ANALYSIS OF SHEAR WALL-FRAME SYSTEMS

by STEPHEN CHRISS*

ABSTRACT: Methods of analysis of interconnected shear and shear wall-frame system subjected to horizontal static loading, are reviewed and summarized. All of the reviewed methods consider structures whose shear walls and frames form rectangular grid plans, are subjected to horizontal loading parallel to the grid, and undergo uniaxial displacements, also parallel to the grid.

A method based on the theorem of minimum total potential, is proposed, resulting in a system of simultaneous linear equations containing displacements as unknowns. The method employing matrix formulation, can be applied in the "general case" to structures undergoing biaxial and rotational displacements (u, v, ϕ). Only horizontal components of external static loads are considered, but need not be parallel. Shear walls and frames need not occupy a rectangular grid plan.

KEY WORDS: analysis; biaxial displacements; frames; interaction; minimum total potential; review; rotational displacements; shear walls.

INTRODUCTION

In present building terminology, the term "shear wall" signifies a structural system in the form of walls or cores capable of withstanding lateral forces. Walls may be flat or curved, while cores may be of the open or closed box type. Cores are becoming increasingly important in contemporary multistory technology because of their twofold function, namely: stiffening of the building against lateral loading, and providing vertical passages for services, stairwells and elevators. When shear walls act in conjunction with beam-column frames, the resulting building behavior under lateral wind or seismic loading constitutes generally an improvement over the behavior resulting from either frames or shear walls acting alone. Therefore, incorporation of both structural components into the system appears advantageous. However, the difference in deflection characteristics between shear walls and frames generates an interaction affecting the system response to direct as well as torsional loading. The latter loading can be generated due to nonsymmetrical loading or nonsymmetrical building configuration. In fact shear wall arrangements resulting in building torsion have been incorporated into several actual buildings,[1,2,3] and further hypothetical but possible situations have also been shown.[1]

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As the effects of torsion on buildings subjected to dynamic loading become apparent,[4,5] and building codes recognize eccentricities, the ability to predict such torsional effects is becoming increasingly important. Accordingly, an investigation into the torsional analysis seems appropriate; and it is mainly due to this consideration that this paper was undertaken.

The present paper is divided into three main parts. In the first, the work of various investigators is reviewed, and the analysis steps are summarized for the purpose of providing the fundamental information needed from a design engineer's point of view. Only a small number of papers sufficiently representing the analysis spectrum of systems subjected to lateral loading is included in this review. Since most of the published literature deals with static loading, only static loading papers were considered. A considerably more extensive bibliography of the papers published up to 1966 is given elsewhere.[6] All of the papers reviewed herein treat cases in which structural components form rectangular grid plans, and loading and displacements are parallel to the grid.

In the second part of the paper, an energy method is developed applicable to building systems that are non-rectangular in plan or irregular in elevation; are subjected to horizontal static but otherwise irregular loading; and are free to undergo biaxial and rotational (torsional) displacements. A method is also developed for analysis for uniaxial displacements.

In the third part, a numerical example is presented employing a Y shaped (in plan) low building of average complexity, acted upon by earthquake loading.

PART I
REVIEW OF METHODS OF ANALYSIS

1. Review Objectives*

Reviews are carried out from the point of view of providing basic information for design purposes. Thus, the implied assumptions are revealed in addition to the stated ones, the analysis steps are summarized, and a broad view of the applicability range of the particular method is given whenever possible. Comments pertaining to a particular analysis method are stated with the method, but comments common to two or more methods are presented at the end of this review part.

*Doubly spaced headings correspond to italics headings of the Journal of the Structural Division, ASCE.

2. General Considerations

A. *Nature of Systems Analyzed*

Analyses have been formulated for systems consisting of (a) perforated shear walls constituting in fact wide column-wide beam bents, (b) two or more shear walls, interconnected through floor slabs or shallow beams, (c) shear walls interconnected with frames.

The various methods of analysis may be classified either according to the type of mathematical treatment, or according to the sssigned physical system behavior. Classification according to the type of mathematical treatment generates four principal categories as follows: (a) portal method, (b) differential equation methods, (c) iteration methods, and (d) simultaneous linear equations and digital computer methods. Classification according to the assigned physical system behavior produces the groups of (a) continuous system methods, and (b) discrete system methods, depending on whether the stress resultants of the system are treated as continuous or point variables. The first classification system, i.e. that pertaining to the type of mathematical treatment, is used in the subsequent sections of this review part.

B. *Assumptions Common to All Methods*

The methods to be described are all based on a number of common assumptions as follows:

1. The system to be analyzed is linearly elastic or can be reduced into constituent linearly elastic systems.
2. The principle of superposition holds.
3. The system consists of frame and shear wall configurations lying totally on vertical planes and forming in plan a rectangular grid pattern.
4. Frames and shear walls are interconnected at floor levels by means of rigid floor diaphragms, except for the treatment by Goldberg[23] in which deformable floor media are considered.
5. The external static loads acting on the system are horizontal, parallel to the grid, and they are applied at floor levels.
6. The system undergoes lateral displacements in the direction of the externally applied loads only.

The additional particular assumptions, stated or implied that are applicable to each method, are presented with the method description in the following sections of this part.

3. Portal Method

Green[7] developed an analysis method applicable to shear wall systems consisting of wide column-wide beam multiple-bay bents. The method, constituting one of the earliest analysis attempts, is a modification of the classical portal method taking into account the effects of finite dimensions of column-beam joints.

The additional particular assumptions used in the analysis are:

a. Axial deformations of beams and columns are negligible.
b. The column-beam joints are rigid rectangles of finite dimensions.
c. Shears delivered from the beams and columns into joints are acting at the center of the joints.
d. Points of inflection occur in the middle of the clear height and the clear span in columns and beams respectively.

