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JET DIFFUSION AND CAVITATION

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Presentation of this first John R. Freeman Memorial Lecture has meant a very great deal to the writer, for Dr. Freeman was a man whom he knew and deeply respected. An able experimenter in his own right, Freeman spoke with the voice of a prophet in behalf of experimental hydraulics, and he inspired and supported the study and work of many a young follower. All in all, his influence upon American hydraulics has probably been more far-reaching than that of any predecessor or successor [1].

Freeman himself would probably agree, if he could see things today, that not everything he began had turned out as he originally intended. Of the several dozen Freeman Scholars whom his engineering-society endowments sent to Europe and around the States, some are no longer alive, some were poor choices, and some went into other professions, but many have done their sponsor credit, and a few have even exceeded his expectations. (The writer can speak quite freely, for he was not a Freeman Scholar, but rather one of three M.I.T. students who held in succession an Institute fellowship which Freeman had arranged to be diverted temporarily to hydraulics.) The National Hydraulic Laboratory, founded within the Bureau of Standards at Freeman's instigation, has unfortunately just about reached its last days. For some reason there was little connection between these two undertakings: as far as the writer knows, only two of the forty-two Freeman Scholars have ever been on the Bureau's laboratory staff. Most interesting, indeed, is the fact that the man who made such a

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name as the laboratory acquired was not even an experimental hydraulician, but a mathematical physicist.

So far as Freeman's own investigations are concerned, his finest work—on the resistance of smooth pipes—was conducted years before that of Stanton and Nikuradse and was of fully comparable quality, yet was not published until so many years thereafter [2] that it played essentially no role in the advancement of the subject. For his work on fire streams, however, he received much acclaim, including the highest award of the American Society of Civil Engineers [3]. It is somewhat ironical that the principal factor involved in the poor performance of fire streams—the turbulence of the issuing water—was scarcely suggested in Freeman's papers. Probably the world of fire fighters and their equipment manufacturers is still not ripe for talk of turbulence, for though a paper of the last decade was written to clarify its role in jet diffusion [4], nozzles and monitors are still made much as they were at Freeman's time.

The tremendous impetus that he gave to American hydraulics may not have carried in the direction that he intended, but there is little doubt that he would have approved the healthy condition in which the subject presently finds itself. The writer had his mentor especially in mind, in fact, when he selected the topic of this paper, for it not only deals with jets and turbulence, but it carries their consideration through territory not then foreseen, and it finally brings the subject matter to useful application, just as Freeman believed that research as a whole should do.

A submerged jet differs from a free jet, on the one hand in the lack of gravitational influence, and on the other hand in the interaction between the jet and the surrounding fluid. In fact, from the latter point of view, the submerged jet can be regarded as the limit of the free jet as the density of the fluid stream approaches that of the fluid into which it is ejected. Except at low Reynolds numbers, the intense shear between the two regions results in the formation of turbulence throughout the shear zone, the effect of which is to entrain the surrounding fluid, diffuse the jet, and gradually dissipate its energy. The first mean-flow analysis of the phenomenon was published by Tollmien [5] just a year before the first Freeman Scholar reached Europe, and less than a decade ago Townsend [6] dealt at length with the jet in his general analysis of turbulent shear flow. A good digest of the many intervening papers is given by Hinze [7]. Essentially all of these studies, it should be remarked, were contributed by physicists and aeronautical engineers. While the behavior of a submerged jet surely does not vary with the observer, the point of view and the ultimate use of the observations definitely do. Not only is the writer a hydraulician, but work in certain aspects of jet diffusion has been taking place in his laboratories for the past two decades. He must hence be pardoned for emphasizing his own point of view and observations in the subsequent discussion.

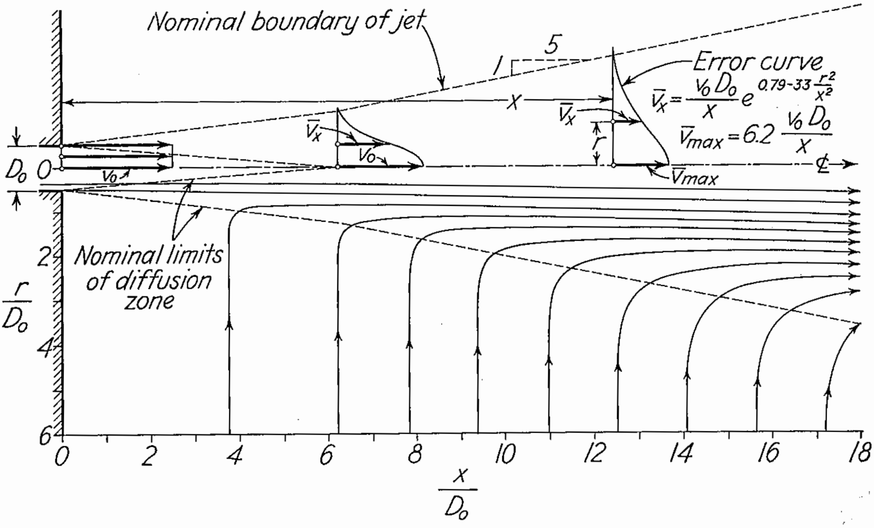

FIG. 1.—MEAN-VELOCITY CHARACTERISTICS OF JET DIFFUSION [8]

