{ft}$. test plate. $\eta_{i}$ is the size effect factor. The ratio $\rho_{i} / \rho_{o}$ is plotted against $B_{i}$ in Fig. 2. The diagram shows that for small footings only (up to about 6 feet) the size effect is of great importance, while above that limit settlement shows smaller sensitivity to size changes.

The basic differential settlement can be eliminated (for $x=1$ and, of course, theoretically only) by designing the footings with adjusted contact pressures (see numerical example). In some cases, assuming that all footings exceed 6 ft ., settlement differences due to the size effect may be disregarded, so that the difference would only be due to deviations of K in homogeneous soil.

As already mentioned, the deviations of K are represented by the ratio

$$
\begin{equation*}
x=\mathrm{K}_{\mathrm{max}} / \mathrm{K}_{\mathrm{min}} \tag{2}
\end{equation*}
$$

and assuming

$$
\mathrm{K}_{\mathrm{A}}=\mathrm{K}_{\text {max }}=x \mathrm{~K}_{\mathrm{B}}
$$

the formula for the damped differential settlement in sand will be (from (15)):

$$
\begin{equation*}
\Delta_{\rho_{\mathrm{AB}}}=\frac{\mathrm{R}_{\mathrm{B}}{ }^{0} x \mathrm{a}^{2}-\mathrm{R}_{\mathrm{A}^{0}} \mathrm{~b}^{2}}{x \mathrm{~K}_{\mathrm{B}} \mathrm{a}^{2} \mathrm{~b}^{2}+\frac{3 \mathrm{E}_{\mathrm{C}} \mathrm{~J}_{\mathrm{c}}}{l^{3}}\left(2 \times \mathrm{a}^{2}+\mathrm{b}^{2}\right)} \tag{29}
\end{equation*}
$$

Now, with the basic differential settlement due to size effect neglected and footings designed with equal contact pressure $\sigma=\frac{\mathrm{R}_{\mathrm{B}^{0}}}{\mathrm{~b}^{2}}$ $=\frac{\mathrm{R}_{B^{0}}}{\mathrm{a}^{2}}$ the virtual damped differential settlement will be:

$$
\begin{equation*}
\Delta_{\rho_{A B}}=\frac{\mathrm{R}_{\mathrm{B}^{0}}(x-1)}{\mathbf{K}_{\max } \mathbf{b}^{2}+\frac{3 \mathrm{E}_{\mathrm{C}} \mathrm{~J}_{\mathrm{C}}}{l^{3}}\left(2 x+\frac{\mathrm{b}^{2}}{\mathrm{a}^{2}}\right)} \tag{30}
\end{equation*}
$$

If the size effect is to be taken into consideration in designing the footings, the settlements of $x=1$ will be equal.

$$
\rho_{\mathrm{A}}=\rho_{\mathrm{B}}=\frac{\mathrm{R}_{\mathrm{A}}{ }^{0}}{\mathrm{a}^{2} \mathrm{~K}_{\mathrm{oA}} \eta_{\mathrm{A}}}=\frac{\mathrm{R}_{\mathrm{B}^{0}}}{\mathrm{~b}^{2} \mathrm{~K}_{o \mathrm{~B}} \eta_{\mathrm{B}}}
$$

from where:

$$
\begin{equation*}
\mathbf{R}_{A^{o}}=\mathbf{R}_{\mathbf{B}^{o}} \frac{\mathbf{a}^{2} \eta_{A}}{\mathrm{~b}^{2} \eta_{\mathrm{B}}} \tag{31}
\end{equation*}
$$

Now, using the formula (15) and assuming $x=\frac{\mathrm{K}_{0 \mathrm{~A}}}{\mathrm{~K}_{\mathrm{OB}}}>1$ the virtual, damped differential settlement will be:

$$
\begin{equation*}
\Delta \rho_{A B}=\frac{R_{B}{ }^{0} K_{o A} \eta_{\mathrm{A}} a^{2}-\left(\mathrm{R}_{\mathfrak{B}^{\circ}} \frac{\mathrm{a}^{2}}{\mathrm{~b}^{2}} \frac{\eta_{\mathrm{A}}}{\eta_{\mathrm{B}}}\right) \mathrm{K}_{\mathrm{oB}} \mathrm{~b}^{2} \eta_{\mathrm{B}}}{\mathrm{~K}_{\mathrm{OA}} \eta_{\mathrm{A}} \mathrm{~K}_{o \mathrm{~B}} \eta_{\mathrm{B}} \mathrm{a}^{2} \mathrm{~b}^{2}+\frac{3 \mathrm{E}_{\mathrm{C}} \mathrm{~J}_{\mathrm{C}}}{l^{3}}\left(2 \mathrm{~K}_{\mathrm{oA}} \eta_{\mathrm{A}} \mathrm{a}^{2}+\mathrm{K}_{o \mathrm{~B}} \eta_{\mathrm{B}} \mathrm{~b}^{2}\right)} \tag{32}
\end{equation*}
$$

from where:

$$
\begin{equation*}
\Delta \rho_{\mathrm{AB}}=\frac{\mathrm{R}_{\mathrm{B}^{o}}(x-1)}{\mathrm{K}_{\mathrm{o} \max } \eta_{\mathrm{B}} \mathrm{~b}^{2}+\frac{3 \mathrm{E}_{o \mathrm{o}} \mathrm{~J}_{\mathrm{O}}}{l^{3}}\left(2 \varkappa+\frac{(\mathrm{b}+1)^{2}}{(\mathrm{a}+1)^{2}}\right)} \tag{33}
\end{equation*}
$$

By assuming $\eta_{\mathrm{A}}=\eta_{\mathrm{B}}$ (and so neglecting size effect differences at footings larger than 6 feet) the formula (30) is obtained again.

The proposed way would be to design the footings according to equal settlements (with size effect influence), but to neglect this factor in the virtual damped differential settlement formula, using (30) instead of (33), as it is shown in the numerical example. Where pres-sure-settlement relations are incapable of expression in such simple terms, the formula lends itself to certain refinements, but the idea remains the same.

It can be seen that for a completely flexible structure ( $\mathrm{E}_{\mathrm{C}} \mathrm{J}_{\mathrm{C}}=0$ )

$$
\begin{equation*}
\Delta \rho_{\mathrm{AB}}=\frac{\mathrm{R}_{\mathrm{B}^{o}}}{x \mathrm{~K}_{\mathrm{B}} \mathbf{b}^{2}}(x-1)=\rho_{\mathrm{B}}{ }^{\circ} \frac{x-1}{x}=\rho_{\max } \frac{\dot{x}-1}{x} \tag{34}
\end{equation*}
$$

or, for:

$$
\begin{align*}
& x=2 \quad\left(\mathrm{~K}=\mathrm{K}_{\mathrm{av}} \pm 33 \%\right) \quad \Delta \rho_{\mathrm{AB}}=\frac{1}{2} \rho_{\max } . \\
& x=3 \quad\left(\mathrm{~K}=\mathrm{K}_{\mathrm{av}} \pm 50 \%\right) \quad \Delta_{\rho_{\mathrm{AB}}}=\frac{2}{3} \rho_{\text {max }} . \tag{35}
\end{align*}
$$

i.e., values close to those given empirically by Terzaghi \& Peck (5) for natural differential settlement and in agreement with the order of magnitude obtained for $\Delta_{\rho} / \rho_{\max }$ by Skempton \& MacDonald (1) in their comprehensive empirical treatment of this subject.

## More Complicated Cases

To adapt the formula for more complicated cases (from the structural point of view), certain modification of the coefficients is required, and the expressions (from (26) and from (30)):

$$
\frac{3 \mathrm{E}_{\mathrm{c}} \mathrm{Jo}_{\mathrm{o}}}{l^{3}}\left(2 x+\frac{\mathrm{b}}{\mathrm{a}}\right) \text { for clays, or } \frac{3 \mathrm{E}_{\mathrm{o}} \mathrm{~J}_{\mathrm{o}}}{l^{3}}\left(2 x+\frac{\mathrm{b}^{2}}{\mathrm{a}^{2}}\right) \text { for sands }
$$

become, in more general terms:

$$
\frac{\beta \mathrm{E}_{\mathrm{o}} \mathrm{Jo}^{\Sigma}}{l^{3}}\left(2 x+\frac{\mathrm{b}}{\mathrm{a}}\right) \text { or } \frac{\beta \mathrm{E}_{\mathrm{c}} \mathrm{Jo}^{\Sigma}}{l^{3}}\left(\gamma x+\frac{\mathrm{b}^{2}}{\mathrm{a}^{2}}\right)
$$

where $\mathbf{B}$ and $\gamma$ are statical factors of the structure (see Table 1) and $\mathrm{J}^{\Sigma}$ is the overall moment of inertia of the whole section of a multi-story

## TABLE I

| Number of bays and position of settling support | $\beta$ | $\gamma$ |
| :---: | :---: | :---: |
| Tro-bsy, internal support | 3 | 2 |
| Nulti-bay, intermal support | 20.8 | 1.35 |
| Tro-bay, end supiort | 1.5 | 2.0 |
| Multi-bey, end support | 1.6 | 2.3 |

frame. The two formulae (26) and (30) can thus be written in the general form

$$
\begin{equation*}
\Delta \rho_{\mathrm{AB}}=\frac{\mathbf{R}_{\mathrm{B}^{o}(x-1)}}{\alpha \mathrm{E}_{\max }^{\prime} \mathbf{b}+\frac{\beta \mathrm{E}_{\mathrm{o}} \mathrm{~J}_{\mathrm{o}}}{l^{3}}\left(\gamma x+\frac{\mathrm{b}}{\mathbf{a}}\right)} \quad \text { for clays } \tag{36}
\end{equation*}
$$

and

$$
\begin{equation*}
\Delta_{\rho_{\mathrm{AB}}}=\frac{\mathrm{R}_{\mathrm{B}^{0}}(x-1)}{\mathrm{K}_{\text {max }} \mathrm{b}^{2}+\frac{\beta \mathrm{E}_{\mathrm{c}} \mathrm{Jo}^{2}}{l^{3}}\left(\gamma x+\frac{\mathrm{b}^{2}}{\mathrm{a}^{2}}\right)} \text { for sands } \tag{37}
\end{equation*}
$$

Calculation of the equivalent moment of inertia presents no difficulties in a multi-story and multi-bay frame. The question is, what degree of accuracy is justified in such a calculation. In most cases approximate methods are quite adequate for the order of magnitude of the influence on the differential settlement.

As a general rule, it can be stated that the equivalent moment of inertia is

$$
\begin{equation*}
\mathrm{J}_{\mathrm{c}}{ }^{\mathrm{L}}=0.6 \Sigma \mathrm{~J}_{\mathrm{c}} \text { to } 1.0 \Sigma \mathrm{~J}_{\mathrm{c}} \tag{38}
\end{equation*}
$$

where $\Sigma \mathrm{J}$ is the sum of J for all beams. But, bearing in mind the increase in the rigidity of the structure due to secondary elements (floors and partitions), it would be justified for this order of accuracy to take the overall moment in view of the fact that measured stresses and deflections are usually about 0.4 to 0.6 of the calculated ones. It thus seems logical to take the overall rigidity of the structure.

Kany (1959) (11) recommends the full $\Sigma \mathrm{J}$ with the rigidity of partitions or walls allowed for.

The influence of frame supported by the same column in the perpendicular direction may be disregarded. It is assumed that the latter settles in such a way as not to allow transfer of loads between them, causing the virtual worst possible case.* This enables a treatment of the discussed frame as a one-dimensional one.

The footnote from page 204 should also be in mind when discussing the accuracy of the formulae and more sophisticated methods.

## Conclusion

Using simplified statical assumptions and extreme soil-elasticity values, a virtual damped differential settlement can be determined separately for each footing of a framed structure foundation. The formulae are flexible enough for adjustment and refinement with a view to a higher degree of accuracy, if required.

This damped differential settlement should be compared with the allowable differential settlement proposed by Terzaghi \& Peck (5) or,

[^8]more recently by Skempton \& MacDonald (1), as well as checked by statical calculations as to its influence on the stresses in the structure.

It is suggested here that soil laboratories should provide the necessary data for the foundation engineer, including extreme soil properties rather than their averages.

## Acknowledgment

The writer is grateful to Dr. Robert V. Whitman, Professor of Civil Engineering at M.I.T. for reading the paper and for helpful comments.

