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HYDRAULICS OF MIXING TANKS

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Introduction

Mixing is widely used in the chemical and pharmaceutical industries to disperse additives throughout liquids or to homogenize two or more liquids. A measure of effectiveness of such mixing is the uniformity of dispersion. A frequent purpose of mixing is to promote chemical or biochemical reactions.

In the treatment of water or wastewater in continuous-flow plants, mixing is used for all of the above purposes, but one of the most important functions is the flocculation of impurities with coagulants so that they may be removed by settling or filtration. It has become common practice to provide a rapid initial or flash mix for dispersion of the coagulants or other chemicals throughout the influent followed by two or more larger chambers in series where floc growth takes place. Mixing propellers or flat blade turbines are usually used in the flash mix, and rotor reels in the floc chambers.

In 1943, the author and P. C. Stein[1] demonstrated that the time rate of flocculation (i.e., agglomeration into larger particles) is directly proportional to the velocity gradient (i.e., space rate of change of velocity) at a point. The velocity gradients vary considerably throughout a mixing tank, but under steady conditions of mixing, a mean velocity gradient related to the energy dissipation per unit of tank volume may be used. The relation between the velocity gradient and the energy dissipated per unit volume was derived by the authors from the original analysis by Sir G. G. Stokes[2] in 1845.

All of the energy dissipated in a moving or stirred liquid is by shear within the liquid, and heat is evolved to increase the temperature of the liquid. The instantaneous shearing stress at a point is

$$\tau = \mu \frac{dv}{ds} \qquad (1)$$

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where $\frac{dv}{ds}$ is the *absolute velocity gradient* at the point and the proportionality constant μ is the *absolute viscosity* of the liquid. The *root-mean-square velocity gradient* for a container is

$$G = \sqrt{\frac{W}{\mu}} \tag{2}$$

where W is the energy dissipated per unit volume, called the *dissipation function* by Stokes. The author[3] has recently demonstrated by extensive experiments with hydrous ferric oxide floc that the floc size and volume concentration may be varied over a wide range by changes in G. The principal constituent of floc (less than 85 to more than 99%) is loosely bound water, the amount of which is controlled by G.

The value of W depends upon the geometry of the rotors, stators and container, and upon the speed of the rotors. Accurate values of W can be determined only by measurement of the torque input to the liquid at various speeds and liquid temperatures. The value of W in terms of the speed and measured torque is

$$W = \frac{2\pi ST}{V} \tag{3}$$

where S is the measured rotor speed in rps, T is the measured torque input and V is the liquid volume.

The calibration curves for G shown in Fig. 1 were computed by means of Eqs. (2) and (3) from numerous measurements of torque input at the water temperatures shown. These curves are for 2 liters of water in a 2-liter beaker, both with and without stators. All dimensions are shown on the figure. The torque measurements were made by supporting the beaker on a platform suspended on a piano wire, and by direct connection of the rotor shaft to a motor mounted on a thrust ball bearing ring. Torque measurements may be made for a large mixing tank by inserting a torque meter in the rotor shaft above the water surface for vertical shafts or in a dry well for horizontal shafts, provided the tank is designed to permit the insertion. Accurate torque meters are commercially available for this purpose. Since the purpose of calibration of a mixing tank is to determine the power dissipated *in the liquid*, torque measurements at each speed must be made with the tank both full and empty. The difference is the net torque for the liquid.

The computed points on Fig. 1 indicate some errors in measurements of torque or speed. The curves were adjusted for a smooth fit of the plotted points. For fully turbulent drag, the ordinates for a particular speed should be inversely proportional to the square roots of the viscosities as required

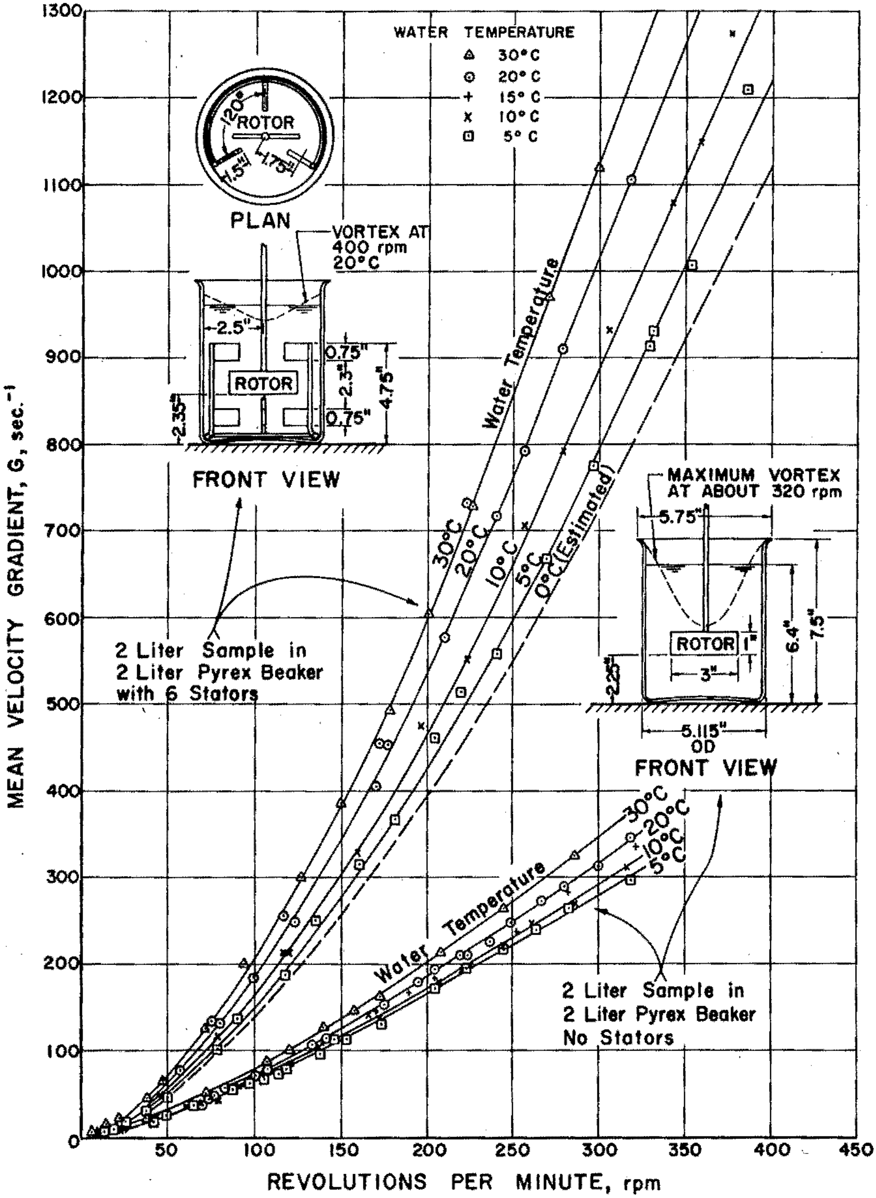

Fig. 1 — Velocity gradient calibration curves for water.

by Eqs. (2) and (4). This relationship is met for the case with stators at all speeds above about 20 rpm, as later shown by Fig. 3, but for the case without stators, the drag is not fully turbulent until the speed is greater than 300 rpm. For fully streamline drag at a particular speed, there is only one value of G for all viscosities as required by Eqs. (2) and (5). This condition is met at speeds less than 5 rpm for the case with stators, and 10 rpm for the case without stators.

The speed without stators was limited to about 320 rpm, which produced a vortex extending down almost to the top of the rotor blade, as shown in Fig. 1. The effect of the stators is to reduce the vortex in all cases and greatly increase the values of G at all speeds. Fig. 1 shows an increase of G by the stators of more than 3-fold at the higher speeds, which corresponds with about a 10-fold increase in the power dissipated. It is obvious from Fig. 1 that mixing tanks and equipment cannot be designed accurately solely on the basis of rotor drag, as assumed by the author in an earlier paper[4] (1955), and that stator and wall drag must also be included.