If it is assumed that the zone of jet diffusion is one of hydrostatic pressure distribution, then it follows for the boundary geometry shown in Fig. 1 [8] that the same momentum flux must occur past all successive sections, since there is no external force at hand to change it. If it is further assumed that the Reynolds number is so high that at all sections beyond the initial region of establishment a state of similarity exists for both the mean flow and the turbulence, it will follow that the jet must expand linearly and that the velocity along any line will vary inversely with distance from the nozzle [9]. Assumption of a velocity distribution like the error curve shown in the figure will then permit evaluation of the changing rates of volume flux and energy flux past successive sections—except for a single numerical factor, which must be determined empirically. Certain gross aspects of the turbulence can be approximated from the phenomenological relationships of Prandtl; indeed, the reverse process—assumption of the turbulence characteristics and evaluation of the corresponding velocity distribution—was the approach originally used by Tollmien. However, whereas the mean flow is relatively insensitive to the assumed distribution of the turbulence, the reverse is not true, and, as will be illustrated later, information as to the actual turbulence characteristics has continued to depend upon direct measurement.

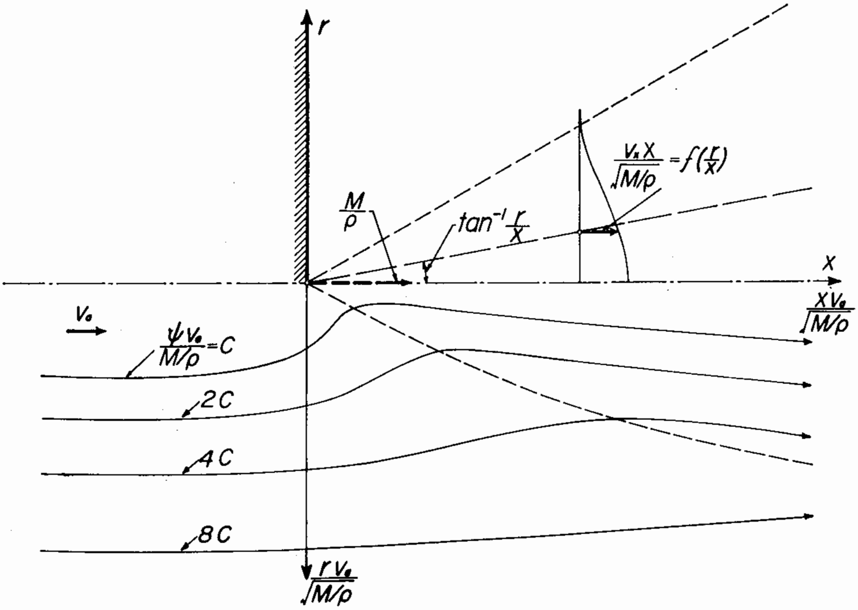

FIG. 2.—POINT SOURCE OF MOMENTUM FLUX: ABOVE, IN A NORMAL WALL; BELOW IN AN AMBIENT FLOW

Consideration of the foregoing discussion will lead to the conclusion that at considerable distances from the efflux section it is neither the efflux velocity nor the outlet diameter which is significant, but the momentum flux $M = \rho v_0^2 \pi D_0^2/4$. In other words the quantity M/ρ (with the dimension $[L^4/T^2]$) becomes the sole independent parameter characterizing the jet as a whole, and the efflux section is reduced to a point source of momentum in the axial direction. As with any source flow, there is no reference length, and all cases are perforce dynamically similar and without linear scale. As indicated schematically in the upper half of Fig. 2, every section is similar to every other one in such a rendition, for mean-flow (the actual values of which can be obtained from those of Fig. 1) and turbulence characteristics alike. This intuitive dimensional method of approach can readily be extended to flow from a nozzle in an ambient stream. The normal wall containing the outlet must then be eliminated, of course, which necessarily changes the direction of the approaching fluid even for the case of entrainment without ambient flow. If it is now realized that the nondimensional coordinates must incorporate both the kinematic momentum flux M/ρ and the velocity of the ambient flow, they will be found to take the form $rv_a/\sqrt{M/\rho}$ and $xv_a/\sqrt{M/\rho}$. Further thought will show that large values of the radial coordinate will correspond to small rates of momentum flux (or large ambient velocities), and vice versa. The nondimensional pattern of streamlines must hence have a form much like that shown schematically in the lower half of Fig. 2, the streamlines