## Numerical example

The plan and two sections with spans, beam dimensions and typical column loads are given in Fig. 3. The subgrade is medium-loose sand with allowable bearing pressure of about 1.5 ton $/ \mathrm{sq}$. ft. The average modulus of subgrade reaction (Terzaghi '55) (12) is:

$$
\begin{equation*}
\mathrm{K}_{\mathrm{o}}=\overline{\mathrm{K}}_{\mathrm{sl}}=100 \mathrm{t} / \text { cub. ft. } \quad \mathrm{K}_{\mathrm{i}}=\mathrm{K}_{\mathrm{o}}\left(\frac{\mathrm{~B}_{\mathrm{i}}+1}{2 \mathrm{~B}_{\mathrm{i}}}\right)^{2} \tag{28}
\end{equation*}
$$

The square footings and their settlements, calculated for equal allowable pressure would be:

1) $5^{\prime}-2^{\prime \prime} \quad \rho_{1}=0.51^{\prime \prime}$
2) $7^{\prime}-6^{\prime \prime} \quad \rho_{2}=0.56^{\prime \prime}$
3) $6^{\prime}-7^{\prime \prime} \quad \rho_{3}=0.54^{\prime \prime}$
4) $8^{\prime}-9^{\prime \prime} \quad \rho_{4}=0.58^{\prime \prime} \quad \rho=\frac{\sigma}{\mathrm{K}}$
5) $6^{\prime}-4^{\prime \prime} \quad \rho_{5}=0.54^{\prime \prime}$
6) $8^{\prime}-2^{\prime \prime} \quad \rho_{6}=0.57^{\prime \prime}$

Footing dimensions, adjusted to equalize all settlements to $0.51^{\prime \prime}$ using Terzaghi's formula (27) will be; $\left(B_{1}=\sqrt{\frac{4 R_{1}}{K_{o \rho}}}-1\right)$

1) $5^{\prime}-2^{\prime \prime}\left(5.17^{\prime}\right) \quad\left(\sigma_{1}=1.5\right.$ ton/sq. ft. $)$
2) $7^{\prime}-11^{\prime \prime}\left(7.92^{\prime}\right) \quad\left(\sigma_{2}=1.36 \mathrm{ton} / \mathrm{sq} . \mathrm{ft}\right.$. $)$
3) $6^{\prime}-9^{\prime \prime}\left(6.75^{\prime}\right) \quad\left(\sigma_{3}=1.42 \mathrm{ton} / \mathrm{sq} . \mathrm{ft}\right.$. $)$
4) $9^{\prime}-4^{\prime \prime}\left(9.33^{\prime}\right) \quad\left(\sigma_{4}=1.30 \mathrm{ton} / \mathrm{sq} . \mathrm{ft}\right.$. $)$
5) $6^{\prime}-6^{\prime \prime}\left(6.50^{\prime}\right) \quad\left(\sigma_{5}=1.43 \mathrm{ton} / \mathrm{sq} . \mathrm{ft}\right.$. $)$
6) $8^{\prime}-8^{\prime \prime}\left(8.67^{\prime}\right) \quad\left(\sigma_{6}=1.33 \mathrm{ton} / \mathrm{sq} . \mathrm{ft}\right.$. $)$

Now, assuming that $\mathrm{K}_{0}$ is the average of a series of tests with scatter limits of $\pm 50 \%, x$ will be:

$$
\begin{equation*}
x=\mathrm{K}_{\max } / \mathrm{K}_{\min }=150 \% / 50 \%=3 \tag{2}
\end{equation*}
$$

Each footing should be checked along both the longitudinal and lateral direction of the structure.

Table II contains the data for computation of the virtual differential settlements and the virtual settlements proper using the formula

$$
\begin{equation*}
\Delta \rho_{\mathrm{AB}}=\frac{\mathrm{R}_{\mathrm{B}^{0}(x-1)}}{\mathrm{K}_{\max } \mathbf{b}^{2}+\frac{\beta \mathrm{E}_{\mathrm{C}} \mathrm{~J}^{\Sigma}}{l^{3}}\left(\gamma \varkappa+\frac{\mathrm{b}^{2}}{\mathrm{a}^{2}}\right)} \tag{37}
\end{equation*}
$$

TABLE II
Computation of Virtual Differential Settlements

| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (9) |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Foot- |  |  |  |  |  |  |  | $\mathrm{E}_{\mathrm{c}} \mathrm{I}_{\mathrm{c}}$ | (10) |
| ing | $\mathrm{R}_{\mathrm{B}}{ }^{\text {o }}$ | b | $\mathrm{K}_{\text {max }}$ | Direc- | (b) ${ }^{2}$ |  | (8) | $1^{3}$ | $\Delta \rho$ |
| No. | tons | ft. | t/c.ft. | tion | (a) ${ }^{2}$ | $\beta$ | $\gamma$ | $\mathrm{t} / \mathrm{ft}$. | inch |
| 1 | 40 | 5.17 | 53 | lat. | 0.43 | 1.5 | 2.0 | 135 | 0.35 |
|  |  |  |  | long. | 0.58 | 1.6 | 2.3 | 203 | 0.25 |
| 2 | 85 | 7.92 | 48 | lat. | 2.35 | 3.0 | 2.0 | 135 | 0.32 |
|  |  |  |  | long. | 0.72 | 1.6 | 2.3 | 203 | 0.37 |
| 3 | 65 | 6.75 | 50 | lat. | 0.53 | 1.5 | 2.0 | 135 | 0.43 |
|  |  |  |  | long. | 1.32 | 10.8 | 1.35 | 203 | 0.11 |
| 4 | 113 | 9.33 | 46 | lat. | 1.90 | 3.0 | 2.0 | 135 | 0.38 |
|  |  |  |  | long. | 1.26 | 10.8 | 1.35 | 203 | 0.17 |
| 5 | 60 | 6.50 | 50 | lat. | 0.56 | 1.5 | 2.0 | 135 | 0.42 |
|  |  |  |  | long. | 1.0 | 10.8 | 1.35 | 203 | 0.11 |
| 6 | 100 | 8.67 | 46 | lat. | 1.78 | 3.0 | 2.0 | 135 | 0.36 |
|  |  |  |  | long. | 1.0 | 10.8 | 1.35 | 203 | 0.17 |

For footing 1, the values in the lateral direction are obtained as follows:

$$
\begin{align*}
\mathrm{K}_{\max } & =150\left(\frac{5.17+1}{2 \times 5.17}\right)^{2}=53 \mathrm{t} / \mathrm{cub} . \mathrm{ft} \\
\mathrm{~b}^{2} / \mathrm{a}^{2} & =(5.17 / 7.92)^{2}=0.43 \tag{28}
\end{align*}
$$

$\beta$ and $\gamma$ are respectively (Table I): 1.5 and 2.0

$$
\frac{\mathrm{E}_{0} \mathrm{Jc}^{\Sigma}}{l^{3}}=\frac{2 \times 10^{6} \times 1.57 \times 10^{6}}{20^{3} \times 12^{3}}=22500 \mathrm{lb} / \mathrm{in}=135 \mathrm{ton} / \mathrm{ft}
$$

where: $\mathrm{E}_{\mathrm{C}}=2,000,000 \mathrm{lb} / \mathrm{sq} . \mathrm{in}$.

$$
\begin{equation*}
\mathrm{J}_{\mathrm{o}}{ }^{\Sigma}=\sum_{1 .}^{5} \frac{\mathrm{~b} \mathrm{~d}^{3}}{5}=5 \times \frac{10 \times 25^{3}}{5}=157,000 \mathrm{in}^{4} \tag{38}
\end{equation*}
$$

(In longitudinal direction: $\quad \mathrm{J}^{\Sigma}=5 \times \frac{8 \times 20^{3}}{5}=64,000 \mathrm{in}^{4}$


Fig. 1


Fig. 2


Fig. 3a

and $\left.\quad \frac{\mathrm{E}_{\mathrm{C}} \mathrm{J}_{\mathrm{c}}{ }^{\Sigma}}{l^{3}}=\frac{2 \times 10^{6} \times 6,4 \times 10^{4}}{13^{3} \times 12^{3}}=33,700 \mathrm{lb} / \mathrm{in}=203 \mathrm{ton} / \mathrm{ft}.\right)$
The virtual differential settlement of footing 1 in the lateral will be:

$$
\begin{aligned}
\Delta_{\rho_{A \mathrm{~B}}} & =\frac{40(3-1)}{53 \times 5.17^{2}+1.5 \times 135(2 \times 3+0.43)} \\
& =0.0296 \mathrm{ft} .=0.35 \mathrm{inch} .
\end{aligned}
$$

It can be seen from comparison (Table II) that although none of them is likely to endanger the structure, it would be reasonable, designwise, to increase the rigidity of the lateral beams. This would help to keep differential settlements in both directions proportionate to the spans, and to keep the virtual angular distortion within the same order of magnitude.

For more exact values of the differential settlements the formula (33) may be used.

Notation

1. A, B, C, -Footings
2. $\mathrm{a}, \mathrm{b}, \mathrm{c}$, -Size of square footings $\mathrm{A}, \mathrm{B}$, and C respectively
3. $\mathrm{B}_{\mathbf{i}} \quad$ - Width of footing (in general)
4. $\mathrm{E}_{\mathrm{c}} \quad$ - Modulus of elasticity of (concrete) beam
5. $\mathrm{E}_{\mathrm{S}}$ - Modulus of elasticity of soil
6. $\mathrm{E}_{\mathrm{S}^{\prime}}, \mathrm{E}_{\mathrm{A}^{\prime}}, \mathrm{E}_{\mathrm{B}}{ }^{\prime} \quad-\frac{\mathrm{E}_{\mathrm{S}}}{1-\mu^{2}}$ Modulus of elasticity of soil including Poisson influence
7. $j_{c} \quad-$ Moment of inertia of (concrete) beam
8. $j_{c} \Sigma \quad$-Overall moment of inertia of the section of a multistory frame
9. $\mathrm{K}_{0}, \mathrm{~K}_{\mathrm{oA}}, \mathrm{K}_{\mathrm{oB}}$ - Coefficient of subsoil reaction of a 1 sq. ft . test plate
10. $\mathrm{K}, \mathrm{K}_{\mathrm{i}}, \mathrm{K}_{\mathrm{A}}, \mathrm{K}_{\mathrm{B}}, \mathrm{K}_{\mathrm{c}}-$ Coefficient of subsoil reaction
11. $l$ - Span
12. $\mathrm{M}_{\mathrm{B}}{ }^{(-)} \quad$ - Moment at support before settlement
13. $\Delta \mathrm{M} \quad$ - Change in moment at support, due to differential settlement
14. $P_{A}, P_{B}, P_{c}$-Loads, independent of the differential settlement
15. g - Load on beam per unit length
16. $\mathrm{R}^{0}{ }_{A}, \mathrm{R}^{0}{ }_{B}, \mathrm{R}^{\mathrm{o}}{ }_{\mathrm{c}} \quad-$ Loads on footings $\mathrm{A}, \mathrm{B}$, and C , before settlement
17. $R_{A}, R_{B}, R_{c} \quad$ - Final loads on footings $A, B$, and $C$ after settlement
18. $\alpha$

- Shape effect factor on footings

19. $\beta, \gamma$

- Factors of statical conditions

20. $x \quad$ - Maximum-minimum ratio of $\mathrm{E}_{\mathrm{S}^{\prime}}$ or K
21. $\mu$ - Poison coefficient
22. $\rho, \rho_{i}$ - Settlement (in general)
23. $\rho_{\mathrm{A}}{ }^{0}, \rho_{\mathrm{B}}{ }^{0}, \rho_{\mathrm{c}}{ }^{0}$ - Basic settlements (before damping by structure)
24. $\rho_{\mathrm{A}}, \rho_{\mathrm{B}}, \rho_{\mathrm{c}} \quad$ - Final damped settlements
25. $\Delta \rho_{\mathrm{A}}{ }^{\circ} \mathrm{B}$ — Basic differential settlement between footings A and B
26. $\Delta \rho_{\mathrm{AB}} \quad$ - Damped differential settlement between footings $A$ and $B$
27. $\Delta \rho_{A}, \Delta \rho_{B}$ - Change in settlement due to the influence of the rigidity of the structure
28. $\rho_{0} \quad$ - Settlement of a 1 sq . ft. test plate
29. $\sigma, \sigma_{A}, \sigma_{B}, \quad$ - Stress in soil
30. $\Delta \sigma, \Delta \sigma_{\mathrm{A}}, \Delta \sigma_{\mathrm{B}} \quad$ - Change in stresses due to differential settlement
31. $\eta_{i}, \eta_{A}, \eta_{B} \quad$-Size effect factors $\left[\left(\frac{B_{i}+1}{2 B_{i}}\right)^{2}\right]$