The analysis includes the effect of shear deformations of the columns and of course the effect of flexural deformations of columns and beams. The method considers a free body separated from the bent by two slicing planes, positioned at column midheights above and below the floor considered; and the analysis is carried out on a trial basis. The method as presented by Green[7] is limited to systems whose physical dimensions conform to the mechanics of analysis. Furthermore, in the analysis, the horizontal external load acting on the floor of the free body under consideration is neglected. Inclusion of this load would result in shearing stress resultants different for the top and bottom halves of the columns, yielding in turn, different opening spacing requirements from floor to floor. In addition, the assumption of negligible column axial deformations should be treated with caution as explained in the discussion later in this part. The applicability of this method is, therefore, severely limited.

4. Differential Equation Methods

A. *Interconnected Shear Walls*

Interconnected shear wall systems have been analyzed by Beck,[8] by Frischmann, Prabhu and Toppler,[9] and by Rosman.[10, 11, 12, 13]

The systems considered by the first two investigators[8, 9] consisted of two identical shear walls acting as parallel vertical cantilevers interconnected at floor levels by means of horizontal beams. The additional particular assumptions used by the investigators[8, 9] are:

a. The externally applied horizontal loading is uniformly distributed throughout the height of the building.
b. Interconnecting horizontal beams are fixed at each end to the shear wall.
c. Shear walls and tying beams possess constant elastic characteristics, and the story height is constant.

d. There are sufficient stories permitting localized effects to be neglected.
e. Shear deformations of the shear walls are negligible.
f. The horizontal tying beams possess negligible flexural stiffness as compared to the flexural story stiffness of shear walls; this assumption permits absence of inflection points in shear walls.
g. The midspan of the tying beams coincides with their inflection point.
h. Shear walls are fixed to a rigid foundation.

Beck[8] considered a free body obtained by bisecting the two shear wall system by means of a vertical plane passing through the midspan of the horizontal tying beams. The free body is considered under the influence of (a) half the external horizontal loading which tends to deflect it, and (b) vertical shearing forces acting on a hypothetical medium consisting of equivalent laminae replacing the finite tying beams, which tend to restore it.

The analysis takes into account the effects of (a) axial deformations of the shear walls, (b) shear deformations of the interconnection beams, and (c) flexural deformations of all components. As a starting point in the analysis, Beck[8] used compatibility of deflections of the hypothetical laminae in the free body, and thus obtained a second order differential equation relating vertical shears of the laminae to the moment of the shear wall due to external loading. Solution of the differential equation yields values for the vertical shears in the interconnected beams, from which stress resultants in the shear walls and system deflections may be determined.

Frischmann *et al.*[9] lumped together the two shear walls into one vertical member whose rigidity is equal to the sum of the rigidities of the component members. This vertical member, termed the "equivalent column", is considered under the influence of the deflecting total externally applied load, and restoring bending moments induced due to distortions of the interconnecting beams. The analysis takes into account only the effect of flexural deformations of vertical and horizontal members.

Frischmann *et al.*[9] through consideration of a moment-slope relationship arrived at a second order differential equation relating the total moment of the equivalent column to the externally applied load. Solution of the differential equation yields values for the total bending moments of the equivalent column, from which system deflections and bending moments in the interconnecting beams may be calculated. The approach used by the authors[9] allows extension of the method to cases of three or more interconnected shear walls of different rigidities.

Rosman[10, 11, 12, 13] developed solutions for two nonsymmetrical wall, and three symmetrical wall configurations, subjected to uniformly distributed, top concentrated, and trapezoidal loads. He also considered a variety of shear wall support conditions. The additional particular assumptions are essentially similar to those used by Beck.[8] Through considerations similar to those used by Beck, but of more general character, Rosman arrived at a second order differential equation relating the integral of vertical shears in the hypothetical laminae to the physical characteristics of the system and the external loading. It is worth noting that the above-mentioned governing differential equation can be established independently of support conditions, whose characteristics are only affecting the integration constants.

Rosman[14] also discussed the possibility of solutions to systems consisting of four or more interconnected shear walls, by consideration of equality of vertical shears in the hypothetical laminae (plastic hinge formation) and application of the Ritz method.

B. *Shear Walls Interconnected with Frames*

Shear wall-frame systems have been analyzed by Rosenblueth and Holtz,[15] by Cardan,[16] and by Rosman.[17] All of the above authors are basing their analyses on the following additional particular assumptions:

a. Shear walls and frames possess constant elastic characteristics and the story height is constant.
b. There are sufficient stories to permit localized effects to be neglected.
c. Shear walls and frames terminate at the same floor level.
d. Behavior of the frame under lateral loading is analogous to the behavior of a discrete spring-type system, the particulars of which are discussed in the section "Discussion of Assumptions", further on in the paper.

In the analysis by Rosenblueth and Holtz[15] and by Cardan,[16] the shear walls are considered as cantilevers subjected to (a) the deflecting total external horizontal loading, and (b) restoring bending moments and horizontal forces induced due to deformation of the interconnected girders and frames. The analysis takes into account the effect of shear and flexural deformations of shear walls. The authors,[15, 16] by consideration of flexural and shearing deformation relationships, arrived at a second order differential equation relating the total slope of the shear wall to the known external lateral loading. From the solution of the differential equation, slopes and subsequently stress resultants may be determined. Cardan,[16] in addition to external uniformly distributed loading, considered the possibilities of triangularly distributed load, top point load, and foundation rotation.