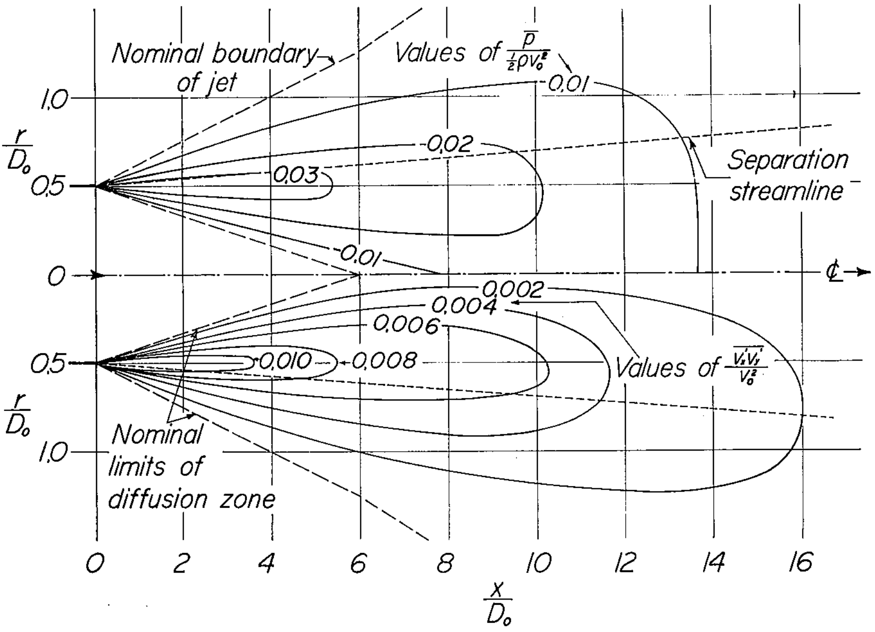

FIG. 3.—DISTRIBUTION OF PRESSURE (ABOVE) AND SHEAR (BELOW) IN A DIFFUSING JET (NOTE SCALE DISTORTION)

becoming more and more nearly parallel to the axis with increasing relative radius; that is, a single picture will represent all possible conditions ranging from those of relatively high jet strength and weak ambient flow in an enlargement of the pattern (i.e., close to the axis) to those of relatively strong ambient flow and low jet strength in a reduction of the pattern (i.e., far from the axis). To the best knowledge of the writer, this manner of composite presentation of the entire range of conditions from pure jet flow to pure ambient flow is new; he leaves determination of the exact geometry of the pattern to an interested thesis student.

So far as the writer is presently concerned, it is the details of the zone of flow establishment rather than of the zone of established flow which are of special interest. This zone has the distinguishing feature of a central, essentially irrotational core into which the vorticity generated in the surrouding shear zone gradually diffuses. If the distinction between efflux from a cylindrical outlet in a normal boundary and efflux from a nozzle (the difference being restricted largely to the surrounding low-velocity flow) is ignored, the distribution of the mean velocity shown in Fig. 1 can be regarded as typical of either. Although the usual simplified analysis rests upon the assumption of hydrostatic pressure distribution, in actuality there are appreciable departures therefrom: to a minor degree because of the necessary radial change in the velocity of the entrained fluid, and to a greater degree because of the variation in the radial component of turbulence intensity. The two factors can be evaluated through the momentum equation for the radial direction [10]. As-yet-unpublished measurements made at Iowa by Thomas Carmody with a static-pressure probe yield the contours of piezometric head shown in the upper half of Fig. 3 [11]. The head is seen to be lowest in the zones of most-pronounced turbulence, as will immediately be delineated. Far more important than the distribution of pressure or normal stress accomplishing the diffusion of the jet fluid, however, is the distribution of shear or tangential stress. At sufficiently high Reynolds numbers, the viscous contribution to the shear can be neglected in comparison with the so-called Reynolds stresses of the turbulence. Contours of the Reynolds stress $-\rho\overline{v'_x v'_y}$ acting on coaxial cylindrical surfaces, from measurements at Iowa by Sedat Sami, are shown in the lower half of Fig. 3 [11]. In general the maximum intensities of shear would be found to occur at points of maximum mean-velocity gradient and turbulence intensity. Sami's hot-wire data for the several components of the intensity have been combined to yield the contours of $\sqrt{\overline{v'^2}} = \sqrt{\overline{v'^2_x} + \overline{v'^2_y} + \overline{v'^2_z}}$ shown at the top of Fig. 4. The geometrically similar contour plot shown at the bottom of the figure represents Sami's results for the fluctuating pressure obtained by means of a cylindrical piezoelectric crystal flush-mounted at the usual piezometer location on a ⅛-inch-diameter static-pressure probe. Careful comparison of the two contour families would show that at essentially all points of pronounced turbulence $\sqrt{\overline{p'^2}} = 3.6\,\rho\overline{v'^2}/2$.