Gross Drag Coefficients

In order to design a mixing chamber to produce a desired range of values for G or W within a desired range of speeds, it is convenient to express the dissipation function W in terms of the drag on the rotors, stators and walls. Extensive velocity measurements in vertical cylindrical vessels with turbine blades on vertical shafts by Nagata *et al*, Aiba and others, as reported by Gray[5], indicate that the radial and vertical velocity components are small as compared to the tangential velocity components, and that the tangential velocity is directly proportional to the radial distance out from the center of the shaft almost to the tip of the turbine blades. Beyond the tip of the blades, the tangential velocity decreases in some cases and is approximately constant in others. The radial distribution of the tangential velocity is about the same throughout the depth. The tangential velocity of the liquid is reduced with stators.

To simplify the theoretical development, the author has assumed that all of the energy dissipation is associated with the tangential velocity components of the liquid, and that this velocity is directly proportional to the radial distance out from the center of the shaft to a maximum at the tips of the rotor paddles and is constant beyond the paddle tips. The assumed distribution of tangential velocity is shown in Fig. 2. The velocity, v, of the liquid at any radial distance r less than r_r at the tip of the rotor is $2\pi rkS$, and beyond the rotor tip is $2\pi r_r kS$; where kS is the speed of the water in

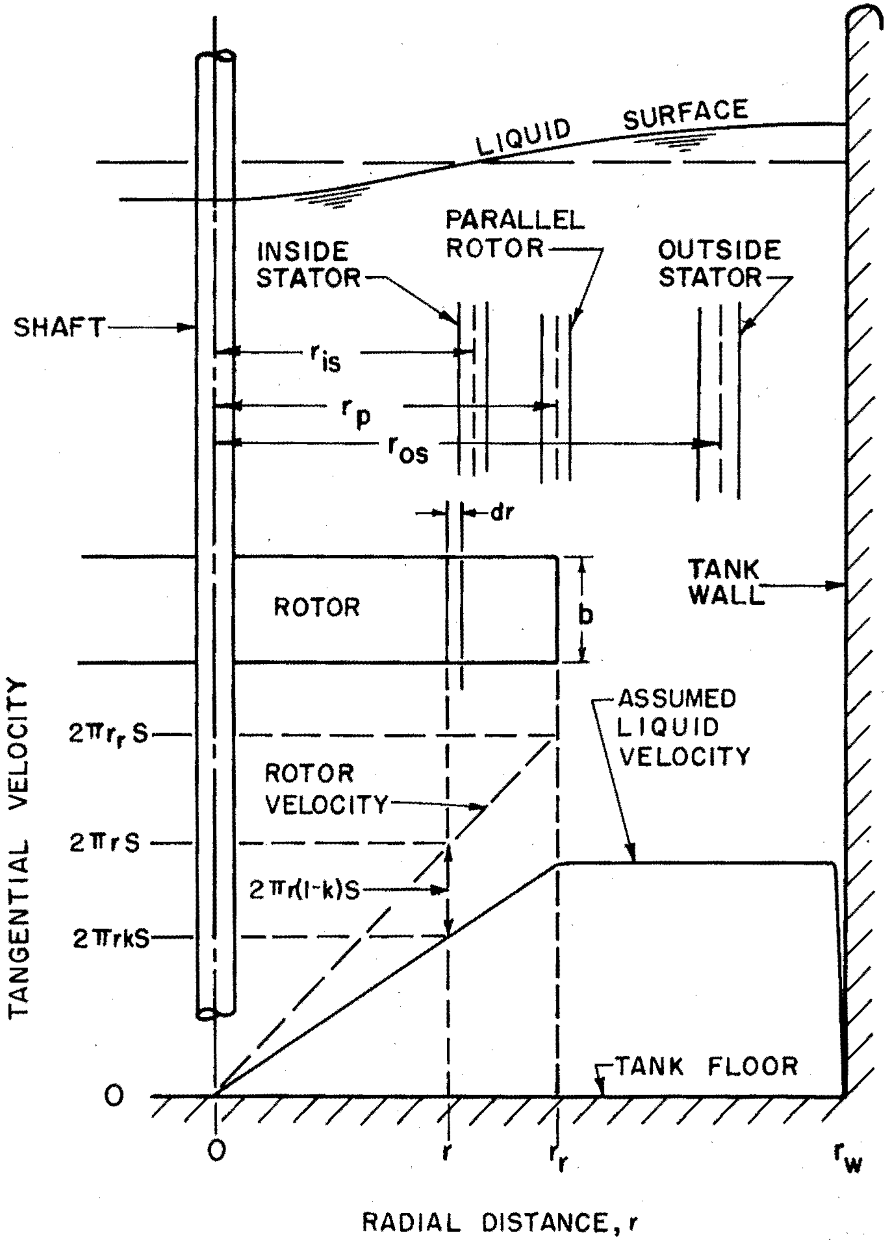

Fig. 2 — Assumed distribution of Tangential Liquid Velocity in mixing tank.

rps. Since the energy dissipated in turbulent drag is proportional to $\rho \frac{v^3}{2}$, the value of the dissipation function W in terms of turbulent drag on rotors, stators and walls is:

$$W = 124\rho a C_t S^3 \tag{4}$$

where 124 is $\frac{(2\pi)^3}{2}$, ρ is the mass density of the liquid; a is the projected area of the rotor blades, and C_t is a dimensionless *turbulent gross drag coefficient* determined by the geometry of the system. The term a is introduced solely for the purpose of making C_t dimensionless.

For any speed S, C_t may be computed by means of Eqs. (3) and (4) from the corresponding measured value of the torque T, or by means of Eq. (2) from the corresponding value of G on one of the calibration curves for velocity gradient. The better procedure is to use a calibration curve for G for a selected temperature because these curves are already adjusted to reduce experimental error. Fig. 3 is a log-log plot of the gross drag coefficients, C_t against rpm computed in this manner for the 2-liter beaker with water at 30°C. For lower water temperatures, the curves will be approximately in the same position for fully turbulent drag and will be higher in proportion to the viscosity for fully streamline drag.

Fig. 3 is enlightening in several respects. First, it is remarkably similar to well known plots of friction factor or drag coefficient against Reynolds number. Second, it clearly shows the existence of a streamline region, at speeds below 5 rpm, with stators, and below 10 rpm without stators, for the particular example. Third, the drag in the example is fully turbulent with stators at about 30 rpm but, without stators, is not fully turbulent at speeds up to about 300 rpm. Since the effectiveness of mixing depends upon the degree of turbulence, it is evident that the stators in the 2-liter beaker make an important contribution. For example, full turbulence is present with the stators at 30 rpm and a G of about 30, whereas, without stators, full turbulence is not present at 300 rpm even though G is about 350.

The curves for C_t, illustrated on Fig. 3, are based on fully turbulent drag which is assumed to be proportional to v^2. This proportionality does not hold except where the curves are nearly horizontal. In the streamline region, the drag is proportional to v and C_t becomes inversely proportional to the Reynolds number. Since the speed of the rotors is the only changeable factor in the Reynolds number for a particular mixing tank and water temperature, the curves for C_t in the streamline region slope downwards on the log-log plot at 45°. It becomes apparent that the dissipation function W

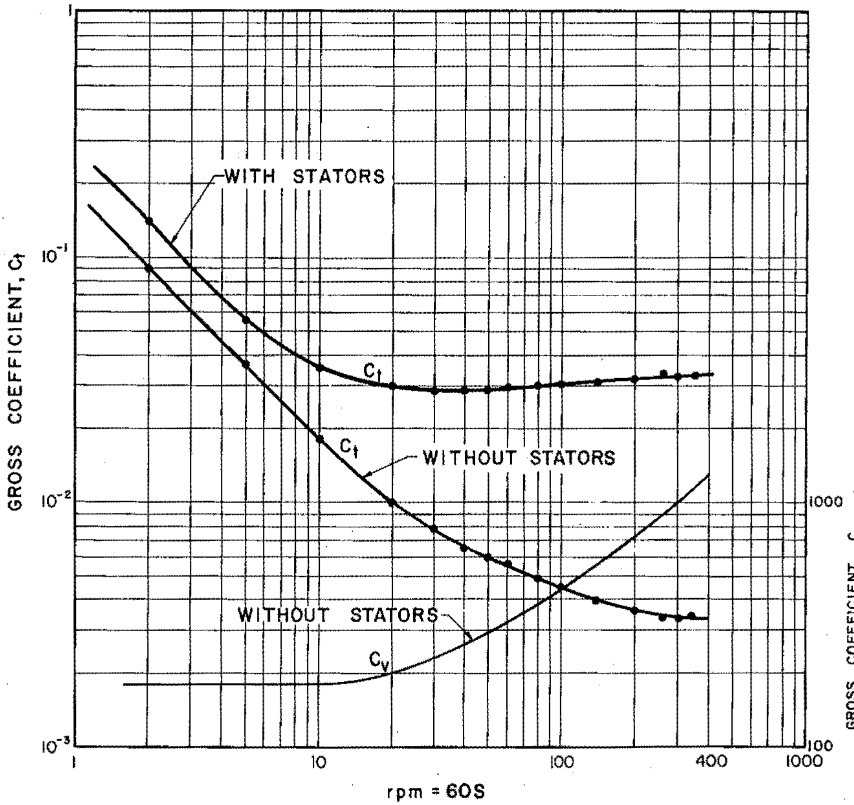

Fig. 3 — Gross drag coefficients for 2-Liter Beaker with water at 30°C.