Rosman's[17] analysis takes into account the effects of flexural deformations of shear walls and of rotation of their foundation. The frames and shear walls are considered interconnected by means of a hypothetical medium consisting of equivalent laminae replacing the finite hinging inextensible links. An expression of the total strain energy of the system is formulated and, through the application of calculus of variation, a differential equation is established relating the bending moment in the shear wall to system characteristics. Uniformly and triangularly distributed lateral loadings are considered.

5. Iteration Methods

Iteration analyses have been applied to shear wall-frame systems by Rosenblueth and Holtz,[15] by Khan and Sbarounis,[18] and by Khan.[19]

In the analysis of Rosenblueth and Holtz[15] the shear walls are considered as cantilevers subjected to external deflecting and internal restoring loading. The authors'[15] analysis is based on the following additional particular assumptions:

a. Shear walls and frames possess constant elastic characteristics.
b. The behavior of the frame under lateral loading is analogous to the behavior of a discrete spring type system.
c. The frame columns exhibit negligible axial deformation.

The analysis of the authors[15] takes into account the effects of (a) shear and flexural deformations of the shear walls, and (b) variations of story height. The iterative process consists of the following operations:

1. Initially, a trial slope distribution is assumed.
2. On the basis of the above slope distribution, the shear taken by the frame is calculated and subtracted from the known external lateral shear to give the shear taken by the shear wall.
3. The wall shear is integrated and the resultant moment is combined with the bending moments induced due to deformation of the interconnecting girders, thus yielding the resultant bending moment on the shear wall.
4. The resultant bending moment is divided by the flexural rigidity E1 of the shear wall, and integrated to give the calculated slope distribution which is compared to the slope distribution assumed at the beginning of the cycle.
5. Subsequent iteration cycles are then performed until the assumed and calculated slope distribution are within acceptable bounds.

From the finally established slope distribution, deflections can be evaluated by intergration, while stress resultants of shear walls and frames will be those pertaining to the last cycle of iteration.

F. Khan and J. Sbarounis[18] followed a different and more generalized approach. The analysis of the authors[18] is based only on the common assumptions cited at the beginning of this part. The analysis takes into account the effects of (a) shear deformations of shear walls and axial deformations of the frame column, (b) flexural deformations of all components, (c) plastic hinge formation in the shear walls, (d) foundation rotation, and (e) variation of story height and system characteristics from story to story.

The iterative process is carried out as follows:

1. Initially the known external horizontal loading is applied to the shear wall and its deflected shape computed.
2. The forces required to hold the frame in a deflected configuration conforming to that of the shear wall are calculated and applied to the shear wall.
3. The horizontal deflections of the shear wall due to the above set of forces are then determined and combined with those determined in step 1, yielding resultant deflections.
4. The above operations constitute the first cycle of iteration.

Initial deflection values to be used in subsequent iteration cycles are given by forced convergence formulas.

According to the authors[18] the iterative process requires three to twelve cycles depending on the frame to wall stiffness ratio. Deflections and stress resultants of the system are those pertaining to the last cycle.

The authors[18] presented a large number of graphs from which a quick evaluation of shear wall and frame shears and deflections can be made for preliminary designs. The method appears to be of sufficiently broad character to cover a variety of actual building configurations.

Khan[19] carried out an analysis pertaining to the interaction of shear walls rigidly interconnected to perpendicular frames, with the system being subjected to lateral loading acting on the plane of the shear wall. An iteration is performed on principles similar to those used by Khan and Sbarounis[18] in order to determine the effective column area to be considered as the flange of an I beam whose web is simulated by the shear wall. The method takes into account the effect of axial deformations of the columns, and flexural and shear deformations of the spandrel beams. Naturally the flange contribution is significant for relatively stiff spandrel beams, and in those cases its effect should definitely be included in the analysis.

6. Simultaneous Linear Equations and Digital Computer Methods

Analysis methods in this category have been presented by Gould,[20] Clough, King and Wilson,[21] and Goldberg.[23]

Gould [20]considered the shear walls as cantilevers subjected to the forcing loading of the external forces, and to the restoring loading of the interconnected frames. The author's [20]analysis is based on the following additional particular assumptions:

a. The shear deformations of the shear walls and axial deformations of the frame columns are negligible.
b. The behavior of the frame under lateral loading is analogous to that of a discrete spring type system.
c. There are sufficient stories to permit application of the finite difference method.

The analysis takes into account the effect of (a) flexural deformations of the system, and (b) variation of story height and system characteristics from story to story. In the analysis, either a fourth order differential equation, or a second order differential equation relating deflections to loading or moment, are employed, depending on their adaptability to the particular structural system.

The differential equations are modified to contain only horizontal deflections as unknowns. One differential equation can be written for every floor level resulting in a system of linear equations. A computer solution of the linear system appears to be advantageous, and from the determined deflections stress resultants of the system may be computed.

Clough et al[21] presented a digital computer analysis applicable to large multistory buildings containing shear walls and frames, interconnected in planar arrays, and subjected simultaneously to lateral and gravity loads. The authors'[21] method constitutes an extension of a previous presentation[22] in which no shear walls were included. The analysis is only subject to the common assumptions cited at the beginning of this part, and it takes into account the effects of (a) axial and shear deformations of sheer walls and frame columns, (b) shear deformations of the frame beams, and (c) flexural deformations of all components.

The analysis consists of the following operations:

1. The building is sliced vertically down into parallel planar arrays and the stiffness of each array to lateral loading is determined.
2. The arrays are reassembled into the original building form, giving by superposition the total lateral stiffness of the building.

3. From the known lateral loads and building lateral stiffness, the lateral displacements and stress resultants in the members are determined.