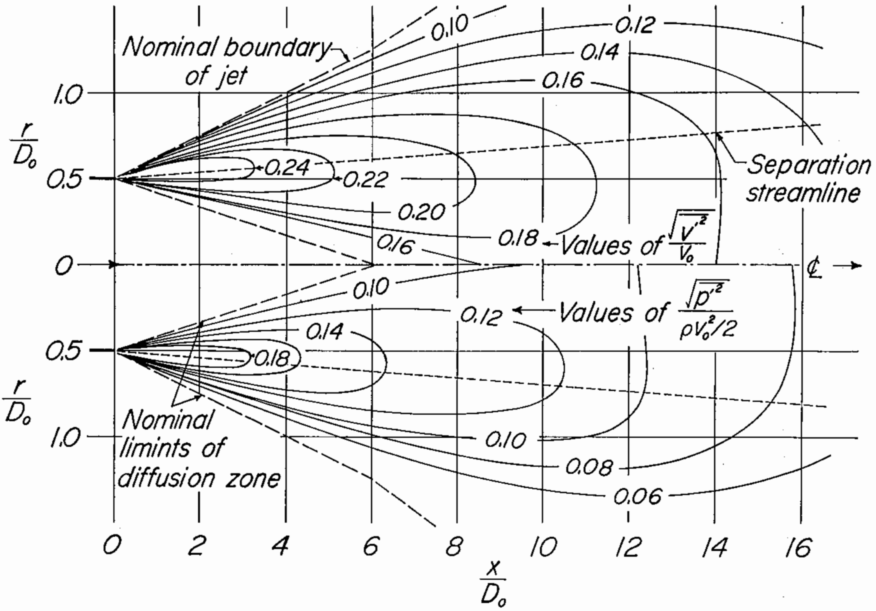

FIG. 4.—DISTRIBUTION OF VELOCITY FLUCTUATION (ABOVE) AND PRESSURE FLUCTUATION (BELOW)

Pressure-fluctuation measurements of this sort are relatively new, and the instrumentation involved has not yet been perfected to the extent of matching the hot-wire anemometer in either size or refinement. However, the nearly perfect correlation between the root-mean-square pressure fluctuation and the mean-square velocity fluctuation would lead one to hope that certain pressure characteristics which are not yet measurable may be represented by their velocity counterparts. Principal among these are their linear scales, i.e., the dimensions of the zones over which individual fluctuations extend. Since it is the eddy structure of the turbulence that is responsible for the two types of fluctuation, it is only reasonable that similar spectral distributions should be characteristic of both. In Fig. 5 are shown contour lines of two scales determined from temporal records of the longitudinal velocity fluctuation [11]: above, the size of the average eddies, determined from the correlation (i.e., degree of agreement) between values at successive instants a variable time interval apart; and below, the average size of the smaller eddies, known as the dissipation length, determined from the time rate of change of the velocity.

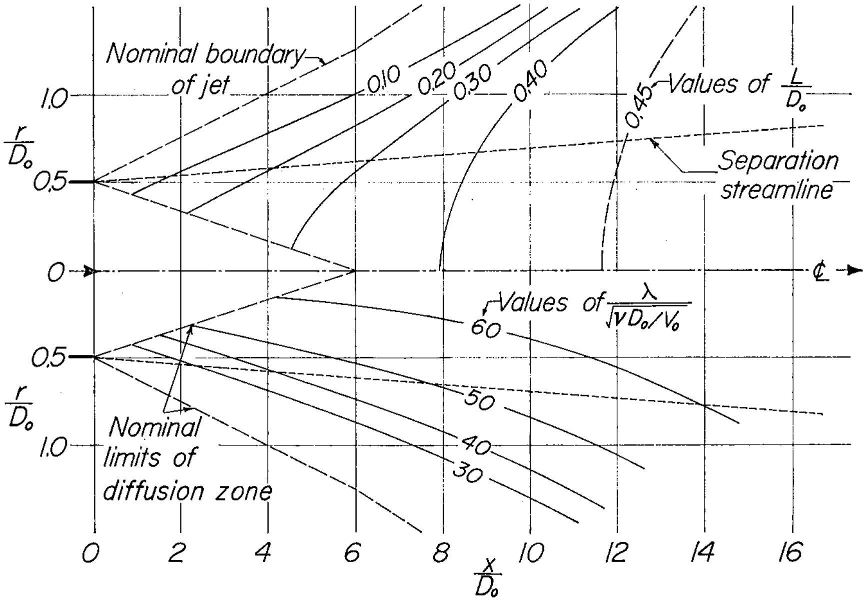

Fig. 5.—Distribution of Mean Eddy Size (Above) and Dissipation Length (Below)

If Freeman could have read the foregoing comments and examined the corresponding illustrations, he might well have lauded the Yankee ingenuity so obviously required by the painstaking and novel experiments that they represent, but he would most certainly have asked what engineering use they could possibly have. As it happens, two primary engineering applications of the information involve phenomena already known at Freeman's time, if not long before. One is the generation of noise. Each pressure fluctuation is a source of sound, whether it occurs in a diffusing jet or along the wall of a conduit, and considerable theoretical study has been given to the phenomenon [12]. However, the fluid compressibility is involved to a considerable degree, and the problem of noise generation is basically one of aerodynamics rather than hydraulics, though it must be granted that plumbing systems are among the most disturbing noise sources in modern civilization.