## References

1. Skempton, A. W. and MacDonaid, D. H. "The Allowable Settlements of Buildings," Proceedings Inst. of Civil Engineers, Part III, Vol. 5, London, Dec. 1956.
2. Bjerrum, L. Lectures in Foundations for Postgraduates, M.I.T., 1961, Unpublished.
3. Terzaghi, K. "The Actual Factor of Safety in Foundations," The Structural Engineer, London, March 1935.
4. Timoshenko, S. and Goodier, J. N. "Theory of Elasticity," McGraw-Hill, 1951.
5. Terzaghi, K. and Peck, R. B. "Soil Mechanics in Engineering Practice," John Wiley \& Sons, 1948.
6. Meyerhof, G. G. "The Settlement Analysis of Building Frames," The Structural Engineer, London, September 1947.
7. Chamecki, S. "Structural Rigidity in Calculating Settlements," Journal Soil Mechanics and Foundation Division, ASCE, January 1956.
8. Yokoo, Y. and Yamagata, K. "A Theoretical Study on Differential Settlements of Structures," Kyoto University, Japan, 1956.
9. Terzaghi, K. "Theoretical Soil Mechanics," John Wiley \& Sons, 1943.
10. Skempton, A. W. "The Bearing Capacity of Clays," Building Research Congress, Division I, 180, London, 1951.
11. Kany, M. "Berechnung von Flächengründungen," Wilhelm Ernst \& Sons, Berlin 1959.
12. Terzaght, K. "Evaluation of Coefficient of Subgrade Reaction." Géotechnique, December 1955.

## Of General Interest

## PROCEEDINGS OF THE SOCIETY

## MINUTES OF MEETING

## Boston Society of Civil Engineers

Jandary 26, 1966:-A Joint Meeting of the Boston Society of Civil Engineers with the Hydraulics Section was held this evening in the Adams Room, United Community Services Building, 14 Somerset Street, Boston, Mass., and was called to order by President Leslie J. Hooper, at 7:00 P.M.

President Hooper stated that the Minutes of the previous meeting held November 17, 1965, would be published in a forthcoming issue of the Journal and that the reading of those Minutes would be waived unless there was objection.

The Secretary announced the names of applicants for membership and that the following had been elected to membership January 26, 1966:-

Grade of Member-Rocco A. Blasi, Alfred A. Gass, Richard D. Moulton, Vincent J. Murphy, Paul N. Narey, Albert W. Parsons, Jr., Araken Silveria
President Hooper called upon the Secretary to present a recommendation of the Board of Government to the Society for action. The President stated that this matter was before the Society in accordance with the provisions of the By-Laws, and that notice of such action
published in the ESNE Journal dated January 10, 1966.

The Secretary presented the following recommendation of the Board to the Society for action to be taken at this meeting:-

MOTION "to recommend to the Society that the Board of Government be authorized to transfer an amount not to exceed $\$ 6500$ from the Principal of the Permanent Fund to the Current Fund for Current Expenditures."
On MOTION duly made and seconded it was VOTED "that the Board of Government be authorized to transfer an amount not to exceed $\$ 6500$ from the Principal of the Permanent Fund to the Current Fund for Current Expenditures."

President Hooper stated that this was the final action on this matter.

President Hooper stated that this was a Joint Meeting with the Hydraulics Section and called upon Peter S. Eagleson, Chairman of that Section to conduct any necessary business for that Section at this time.

President Hooper introduced Robert L. Meserve who gave a short résumé of the activities of the ESNE.

President Hooper stated that this meeting was sponsored by the John R. Freeman Committee as "A John R. Freeman Memorial Lecture," and intro-
duced the guest speaker Dr. Hunter Rouse, Director Iowa Institute of Hydraulic Research, who presented an illustrated talk on "Jet Diffusion and Cavitation."

At the conclusion of the Lecture President Hooper presented Dr. Rouse with an honorarium and a certificate of appreciation.

A discussion period followed the lecture.

Seventy-two members and guests attended the dinner preceding the meeting and ninety-two members and guests attended the meeting.

The meeting adjourned at 9:30 P.M.

> Respectfully submitted, Charles O. Baird, Jr., Secretary

Marcar 23, 1966-The 118th Annual Meeting of the Boston Society of Civil Engineers was held today at the Hotel Vendome, 160 Commonwealth Avenue, Boston, Mass., and was called to order at 4:00 P.M., by President Leslie J. Hooper.

President Hooper announced that the reading of the Minutes of Society Meetings had been omitted during the year. The Minutes of the January and February 1966 meetings would be published in a forthcoming issue of the Journal. The Minutes of the May, Oct., Nov., Dec., 1965 meetings to be declared approved as published.

It was VOTED "to approve the Minutes as published."

The Secretary announced the names of applicants for membership and that the following had been elected to membership March 21, 1966:-

Grade of Member-Carl V. Anderson, Jr., John H. Bousman, Robert D. Buckley, James P. Dunne, Joseph C. Hohmann, Jr.,* James C. Jacobs, Edmund C. Johnson, Edwin S.

Joiner, Robert L. McWilliams, Herbert M. Priluck,* David D. Primmer, Oscar L. Smith
Grade of Student-Christian Wethe
*Trans. from Junior
The Annual Reports of the Board of Government, Treasurer, Secretary and Auditors were presented. Reports were also made by the following committees: -Hospitality, Library, John R. Freeman, Subsoils of Boston, Joint Committee on Legislative Affairs, Education Committee, Professional Conduct, Advertising Committee.

It was VOTED "that these reports be placed on file."

The Annual Report of the various Sections were read and it was VOTED "that the Annual Reports of the various Sections be placed on file."

President Hooper stated that all foregoing reports would be published in the April, 1966 issue of the Journal.

The Report of the Tellers of Election, Robert L. Fuller and Charles E. Fuller was presented and in accordance therewith the President declared the following had been elected Officers for the ensuing year:-

| President | John M. Biggs |
| :--- | :--- |
| V-President | Harl P. Aldrich, Jr. |
| Secretary | Charles O. Baird, Jr. |
| Treasurer | Paul A. Dunkerley |
| Directors | Ernest A. Herzog |
|  | Frank J. Heger |
| Nominating | Max D. Sorota |
| Committece | Darrell A. Root <br>  <br>  Harry S. Feldman |

The retiring President Leslie J. Hooper then gave his address entitled, "A Backsight."

Thirty-seven members and guests attended the business meeting.

The meeting adjourned at 5:30 P.M., to re-assemble at 7:00 P.M. The social

| Award | Recipient | Paper |
| :--- | :--- | :--- |
| Ralph W. Horne Fund | Miles N. Clair |  |
| Award | "Diffusion and Mixing" |  |
| Desmond FitzGerald Medal <br> Clemens Herschel Award | William E. Dobbins <br> Peter S. Eagleson | "Hydraulic Fundamentals <br> $1 \& 2 "$ |
| Hydraulics Section Award | James W. Daily | "Cavitation Phenomena in |
| Hydraulic Systems" |  |  |

hour and annual dinner being held during the interim.

President Hooper called the meeting to order at 7:00 P.M.

Following general remarks and the introduction of the newly elected President John M. Biggs, President Hooper also introduced Walter Jacobson, the Finnish Ambassador to the United Nations, who was a guest of the Society.

President Hooper stated that a number of prizes were awarded annually for worthy papers presented at the Society and Section meetings, and also Scholarship Awards. The Secretary read the names of recipients and asked them to come forward and President Hooper presented the following awards.
Honorarium and prize books were awarded also to speakers in the Soil Mechanics Lecture Series:

Harl P. Aldrich, Jr., "Selection and Performance of Foundations." Arthur Casagrande, "Foundations on Soft Ground with Emphasis on Floating Foundations." Donald T. Goldberg, "Bearing Capacity and Settlement of Footings." James F. Haley, "Shallow Foundations." Ronald C. Hirschfeld, "Introduction-Local Geology." Charles C. Ladd, "Consolidation Behavior of Soils." T. William Lambe, "Analysis of

Settlements." Henry A. Mohr, "Piles and Caisson Foundations." Steve J. Poulos, "Theory of Consolidation." Robert V. Whitman, "Strength of Soils." President Hooper then introduced the guest speaker of the evening. Honorable John F. Collins, Mayor of the City of Boston, who gave a most interesting talk on "Plight of the Cities."

At the conclusion of the talk President Hooper turned the meeting over to President elect, John M. Biggs.

President Biggs presented retiring President Leslie J. Hooper with a certificate of appreciation for services rendered.

Two hundred fifty-one members and guests attended the dinner and meeting.

The meeting adjourned at 9:00 P.M.
Respectfully submitted, Charles O. Batrd, Jr., Secretary

## STRUCTURAL SECTION

Jandary 12, 1966:-A regular meeting of the Structural Section was held this evening in the Society rooms and was called to order by the chairman, Donald T. Goldberg, at 7:35 P.M.

The chairman asked for nominations for the nominating committee. Nominated were Percival S. Rice, Chairman,

Harl P. Aldrich, Jr., and Max D. Sorota.
The chairman introduced the speaker of the evening, Professor Sepp Firnkas of Northeastern University, who spoke on "Prestressed-Precast Concrete Applied to Urban Renewal Projects."

The Speaker wished to illustrate the advances made in this area during recent years. All bearing walls, floors, and stairs were precast as a method to reduce costs as they were mass produced. Means for assembling the units were illustrated. Projects in Boston, New London and Winchester were mentioned, pointing out problems with contractors, various building authorities, and erection procedures. Although the same structural system was used for all projects, different curtain walls and layouts led to apartments with varying appearance. The talk was well illustrated with colored slides. The speaker concluded with some pictures of future projects and a summary of some of the costs. Structural costs were about $\$ 3$ per sq. ft. and finished costs varied from $\$ 10$ to about $\$ 14$ per sq. ft.

After a very extensive question and answer period, the meeting was adjourned at 9:00 P.M.

Attendance was 47.
Respectfully submitted, Charles C. Ladd, Clerk

## STRUCTURAL SECTION

February 9, 1966:-A regular meeting of the Structural Section was held this evening in the Society rooms and was called to order by the vice-chairman, Robert L. Fuller, at 7:12 P.M.

Mr. William Henderson read the report of the nominating committee and conducted the election. Elected were: Chairman, Robert L. Fuller; Vice-Chairman, Charles C. Ladd; Clerk, Fritz F. Hampe, Executive Committee, Richard
C. Jasper, Albert B. Rich and Floyd E. Brown.

The vice-chairman introduced the speaker of the evening, Professor Kentaro Tsutsumi of Tufts University, who spoke on "A Stable Test Station Foundation for Aero Space Projects."

The topic concerned the foundations of structures used to test inertial guidance equipment. These equipment require an extremely stable foundation which needs a quiet environment, although it is impossible to have complete isolation from motions. Some of the sources of disturbance are earth motions, thermal distortions and subsidence of the foundation and/or building, acoustic noise, local variations in temperature, humidity and barometric pressure, and stray electric currents.

The speaker described the test facilities and the results of investigations of foundation movements at several locations (Martin's facility in Denver, several facilities of Honeywell in Minnesota, and M.I.T.'s facility in Cambridge). Measurements of extremely small accelerations (tens of micro g's) and tilts (few arc seconds) were recorded which required the development of new instrumentation. Some of the equipment was so sensitive that it could detect persons walking outside of a building. A servo-control system for controlling motions of the order of one micro inch was described.

After a question and answer period, the meeting was adjourned at 8:35 P.M.

Attendance was 48.
Respectfully submitted, Charles C. Ladd, Clerk

## TRANSPORTATION SECTION

February 24, 1966 :--The Annual Meeting of the Transportation Section of the

Boston Society of Civil Engineers was held in the Society rooms, 47 Winter Street, Boston, Mass., and was called to order by Chairman Benedict J. Quirk, at 7:00 P.M.