The lateral stiffness of the array outlined in step 1 above is determined as follows:

1. A loading-displacement relationship is established for the array resulting in a matrix equation relating gravity and lateral forces to rotational, vertical, and translational displacements.
2. The rotational and translational displacements are eliminated from the array loading-displacement relationship, yielding the lateral frame stiffness and the lateral forces necessary to keep the frame unswayed while subjected to gravity loading.

The method appears to be advantageous for cases in which sidesway effects resulting from gravity loads are not negligible. The method can be improved to take into account the eccentric effect of girder shears acting on the face of finite width shear walls, by introducing the concept or rigid gusset extensions as proposed by MacLeod.[24] Interconnected shear wall systems can then be treated on the same basis as above.

Goldberg[23] considered long narrow multistory buildings containing shear walls and frames and subjected to lateral loading. The analysis is subjected to the common assumptions cited at the beginning of this part with the exception of the rigid floor diaphragm. Thus, the analysis takes into account the effects of (a) flexural and shear deformations of floor slabs and shear walls whether in the form of flat or ribbed plates, and (b) flexural deformations of frame columns and beams. Essentially, the following operations are performed:

1. Moment and shear balancing equations are established for every shear wall-floor slab and frame-floor slab intersection.
2. The unknown moments and shears are then replaced by equivalent slope deflection expressions containing rotational and extensional displacements as unknowns. A system of matrix equations is thus obtained.
3. The system is solved through elimination by substitution, yielding rotational and extensional displacements.

Stress resultants may be obtained by substitution of displacements into the appropriate slope deflection equations. The method is capable of being extended to include secondary effects as well as treatment of nonsymmetrical buildings and loadings.

7. Further Analysis Considerations

Barnard and Schwaighofer[27] undertook model studies to determine the coupling effectiveness of a system consisting of two shear walls interconnected solely through a floor slab. For the particular configuration studied it was concluded that for a continuous system analysis of the Beck or Rosman type, the total slab width is effective as an interconnecting beam, although criticism has been raised.[28, 29]

Michael[30] showed that in systems consisting of wide shear walls interconnected through "non-shallow" beams, the effective beam span to be used in the analysis is equal to the physical clear span plus the beam depth. Such span increase, due to wall-beam interaction, appears to be mostly significant for cases of small beam span to depth ratios.

8. Discussion of Assumptions

A. *Common Assumptions*

All of the common assumptions presented in section 2.B of the present part appear realistic and attainable. In particular, the assumption of rigid floor diaphragms appears realistic for square, or nearly square, floor plans. For floor plans in the form of a rectangle (ratio of sides of 3 or over) it has been shown[23, 25] that neglect of a floor plate deformations leads to considerably underestimated horizontal deflections and causes upsetting of the distribution of horizontal shears between frames and shear walls. Such discrepancies between the results based on a rigid or non-rigid floor plate naturally become smaller for the most favorable shear wall-frame relative positions, and vice versa.

B. *Particular Assumptions*

The particular assumptions that have been most commonly encountered in the preceding analysis methods are discussed briefly.

B. 1 *Assumption of Constant Characteristics*

In most medium height buildings (eight to fifteen stories), and almost all tall buildings (over twenty stories), the rigidity of the columns and bearing shear walls is decreased with increasing floor level, due to reduction in member load carrying capacity. In addition, there are variations in story heights between the first few and the typical stories, dictated by architectural and utilitarian considerations. Moreover, special structural features, although predominant in typical floors, are absent in the floors at or near ground level. It, therefore, appears that the assumptions of constant characteristics and constant story height must be introduced with caution.

B. 2 *Assumption of Discrete Spring System Frame*

A discrete spring type system is, for the purpose of this investigation, considered to consist of the assembly shown in Fig. 1, originally introduced by Gould.[20] To test the validity of this assumption, the frame section shown in Fig. 2 was used to determine the load-displacement relationship.

The frame pertains to a building with a spandrel beam to column stiffness ratio of 3. Relative reaction values as determined by analysis as well as those pertaining to the discrete spring type system are shown in Fig. 2.

It is seen that a sizeable difference exists between the two sets of forces. It, therefore, appears that the analogy of a discrete spring type system is only justified for high spandrel beam to column stiffness ratios.

B. 3 *Assumption of Negligible Axial Deformations*

MacLeod[24] has shown that for stiff interconnecting beams, neglect of axial deformations of the shear walls leads to significantly underestimated horizontal deflections and moderately overestimated shears in the connecting beams. Rosman[12] has also emphasized that neglect of axial deformations especially in high, interconnected shear walls, leads to erroneous results. Bandel[26] has shown that neglect of axial deformation, in the columns of narrow tall frames leads to considerably underestimated deflections. Therefore, it appears that the assumption of negligible axial deformations becomes unreliable under certain conditions; however, no general criteria regarding the range of applicability of this assumption have been established.

9 Method Evaluation

Differential equation methods are relatively easy to manipulate, in general do not require computer facilities, and inherently yield results in a formulated manner. However, they require imposition of limiting assumptions which may result in misrepresentation of the state of stress and deformation of the system. Their use is recommended only for cases of multistory buildings in which variations of system characteristics are kept at a minimum.