may also be expressed in terms of the viscous drag, where energy is dissipated in proportion to μv^2, as follows:

$$W = 4.92\mu C_V S^2 \qquad (5)$$

where 4.92 is $\frac{(2\pi)^2}{8}$ and C_V is a dimensionless *viscous gross drag coefficient* determined by the geometry of the system. Fig. 3 shows a curve for values of C_V for the 2-liter beaker without stators, which illustrates that the curve is horizontal in the streamline region and slopes upward at 45° in the fully turbulent region.

Plots of C_V are not necessary because the value of C_V may be computed for any rotor speed from the corresponding value of C_t taken from the curve. The value of C_V in terms of C_t may be obtained by equating the values of W in Eqs. (4) and (5) as follows:

$$C_V = \frac{25.1}{\nu} a C_t S \qquad (6)$$

where 25.1 is 8π and υ is the kinematic viscosity of the liquid or $\frac{\mu}{\rho}$.

The value of the turbulent gross drag coefficient, C_t in terms of the drag on the rotors, stators and walls is

$$C_t = \frac{C_D}{Va}\left[(1-k)^3 A + k^3 (B + \frac{f}{C_D} C)\right] \qquad (7)$$

where C_D is the conventional turbulent drag coefficient applied to the system of rotors and stators, f is the Weisbach-Darcy wall friction factor, and A, B and C are the dimensional turbulent work parameters for the rotors, stators and walls, respectively.

The value of the viscous gross drag coefficient, C_V, in terms of the drag on the rotors, stators and wall is

$$C_V = \frac{C_D'}{V^2}\left[(1-k)^2 A' + k^2 (B' + \frac{f'}{C_D'} C')\right] \qquad (8)$$

where C_D' is C_D times the Reynolds number, f′ is f times the Reynolds number, and A′, B′ and C′ are the dimensional viscous work parameters for the rotors, stators and walls, respectively.

The dimensional work parameters are derived hereinafter in the Appendix for various types of rotors and stators in terms of the areas and radii. The dimensions of the turbulent work parameters are $(\text{length})^5$, and the dimensions of the viscous work parameters are $(\text{length})^6$.

Eq. (7) for C_t is applicable only where turbulence is fully developed and does not change appreciably with change in speed. Eq. (8) for C_V is applicable only where flow is streamline and C_V does not change appreciably with change in speed. It is proposed to use these equations in design to estimate the position of the two tangents to the C_t curve on log-log paper.

Evaluation of Coefficients

In order to use the gross drag coefficients for design, torque measurements were made on existing tanks to evaluate friction factors and drag coefficients. The factor k may be estimated by equating the moment of the drag on the rotors to the moment of the drag on the stators and walls, each of which is equal to the torque. The drag and friction coefficients may be estimated by assuming that at the same speed they have the same values without the stators in place as with the stators. This requires evaluation of C_t for both cases, as illustrated on Fig. 3.

Drag Moments

The moments of the turbulent drag are proportional to $\rho \frac{v^2}{2} r$, as follows:

Rotors $$M_r = 19.7\, \rho\, C_D (1-k)^2 A_m S^2 \tag{9}$$

Stators $$M_s = 19.7\, \rho\, C_D k^2 B_m S^2 \tag{10}$$

Walls $$M_w = 19.7\, \rho\, C_D k^2 \frac{f}{C_D} C_m S^2 \tag{11}$$

where 19.7 is $\frac{(2\pi)^2}{2}$ and A_m, B_m and C_m are the dimensional moment parameters for turbulent drag on rotors, stators and walls, respectively.

The moments of the viscous drag are proportional to $\mu v r$, as follows:

Rotors $$M'_r = 0.785 \mu \frac{C'_D}{V} (1-k) A'_m S \tag{12}$$

Stators $$M'_s = 0.785 \mu \frac{C'_D}{V} k B'_m S \tag{13}$$

Walls $$M'_w = 0.785 \mu \frac{C'_D}{V} k \frac{f'}{C'_D} C'_m S \tag{14}$$

where 0.785 is $\frac{2\pi}{8}$ and A'_m, B'_m and C'_m are the dimensional moment parameters for viscous drag on rotors, stators and walls, respectively.

The dimensional moment parameters are derived hereinafter in the Appendix for various types of rotors and stators in terms of the areas and radii. The dimensions of the turbulent moment parameters are $(\text{length})^5$ and of the viscous moment parameters are $(\text{length})^6$.

Turbulent Coefficients

Equating the moments of turbulent drag,

$$k^2 (B_m + \frac{f}{C_D} C_m) = (1-k)^2 A_m \tag{15}$$

from which,

$$k = \frac{1}{1 + \sqrt{\frac{B_m + f/C_D C_m}{A_m}}} \tag{16}$$

and

$$\frac{f}{C_D} = \frac{(1-k)^2}{k^2} \frac{A_m}{C_m} - \frac{B_m}{C_m} \tag{17}$$

Since $\frac{f}{C_D} > 0$

$$\frac{(1-k)^2}{k^2} A_m > B_m \tag{18}$$

and

$$k < \frac{1}{1 + \sqrt{\frac{B_m}{A_m}}} \tag{19}$$

From (19), for the case without stators, B_m is zero and k is less than 1.0. For the case with stators, k_s is less than a fraction of 1.0. For example, for the 2-liter beaker with stators where B_m, A_m and C_m are 10.67×10^{-5}, 1.017×10^{-5} and 67.5×10^{-5} ft^5,

$$k_s < \frac{1}{1 + \sqrt{\frac{10.67 \times 10^{-5}}{1.017 \times 10^{-5}}}} < 0.236$$

If from Eq. (17) the values of $\frac{f}{C_D}$ for the cases with and without stators are equated, the following results:

$$\frac{(1-k_s)^2}{k_s^2} - \frac{(1-k)^2}{k^2} = \frac{B_m}{A_m} \tag{20}$$

where k is for the case without stators and k_s is for the case with stators.

If the moment of the drag on the rotors of Eq. (9) is set equal to the torque of Eq. (3), the following results:

$$W = 124 \rho a \frac{C_D}{Va} (1-k)^2 A_m S^3 \tag{21}$$

in which, by comparison with Eq. (4),

$$C_t = \frac{C_D}{Va}(1-k)^2 A_m \tag{7a}$$

It is evident from Eq. (7a) that for fully turbulent drag,

$$\frac{(1-k_s)^2}{(1-k)^2} \doteq \frac{C_t \text{ with stators}}{C_t \text{ without stators}} \tag{22}$$

With known values of C_t for the two cases, the values of all coefficients can be determined by trial-and-error solutions of Eqs. (22), (19), (20), (17), (16), and (7a). For example, for the 2-liter beaker at 300 rpm, C_t is 3.4×10^{-2} with stators and 0.33×10^{-3} without stators. From (22),

$$\frac{(1-k_s)^2}{(1-k)^2} = \frac{3.4}{0.33} = 10.3 \text{ and } \frac{1-k_s}{1-k} = 3.21$$

From (19), k_s is less than 0.236, and by successive trials by (20) k_s is equal to 0.235, from which $1-k_s$ is 0.765. From the above ratio, 1-k is $\frac{0.765}{3.21}$ or 0.238, from which k is 0.762. Check by Eq. (20)

$$\left(\frac{0.765}{0.235}\right)^2 - \left(\frac{0.238}{0.762}\right)^2 = 10.585 - 0.098 = 10.49 = \frac{10.67}{1.017}$$