Chairman Quirk dispensed with the reading of the Minutes and nominations for the officers for the coming year were presented from the floor. The following officers were elected for 1966-67:-

Chairman Casimir J. Kray
V-Chairman Warren D. McLea
Clerk Robert J. Kiley
Executive
Committee Alexander J. Bone
Charles H. Flavin, Jr. James F. Orpin, Jr.
Incoming Chairman Kray then introduced the speaker of the evening Robert J. Hansen, Professor of Civil Engineering, M.I.T., Deputy Director of Project Transport, who spoke on "High Speed

Ground Transportation 120 MPH and 200 to 400 MPH-Boston-New York -Washington."

The speaker described the progress to date on the Federal Government's high speed ground transportation program for the Northeast Corridor and the role that M.I.T. and other research and development agencies are playing in this program. He described many of the systems and component parts that are now being studied and developed.

After a lively question period, Chairman Kray commended last year's committee for a job well done and requested that all members actively participate in future meetings.

The meeting was adjourned at 8:00 P.M.

Attendance was 70.
Respectfully submitted, Robert J. Kiley, Clerk
ANNUAL REPORTS
REPORT OF THE BOARD OF GOVERNMENT FOR YEAR ..... 1965-1966Boston, Mass., March 16, 1966
To the Boston Society of Civil Engineers:Pursuant to the requirements of the By-Laws the Board of Government pre-sents its report for the year ending March 23, 1966.The following is a statement of the status of membership in the Society:
Honorary ..... 11
Members ..... 1024
Associates ..... 5
Juniors ..... 42
Students ..... 12
Total ..... 1094
Student Chapters ..... 2
Summary of Additions
New Members ..... 37
New Juniors ..... 9
New Students ..... 5
Reinstatements
Members ..... 0
Summary of Transfers
Junior to Member ..... 6
Summary of Loss of Members
Deaths ..... 13
Resignations ..... 13
Dropped for non-payment of dues ..... 18
Dropped for failure to transfer ..... 2
Life Members ..... 108
Members becoming eligible today for Life Membership ..... 5
Applications pending on March 16, 1966 ..... 23
Honorary Membership is as follows:

Arthur R. Casagrande, elected, February 1, 1965
E. Sherman Chase, elected, February 3, 1964

Gordon M. Fair, elected, February 3, 1964
Frank M. Gunby, elected, February 15, 1950
Ralph W. Horne, elected, February 1, 1965
Karl R, Kennison, elected, February 7, 1951
Frank A. Marston, elected, February 15, 1960
Howard M. Turner, elected, February 18, 1952
Frederic N. Weaver, elected, February 1, 1965
The following members have been lost through death:
Albert E. Abruzzese, September 27, 1965
Henry W. Buck, September 16, 1965
Edward H. Cameron, November 28, 1964
Frank M. Carhart, June 24, 1965
Frederick O. Clapp, 1966
William D. Jordan, January 17, 1966
Edward L. Lockman, September 24, 1965
Charles A. McManus, February 20, 1965
Arthur F. McVarish, May 31, 1965
Herman S. Price, May 7, 1965
Allen E. Rucker, December 21, 1965
Harrison E. Schock, August 1965
Heald, Harold F., September 3, 1965

## Meetings of the Society

March 18, 1965. Address of the retiring President William A. Henderson. "Whither the Boston Society of Civil Engineers."

May 17, 1965. Joint Meeting with the Transportation Section. Frank P. Davidson, President, Technical Studies, Inc., "Channel Tunnel: The View from Dover Castle."

October 27, 1965. Joint Meeting with Massachusetts Section, American Society of Civil Engineers (Student Night). George R. Rich, Vice President of Chas. T. Main, Inc., and Partner of Uhl, Hall \& Rich. "Challenge of Civil Engineering."

November 17, 1965. Joint Meeting with Construction Section. Mr. Gary Kallmann, Mr. William J. LeMessurier, Mr. David Morton. "Design and Construction of the New Boston City Hall."

January 26, 1966. Joint Meeting with the Hydraulics Section. John R. Freeman Memorial Lecture. Dr. Hunter Rouse. "Jet Diffusion and Cavitation."

February 14, 1966. Joint Meeting with Massachusetts Section, American Society of Civil Engineers. John W. Leslie. "Upper Saint John River Power Project."

Twenty meetings were held by the Sections of the Society during the year. These meetings of the Sections offering opportunity for more detailed discussions continue to demonstrate their value to their members and to the Society. A wide variety of subjects were presented. The Annual Reports of the various Sections will

Attendance at Meetings

| Daie | Place | Meetings | Dinner |
| :--- | :--- | :---: | :---: |
| March 18, 1965 | Hotel Vendome | 60 | 227 |
| May 17, 1965 | Society Rooms | 35 | 155 |
| October 27, 1965 | Worcester Polytechnic <br> Institute | 155 |  |
| November 17, 1965 | United Community Services <br> Building | 165 | 115 |
| January 26, 1966 | United Community Services <br> Building | 72 | 92 |
| February 14, 1966 | Red Coach Grill | 140 | 140 |

be presented at the Annual Meeting and will be published in the Journal of the Society.

## Funds of the Society

Permanent Fund. The Permanent Fund of the Society has a present value of $\$ 78,836.26$. The Board of Government authorized the use of as much as necessary of the current income of this fund in payment of current expenses. By vote of the Society (as prescribed by the By-Laws) at the November 17, 1965, and January 26, 1966 meetings, the Board of Government was authorized to transfer an amount not to exceed $\$ 6,500$ from the principal of the Permanent Fund for current expenditures. The amount necessary to transfer from the principal of the Permanent Fund for current expenditures was $\$ 3,418.17$.

John R. Freeman Fund. In 1925, the late John R. Freeman, a Past President and Honorary Member of the Society, made a gift to the Society of securities which was established as the "John R. Freeman Fund." The income from this fund is to be particularly devoted to the encouragement of young engineers. Mr. Freeman suggested several uses, such as, the payment of expenses for experiments and compilations to be reported before the Society; for underwriting meritorious books or publications pertaining to hydraulic science or art; or a portion be devoted to a yearly prize for the most useful paper relating to hydraulics contributed to the Society; or establishing a traveling scholarship every third year open to members of the Society for visiting engineering works, a report of which would be presented to the Society. This year a "John R. Freeman Memorial Lecture" was given. Dr. Hunter Rouse presented a paper on "Jet Diffusion and Cavitation." The expenditure from this fund during the year was $\$ 433.00$.

Edmund K. Turner Fund. In 1916 the Society received a bequest of $\$ 1,000$ from Edmund K. Turner, a former member of the Society, the income of which "is to be used for library purposes." The Board voted to use $\$ 100$ of the income of this fund for the purchase of books for the library. The expenditure from this fund during the year was \$94.49.

Alexis H. French Fund. The Alexis H. French Fund, a bequest of $\$ 1,000$ was received in 1931, from the late Alexis H. French, a former Past President of the Society. The income of this fund is "to be devoted to the library of the Society." The Board voted to use $\$ 100$ of the income of this fund for the purchase of books for the library. The expenditure from this fund during the year was $\$ 60.99$.

Tinkham Memorial Fund. The "Samuel E. Tinkham Fund," established in 1921 at Massachusetts Institute of Technology, by the Society, "to assist some worthy student of high standing to continue his studies in civil engineering," had a value of $\$ 2,922.70$ on June 30, 1965. Roger Allen Samuel, a student in civil engineering, class of 1966, was awarded this Scholarship of \$100 for year 1965-1966.

Desmond FitzGerald Fund. The Desmond FitzGerald Fund established in 1910, a bequest of $\$ 2,000$ from the late Desmond FitzGerald, a Past President and Honorary Member of the Society, provided that the income from this fund shall "be used for charitable and educational purposes." The Board voted on April 13, 1964, to appropriate from the income of this fund the sum of $\$ 100$ to be known as the "Boston Society of Civil Engineers' Scholarship in Memory of Desmond FitzGerald," and be given to a student in civil engineering, at Northeastern University. It was voted on February 18, 1965 "to adopt the recommendation of the Committee at Northeastern University, namely, a $\$ 100$ scholarship be given to Richard A. Easler. Presentation was made at the Annual Meeting of the Society on March 18, 1965.

Clemens Herschel Fund. This fund was established in 1931, by a bequest of $\$ 1,000$ from the late Clemens Herschel, a former Past President and Honorary Member of the Society. The income from this fund is "to be used for presentation of prizes for papers which have been particularly useful and commendable and worthy of grateful acknowledgment." The Board voted April 22, 1965 "that no appropriation be made from this fund this year."

Edward W. Howe Frund. This fund is a bequest of $\$ 1,000$ received in 1933, from the late Edward W. Howe, a former Past President of the Society. No restrictions were placed on the use of this bequest, but the recommendations of the Board of Government were "that the fund be kept intact, and that the income be used "for the benefit of the Society or its members." The Board voted April 22, 1965 "that a sum not to exceed $\$ 127$ be appropriated from the income of this fund to be used to pay half of the cost of prize awards and Honorary Membership Certificates." The expenditure from this fund during the year was $\$ 119.07$.

William P. Morse Fund. This fund, a bequest of $\$ 2,000$, was received in 1949, from the late William P. Morse, a former member of the Society. No restrictions were placed on the use of this bequest, but the recommendations of the Board of Government were "that this fund be kept intact and that the income be used for the benefit of the Society or its members." Upon recommendation of the Committee appointed by the President, the Board voted on April 5, 1954, "to appropriate from the income of this fund the sum of $\$ 100$, to be known as the "Boston Society of Civil Engineers' Scholarship in Memory of William P. Morse," and be given to a civil engineering student at Tufts University." It was voted on February 18, 1965, "to adopt the recommendation of the Committee at Tufts University, namely, a
$\$ 100$ scholarship be given to Allen B. Potvin. Presentation was made at the Annual Meeting of the Society on March 18, 1965.

Frank B. Walker Fund. This fund, a bequest of $\$ 3,000$, was received in 1961, from Mary H. Walker, wife of Frank B. Walker, a former Past President of the Society. No restrictions were placed on the use of this bequest, but the recommendations of the Board of Government were "that this fund be kept intact and that the income be used for the benefit of the Society or its members." The Board voted on April 22, 1965 "that a sum not to exceed $\$ 127$ be appropriated from the income of this fund to be used to pay half of the cost of prize awards and Honorary Membership Certificates." The expenditure from this fund during the year was $\$ 119.07$.

Ralph W. Horne Fund. This fund, a bequest of $\$ 3,000$, was received June 29, 1964, from the Directors of Fay, Spofford \& Thorndike, Inc., the income from which shall be devoted to a prize or certificate to be awarded annually to a member designated by the Board of Government as having been outstanding in unpaid public service in municipal, state, or federal elective or appointive posts; or in philanthropic activity in the public interest. Members of BSCE only, are eligible for the Award. The Board of Government voted unanimously January 26, 1966 "to approve recommendation of the Ralph W. Horne Fund Award Committee, namely, "that Miles N. Clair be the second recipient to receive the Ralph W. Horne Fund

| Prizes |  |
| :---: | :---: |
| Award | Recipient Paper |
| Desmond FitzGerald Medal | William E. Dobbins "Diffusion and Mixing" |
| Clemens Herschel Award | Peter S. Eagleson "Hydraulic Fundamentals |
| Hydraulics Section Award | James W. Daily <br> "Cavitation Phenomena in Hydraulic Systems" |
| Recipient | Paper |
| Ronald C. Hirschfeld | "Introduction-Local Geology" |
| Steve J. Poulos | "Theory of Consolidation" |
| Charles C. Ladd | "Consolidation Behavior of Soils" |
| T. William Lambe | "Analysis of Settlements" |
| Robert V. Whitman | "Strength of Soils" |
| Donald T. Goldberg | "Bearing Capacity and Settlement of Footings" |
| James F. Haley | "Shallow Foundations" |
| Henry A. Mohr | "Piles and Caisson Foundations" |
| Arthur Casagrande | "Foundations on Soft Ground with Emphasis on Floating Foundations" |
| Harl P. Aldrich, Jr. | "Selection and Performance of Foundations" |

Award." Presentation to be made at the Annual Meeting of the Society on March 23, 1966.

The Board of Government voted November 15, 1965 "that an honorarium be given to each of the speakers in the Soil Mechanics Lecture Series, of a book or books not to exceed $\$ 25$, also that a stipend of $\$ 50$ be given to each of the speakers."

## Library

The report of the Library Committee contains a complete account of the Library Committee's activities during the past year.