As a matter of fact, it is cavitation that is usually the cause of noise in plumbing systems, just as it is cavitation that is the other phenomenon involved in applying the foregoing information about pressure fluctuation in submerged jets. The study of jet cavitation as such seems to have originated with the writer some fifteen years ago in connection with the proposed jet propulsion of ships—i.e., the placing of enlarged propellers in pressure passages within the hulls to prevent blade cavitation. It was readily demonstrated that the low-pressure cores of the eddies generated in the shear zone around the emerging jet could well produce cavitation considerably in advance of the ducted propeller. Although the propeller would thus be protected from cavitation damage, the elimination of cavitation as a source of noise and bubbles in the wake would not necessarily be realized. The original studies conducted on the phenomenon at Iowa yielded a magnitude of about 0.6 for the cavitation index $\sigma = (h_0 - h_v)/(V_0^2/2g)$ under conditions of incipiency, as well as the first measurements that appear to have been made of the distribution of pressure fluctuations in the zone of turbulence generation, and the two photographs of vapor bubbles shown in Fig. 6 [13].

Because it was still impossible to predict other than empirically either the magnitude of the cavitation index for incipiency or the position at which collapse of the vapor bubbles could be expected to occur, i.e., the location of the noise source, the two phases of the investigation were continued independently. On the one hand, detailed measurements of the mean-flow and turbulence characteristics of submerged jets without cavitation (for convenience, air into air) were undertaken. These required not only the development of improved instrumentation but the perfection of measurement techniques to the state that the resulting values satisfied the equations of motion in detail as well as in gross. Only now being readied for publication elsewhere [11], the information thus obtained was used by S. T. Hsu of the Iowa Institute staff in preparing Figs. 3, 4, and 5 for the present paper. On the other hand, detailed sonic studies were made by Appel [14] of a cavitating jet at various values of the cavitation index, not only to define the location of the collapse zone but also to clarify a number of apparent anomalies in the earlier study.

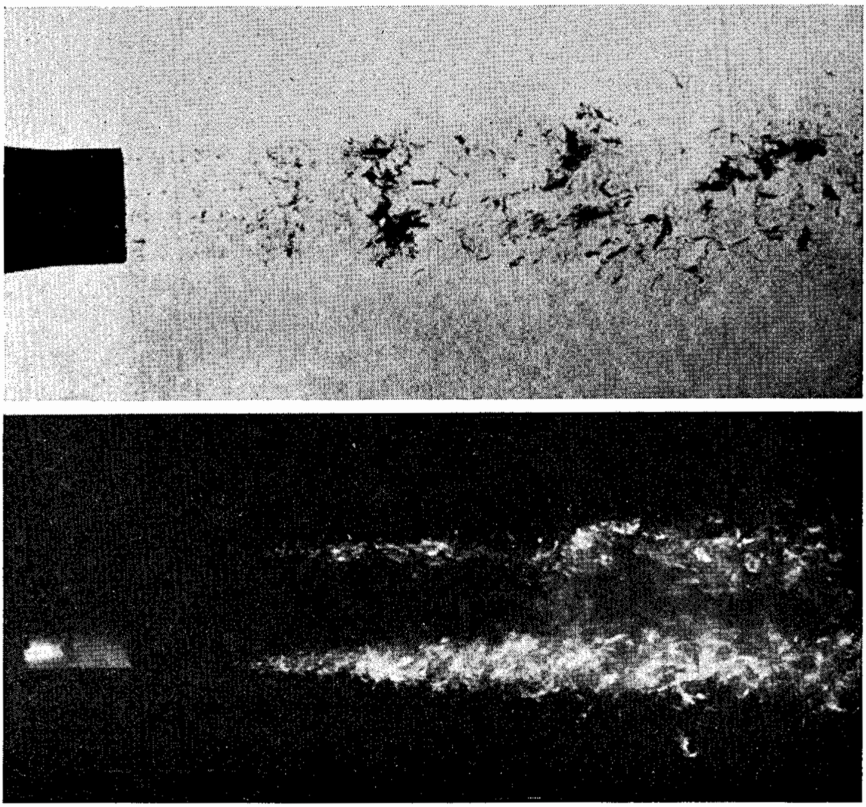

FIG. 6.—HIGH-SPEED PHOTOGRAPHS OF EDDY CAVITATION IN A DIFFUSING JET: ABOVE, SINGLE EXPOSURE, REAR ILLUMINATION; BELOW, 25 EXPOSURES, VERTICAL SHEET ILLUMINATION [13]

As is evident from Fig. 6, the intensity of cavitation-bubble concentration is distributed in much the same manner as the intensity of turbulence, shown by the contours of velocity and pressure fluctuation in Fig. 4. The smaller the cavitation index, moreover, the greater do experiments show the longitudinal and radial range of the visible bubbles to be, as one would expect from the general eddy-cavitation concept. Somewhat surprising, in this regard, was Appel's observation that the maximum source of noise was invariably at the end of the zone of flow establishment, approximately five nozzle diameters downstream from the efflux section. Although bubble collapse must evidently occur beyond any section at which bubbles are still visible, for some reason not yet clear the greatest rate of collapse thus coincides with the zone in which the turbulence reaches the jet axis. Appel's sonic measurements further revealed that at any point the variation in audible noise with the cavitation index involved its frequency but not its amplitude. In other words, each individual bubble collapse produces a pressure fluctuation of essentially the same magnitude, and the relative level of noise is a measure of the relative number of such collapses occurring per unit time. Moreover, the frequency of collapse was found to be quite randomly distributed. Definition of the point of incipiency is thus rather arbitrary. The writer's previouse hope [13] of being able to predict at least the point of incipiency from the turbulence data presented herein is now seen to meet yet another obstacle: whereas the distribution of eddy scale shown in Fig. 5 permits the relative magnitude of