Iteration methods allow complete freedom in variation of system characteristics and loading, but they are also subject to judicious assumptions regarding the grouping together of frames and shear walls for easier manipulation. The possibility of arithmetical errors is kept at a minimum through step by step comparisons. However, methods in this category are laborious and unless quick convergence is obtained, manual manipulation of the operations appears to be of little merit. Digital computer methods inherently

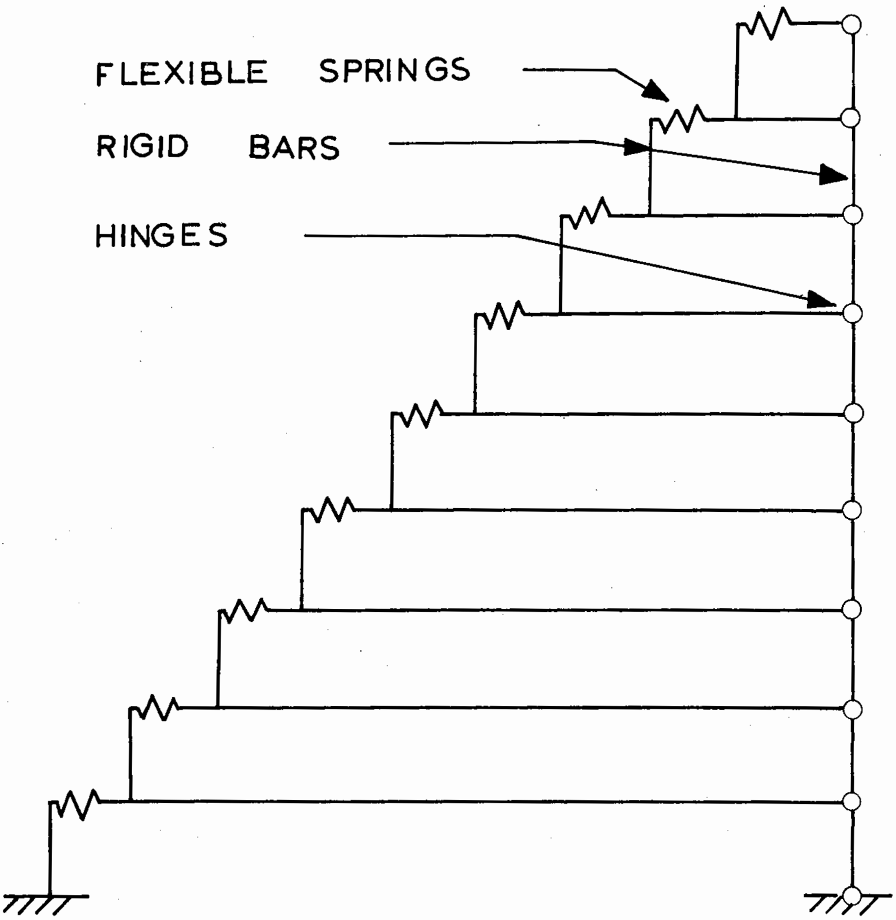

Fig. 1 — Discrete Spring Type System

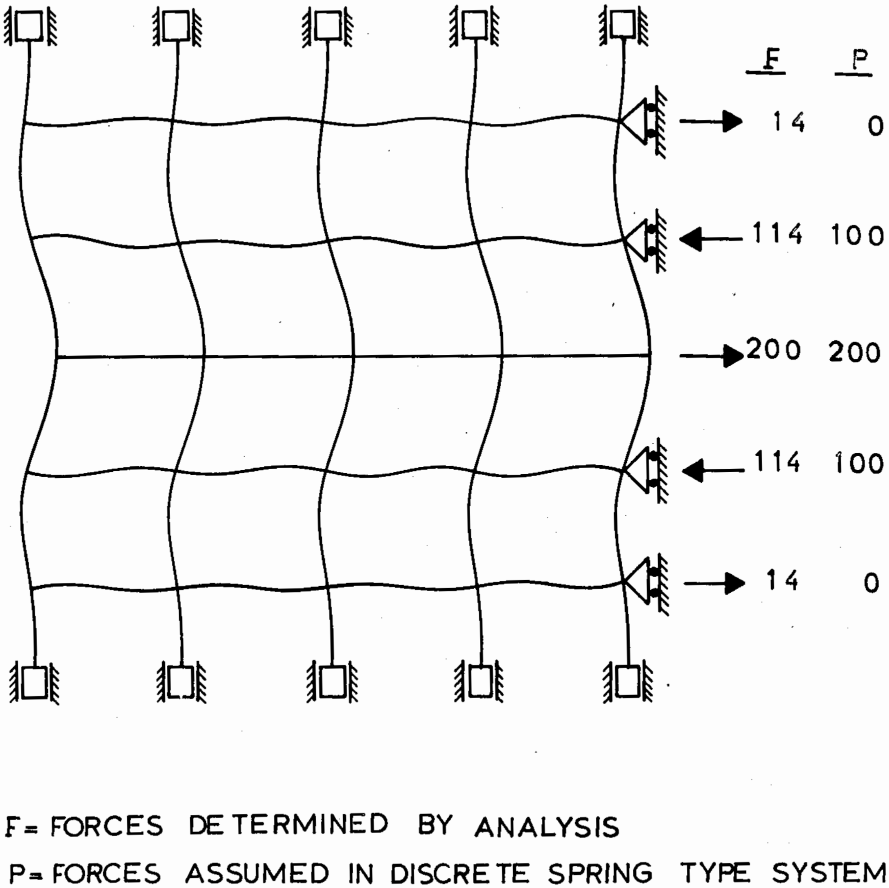

Fig. 2 — Determined and Assumed Forces in Typical Frame Action

allow extensive flexibility in system characteristics. However, one may be faced with the lack of a suitable program, excessive programming and processing costs, or even limited computer capacity.

PART 2
METHOD OF ANALYSIS FOR BIAXIAL AND ROTATIONAL DISPLACEMENTS

1 General Considerations

The method of analysis presented in this paper is based on the theorem of minimum total potential,[31, 32] and it results in a system of simultaneous linear equations for displacements. Computational work at various stages of the analysis may be carried out by means of a slide rule, desk calculator or digital computer, depending on the size of the work and the accuracy required.