## Committees

The usual special committees dealing with the activities and conduct of the Society were appointed. The membership of these committees is published in the Journal and the reports of the committees will be presented at the Annual Meeting.

Your Board in conclusion, wishes to express its appreciation of the excellent work done by the officers of the Sections and by the committees of the Society.

Leslie J. Hooper, President

## REPORT OF THE SECRETARY

Boston, Mass., March 23, 1966

## To the Boston Society of Civil Engineers:

The following is a statement of cash received by the Secretary and of the expenditures approved by the President in accordance with the budget adopted by the Board of Government.

For the Year Ending March 23, 1966

|  | Expenditures | Receipts |
| :--- | ---: | ---: |
| Office |  |  |
| Secretary's Salary \& Expeinse | $\$ 1,169.48$ |  |
| Treasurer's Honorarium | 746.65 |  |
| Stationery, Printing \& Postage | 845.72 |  |
| Incidentals \& Petty Cash | 103.84 |  |
| Insurance \& Treasurer's Bond | 148.00 |  |
| Quarters, Rent, Tel. \& Light | $5,377.90$ |  |
| Office Secretary | $6,007.00$ |  |
| Social Security | 266.91 |  |


| Meetings |  |  |
| :---: | :---: | :---: |
| Rent of Halls, etc. | 80.00 |  |
| Stationery, Printing \& Postage | - |  |
| Hospitality Committee | 1,322.40 | \$ 1,147.75 |
| Reporting \& Projection | 7.50 |  |
| Annual Meeting, March, 1965 | 1,282.75 | 1,040.00 |
| Sections |  |  |
| Sanitary Section | 34.50 |  |
| Structural Section | 54.32 |  |
| Transportation Section | 21.50 |  |
| Hydraulics Section | 9.05 |  |
| Construction Section | 12.50 |  |
| Journal |  |  |
| Editor's Salary \& Expense | 748.92 |  |
| Printing \& Postage | 5,641.67 |  |
| Advertisements | - | 1,303.40 |
| Sale of Journals | - | 2,616.00 |
| Reprints | 370.62 | 293.84 |
| Copyright | 14.00 |  |
| Library |  |  |
| Periodicals | 51.00 |  |
| Binding | 41.05 |  |
| Miscellaneous |  |  |
| Binding Journals for Members | 17.00 | 17.00 |
| Badges | - | 15.00 |
| Bank Charges | 3.32 |  |
| Miscellaneous | 325.99 | 181.85 |
| Engineering Societies Dues and |  |  |
| - Charge for Journal Space | 1,196.93 |  |
| Public Relations Committee | 54.75 |  |
| Dues from B.S.C.E. Members |  | 11,789.97 |
| Trans. Income Perm. Fund |  | 4,132.29 |
| Trans. Prin. |  | 3,418.17 |
|  | \$25,955.27 | \$25,955.27 |

Entrance Fees to Permañent Fund $\$ 430$.
37 New Members; 9 New Junior Members; Juniors Trans. to Members 6; 5 New Student Members.

The above receipts have been paid to the Treasurer whose receipt the Secretary holds. The Secretary holds cash amounting to $\$ 30$ included as payment under item 36 (Petty Cash) to be used as a fixed fund or cash on hand. $\$ 279.24$ withholding
tax and $\$ 140.72$ social security, which is payable to Collector of Internal Revenue and State of Massachusetts in April, 1966, is not included in the above tabulation.

Respectfully submitted, Charles O. Batrd, Jr., Secretary

## REPORT OF THE TREASURER

Boston, Mass., March 23, 1966

## To the Boston Society of Civil Engineers:

This report covers the fiscal year beginning March 1, 1965 and ending at the close of business on March 1, 1966.

The Boston Safe Deposit and Trust Company continues to hold the investment securities and to serve as custodian and financial advisor. All security transactions are made by the custodian upon a vote of approval of the Board of Government of this Society. In accordance with the terms of the agreement, the custodian has furnished the Treasurer of the Boston Society of Civil Engineers with a certified audit of the principal and income accounts. This audit, the receipts from the Secretary, the bills paid by the Treasurer, the savings bank passbook, and the checkbook have all been reviewed by the Auditing Committee of the Society and the information in this report verified.

The Boston Safe Deposit and Trust Company reviews the portfolio of securities twice a year. After each review the custodian usually recommends changes. The Investment Committee of the Society studies the proposed changes and submits them to the Board of Government in the form of a motion. The bank is advised of the vote taken by the Board and proceeds with the appropriate action.

The following is a record of security changes accomplished by the custodian during the fiscal year:

Jewel Tea Co., Inc.
Received in stock split 62 shares
Illinois Power Co.
Bought 200 shares
Pacific Gas \& Electric Co. (Common)
Sold 315 shares
U.S.A. Treasury Bonds

Bought $\$ 3,000$ par value
Monsanto Co.
Received in stock dividend $2-4 / 100$ shares
Bought 96/100 shares
Received in consolidation 1 share
International Business Machine Corp.
Bought 9 shares
American Telephone and Telegraph Co.
Sold 70 shares

The financial standing of the Society as of March 1,1966 is summarized in the seven tables accompanying this report. These tables are as follows:

Table I Distribution of Funds
Table II Distribution of Funds-Receipts and Expenditures
Table III Record of Investments-Bonds
Table IV Record of Investments-Stocks
Table V Record of Investments-Savings Bank
Table VI Comparison of Book and Market Values of Stocks, Bonds, Savings Bank, and Investment Cash
Table VII Comparison of Book and Market Values of Funds
In order to have enough cash to pay the bills it was necessary to transfer $\$ 10,000.00$ to the checking account. Of this, $\$ 2,000.00$ was withdrawn from the savings bank, and $\$ 8,000.00$ from the Income Account of the Custodian Account. On March 1, 1966 the checkbook had a balance of $\$ 2,750.77$.

Receipts from the Secretary including membership dues, advertisements in the Journal, sale of Journals, etc., amounted to $\$ 18,404.81$ and was credited to the Current Fund. The Secretary also received $\$ 430.00$ in entrance fees which was credited to the Permanent Fund.

## Permanent Fund

The income from interest and dividends which was credited to the Permanent Fund amounted to $\$ 4,651.04$. That portion of custodian charges which is attributable to the Permanent Fund came to $\$ 518.75$. The remainder of the income to the Permanent Fund ( $\$ 4,132.29$ ) was transferred by vote of the Board of Government to the Current Fund to meet current expenses.

At the November 1965 and January 1966 meetings of the Society it was voted that a sum not to exceed $\$ 6,500.00$ might be transferred from the principal of the Permanent Fund to the Current Fund to meet current expenses.

It became necessary to transfer $\$ 3,418.17$ from the principal of the Permanent Fund to the Current Fund.

The total expenditure from the Permanent Fund was $\$ 8,069.21$.
The January 1966 issue of the Journal was not ready for publication by the end of the fiscal year. The cost of the issue (approximately $\$ 1,500$ ) was removed from the budget of the 1965 fiscal year and will be included in the budget for the 1966 fiscal year.

## John R. Freeman Fund

The Freeman Fund Committee has inaugurated an annual series of lectures in memory of John R. Freeman. It is intended to invite distinguished investigators and lecturers in the field of fluid mechanics and hydraulics to deliver a lecture or series of lectures using the income of the John R. Freeman Fund. Dr. Hunter Rouse was the first invited guest to deliver a Freeman Memorial Lecture. The honorarium and part of the expenses paid during the fiscal year amounted to $\$ 433.00$.

## Edmund K. Turner Fund

The sum of $\$ 94.49$ was expended from the income of the Edmund K . Turner Fund for new books for the Library.

## Desmond FitzGerald Fund

A scholarship of $\$ 100.00$ from the income of the Desmond FitzGerald Fund was awarded at the last annual meeting of the Society to Richard A. Easler, a student at Northeastern University.

## Alexis H. French Fund

The income of the Alexis H. French Fund was used to provide new books for the Library in the amount of $\$ 60.99$.

Edward W. Howe Fund
Part of the prize awards and honorary membership certificates awarded in March 1965 were paid for, using income of the Edward W. Howe Fund. The sum of $\$ 119.07$ was expended.

## William P. Morse Fund

Mr. Allen B. Potvin, a student at Tufts University was awarded a $\$ 100$ scholarship from the income of the William P. Morse Fund. The award was made at the annual meeting of the Society in March 1965.

## Frank B. Walker Fund

The income of the Frank B. Walker Fund was depleted by $\$ 19.07$ to pay for the other half of the prize awards and honorary membership certificates which were presented at the last annual meeting.

## Ralph W. Horne Fund

An expenditure of $\$ 232.50$ was made from the income of the Ralph W. Horne Fund to provide the certificates awarded to Professor Charles O. Baird (the Secretary) at the annual meeting of March 1965, and to the recipient of the award at this annual meeting, Mr. Miles N. Clair.

## Surveying Lectures Fund

At the May 17, 1965 meeting of the Board of Government it was voted to transfer to the Structural Lectures Fund the unexpended balance (\$628.68) of the Surveying Lectures Fund.

## Structural Lectures Fund

In the late fall of 1965 and early in 1966 a series of Soil Mechanics Lectures was sponsored by the Structural Section. A nominal fee was charged those attending. The number of persons attending was so large that $\$ 2,895.50$ was realized as income. Many of those attending were not members of the Society and have since become members. Part of the fee to nonmembers has been transferred to dues when those nonmembers joined the Society. $\$ 237.00$ from those receipts has already been transferred to dues and additional amounts will be transferred in fiscal 1966.

MIT and Harvard University both provided quarters for this series of lectures for which the B.S.C.E. is deeply grateful.

Printing, postage, slides, projection costs plus awards to the individual lecturers in recognition of their services amounted to $\$ 1,470.46$.

## Boring Data Fund

The sale of Boring Data books realized an income of $\$ 35.00$ to this fund.

## Soil Mechanics Fund

In November 1965 the third volume of the Soil Mechanics Series was published and dedicated to the memory of Dr. Karl Terzaghi. The total bill for publication of this volume was $\$ 2,697.00$. Volumes I and II were reprinted last year and the demand for them remains high. One order alone filled for a university in California amounted to $\$ 351.10$. These are extremely valuable publications and widely used as textbooks. The usual student reaction is "Where can you buy a textbook today for $\$ 4.50$ ?" These volumes are being distributed at about cost, and this is typical of the projects which this Society endeavors to undertake.

## Sanitary Lectures Fund

The sale of Sanitary Lectures Volumes provided an income of $\$ 37.50$ during the fiscal year.

## Journal of the Boston Society of Civil Engineers

Expenditures required to publish the Journal are made from the Current Fund. A summary of the figures associated with the publication of three issues of the Journal this fiscal year is as follows:

| Printing, Postage, and Copyright | $\$ 5,655.67$ |
| :---: | ---: |
| Receipts from Advertisements | $1,303.40$ |
| Net expenditure | $\$ 4,352.27$ |
| Withholding Taxes and Social Security |  |

Of the cash on hand indicated in the accompanying tables, $\$ 419.97$ is held in escrow for Federal and Massachusetts withholding taxes, and for social security payments.