From 17,

$$\frac{f}{C_D} = 0.098 \frac{1.017}{67.5} = 0.00147 \qquad \text{Check by (16)}$$

$$k_s = \frac{1}{1 + \sqrt{\frac{10.67 + 0.00147 \times 67.5}{1.017}}} = 0.235$$

and

$$k = \frac{1}{1 + \sqrt{\frac{0.00147 \times 67.5}{1.017}}} = 0.762$$

For V of 0.0706 ft^3, from Eq. (7a),

$$C_D = \frac{C_t Va}{(1-k_s)^2 A_m} = \frac{3.4 \times 10^{-2} \times 0.706 \times 10^{-1} \times 2.08 \times 10^{-2}}{0.765^2 \times 1.017 \times 10^{-5}} = 8.4$$

and $f = 0.00147 \times 8.4 = 0.0123$

Viscous Coefficients

Equating the moments of the viscous drag,

$$k\left(B_m' + \frac{f'}{C_D'}\,C_m'\right) = (1 - k)\,A_m' \tag{23}$$

From which
$$k = \cfrac{1}{1 + \cfrac{B_m' + f'/C_D' C_m'}{A_m'}} \tag{24}$$

and
$$\frac{f'}{C_D'} = \frac{1-k}{k}\,\frac{A_m'}{C_m'} - \frac{B_m'}{C_m'} \tag{25}$$

Since $\dfrac{f'}{C_D'} > 0$
$$\frac{1-k}{k}\,A_m' > B_m' \tag{26}$$

and
$$k < \cfrac{1}{1 + \cfrac{B_m'}{A_m'}} \tag{27}$$

From (27), for the case without stators, B_m' is zero and k is less than 1.0. For the case with stators, k is less than some fraction of 1.0. For example, for the 2-liter beaker with stators, where B_m', A_m' and C_m' are 40.0×10^{-6}, 2.26×10^{-6} and 4.49×10^{-3} ft^6.

$$k_s < \cfrac{1}{1 + \cfrac{40.0 \times 10^{-6}}{2.26 \times 10^{-6}}} < 0.0535$$

If from (25), the values of $\dfrac{f'}{C_D'}$ for the cases with and without stators are equated, the following results:

$$\frac{1-k_s}{k_s} - \frac{1-k}{k} = \frac{B_m'}{A_m'} \tag{28}$$

where k is for the case without stators and k_s is for the case with stators.

If the moment of the drag on the rotors of Eq. (12) is set equal to the torque of Eq. (3), the following results:

$$W = 4.92\mu\,\frac{C_D'}{V^2}\,(1-k)\,A_m' S^2 \tag{29}$$

in which, by comparison with Eq. (5),

$$C_V = \frac{C_D'}{V^2} (1 - k) A_m' \tag{8a}$$

It is evident from Eq. (8a) and Eq. (6) that for streamline drag at the same rpm,

$$\frac{1 - k_s}{1 - k} = \frac{C_V \text{ with stators}}{C_V \text{ without stators}} = \frac{C_t \text{ with stators}}{C_t \text{ without stators}} \tag{30}$$

With known values of C_t for the two cases, all coefficients can be evaluated by trial-and-error solutions of Eqs. (30), (27), (28), (25), (24), and (8a). For example, for the 2-liter beaker at 3 rpm, C_t is 9.32×10^{-2} with stators and 5.96×10^{-2} without stators. From (30),

$$\frac{1 - k_s}{1 - k} = \frac{9.32}{5.96} = 1.56$$

From (27), k_s is less than 0.535, and by successive trials by (28) k_s is equal to 0.049 from which $1 - k_s$ is 0.951. From the above ratio, 1-k is 0.609, from which k is 0.391. Check by (28),

$$\frac{0.951}{0.049} - \frac{0.609}{0.391} = 19.28 - 1.56 = 17.72 \quad \frac{B_m'}{A_m'} = \frac{40}{2.26} = 17.7$$

From (25), $\frac{f'}{C_D'} = 1.56 \frac{2.26}{4490} = 0.785 \times 10^{-3}$ Check by (24),

$$k_s = \frac{1}{1 + \frac{40 \times 10^{-6} + 0.785 \times 4.49 \times 10^{-6}}{2.26 \times 10^{-6}}} = 0.049$$

and

$$k = \frac{1}{1 + \frac{0.785 \times 4.49 \times 10^{-6}}{2.26 \times 10^{-6}}} = 0.392$$

For water at 30°C, $\upsilon = 0.86 \times 10^{-5}$ ft²/sec, and from Eq. (6) for 3 rpm

$$C_V = \frac{25.1}{0.86 \times 10^{-5}} \times 2.08 \times 10^{-2} \times 5.96 \times 10^{-2} \times \frac{3}{60} = 181 \text{ without stators}$$

For V of 0.0706 ft³, from Eq. (8a),

$$C_D' = \frac{C_V V^2}{(1-k)A_m'} = \frac{181 \times 0.706^2 \times 10^{-2}}{0.609 \times 2.26 \times 10^{-6}} = 6.57 \times 10^5$$

and $$f' = 0.785 \times 10^{-3} \times 6.57 \times 10^5 = 516$$

Principle of Least Work

The author believes that the Principle of Least Work is a natural law universally applicable. This principle holds that nature is lazy and follows the path of least resistance with minimum work in all of its actions. It has been used in analyses of indeterminate structures where it is assumed that under an applied load the stresses will be distributed so that the total work done in deformation is a minimum. Since this principle has been badly neglected in fluid dynamics, the author has studied it as another means for the evaluation of k.

If the partial derivative with respect to k of the work W, and therefore of C_t and C_V in Eqs. (7) and (8), is set to equal to zero, equations similar to (15) and (23) result. The dimensional parameters, however, are the work parameters instead of the moment parameters. The work parameters are the same as the moment parameters for the rotors and inside stators where the same value of r is used for the moment arm and tangential velocity, as shown on Fig. 2. The parameters differ, however, for outside stators and the walls, because the velocity is determined by r_r and the moment arms are r_{os} and r_W, respectively.

The value of k determined by the least work method does correspond with the minimum value of W, but the method is incorrect because the requirement that the input moment must equal the resisting moment is not met for the assumed distribution of the velocity. Nevertheless, solutions by least work are approximately correct. For example, for the 2-liter beaker, the error in C_D is +3.6% and in C_D' is +0.3%, and the errors in k_s are +6.0% for turbulent drag and +14% for viscous drag. It is evident that the Principle of Least Work does apply to this case, provided the drag moments are also equal.

Dimensional Effects

In order to study the effects on drag and wall friction coefficients of tank shape and paddle arrangements, extensive exploratory torque measurements were made with water in the units of a small pilot plant consisting of a rapid mix tank 4.4 in. sq. by 3 ft. deep, followed by 3 tanks each 1.0 ft. sq. by 3 ft. deep with vertical shafts. The rapid mix tank was designed to produce G-values of about 30 to 1000 with water up to 30°C and fully turbulent mixing. The first 1.0 ft. sq. tank was designed to produce, with fully turbulent mixing, G-values of about 30 to 750, and the second and third tanks, G-values of about 10 to 100. One of the larger tanks was filled to a depth of 1 ft. of water to study the effects of a cubic-shaped tank, and the paddles were designed to produce G-values of about 30 to 500 with full turbulence.

The results of the torque measurements are shown in Table 1. Column 1 shows the data for the 2-liter beaker, column 2 for the 4.4 in. sq. rapid mix tank, column 3 for the first 1 ft. sq. by 3 ft. deep tank, column 4 for the second and third 1 ft. sq. by 3 ft. deep tank, and column 5 for the 1 ft. sq. by 1 ft. deep tank.

The first measurements, made with rotor blades vertical, resulted in large vortices at high speeds. The vortices were eliminated by pitching the blades 45° with the vertical, and placing the top rotor assemblies to pump upward. Hoerner[6] has published the results of experiments which show that the turbulent drag coefficient is approximately constant with plates tilted against the direction of flow from 90° to 45°. It is therefore permissible to use the projected plate area for pitched blades, provided the pitch is not less than 45° against the direction of flow. Experiments conducted with the 4.4 in. sq. tank with rotor blades vertical and tilted 45°, but without stators, showed that the turbulent torque with the pitched blades was about 75.5% of the torque with blades at 90° with the direction of flow. Since the projected area was reduced to 70.7% of the area of the vertical blades, the difference indicates an increase of about 7% in the value of $(1\text{-}k)^2$ in Eq. (9).