2 Assumptions and Limitations

The proposed method is based on the following assumptions and limitations:

1. The structure may contain any number of shear walls and frames which remain linearly elastic during deformation. Formation of plastic hinges may be taken into account by reducing the original structure to constituent linearly elastic systems.
2. Foundations of the structure may rotate in a linearly elastic manner.
3. Frames and shear walls are interconnected by means of floor diaphragms which are extensionally rigid in their own horizontal plane, but flexurally or torsionally flexible about horizontal axes lying in their plane.
4. External loads acting on the system are horizontal and are applied only at floor levels.

3 System Characteristics

A building containing several shear walls and frames arranged in a nonrectangular grid is shown in Fig. 3. For purposes of identification, frames and shear walls are assigned characteristics f1, f2 etc. and w1, w2 etc., in a prescribed order. In formulating one class of energy equations, columns are considered as isolated elements regardless of whether or not they belong to frames, and thus they are also assigned characteristics c1, c2 etc.

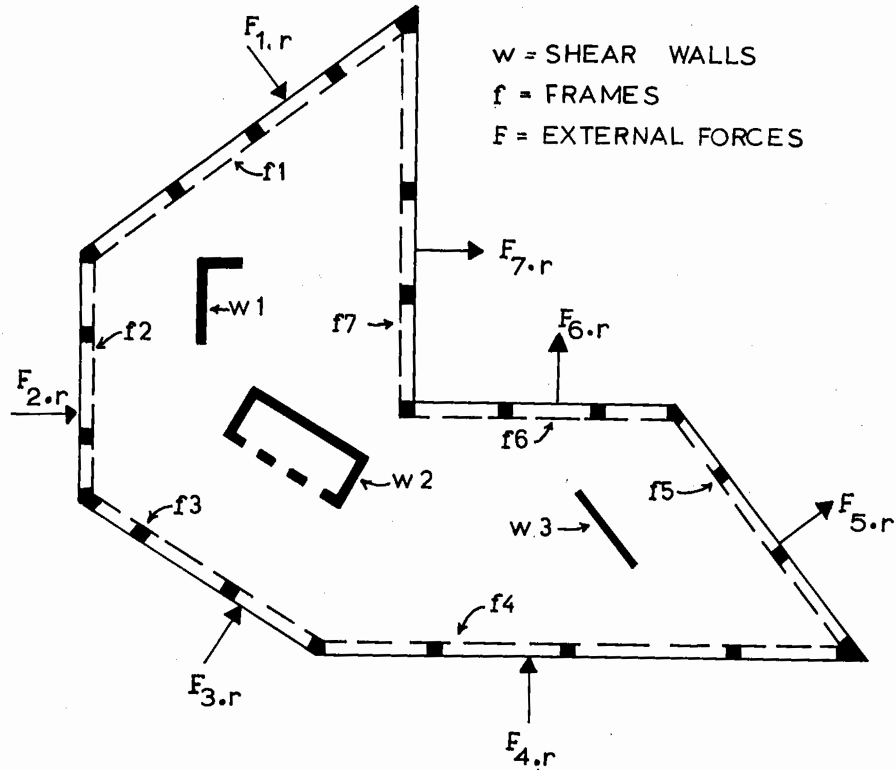

Fig. 3 — Shear Walls and Frames in Nonrectangular Grid

The building is subjected to external horizontal loads F, which for the case of wind loading will act perpendicularly on its faces. The resultant wind force on each face may be located at a position other than the center of the face, and the resultants pertaining to each floor level on the same face do not have to lie on the same vertical line.

For purposes of analysis, the building is assumed to undergo successive displacements; the position of each of the building floors can then be specified as shown in Fig. 4 in terms of (a) rotational displacements about a specific point C in the plane of the floor, and (b) translational displacements u and v with respect to a fixed coordinate system XOY. Points C for the various floors lie on the same vertical line.