## Karl R. Kennison Fund

On March 11, 1966, Mr. Richard Claybourne reported to the Treasurer the status of the irrevocable trust involving shares in Massachusetts Hospital Life Insurance Company established by Mr. Karl R. Kennison.

| Trust \#4315 | 356.650 shares | $\$ 4,565.00$ |
| :---: | :---: | ---: |
| Trust \#4444 | 382.552 shares | $4,897.00$ |
|  | Total | 739.202 |
| shares | $\$ 9,462.00$ |  |

Paul A. Dunkerley, Treasurer
TABLE I
Distribution of Funds

|  |  |  |  | Net Pr | Loss | Transfer | Funds |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Book Value | Interest an | Dividends | at Sale | turity | Purchased | Sold | Book Value |
|  | Mar. 1, 1965 | $\begin{gathered} \text { Cash } \\ 2 \end{gathered}$ | $\begin{gathered} \text { Credit } \\ 3 \end{gathered}$ | $\begin{aligned} & + \\ & 4 \end{aligned}$ | $\overline{5}$ | $\begin{gathered} + \\ 6 \end{gathered}$ | $\overline{7}$ | $\begin{aligned} & \text { Mar. } 1,1966 \\ & 8 \end{aligned}$ |
| Bonds | \$ 72,199.09 | \$2,881.25 |  |  |  | \$ 2,998.13 |  | \$ 75,197.22 |
| Savings Bank | 5,231.44 |  | \$139.54 |  |  |  | \$2,000.00 | 3,370.98 |
| Stocks | 61,813.37 | 5,620.82 |  | \$10,575.64 |  | 14,260.39 | 4,856.59 | 71,217.17 |
| Available for Investment |  |  |  |  |  |  |  |  |
|  | 3,772.21 |  |  |  |  | 554.21 |  | 3,218.00 |
|  | \$143,016.11 | \$3,502.07 | \$139.54 | \$10,575.64 |  | \$17,258.52 | \$7,410.80 | \$153,003.37 |
| Columns $1+3+6-7=8$ |  |  |  |  |  |  |  |  |

TABLE II
Distribution of Funds-Receipts and Expenditures

| Funds | Book Value Mar. 1, 1966 | Allocation of IncomeProfit and Loss |  | Received | Expended | Book Value <br> Mar. 1, 1966 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Income Col. 2\&3 | Net Profit Col. 4\&5 |  |  |  |
| Permanent | \$76,112.32 | \$4,651.04 | \$+5,712.11 | \$430.00 | \$8,069.21 | \$78,836.26 |
| John R. Freeman | 44,342.95 | 2,703.52 | 3,312.69 |  | 734.90 | 49,624.26 |
| Edmund K. Turner | 1,837.25 | 111.43 | 135.81 |  | 106.99 | 1,977.50 |
| Desmond FitzGerald | 3,587.91 | 214.10 | 261.19 |  | 124.08 | 3,939.12 |
| Alexis H. French | 1,815.75 | 109.96 | 134.29 |  | 73.31 | 1,986.69 |
| Clemens Herschel | 1,385.21 | 84.50 | 103.71 |  | 9.43 | 1,563.99 |
| Edward W. Howe | 2,006.64 | 116.73 | 141.39 |  | 132.31 | 2,132.45 |
| William P. Morse | 3,741.64 | 223.47 | 272.70 |  | 125.11 | 4,112.70 |
| Frank B. Walker | 1,659.83 | 95.58 | 115.43 |  | 129.95 | 1,740.89 |
| Ralph W. Horne | 3,223.78 | 191.16 | 232.81 |  | 254.04 | 3,393.71 |
| Surveying Lectures | 622.66 | 8.03 |  |  | 630.69 | 0.00 |
| Transportation Lectures | 395.01 | 24.10 | 29.57 |  | 2.70 | 445.98 |
| Structural Lectures | 601.79 | 107.99 | 123.94 | 3,524.18 | 1,717.02 | 2,640.88 |
| Boring Data | 270.50 |  |  | 80.00 |  | 350.50 |
| Soil Mechanics | 3,548.74 |  |  | 1,567.18 | 2,759.11 | 2,356.81 |
| Sanitary Lectures | -2,135.87 |  |  | 37.50 |  | -2,098.37 |
| Subtotal | \$143,016.11 | \$8,641.61 | \$10,575.64 | \$5,638.86 | \$14,868.85 | \$153,003.37 |
| Current | 1,500.00 | 4,132.29 |  | 21,822.98 | 25,955.27 | 1,500.00 |
| Totals | \$144,516.11 | \$12,773.90 | \$10,575.64 | \$27,461.84 | \$40,824.12 | \$154,503.37 |
| Secretary's change fund of $\$ 30.00$ should be added to show total cash Cash Balance Investment Fund $\$ 3,218.00$ Current Fund $\quad 1,500.00$ |  |  |  |  |  |  |
|  | \$7,550.46 Trans \$4,132.29 Trans | erred from erred from | Fund Permanent Fund ncome to Per | 718.00, <br> nent Fund |  |  |


TABLE III
Record of Investments-Bonds

| Bonds | Date of Maturity | Interest <br> Rate | Interest Received | Par Value | Book Value Mar. 1, 1966 | Market Value Mar. 1, 1966 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Aluminum Company of America | Apr. 1, 1983 | 3-7/8 | \$193.75 | \$5,000.00 | \$5,037.50 | \$4,375.00 |
| Associates Investment Co., Dcb. | Aug. 1, 1979 | 5-1/8 | 307.50 | 6,000.00 | 6,000.00 | 5,760.00 |
| Columbia Gas Systems Inc., Dcb., Series D | July 1, 1979 | 3-1/2 | 70.00 | 2,000.00 | 2,066.17 | 1,680.00 |
| Consumers Power Co., 1st Mortgage | Sept. 1, 1975 | 2-7/8 | 86.25 | 3,000.00 | 3,140.35 | 2,475.00 |
| Flintkote Co. | Apr. 1, 1981 | 4-5/8 | 462.50 | 10,000.00 | 10,450.00 | 9,400.00 |
| Florida Power Co., 1st Mortgage | July 1, 1984 | 3-1/8 | 31.25 | 1,000.00 | 1,017.50 | 783.75 |
| Florida Power Co., 1st Mortgage | July 1, 1986 | 3-7/8 | 193.75 | 5,000.00 | 5,037.59 | 4,331.25 |
| General Motors Acceptance Corp. | Sept. 1, 1975 | 3-5/8 | 181.25 | 5,000.00 | 5,101.80 | 4,318.75 |
| Georgia Power Co., 1st Mortgage | Dec. 1, 1977. | 3-3/8 | 168.75 | 5,000.00 | 5,162.50 | 4,250.00 |
| Marine Midland Corp., Dcb. | July 1, 1989 | 4-1/2 | 225.00 | 5,000.00 | 5,000.00 | 4,600.00 |
| Province of Ontario | Sept. 1, 1972 | 3-1/4 | 97.50 | 3,000.00 | 2,936.25 | 2,673.75 |

TABLE III (Continued)

| Bonds | Date of Maturity | Interest Rate | Interest <br> Received | Par Value | Book Value Mar. 1, 1966 | Market Value <br> March 1, 1966 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Public Service Electric and Gas Co. | June 1, 1979 | 2-7/8 | 115.00 | 4,000.00 | 4,097.50 | 3,100.00 |
| So. Pacific, 1st Series A, Oregon Lines | Mar. 1, 1977 | 4-1/2 | 180.00 | 4,000.00 | 4,191.30 | 3,720.00 |
| Superior Oil Co., Dcb. | July 1, 1981 | 3-3/4 | 150.00 | 4,000.00 | 4,000.00 | 3,600.00 |
| Tidewater Oil Co., Dcb. | Apr. 1, 1986 | 3-1/2 | 70.00 | 2,000.00 | 2,032.50 | 1,642.50 |
| U.S.A. Treasury Bonds | May 15, 1974 | 4-1/4 | 85.00 | 2,000.00 | 2,000.00 | 1,883.12 |
| U.S.A. Treasury Bonds | Aug. 15, 1973 | 4 | 200.00 | 5,000.00 | 4,928.13 | 4,650.00 |
| U.S.A. Treasury Bonds | May 15, 1974 | 4-1/4 | 63.75 | 3,000.00 | 2,998.13 | 2,824.68 |
| Totals |  |  | \$2,881.25 | \$74,000.00 | \$75,197.22 | \$66,067.80 |

TABLE IV
Record of Investments-Stocks
March 1, 1965 to March 1, 1966

| Stocks | Classifi- <br> cation | Number <br> of <br> shares | Dividend <br> Received | Book Value <br> Mar. 1, 1966 | Market Value <br> Mar. 1, 1966 |
| :--- | :---: | :---: | :---: | ---: | ---: |
| American Telephone \& Telegraph Co. | Common | 250 | $\$ 656.00$ | $\$ 4,505.87$ | $\$ 14,859.50$ |
| Commercial Credit Co. | Common | 240 | 432.00 | $9,963.21$ | $7,515.12$ |
| General Electric Co. | Common | 150 | 360.00 | $2,341.47$ | $16,228.20$ |
| General Motors Corp. | Common | 126 | 661.50 | $5,576.32$ | $12,513.44$ |
| Hartford Fire Insurance Co. | Common | 107 | 155.15 | $1,534.39$ | $7,744.13$ |
| Illinois Power Co. | Common | 200 | 160.00 | $9,686.00$ | $7,837.60$ |
| International Business Machine Corp. | Common | 9 | 283.50 | 0.00 |  |
| Jewel Tea Co. | Common | 187 | 212.20 | $4,976.81$ | $7,304.78$ |
| Monsanto Co. | Common | 105 | 147.90 | $7,260.28$ | $8,564.12$ |
| National Dairy Products Corp. | Common | 100 | 260.00 | $1,154.74$ | $7,906.30$ |
| New England Electric System | Common | 208 | 253.76 | $3,332.89$ | $5,564.00$ |
| Pacific Gas and Electric Co. | Common | 0 | 0.00 | $4,497.75$ | $4,578.75$ |
| Scott Paper Co. | Common | 263 | 243.29 | $5,944.04$ | $10,257.00$ |
| Southern California Edison Co. | Common | 177 | 216.82 | $1,932.99$ | $6,150.75$ |
| Standard Oil of New Jersey | Common | 200 | 630.00 | $2,012.76$ | $15,225.00$ |
| Texaco Inc. | Common | 236 | 578.20 | $1,515.72$ | $18,275.37$ |
| Pacific Gas and Electric Co. | Preferred | 100 | 150.00 | $2,704.89$ | $3,018.80$ |
| Southern California Edison Co. | Preferred | 120 | 145.50 | $1,140.24$ | $3,952.56$ |
| Southern Railways Co. | Preferred | 75 | 75.00 | $1,136.80$ | $1,425.00$ |
| Totals |  |  | $\$ 5,620.82$ | $\$ 71,217.17$ | $\$ 158,920.42$ |

TABLE V
Record of Investments-Savings Banks

| Bank | Savings <br> Account Number | Interest Received | Book Value Mar. 1, 1966 | Market Value Mar. 1, 1966 |
| :---: | :---: | :---: | :---: | :---: |
| First Federal Savings and Loan |  |  |  |  |
| Assoc. of Boston | 1S-631 | \$139.54 | \$3,370.98 | \$3,370.98 |

TABLE VI
Comparison of Book Values and Market Values of Stocks, Bonds, Savings Bank, and Investment Cash

|  | Book Value <br> March 1, 1966 | Market Value <br> March 1, 1966 |
| :--- | ---: | ---: |
| Bonds | $\$ 75,197.22$ | $\$ 66,067.80$ |
| Stocks | $71,217.17$ | $158,920.42$ |
| Savings Bank | $3,370.98$ | $3,370.98$ |
| Available for Investment | $3,218.00$ | $3,218.00$ |
| Total March 1, 1966 | $\$ 153,003.37$ | $\$ 231,577.20$ |
| Total March 1, 1965 | $143,016.11$ | $245,225.12$ |
| Increase or Decrease | $+\$ 9,987.26$ | $-\$ 13,647.92$ |

TABLE VII Comparison of Book and Market Value of Funds

| Funds | Book Value <br> March 1, <br> Permanent | Market Value <br> March 1, 1966 |
| :--- | ---: | ---: |
| John R. Freeman | $78,836.26$ | $\$ 119,483.86$ |
| Edmund K. Turner | $49,624.26$ | $75,210.29$ |
| Desmond FitzGerald | $1,977.50$ | $2,997.09$ |
| Alexis H. French | $3,939.12$ | $5,970.11$ |
| Clemens Herschel | $1,986.69$ | $3,011.02$ |
| Edward W. Howe | $1,563.99$ | $2,370.37$ |
| William P. Morse | $2,132.45$ | $3,231.93$ |
| Frank B. Walker | $4,112.70$ | $6,233.19$ |
| Ralph W. Horne | $1,740.89$ | $2,638.49$ |
| Transportation Lectures | $3,393.71$ | $5,143.49$ |
| Structural Lectures | 445.98 | 675.92 |
| Boring Data* | $2,640.88$ | $4,002.50$ |
| Soil Mechanics* | 350.50 | 350.50 |
| Sanitary Lectures* | $2,356.81$ | $2,356.81$ |
| Subtotal | $-2,098.37$ | $-2,098.37$ |
| Current* | $\$ 153,003.37$ | $\$ 231,577.20$ |
| Total | $1,500.00$ | $1,500.00$ |

[^9]
# REPORT OF THE AUDITING COMMITTEE 

Boston, Mass., March 23, 1966

## To the Boston Society of Civil Engineers:

We have reviewed the records and accounts of the Secretary and Treasurer of the Boston Society of Civil Engineers, and we have compared the bank statement of securities held by the Boston Safe Deposit and Trust Company with the enumeration submitted by the Treasurer.