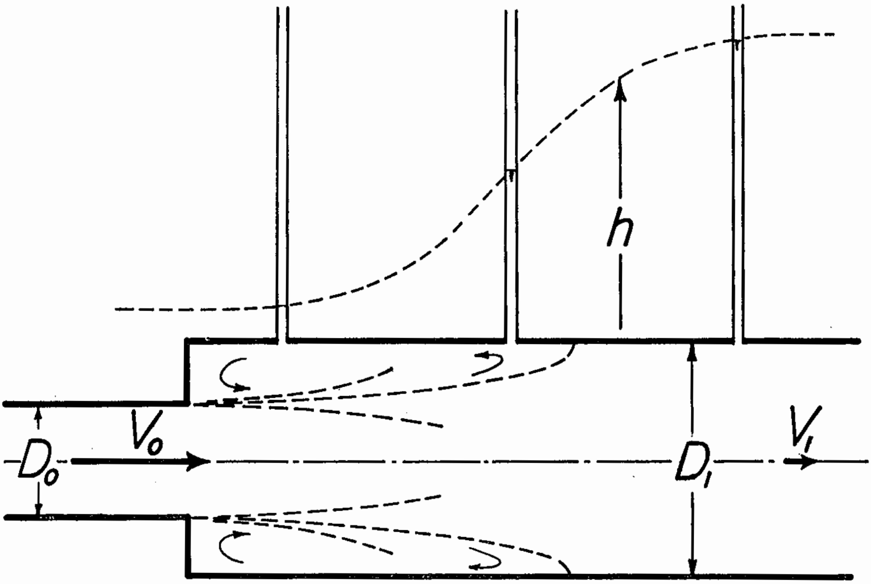

FIG. 7.—JET DIFFUSION AND PRESSURE VARIATION AT A CONDUIT EXPANSION

average eddy size to be predicted at any point (and hence the relative number of cavitation bubbles present at any instant), not only is this a statistical average, but an unknown coefficient of proportionality is also involved. As a result, although two conditions can now be compared quantitatively, one or the other still requires experimental evaluation.

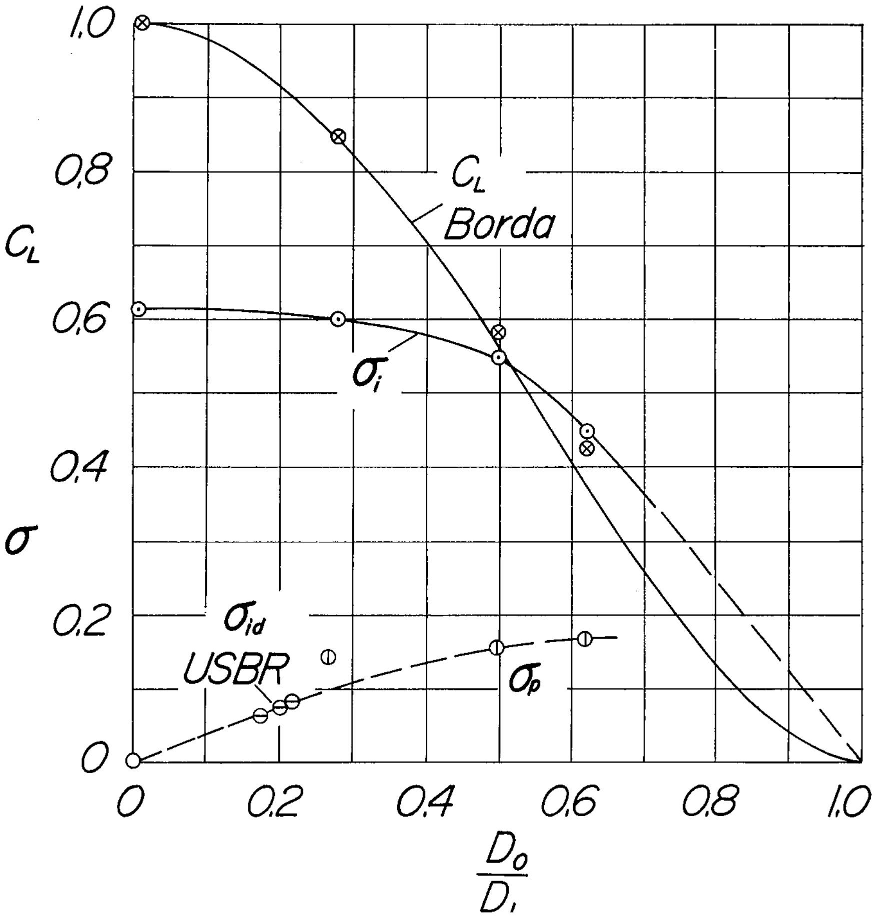

FIG. 8.—COEFFICIENTS OF LOSS, INCIPIENT CAVITATION, AND CAVITATION DAMAGE VERSUS EXPANSION RATIO [15]