The small tank was equipped with 4 rotors 3 inches in diameter at depths of 12, 19, 26 and 33 inches below the water surface. Each rotor has 6 radial blades attached to its hub at 60° intervals and pitched 45° with the vertical. Each blade is 1 in. wide by 1-1/16 in. long. The value of a is 0.125 sq. ft. The top and third rotors pump upwards, and the other two downwards. The stators consist of 12 blades, each 2-1/2 in. high by 1-1/2 in. wide, placed in radial vertical planes through the corners with the inside edges 0.9 in. from the center of the shaft. Three blades are in each corner spaced vertically between rotors.

The first large tank has 4 rotors 8 in. in diameter at depths of 3-3/8, 16-1/4, 19-3/4 and 32-5/8 in. below the water surface. Each rotor has 6 radial blades attached to its hub at 60° intervals and pitched 45° with the vertical. Each blade is 1 in. wide by 3-5/8 in. long. The rotor area, a, is

TABLE I. EFFECTS OF DIMENSIONS ON COEFFICIENTS, WATER AT 30° C.

CONTAINER	(1) 5" DIA. 6.4" DEEP	(2) 4.4" SQ. 3' DEEP	(3) 1.0' SQ. 3' DEEP	(4) 1.0' SQ. 3' DEEP	(5) 1.0' SQ. 1' DEEP
TURBULENT					
A_m ($ft^5 \times 10^4$)	0.1017	0.855	45.21	5.16	22.3
B_m ($ft^5 \times 10^4$)	1.067	6.102	324.4	223.3	147.0
C_m ($ft^5 \times 10^4$)	6.75	32.4	1810.0	1018.0	695.0
C_{t_s}/C_t	10.3	1.44	1.70	1.55	1.56
k_s	0.235	0.2397	0.2498	0.1252	0.2514
k	0.762	0.3689	0.4240	0.2976	0.3989
f	0.0123	0.1365	0.0804	0.0472	0.1072
C_D	8.4	1.76	1.735	1.632	1.469
STREAMLINE					
A_m' ($ft^6 \times 10^4$)	0.0226	1.12	76.5	3.18	18.56
B_m' ($ft^6 \times 10^4$)	0.400	8.75	516.2	693.2	86.8
C_m' ($ft^6 \times 10^4$)	44.90	1150.0	70500.0	52800.0	10420.0
C_{t_s}/C_t	1.56	1.13	1.34	1.44	1.38
k_s	0.049	0.0941	0.1025	0.0045	0.1393
k	0.391	0.3554	0.3329	0.3094	0.3996
f'	516.0	918.0	2170.0	511.0	1395.0
$C_D' \times 10^{-5}$	6.57	5.18	9.96	37.9	5.23

0.433 sq. ft. The third rotor from the top pumps down and the other three upwards. There are 8 stators, each 6 in. high by 3 in. wide, placed in radial vertical planes through the corners with the inside edges 2.5 in. from the center of the shaft. Two blades are in each corner spaced vertically between the two top and two bottom rotors, respectively.

The second large tank (and the third) has 3 rotors 6 in. in diameter at depths of 3-3/8, 18 and 32-5/8 in. below the water surface. Each rotor has 3 radial blades attached to its hub at 120° intervals and pitched 45° with the vertical. Each blade is 1 in. wide by 2-5/8 in. long. The rotor area, a, is 0.116 sq. ft. The middle rotor pumps down and the other two upwards. The stators are identical with those in the first large tank.

The cubic tank has 2 rotors 8 in. in diameter at depths of 4 and 8 in. below the water surface. Each rotor has 6 radial blades attached to its hub at 60° intervals and pitched 45° with the vertical. Each blade is 1 in. wide by 3-5/8 in. long. The projected rotor area, a, is 0.2135 sq. ft. The top rotor pumps upwards and the bottom rotor downwards. There are 12 stators, each 1-1/2 in. high by 3 in. wide, placed in radial vertical planes through the corners with the inside edges 3 in. from the center of the shaft. Three blades are in each corner, 1.9, 6 and 10.1 in. below the water surface.

Table 1 shows very significant differences between cylindrical and square tanks in the region of turbulent drag. The drag coefficient C_D is much greater and the wall friction factor f much less for the cylindrical tank than for the square tanks, which results in a much higher value for $C_\uparrow / C_{\uparrow s}$ for the cylindrical tank. Figs. 4 to 7 show that turbulent drag can be obtained over a much wider speed range in square tanks both with and without stators than is the case with cylindrical tanks.

Design of Tanks

The size and dimensions of a mixing tank are usually determined by the volume desired or retention period for continuous flow units. The design procedure thereafter is best illustrated by an example: a rapid-mix tank 10 ft. by 12 ft. in plan with a 13 ft. water depth constructed in an expansion of the Billerica (Massachusetts) water treatment plant. It is proposed to dissipate 10 hp, or 3.53 ft. lb. per sec per cf for a volume of 1560 cf. The maximum value of G for this value of W will range from 311 sec^{-1} at 2°C. to 471 at 30°C.

Two six-blade rotors, 6 ft. in diameter and 5 ft. on centers, are mounted on a vertical shaft; the top rotor to pump upward, and the bottom downward. Each blade is 8 in. wide by 2.62 ft. long and pitched 45° with the vertical. The projected area of the rotor blades, a, is 14.8 sq. ft. There

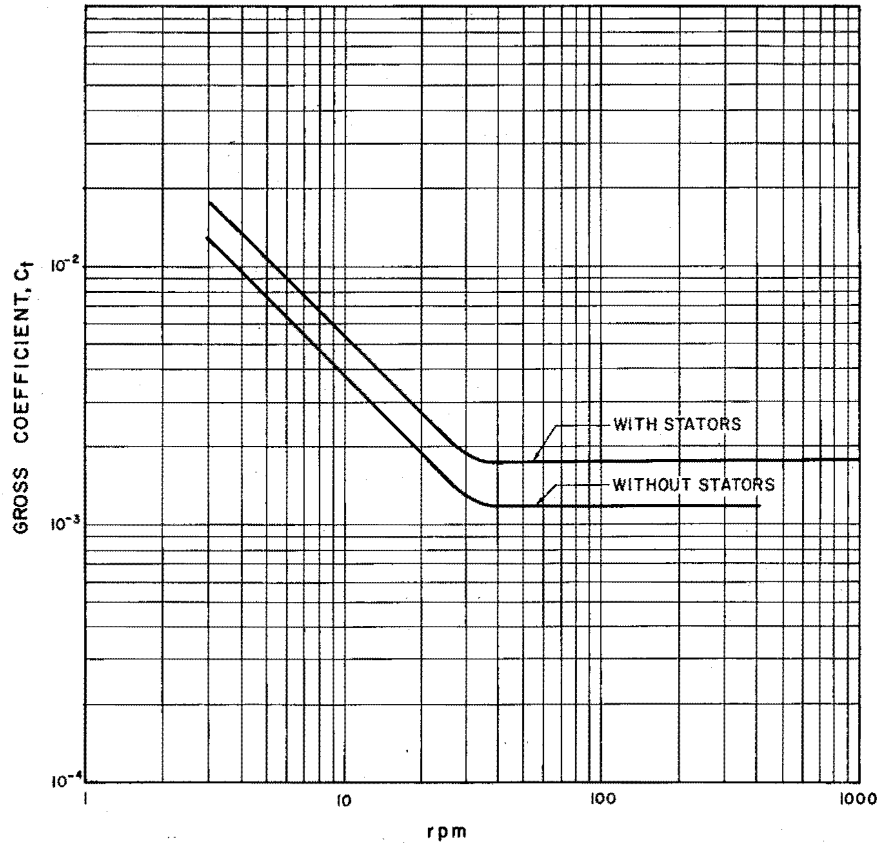

Fig. 4 — Gross drag coefficients for 4.4 in. sq. Rapid Mix Tank with water at 30°C.