We have found them to be in order and to account accurately for the Society's Funds.

Respectfully submitted, Harl P. Aldricti, Jr. Myle J. Holley, Jr.

## REPORT OF THE EDITOR

Boston, Mass., March 23, 1966

## To the Board of Government Boston Society of Civil Engineers

The Journal was issued quarterly, for the months of April, July, and October, 1965, and January, 1966, as authorized by the Board of Government on December 20, 1936.

Published during the year were 17 papers and one discussion; of the papers published, six were presented in the John R. Freeman Hydraulics Lecture Series, eight were presented at other meetings of the Society and Sections, one was presented at the Engineers' Week Luncheon in 1965, one was presented at the First New England Conference on Urban Planning for Environmental Health, and the remaining paper was a contribution to the Journal.

The four issues of the Journal contained about 380 pages of papers and proceedings, 10 pages of index, and 36 pages of advertising, a total of about 426 pages.*

The cost of printing the Journal was as follows:

## Expenditures

Composition and printing ............................................. . . . \$4,244.56**
Cuts ........................................................................ . . . 856.63
Wrapping, mailing and postage . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 328.64
Editor . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 748.92
Copyright . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . 14.00

Envelopes ............................................................. 211.84
\$6,775.21

## Receipts

| From Sale of Journals | \$2,616.00 |
| :---: | :---: |
| From Sale of Reprints | 293.84 |
| From Advertising | 1,303.40 |
|  | \$4,213.24 |
| Net cost of Journal to be paid from Current Fund | \$2,561.97 |
| ${ }_{* *}^{*}$ These figures and those for costs are estimated pending final pu <br> ** Includes a balance of $\$ 347.61$ for January, 1964 . | nuary issue. |

Until very recently it has been difficult to obtain a satisfactory backlog of papers for publication. Section Chairmen are again encouraged to obtain manuscript from speakers at the time of presentation so that they may be considered for publication. Other contributions to the Journal are desirable also, and a "Guide for Authors" prepared by the Publications Committee, has been published in several issues (the most recent being in the October 1965 issue) in order that interested engineers might submit papers for consideration in an acceptable format.

Advertising continues to diminish. It would seem advisable either to develop a determined and vigorous advertising solicitation committee or to establish a contract with an advertising agency. In any event advertising revenue should be tripled in order for the Journal to pay for itself.

Respectfully submitted, Robert L. Meserve, Editor

## REPORT OF THE LIBRARY COMMITTEE

Boston, Mass., February 23, 1966

## To the Boston Society of Civil Engineers:

The Library Committee met at the Society Rooms on November 9, 1965. The meeting was devoted to preparing a list of books to be purchased and a general discussion on ways to encourage more extensive use of the library.

The following is a list of books purchased for the year at a cost of $\$ 157.28$.
Unit Processes of Sanitary Engineering-1963, L. G. Rich
Unit Processes of Sanitary Engineering-1961, L. G. Rich
National Technical Information Services Worldwide Dictionary
Urban Transportation-The Federal Role, G. M. Smech
Handbook of Applied Hydrology, V: T. Chow
The Critical Path Method, L. R. Shaffer, J. B. Ritter, W. L. Mayer
Structural Effects of Impact, M. Komhauser
Corrosion and Corrosion Control, Uhlig
Building Construction: Information Sources, H. B. Bentley
Water Atlas of the United States, D. W. Miller
Urban Land Use Planning, F. S. Chapin, Jr.
The Analysis of Braced Domes, B. S. Benjamin

Dynamics of Bases and Foundations, Barkam
Linear Programming, S. I. Gass
Ideas, Inventions and Patents, Buckles
The Committee, on behalf of the BSCE, wishes to thank the Portland Cement Association for the following ACI publications:

Symposium on Fire Resistance of Concrete
Symposium on Creep of Concrete
ACI Standard Recommended Practice for Evaluation of Compression Test Results of Field Concrete

The following books were donated by Pergamon Press:
Elementary Mechanics of Solids, P. P. Benham
Problems in Strength of Materials, N. M. Belyayer
The Design of Structures of Least Weight, M. L. Cox
Tables for Computing Elevations in Topographic Leveling, L. S. Khrenov
Robert L. Fuller, Chairman

## REPORT OF THE HOSPITALITY COMMITTEE

Boston, Mass., February 23, 1966
To the Boston Society of Civil Engineers:
The Hospitality Committee submits the following report for the year 19641965.

A total of six meetings of the Society were held during the past year. Included in this total were the annual dinner, a student night meeting, a joint luncheon meeting with the American Society of Civil Engineers, and three regular meetings.

Catered dinners were served prior to five of the six meetings.
SUMMARY OF MEETINGS

|  |  | Attendance |  |
| :--- | :--- | :---: | :---: |
| Date | Place | Meeting | Dinner |
| March 18, 1965 | Hotel Vendome | 227 | 227 |
| May 17, 1965 | Society Rooms | 35 | - |
| October 27, 1965 | Worcester Polytechnic |  |  |
| November 17, 1965 | Inst. | 155 | 155 |
| United Community |  |  |  |
| January 26, 1966 | $\quad$ Service Building | 155 | 115 |
| February 14, 1966 | United Community |  |  |

The average attendance of members and guests for all six meetings or dinners (using the larger attendance figure) was 134, as compared to last year's average of 111 .

Attendance at regular meetings of the Society during the past year was 94 persons per meeting. This represents a $62 \%$ increase in attendance over last year.

Respectfully submitted,
Robert L. Fuller, Chairman

## REPORT OF COMMITTEE ON SUBSOILS.OF BOSTON

Boston, Mass., March 10, 1966
To the Boston Society of Civil Engineers:
The Committtee continued giving cooperation and encouragement to the U.S. Geological Survey group that is preparing subsurface structure maps of Greater Boston. These maps are based on all available borings and excavation data including the published and unpublished boring information gathered by the Boston Society of Civil Engineers. It is recommended that the Committee continue helping the U.S. Geological Survey where possible on this project.

Respectfully submitted,
Donald G. Ball, Chairman
BSCE Subsoils Committee

## REPORT OF ADVERTISING COMMITTEE

March 11, 1966
To the Boston Society of Civil Engineers:
The Advertising Committee held no formal meetings during 1965.
Review of the number of advertisements printed in the Journal over the past five years indicates the following:

1. Professional Cards decreased from 39 to 29.
2. Full Page Advertisements ran from 1 in 1960 to 2 in 1964 to 1 in 1965.
3. One-half Page Advertisements increased from 1 in 1960 to 2 in 1965.
4. One-quarter Page Advertisements decreased from 19 in 1960 to 11 in 1965.

It is apparent that action must be taken by the Society and its Advertising Committee to halt this trend.

It is suggested that a campaign be conducted over the next year to improve this phase of the Society's activity.

R. F. Battles, Chairman BSCE Advertising Committee

# REPORT OF JOINT LEGISLATIVE COMMITTEE 

Boston, Mass., March 11, 1966

## To the Boston Society of Civil Engineers:

The following Legislative acts of interest to engineers were enacted during the 1965 session of the Massachusetts General Court:

| Bill No. | Subject | Chapter <br> No. |
| :--- | :--- | :---: |
| HI34 | An act permitting cities and towns to borrow <br> money outside of the debt limit for build- <br> ings for water treatment. |  |
| H3388 | An act relating to the protection of flood plains. <br> An act providing for the establishment of a <br> uniform plumbing code in cities and towns. | 68 |
| H353 | An act extending the time within which the <br> Massachusetts Turnpike Authority may lease <br> air rights in connection with the extension <br> into the city of Boston of the Massachusetts <br> Turnpike. | 358 |
| S1081 | An act further regulating the filing of bids for <br> contracts for the construction, reconstruc- <br> tion, alteration, remodeling, repair and demo- <br> lition of public buildings. | 446 |

A review of bills filed for the 1966 session shows the following items of interest:

| Bill No. | Subject | Report |
| :---: | :---: | :---: |
| S176 | Petition of Philip A. Quinn and Joseph D. Ward for legislation to direct the Department of Public Works to provide for the protection of water resources, fish and wildlife and recreation values in advance planning for highway construction. | No final action on any of the Bills |
| S543 | Petition of Philip A. Graham and Kevin H. Harrington for legislation to provide that graduation from a service academy shall be considered as graduation from an engineering school for the purpose of registration of professional engineers. |  |


| H490 | Petition of Francis W. Hatch, Jr., and other members of the House for extending to public authorities the requirements of competitive bidding for construction, reconstruction, alteration, remodeling, repair or demolition of public buildings. |
| :---: | :---: |
| H1004 | Petition of the Consulting Engineers Council of New England, Inc., for a limitation of three years for the bringing of actions of contract or tort for malpractice, error or mistake against architects, professional engineers and land surveyors. |
| H1093 | Petition of the Consulting Engineers Council of New England, Inc., that provision be made for methods of payment on public consulting services. |
| H2011 | Petition of Charles W. Terenzio and Herbert B. Hollis for legislation to provide for the appointment of an executive secretary for the Board of Registration of Professional Engineers and of Land Surveyors. |
| H2614 | Petition of Raymond F. Rourke and James DeNormandie that the Department of Public Health be authorized to grant assistance to cities and towns for the construction of water pollution abatement facilities. |
| H2776 | Petition of the Massachusetts State Association of Architects for legislation relative to the time within which certain actions must be brought against architects and others. |
| H3042 | Petition of John A. Armstrong relative to the acceptance or approval of plans and specifications for the construction, reconstruction, enlargement, or alteration of buildings or structures. |

In addition, there are other bills of interest to engineers which your Committee will follow to protect the interest of Professional Engineers and Land Surveyors.

Charles A. Parthum<br>Ralph M. Soule<br>James L. Dallas, Chairman

# REPORT OF EDUCATIONAL COMMITTEE 

Boston, Mass., February 14, 1966
To the Boston Society of Civil Engineers:
No scheduled meetings were held by the Education Committee.
No topics requiring the attention of the Committee were submitted during the year 1965-1966.

George W. Hankinson, Chairman

## REPORT OF COMMITTEE ON PROFESSIONAL CONDUCT

Boston, Mass., March 21, 1966

## To the Boston Society of Civil Engineers:

Your Chairman and other members of the Committee on Professional Conduct have attended most of the monthly meetings held during the past year by the Joint Committee on the same subject (ASCE, MSPE, and BSCE).

These meetings have considered competitive bidding for professional services, advertising practices by professional engineers, conflict of interest, and political contributions. In the matter of competitive bidding for aerial survey and mapping, the subject was given considerable attention. In one case a consulting engineering firm had its attention called to the fact that some of its advertising appeared to be contrary to accepted criteria. Political contributions were discussed but without the development of any definite opinion. This was a matter which had been discussed at some Iength in the 1965 report of the Massachusetts Crime Commission and is a somewhat embarrassing question.

Your Chairman took up informally on behalf of the BSCE a matter of alleged conflict of interest which, however, appeared on investigation to be unfounded. He also took up another matter involving what was claimed to have been an attempt to supplant another engineer. In this case the allegation was not pressed by the party claiming injury, and no further action was taken. The Chairman believes that where there is alleged unethical conduct that informal inquiry is advisable as a first step. This at least serves as a warning to members who may be approaching an area of unethical conduct and avoids unpleasantness; a result which is to be desired unless the alleged conduct is clearly demonstrated to be in flagrant violation of Codes of Ethics.

It should be noted that this past year one local member of another society represented in this Joint Committee, according to newspaper accounts, testified that, at the demand of his employer, he falsified invoices for services rendered. This employer recently has been found guilty of larceny in the lower court, but has appealed. Another local member of the society referred to above was cited for conspiracy in connection with indictments brought by the Attorney General. All three of the men are registered professional engineers in Massachusetts.

In view of the scandals involving engineers in the so-called Worcester case and the Boston Common Garage case, it is evident that the image of the engineer has
received some tarnishing in Massachusetts during the past few years. It seems to your committee that the individual members of professional engineering societies should look well to their conduct, not only for their own reputations but for the good repute of the Profession.

Respectfully submitted, Francts S. Harvey
William L. Hyland E. Sherman Chase, Chairman

## REPORT OF PUBLICATIONS COMMITTEE

March 23, 1966
To the Boston Society of Civil Engineers:
This report covers the period from March 1965 to March 1966. It was compiled from reports by the Chairmen of the five section committees: Construction, Hydraulics, Sanitary, Structural and Transportation.