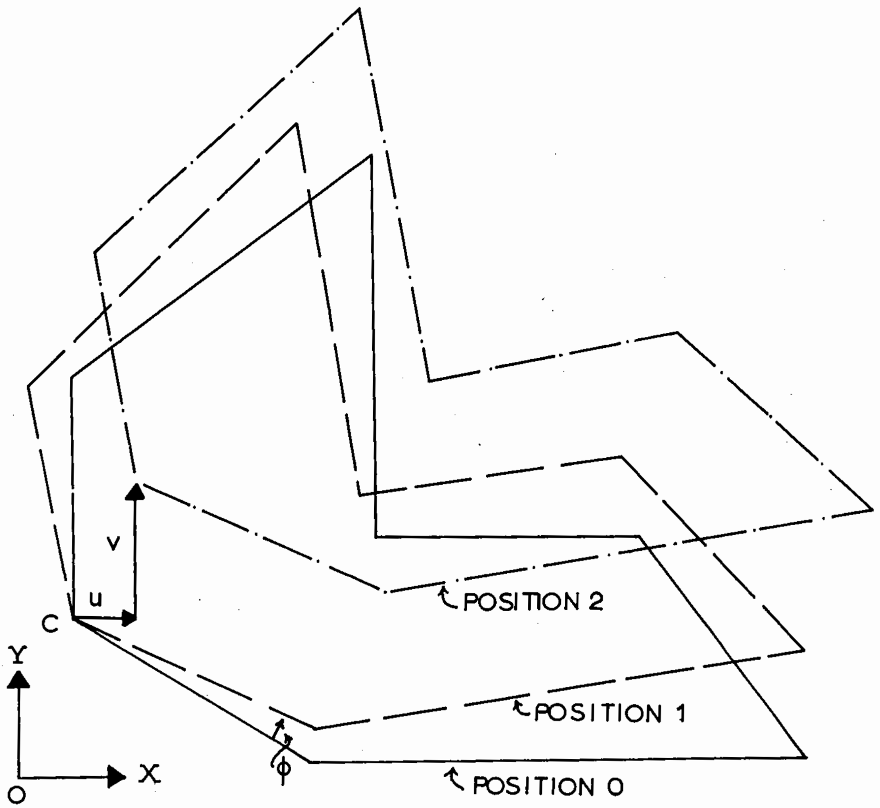

Fig. 4 — Successive Floor Displacements

4 Considerations for Shear Walls

A. *Kinematic Relationships*

The displacement of shear wall w2 shown in Fig.3 is considered in detail. It is assumed for simplicity that the shear center and center of rotation of the wall cross-section coincide with its centroid.

Rotation of the wall centroid about point C, as shown in Fig. 5, and subsequent translation from position 1 to position 2 as shown in Figs. 4 and 6, will cause displacements given by

$$u' = (r \sin \theta)\, \phi + u \qquad \text{(1a)}$$

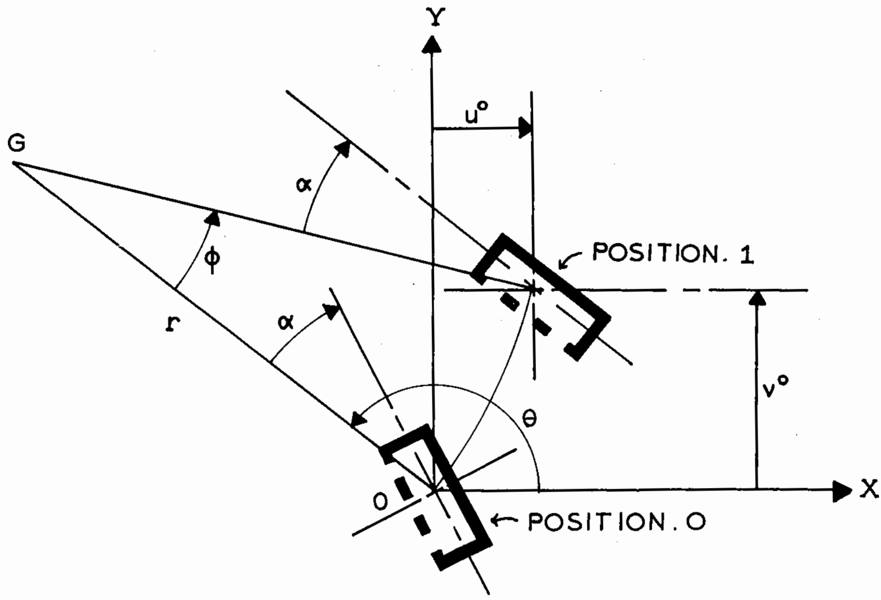

Fig. 5 — Shear Wall Displacements Due to Rotation

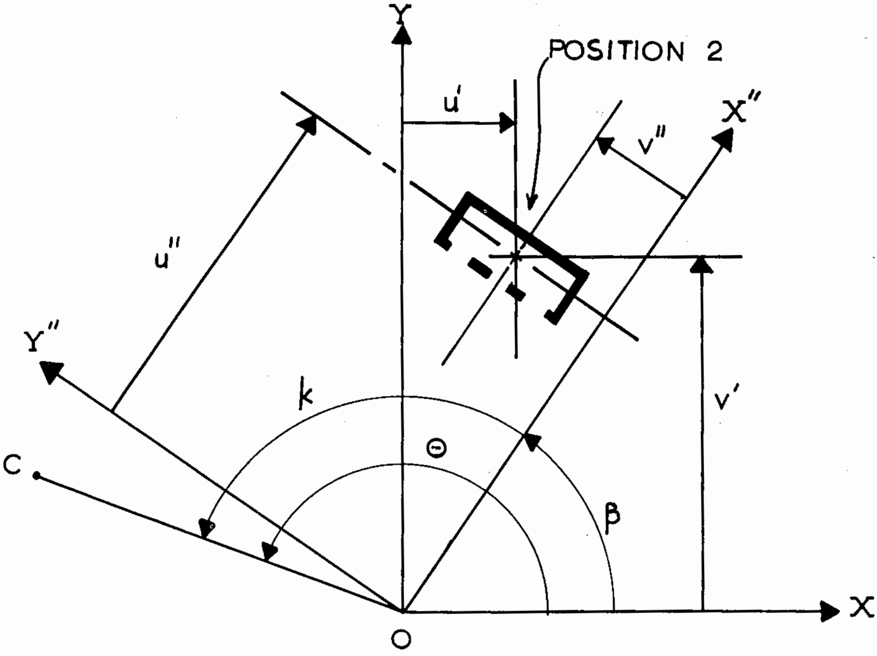

Fig. 6 — Shear Wall Coordinate Transformation

$v' = -(r \cos \theta)\, \phi + v$ . . . . (lb)

$\phi' = \phi$ . . . . (1c)

in which r is the distance from point C to the centroid of the wall; θ is the angle between r and OX axis.

Transforming displacements u', v', ϕ' into displacements u'', v'', ϕ'' pertaining to the principal axes of the wall as shown in Fig. 6, gives

$u'' = au + bv + c\phi$ . . . . (2a)

$v'' = -bu + av - d\phi$ . . . . (2b)

$\phi'' = -\phi$ . . . . (2c)

in which a, b, c, d designate the quantities $\cos\beta$, $\sin\beta$, r sink, r cosk respectively; β is the angle between coordinate systems XOY and X″OY″; k is the difference between angles θ and β.