Whereas the direct application of such knowledge about eddy diffusion and cavitation is apparently restricted to the phenomenon of jet propulsion, small changes in boundary geometry can extend its range of interest greatly. Consider, for example, the current use of conduit expansions as a means of energy dissipation in hydroelectric installations. Since these are simply diffusion zones of limited radial extent, all of the foregoing flow characteristics should again be encountered, plus a few more. Principal among the latter (see Fig. 7) are the region of reverse flow enclosing the main stream and the longitudinal rise in pressure as the jet expands. The rate of energy loss at such an expansion can be expressed in terms of the Borda coefficient

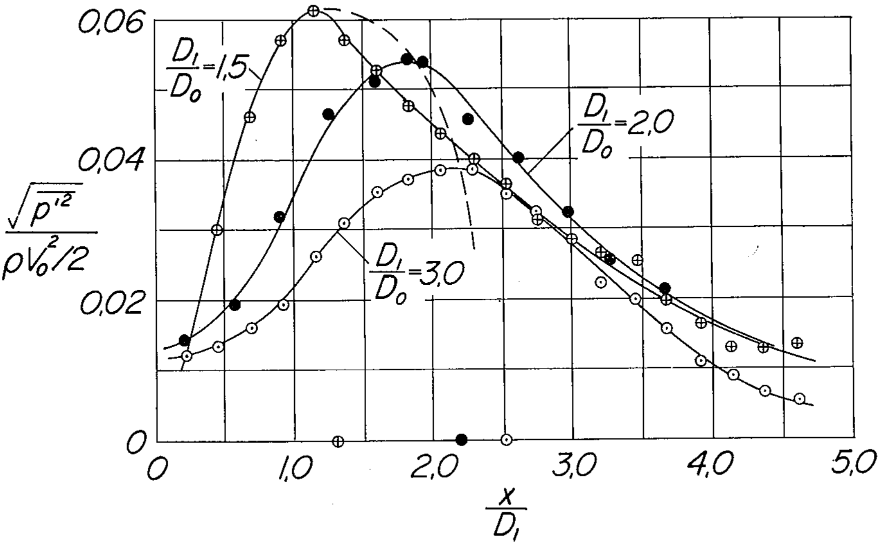

FIG 9.—LONGITUDINAL DISTRIBUTIONS OF WALL-PRESSURE FLUCTATION [17]

$C_L = (V_1 - V_0)^2/V_0^2$ as a function of expansion ratio, as plotted in the uppermost curve of Fig. 8 [15]. Conditions of incipient cavitation, determined experimentally, are seen from the middle curve to agree with those for the unconfined jet at the limiting expansion ratio. While even incipient cavitation is to be avoided in most design, it should be noted that actual danger to the structure from cavitation damage will occur only when the bubble collapse takes place in the vicinity of the wall. Such conditions as determined sonically at Iowa (σ_p) are compared near the bottom of the figure with re-evaluated measurements of actual cavitation damage (σ_{id}) conducted at the Bureau of Reclamation [16].

Since cavitation of this nature results from the eddy structure of the turbulence, the pressure fluctuations which the eddies produce cannot be made to disappear, as can the cavitation itself, simply by raising the ambient pressure. Hence, not only must an energy dissipator of the expansion type be designed to perform without cavitation, but the pressure fluctuations must also be considered in the structural design. Such considerations involve not only the magnitude of the pressure departure from the mean but also the size of the region over which the average fluctuation extends and its degree of vibration-producing regularity. In a much more recent study [17], an initial exploration of these characteristics was made for the three expansions involved in the previous experiments (which, in turn, had been made with the same equipment used in the original investigations of the submerged jet itself).

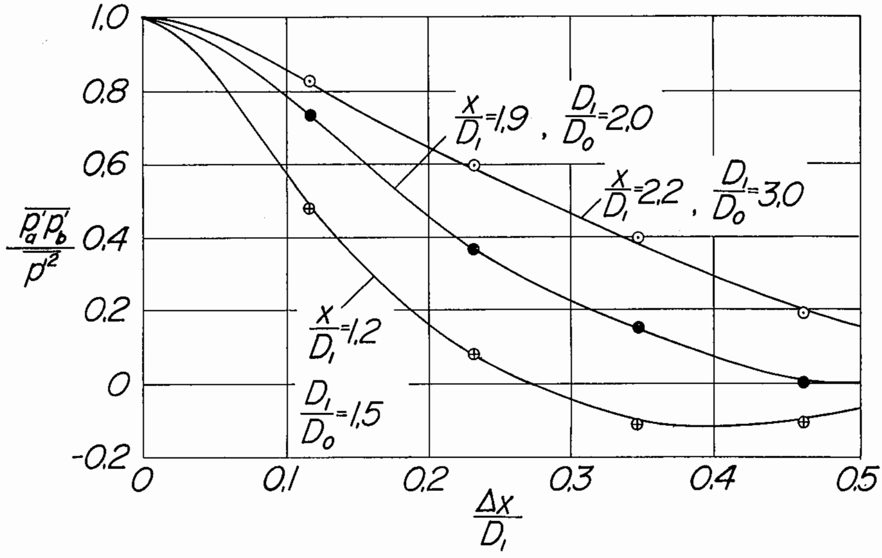

FIG. 10.—LONGITUDINAL WALL-PRESSURE CORRELATIONS [17]