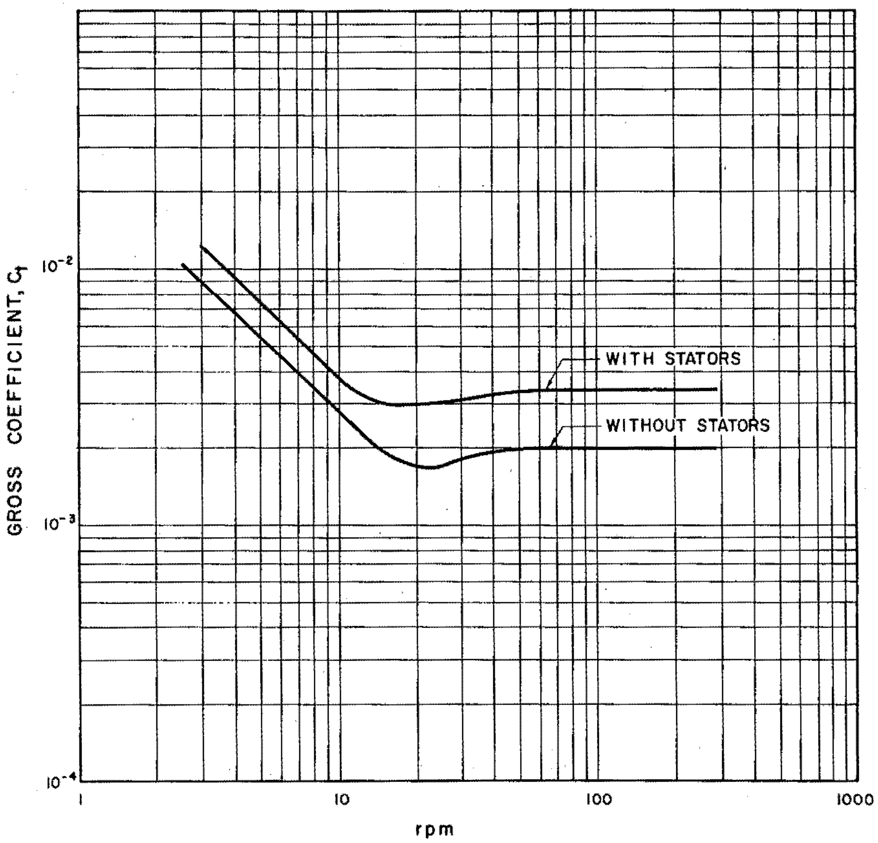

Fig. 5 — Gross drag coefficients for first 1 ft. sq. tank 3 ft. deep with water at 30°C.

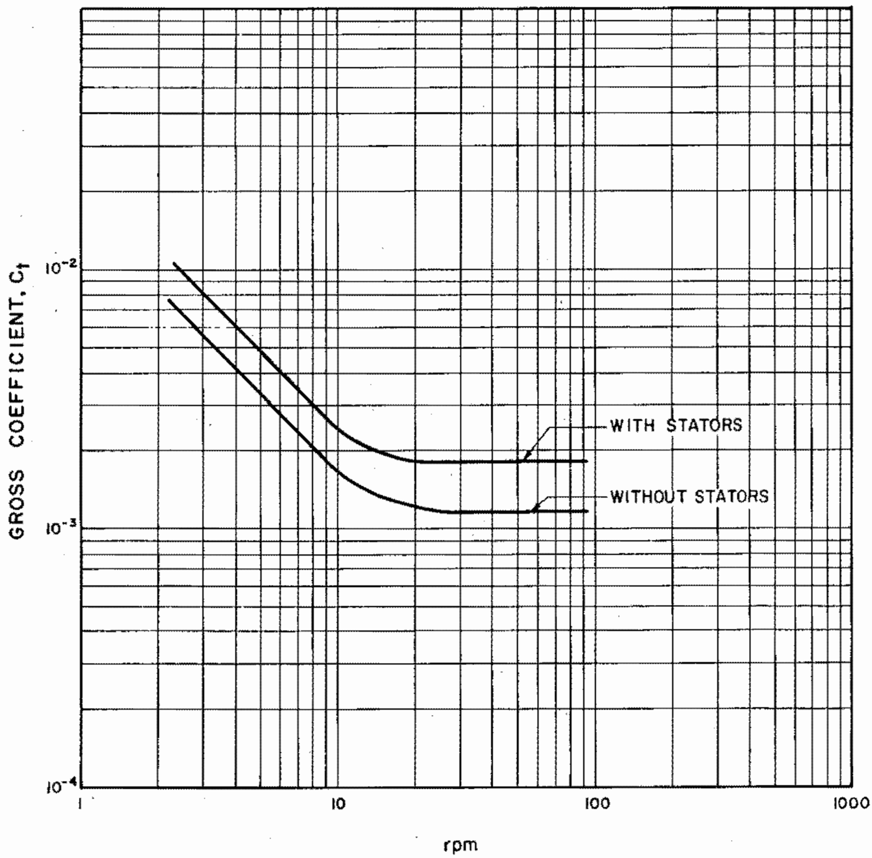

Fig. 6 — Gross drag coefficients for second 8 third 1 ft. sq. tank, 3 ft. deep, watei at 30°C.

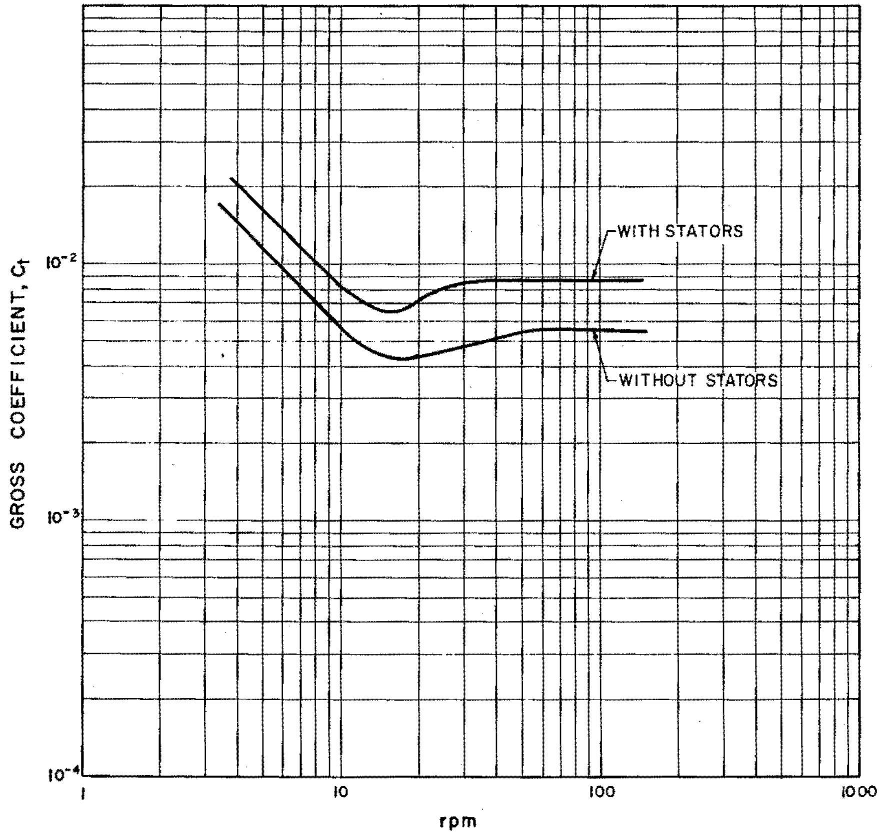

Fig. 7 — Gross drag coefficients for 1 ft. sq. tank 1 ft. deep with water at 30°C.

are 12 stators, placed in 4 groups of 3 each in the radial vertical planes through the corners with the inside edges 2.0 ft. from the center of the shaft. The stators are 3.0 ft. wide with the top and bottom stators 1.5 ft. high and the middle stators 2.0 ft. high. The dimensional moment parameters for turbulent drag, A_m, B_m and C_m are 114.2, 1752 and 7790 ft.[5], respectively; and for viscous drag, A_m', B_m' and C_m' are 0.0752×10^4, 2.17×10^4 and 179.2×10^4 ft.[6], respectively.