22 papers delivered at meetings, including meetings held jointly with the Main Society or other groups.
10 papers of the 22 were of general interest and the other 12 were specifically regarding a single project.
11 papers were requested from speakers.
11 papers were promised.
4 papers were received.
3 of the 4 were approved for publication and one still being reviewed. None of the papers received were rejected.
It can be observed that more papers were published during the year than were received. This is due to the backlog of papers available from preceding years.

The Publication Committee of the previous year prepared a "Guide for Authors" which was published in some of the issues of the 1965 year. It was hoped that the guide would encourage presentation for publication of papers other than those presented at meetings. Although no papers in this category were received, it is still hoped that some will be received in the future. This feature is to help those who have something to say, and worth the reading, but the author cannot for whatever reason present the paper at a formal meeting.

Respectfully submitted, Simon Kirshen, Chairman Publications Committee

## REPORT OF JOHN R. FREEMAN FUND COMMITTEE

During the year 1965-66 this Committee instituted a John R. Freeman Memorial Lecture. Dr. Hunter Rouse presented the first such lecture at a Joint Meeting of the Society with the Hydraulic Section on January 26, 1966. His topic was "Jet Diffusion and Cavitation." The lecture was well attended and set a high standard for such a program.

Respectfully submitted, Leslie J. Hooper, Chairman

# REPORT OF THE EXECUTIVE COMMITTEE OF THE SANITARY SECTION 

Boston, Mass., February 4, 1966
To the Sanitary Section
Boston Society of Civil Engineers:
During the preceding year the Executive Committee has included:
William C. Traquair, Chairman
Robert L. Meserve, Vice Chairman
Walter M. Newman, Clerk
Charles A. Parthum, Executive Committee
Allison C. Hayes, Executive Committee
David A. Duncan, Executive Committee

## Annual Meeting March 3, 1965

Following election of officers, Dr. N. Bruce Hanes, Professor of Civil Engineering, Tufts University, presented an illustrated paper titled, "Relationship Between Escherichia Coli Type I, Coliform, and Enterococci in Water." Meeting attendance was thirty-five.

Annual Outing June 3, 1965
The outing was held in conjunction with the parent society at the Brockton, Massachusetts, Sewage Treatment Plant. Following tours of the plant, Mr. Fozi M. Cahaly, Vice President, Fay, Spofford and Thorndike, Inc., presented a historical background of sewerage facilities in the City of Brockton. Thirty-five persons toured the sewage treatment plant, and forty-five persons attended the dinner and presentation following.

## Meeting October 6, 1965

Dr. Clair N. Sawyer, Senior Associate and Director of Research at Metcalf and Eddy, presented a challenging paper titled, "Engineering Aspects of Problems in the Aquatic Environment Related to Excessive Nutrients." The presentation included a relevant series of slides. Meeting attendance was forty-three persons.

## Meeting December 1, 1965

The speaker at this meeting was Walter M. Newman, Acting Chief, Water Resources Development Section of the Division of Water Supply and Pollution Control. Mr. Newman presented a paper titled, "The Federal Water Pollution Control Effort in New England." Illustrative slides depicting pollutional problems of New England rivers and streams were shown. Thirty-eight persons attended this meeting.

The Executive Committee met four times during the year.
Walter M. Newman, Clerk

## REPORT OF THE EXECUTIVE COMMITTEE OF THE STRUCTURAL SECTION

February 11, 1966

## To the Structural Section

Boston Society of Civil Engineers:
The following meetings were held during the past year:
March 10, 1965-Mr. William A. Milek, Jr., of the American Institute of Steel Construction spoke on "Practical Aspects of Buckling in Structural Steel Design." Attendance was 54.

April 14, 1965-Dr. Russell C. Jones, Assistant Professor of Civil Engineering, M.I.T., spoke on "Structural Metals-Understanding through Material Science." Attendance was 19.

May 10, 1965-Dr. Ralph B. Peck, Professor of Soil Mechanics at the University of Illinois, spoke on the Alaskan earthquake. The attendance was not counted.

October 13, 1965-Mr. Horst Borbereky, Senior Soils Engineer, Howard, Needles, Tannen and Bergendoff, spoke on "Foundation Studies and Pile Loading Tests for the Massachusetts Turnpike Boston Extension." Attendance was 42.

November 10, 1965-Mr. Nomer Gray, Partner of Ammann and Whitney, spoke on the "Verrazano Narrows Bridge." This was a joint meeting with the Main Society. Attendance was about 45.

December 8, 1965-Mr. John A. Gilligan of the United States Steel Corporation spoke on "Suggestions for Including ASTM A514 Steel in the AISC Specifications." Attendance was 17.

January 12, 1966-Professor Sepp Firnkas of Northeastern University spoke on "Prestressed-Precast Concrete Applied to Urban Renewal Projects." Attendance was 47.

February 9, 1966—Professor Kentaro Tsutsumi of Tufts University spoke on "A Stable Test Station Foundation for Aero Space Projects." At this annual meeting of the Structural Section, the following officers were elected for the forthcoming year: Chairman, Robert L. Fuller; Vice-Chairman, Charles C. Ladd; Clerk, Fritz F. Hampe; Executive Committee, Richard C. Jasper, Albert Rich, and Floyd Brown. Attendance was 48.

In addition to the above meetings, the Structural Section sponsored a series of ten lectures on "Soil Mechanics and Foundation Engineering" run between November 4, 1965 and February 3, 1966 under the Co-Chairmenship of Mr. Donald T. Goldberg and Prof. Charles C. Ladd. Over 275 persons registered for this series and the attendance varied from 175 to 225 persons.

The total attendance at seven of the eight meetings was 272 , averaging 39 per meeting.

Charles C. Ladd, Clerk

## EXECUTIVE COMMITTEE REPORT HYDRAULICS SECTION

March 11, 1966
To the Hydraulics Section
Boston Society of Civil Engineers:
The following meetings were held during the past year:-
May 5, 1965--Mr. Philip J. Holton, Jr., Chief Engineer, City of Providence Water Supply Board, spoke on "The Providence Water Works and Proposed Improvements." Attendance- 38.

November 3, 1965—The guest speaker was Mr. Richard F. Dutting, Hydraulic Engineer, Camp, Dresser \& McKee of Boston; who described "The Application of Linear Programming to Water Distribution System Design." Attendance-48.

January 26, 1966-The Annual Meeting was held jointly with the Main Society of the Boston Society of Civil Engineers, in the Adams Room of the United Community Services Building, 14 Somerset Street, Boston, Mass. President Leslie J. Hooper presided. Prof. Eagleson presented the report of the Nominating Committee of the Hydraulics Section, K. Peter Devenis, Chairman, Richard F. Dutting and Lawrence C. Neale, for the following slate of officers for the coming year 1966-1967,

$$
\begin{aligned}
\text { Chairman } & \text { Nicholas Lally } \\
\text { V-Chairman } & \text { Allan Grieve, Jr. } \\
\text { Clerk } & \text { Athanosios A. Vulgaropulos } \\
\text { Executive Committee } & \text { Ronald T. McLaughlin } \\
& \text { Stephen E. Dore, Jr. } \\
& \text { Albert G. Ferron }
\end{aligned}
$$

which were approved by voice vote. The reading of the Minutes of the previous meeting was dispensed with.

President Hooper presented Doctor Hunter Rouse, Director, Iowa Institute of Hydraulic Research, to speak in the first John R. Freeman Memorial Lecture. The speaker reviewed a series of studies of submerged jets that have been made in the Iowa laboratories in recent years.

The meeting was adjourned at 8:45 P.M.
Attendance at dinner preceding meeting 72, attendance at meeting 98.
Respectfully submitted, Allan Grieve, Jr., Clerk

## REPORT OF EXECUTIVE COMMITTEE OF THE TRANSPORTATION SECTION

Boston, Mass., March 1, 1966

## To the Transportation Section <br> Boston Society of Civil Engineers:

The Transportation Section of the BSCE held three meetings during the 196566 season as follows:

May 17, 1965-Mr. Frank P. Davidson, Pres. of Technical Studies, Inc., spoke on the subject "A View from Dover Castle-The English Channel Tunnel." Attendance: 41. Mr. Davidson submitted a paper on his talk which was published in the Journal of the Society.

November 22, 1965-Mr. Earl D. VanReenan: Edgerton, Getmeshausen \& Grier, spoke on the subject: "Geophysical Survey of the English Channel Crossing." Attendance: 35.

February 24, 1966-Annual Meeting. Prof. Robert J. Hansen, Deputy Director of Project Transport and Professor at M.I.T. spoke on the subject "High Speed Ground Transportation." A paper on this subject will be submitted in the near future. Attendance: 48. At this meeting the following members were elected to office for the 1966-67 season:

Chairman Casimir J. Kray<br>V-Chairman Warren D. McLea<br>Clerk Robert J. Kiley<br>Executive Committee Alexander J. Bone<br>James F. Orpin, Jr.<br>Charles H. Flavin, Jr.

During the 1965-66 season the Executive Committee held meetings on March 30, 1965; May 10, 1965 and February 24, 1966.

Respectfully submitted, Benedict J. Quirk, Chairman

## REPORT OF THE EXECUTIVE COMMITTEE OF THE CONSTRUCTION SECTION

Boston, Mass., March 8, 1966

## To the Construction Section <br> Boston Society of Civil Engineers:

The following meetings of the Construction Section were held during the past year:

January 6, 1965-Mr. Christopher J. Murray, Jr., Geo. A. Fuller Co., Inc., presented an illustrated talk on "Construction of the Behavioral Science Building at Harvard University."

The following slate of officers were elected:
Chairman Herman G. Protze
V-Chairman Robert J. Van Epps
Clerk William E. Wiley

March 24, 1965-Mr. Herbert M. Priluck, Construction Planning Management, Inc., using example problems, discussed "Concepts and Misconceptions Concerning the Use and Abuse of CPM for Construction."

November 17, 1965-Joint Meeting with Main Society. An illustrated talk on "Design and Construction of New Boston City Hall" was presented by, Mr. Gary Kallman, Mr. William J. LeMessurier, of LeMessurier Associates, Inc., and Mr. David Morton, J. W. Bateson Co., Inc., spoke respectively from the architect's, engineer's, and contractor's point of view.

The following slate of officers were elected:
Chairman Robert J. Van Epps
V-Chairman William E. Wiley
Clerk Arthur H. Mosher
William E. Wiley, Clerk

# PROFESSIONAL SERVICES 

## AND

## ADVERTISEMENTS

$\left(\begin{array}{l}\text { The advertising pages of the JOURNAL aim } \\ \text { to acquaint readers with Professional and } \\ \text { Contracting Services and Sources of Various } \\ \text { Supplies and Materials. You would find it } \\ \text { of advantage to be represented here. }\end{array}\right)$

# BOSTON SOCIETY OF CIVIL ENGINEERS 

FOUNDED 1848

## PROFESSIONAL SERVICES

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BARNES ENGINEERING
COMPANY, Inc.
Civil Engineers
411 Lexington Street
AUBURNDALE 66, MASS.

## BRASK ENGINEERING COMPANY

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[^2]:    * Professor of Civil Engineering, Hydrodynamics Laboratory, Massachusetts Institute of Technology, Cambridge 39, Massachusetts.

[^3]:    * Designing Engineer M.D.C. Construction Division, Retired.

[^4]:    357.0
    405.0
    6.-Power Plant Elev.

    Fig.

[^5]:    * Sen:or Lecturer, Faculty of Civil Engineering, Technion-Israel Institute of Technology

    Notation.-The letter symbols used in this paper are defined where they first appear, in the illustrations or in the text, and are arranged alphabetically, for convenience of reference, at the end of the paper.

[^6]:    * "Inhomogeneity" in this case refers to deviations from the mean of the properties of so-called "homogeneous" soil.

[^7]:    * If this assumption does not suit the foundation conditions, more sophisticated calculations have to be made, but the same principles and the same method can be used. Here in this paper, for simplicity of presentation, simplifying assumptions have been made.

[^8]:    * Note: An assumption that the perpendicular frame may also at the same time transfer load to the discussed one, though theoretically possible, seems to be exaggerated compared to the approach presented here.

[^9]:    * These are not interest-earning invested funds.