Figure 9 reproduces the distribution of root-mean-square pressure fluctuation along the wall of each expansion. Since the fluctuation of pressure, like that of velocity, approximates the Gaussian probability function, magnitudes greater than the root-mean-square can be expected some 16 percent of the time. The root-mean-square values themselves are seen to increase between the initial section and one slightly upstream from that at which the separation streamline reaches the wall (marked by one of the three points along the abscissa scale), and thereafter to decrease. As must be concluded from the broken curve connecting the maximum values, an optimum condition is reached at about the smallest expansion ratio tested; it is, of course, only logical that this curve should approach zero as the expansion ratio becomes either negligible or very large.

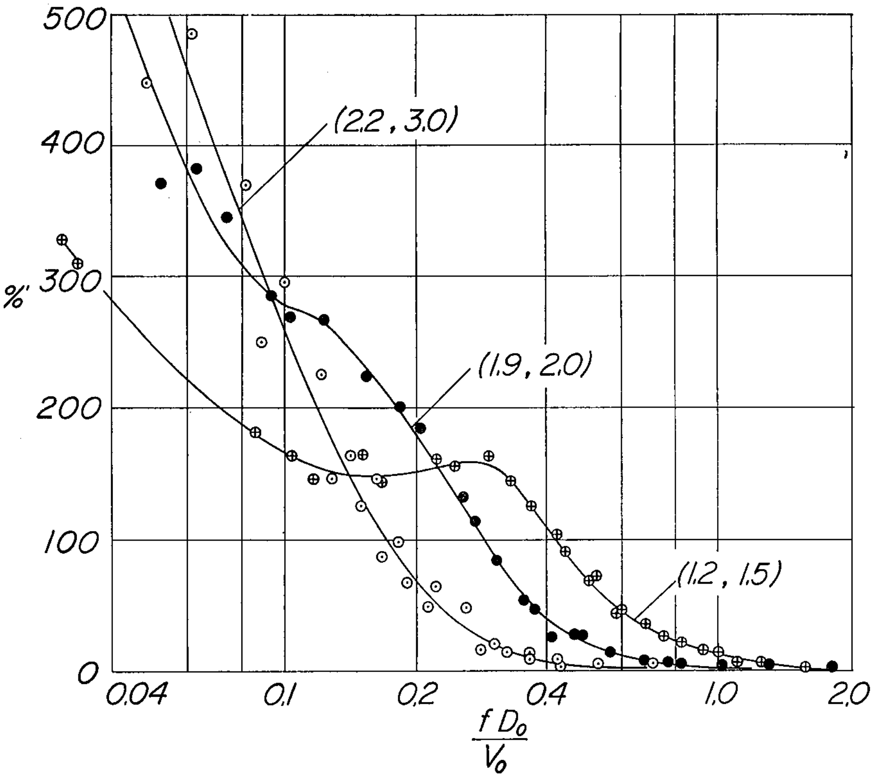

FIG. 11.—FREQUENCY SPECTRA OF WALL-PRESSURE FLUCTUATION [17]

In order to obtain an indication of the average size of the zone over which individual fluctuations extend, measurements were made of the correlation (degree of agreement) between pressures at points a and b varying distances Δx/2 either side of the point of maximum root-mean-square fluctuation. With a negligible separation, the two values should be almost identical, and the correlation coefficient $\overline{p_a'p_b'}/\overline{p'^2}$ (see Fig. 10) should then be very close to unity; if, on the contrary, the separation is greater than the distance over which the individual fluctuation is likely to extend, the correlation should be essentially zero (the tendency of such a curve to dip below the axis is a sign of some degree of regularity of the fluctuations). As is seen from Fig. 10, the size of the fluid elements involved in the individual fluctuations is generally a small fraction of the conduit diameter.

Finally, the spectral analyses of the fluctuations, again at the sections of maximum root-mean-square fluctuation, are shown in Fig. 11. The ordinate scale is such as to yield an area of 100 percent under each curve when plotted arithmetically, and the abscissa scale includes the frequency of the fluctuations passed by the filter. It is actually the form of the curves rather than their elevation which is significant, for local humps (such as that in the curve for the 1.5 expansion ratio) indicate a tendency for fluctuations of a particular frequency to predominate and possibly excite elastic vibrations in the structure. It is evident from the figure that this tendency rapidly diminishes with increasing expansion ratio.

The points that the writer has emphasized are but a few of the conclusions that can be reached from the experimental data that have been presented. Likewise, the boundary forms used for illustration are probably but a few of the many which will eventually be investigated in a comparable manner. Phenomena of turbulence and pressure fluctuation are in the air, hydraulically speaking, both here and abroad, particularly, it should be noted, in the Soviet Union. Dr. Freeman would certainly have relished both the situation and the challenge that it brings.

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