For trial computations of the values of C_t for water at 30°C. on the turbulent and viscous tangents, the values of the drag and wall friction coefficients for the small cubical tank, column 5 of Table 1, will be used. For turbulent drag, f is assumed at 0.1072 and C_D at 1.469. For viscous drag, f' is assumed at 0.01395×10^5 and C_D' at 5.23×10^5. For turbulent drag, the value of k_s, by Eq. (16), is 0.182 and the value of k is 0.31. From (7a), C_{t_s} is 4.85×10^{-3} and C_t is 3.47×10^{-3}. For viscous drag, by Eq. (24), k_s is 0.0276 and k is 0.1365. From (8a), C_{V_S} is 157.3 and C_V is 139.7. For 0.1 rpm, S is 1.667×10^{-3}; and from Eq. (6), C_{t_s} is 2.20×10^{-3} and C_t is 1.95×10^{-3}. The estimated positions of the tangents to the curves for C_t are shown by broken lines on Fig. 8.

For the proposed maximum value of W at 3.53 ft. lbs. per sec per ft.[3], the estimated speeds from Eq. (4) are 0.590 rps or 35.4 rpm with the stators, and 0.66 rps or 39.6 rpm without stators.

Acknowledgements

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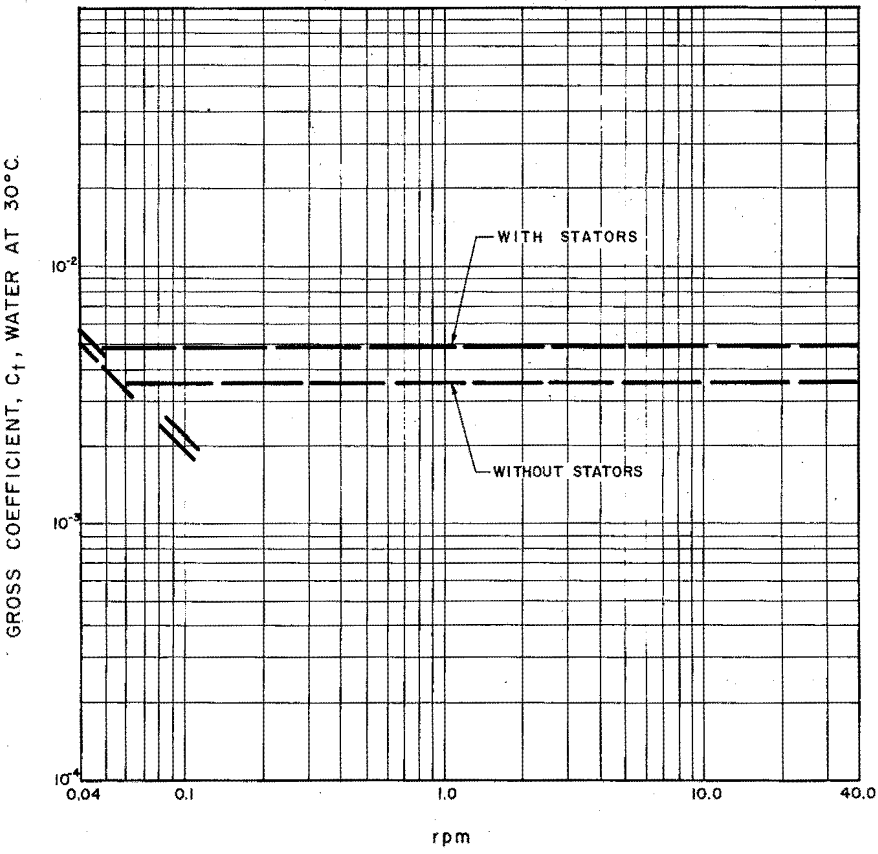

Fig. 8 — Estimated position of tangents to curves for gross drag coefficients for Billerica, (Mass.) rapid mix.

APPENDIX
Dimensional Parameters

The dimensional parameters for the rotors and stators are functions of the projected blade areas perpendicular to the direction of tangential flow and of the radii which determine the velocity and drag moments. As indicated on Fig. 2, the velocity is assumed to increase from zero at the center of the rotor shaft to a maximum at the tip of the radial rotor blade (or to the center of the outermost parallel rotor blade or outer edge for wide blades), and to remain constant from this point almost to the wall. The drag on the walls and floor is assumed to be the same per unit of wetted area and to be controlled by the maximum velocity at r_r ; and the drag moment arm is assumed to be r_w, the radius of circular tanks and the shortest radial distance to a wall for rectangular tanks. For simplicity, the total wall area is used for rectangular tanks with no allowance for corners.

Both radial rotors and wide blade parallel rotors and inside stators require integration of the differential moment and work on differential area, bdr, between r_1 and r_2. The moment arm to an outside stator is to its center, regardless of its width. Some stators may be partially inside and subjected to a velocity increasing with radial distance, and partially outside with constant velocity.

Wall Parameters

The drag per unit of wall area $= \rho \dfrac{fv^2}{8}$ (31)

For turbulent drag, the moment is (32)

$$M_w = \rho \frac{f}{4} A_w \frac{(2\pi)^2}{2} r_r^2 k^2 S^2 r_w = \frac{(2\pi)^2}{2} \rho C_D k^2 \frac{f}{C_D} (A_w \frac{r_r^2}{4} r_w) S^2$$

and the dimensional moment parameter is

$$C_m = A_w \frac{r_r^2}{4} r_w \qquad (33)$$

Since the work is $M_w \dfrac{v}{r_w}$ or $M_w 2\pi \dfrac{r_r}{r_w} kS$,

$$W_w = \frac{(2\pi)^3}{2} \rho a \frac{C_D}{Va} k^3 \frac{f}{C_D} (A_w \frac{r_r^3}{4}) S^3 \qquad (34)$$

and the dimensional work parameter is

$$C = A_w \frac{r_r^3}{4} \qquad (35)$$

For viscous drag,

$$f = \frac{f'}{R} = \frac{f'\nu}{v4R} = \frac{f'\nu A_w}{v4V} \qquad (36)$$

Substituting this value of f in (31), the viscous drag per unit of wall area

$$= \frac{f' \nu A_W}{v 4V} \frac{\rho v^2}{8} = \frac{f'}{32} \mu \frac{A_W}{V} v \tag{37}$$

and viscous drag moment is

$$M_W' = \frac{f'}{32} \mu \frac{A_W^{\ 2}}{V} v r_W = \frac{f'}{32} \mu \frac{A_W^{\ 2}}{V} (2\pi r_r k S) r_W$$

$$= \frac{2\pi}{8} \mu \frac{C_D'}{V} k \frac{f'}{C_D'} (A_W^{\ 2} \frac{r_r}{4} r_W) S \tag{38}$$

and the dimensional moment parameter is

$$C_m' = A_W^{\ 2} \frac{r_r}{4} r_W \tag{39}$$

Since the work is $M_W' \frac{v}{r_W}$ or $M_W' 2\pi \frac{r_r}{r_W} kS$,

$$W_W = \frac{(2\pi)^2}{8} \mu \frac{C_D'}{V^2} k^2 \frac{f'}{C_D'} (A_W^{\ 2} \frac{r_r^{\ 2}}{4}) S^2 \tag{40}$$

and the dimensional work parameter is

$$C' = A_W^{\ 2} \frac{r_r^{\ 2}}{4} \tag{41}$$

Rotor Parameters

The turbulent drag per unit of projected blade area $= C_D \rho \frac{v^2}{2}$ (42)

The turbulent moment for narrow parallel rotors is

$$M_{pr} = C_D A_p \rho \frac{v^2}{2} r_p = \frac{(2\pi)^2}{2} \rho C_D (1-k)^2 (A_p r_p^{\ 3}) S^2 \tag{43}$$

and the dimensional moment parameter is

$$A_m \text{ per pr} = A_p r_p^{\ 3} \tag{44}$$

Since the work is $M_{pr} \frac{v}{r_p}$ or $M_{pr} 2\pi \frac{r_p}{r_p} (1-k) S$,

$$W_{pr} \text{ per pr} = \frac{(2\pi)^3}{2} \rho a \frac{C_D}{Va} (1-k)^3 (A_p r_p^{\ 3}) S^3 \tag{45}$$

and the dimensional work parameter is

$$A \text{ per pr} = A_p r_p^{\ 3} \tag{46}$$

which is the same as the moment parameter.

The turbulent moment for wide rotors, including radial rotors, is

$$M_{wr} = C_D b \int dr \, \rho \frac{v^2}{2} r = \frac{(2\pi)^2}{2} \rho C_D (1-k)^2 (b \int r^3 dr) S^3 \tag{47}$$

and the dimensional moment parameters are

$$A_m \text{ per rr} = b\int_0^{r_r} r^3\,dr = A_r\frac{r_r^{\,3}}{4} \tag{48}$$

and

$$A_m \text{ per wr} = b\int_{r_1}^{r_2} r^3\,dr = A_{wr}\,\frac{r_2^{\,3} + r_2^{\,2}r_1 + r_2r_1^{\,2} + r_1^{\,3}}{4} \tag{49}$$

Since the work is

$$M_{wr}\frac{v}{r} \text{ or } M_{wr}\,2\pi(1-k)\,S,$$

$$W_{wr} \text{ per wr} = \frac{(2\pi)^3}{2}\,\rho a\,\frac{C_D}{Va}\,(1-k)^3\,(b\int r^3\,dr)\,S^3 \tag{50}$$

and the dimensional work parameters are the same as the moment parameters as given by (48) and (49).

The viscous drag per unit of projected blade area =

$$\frac{C_D'}{R}\,\rho\frac{v^2}{2} = \frac{C_D'\,\nu}{v4R}\,\rho\,\frac{v^2}{2} = \frac{C_D'}{8}\,\mu\,\frac{A}{V}\,v \tag{51}$$

The viscous drag moment for parallel rotors is

$$M_{pr}' = \frac{C_D'}{8}\,\mu\,\frac{A_p^{\,2}}{V}\,v\cdot r_p = \frac{2\pi}{8}\,\mu\,\frac{C_D'}{V}\,(1-k)\,(A_p^{\,2}r_p^{\,2})\,S \tag{52}$$

and the dimensional moment parameter is

$$A_m' \text{ per pr} = A_p^{\,2}r_p^{\,2} \tag{53}$$

Since the work is $M_{pr}'\frac{v}{r_p}$ or $M_{pr}'\;2\pi\,(1-k)\,S,$

$$W_{pr}' \text{ per pr} = \frac{(2\pi)^2}{8}\,\mu\,\frac{C_D'}{V}\,(1-k)^2\,(A_p^{\,2}r_p^{\,2})\,S \tag{54}$$

and the dimensional work parameter is $A' \text{ per pr} = A_p^{\,2}r_p^{\,2}$ (55)

which is the same as the moment parameter in (53).

The viscous drag moment for wide rotors, including radial rotors is

$$M_{wr}' = \frac{C_D'}{8}\,\mu\frac{A}{V}\,b\int d\,r\,v\,r = \frac{2\pi}{8}\,\mu\,\frac{C_D'}{V}\,(1-k)\,(Ab\int r^2\,dr)\,S \tag{56}$$

and the dimensional moment parameters are

$$A_m' \text{ per rr} = A_r b\int_0^{r_r} r^2\,dr = A_r^{\,2}\,\frac{r_r^{\,2}}{3} \tag{57}$$

and

$$A_m' \text{ per wr} = A_{wr}b\int_{r_1}^{r_2} r^2\,dr = A_{wr}^{\,2}\,\frac{r_2^{\,2} + r_2r_1 + r_1^{\,2}}{3} \tag{58}$$

The dimensional work parameters A′ per rr and per wr have the same values as the moment parameters in (57) and (58).

The values of the total rotor parameters for a mixing tank are summations of the individual rotor parameters, as follows.

The turbulent moment parameter or work parameter is

$$A_m = A = \sum A_p r_p^3 + \sum A_r \frac{r_r^3}{4} + \sum A_{wr} \frac{r_2^3 + r_2^2 r_1 + r_2 r_1^2 + r_1^3}{4} \quad (59)$$

and the viscous moment and work parameters are

$$A_m' = A = \sum A_p^2 r_p^2 + \sum A_r^2 \frac{r_r^2}{3} + \sum A_{wr}^2 \frac{r_2^2 + r_2 r_1 + r_1^2}{3} \quad (60)$$

In computing values of the parameters, the terms in (59) and (60) which are not applicable are omitted.

The parallel rotor term may be used for wide rotors if r_2 is not greater than $1.5r_1$. For turbulent drag, the value of r_p^3 is 3.7% less than

$$\frac{r_2^3 + r_2^2 r_1 + r_2 r_1^2 + r_1^3}{4}$$

for $\frac{r_2}{r_1}$ of 1.5 and 10% less for $\frac{r_2}{r_1}$ of 2.0. For viscous drag, the value of r_p^2 is 1.2% less than $\frac{r_2^2 + r_2 r_1 + r_1^2}{3}$ for $\frac{r_2}{r_1}$ of 1.5 and 3.4% less for $\frac{r_2}{r_1}$ of 2.0.

Stator Parameters

Using the turbulent drag of (42), the turbulent moment for inside narrow stators and outside stators of width less than $r_w - r_r$ is

$$M_s = C_D A_s \rho \frac{v^2}{2} (r_{is} \text{ or } r_{os}) = \frac{(2\pi)^2}{2} \rho C_D k^2 (A_s r_{is}^3 \text{ or } A_s r_r^2 r_{os}) S^2 \quad (61)$$

and the dimensional moment parameter and work parameter for the inside narrow stators is

$$B_m \text{ per is} = B \text{ per is} = A_s r_{is}^3 \quad (62)$$

The dimensional moment parameter for the outside stator is

$$B_m \text{ per os} = A_s r_r^2 r_{os} \quad (63)$$

Since the work is $M_s \frac{v}{r_{os}}$ or $M_s \, 2\pi \frac{r_r}{r_{os}} k S$, the dimensional turbulent work parameter for the outside stator is

$$B \text{ per os} = A_s r_r^3 \quad (64)$$

For wide inside stators,

$$B_m \text{ per is} = B \text{ per is} = A_s \frac{r_2^3 + r_2^2 r_1 + r_2 r_1^2 + r_1^3}{4} \quad (65)$$

Using the viscous drag of (51), the viscous drag moment for inside narrow stators and for outside stators is

$$M_s' = \frac{2\pi}{8} \mu \frac{C_D'}{V} k (A_s^2 r_{is}^2 \text{ or } A_s^2 r_r r_{os}) S \quad (66)$$

and the dimensional moment parameter and work parameter for the inside narrow stators is

$$B_m' \text{ per is} = B \text{ per is} = A_s^2 r_{is}^2 \quad (67)$$

The dimensional moment parameter for the outside stator is

$$B_m' \text{ per os} = A_s^2 r_r r_{os} \quad (68)$$

and the work parameter is

$$B' \text{ per os} = A_s^2 r_r^2 \quad (69)$$

For wide inside stators,

$$B_m' \text{ per is} = B \text{ per is} = A_s^2 \frac{r_2^2 + r_2 r_1 + r_1^2}{3} \quad (70)$$

The values of the total stator parameters for a mixing tank are summations of the individual stator parameters, as follows.

The turbulent moment parameter is (71)

$$B_m = \sum A_s r_{is}^3 + \sum A_s r_r^2 r_{os} + \sum A_s \frac{r_2^3 + r_2^2 r_1 + r_2 r_1^2 + r_1^3}{4}$$

and the turbulent work parameter is (72)

$$B = \sum A_s r_{is}^3 + \sum A_s r_r^2 r + \sum A_s \frac{r_2^3 + r_2^2 r_1 + r_2 r_1^2 + r_1^3}{4}$$

The viscous moment parameter is (73)

$$B_m' = \sum A_s^2 r_{is}^2 + \sum A_s^2 r_r r_{os} + \sum A_s^2 \frac{r_2^2 + r_2 r_1 + r_1^2}{3}$$

and the vicous work parameter is (74)

$$B' = \sum A_s^2 r_{is}^2 + \sum A_s^2 r_r^2 + \sum A_s^2 \frac{r_2^2 + r_2 r_1 + r_1^2}{3}$$

In computing values of the parameters, the terms which are not applicable are omitted.

The parallel stator term may be used for wide inside stators if r_2 is not greater than $1.5r_1$. The error in using the simpler term is the same as shown above for rotors.