Since the quantities r are identical, and the quantities β and k are practically identical for all floors, the constants in Eqs. 2 are practically identical for all floors.

B. *Energy Relationships*

The strain energy stored in the shear wall is due to biaxial translational and rotational displacements.

The stiffness equation for a cantilever structure is given by

$\{P^W\} = [A^W] \{u^W\}$ . . . . (3)

in which $\{P^W\}$ and $\{u^W\}$ denote the column matrices of load and displacement respectively; $[A^W]$ denotes the stiffness square matrix. Coefficients of the stiffness matrix include the effect of shear as shown in Appendix I. The strain energy $\{U^W\}$ of the structure is given by

$\{U^W\} = \frac{1}{2} [u_r^W] \{P^W\}$ . . . . (4)

in which $\{U^W\}$ is the strain energy in the form of a lxl matrix; $[u_r^W]$ is the row vector of displacements.

Combining Eqs. 3 and 4, considering that the stiffness matrix is symmetric,[33] and differentiation of both sides of the resultant equation, the following is obtained.

$$\left\{\begin{matrix} \dfrac{\partial U^w}{\partial U_1^w} \\ \dfrac{\partial U^w}{\partial u_2^w} \\ \cdot \\ \cdot \\ \dfrac{\partial U^w}{\partial u_n^w} \end{matrix}\right\} = \left\{\frac{\partial U^w}{\partial u^w}\right\} = [A^w] \quad \{u^w\} \quad \ldots\ldots\ldots\ldots\ldots \quad (5)$$

The implication of the above equation is noteworthy: the first derivative of the strain energy matrix for a cantilever structure, is equal to the stiffness matrix postmultiplied by the displacement column matrix. Applying Eq. 5 to the particular case of the shear wall, undergoing biaxial and rotational displacements gives

$$\left\{\frac{\partial U_a^w}{\partial u''}\right\} = [A^w] \quad \{u''\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5a)$$

$$\left\{\frac{\partial U_b^w}{\partial v''}\right\} = [B^w] \quad \{v''\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5b)$$

$$\left\{\frac{\partial U_\phi^w}{\partial \phi''}\right\} = [C^w] \quad \{\phi''\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (5c)$$

in which $\left\{U_{\phi}^{W}\right\}$, $\left\{U_{a}^{W}\right\}$, $\left\{U_{b}^{W}\right\}$ are the strain energy column matrices of shear wall due to rotational displacements and translational displacements parallel to OX″ and OY″ axes respectively; $[C^{W}]$, $[A^{W}]$, $[B^{W}]$ denote the respective stiffness matrices.

The total strain energy stored in the shear wall is given by

$$\left\{U_{T}^{W}\right\} = \left\{U_{a}^{W}\right\} + \left\{U_{b}^{W}\right\} + \left\{U_{\phi}^{W}\right\} \quad \ldots\ldots\ldots\ldots \quad (6)$$

Differentiating $\left\{U_{T}^{W}\right\}$ with respect to u, v and ϕ gives

$$\left\{\frac{\partial U_{T}^{W}}{\partial u}\right\} = \left\{\frac{\partial U_{a}^{W}}{\partial u''}\right\}\left\{\frac{\partial u''}{\partial u}\right\} + \left\{\frac{\partial U_{b}^{W}}{\partial v''}\right\}\left\{\frac{\partial v''}{\partial u}\right\} \quad \ldots\ldots\ldots \quad (7a)$$

$$\left\{\frac{\partial U_{T}^{W}}{\partial v}\right\} = \left\{\frac{\partial U_{a}^{W}}{\partial u''}\right\}\left\{\frac{\partial u''}{\partial v}\right\} + \left\{\frac{\partial U_{b}^{W}}{\partial v''}\right\}\left\{\frac{\partial v''}{\partial v}\right\} \quad \ldots\ldots\ldots \quad (7b)$$

$$\left\{\frac{\partial U_{T}^{W}}{\partial \phi}\right\} = \left\{\frac{\partial U_{a}^{W}}{\partial \phi''}\right\} + \left\{\frac{\partial U_{b}^{W}}{\partial \phi''}\right\} + \left\{\frac{\partial U_{\phi}^{W}}{\partial \phi''}\right\} \quad \ldots\ldots\ldots \quad (7c)$$

Combining Eqs. 7a, 5a, 5b, 2a and 2b

$$\left\{\frac{\partial U_{T}^{W}}{\partial u}\right\} = a[A^{W}]\left\{u''\right\} - b[B^{W}]\left\{v''\right\} \quad \ldots\ldots\ldots\ldots \quad (8a)$$

Introducing Eqs. 2a and 2b into Eq. 8a rearranging, and abbreviating the resulting matrix sums by.

$$a^{2}[A^{W}] + b^{2}[B^{W}] = [K_{1}^{W}] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8b)$$

$$ab[A^{W}] - ab[B^{W}] = [L_{1}^{W}] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8c)$$

$$ac[A^{W}] + bd[B^{W}] = [M_{1}^{W}] \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (8d)$$

The following is obtained

$$\left\{\frac{\partial U_T^W}{\partial u}\right\} = [K_1^W]\{u\} + [L_1^W]\{v\} + [M_1^W]\{\phi\} \ldots (9)$$

Combining Eqs. 7b, 5a, 5b, 2a and 2b

$$\left\{\frac{\partial U_T^W}{\partial v}\right\} = b[A^W]\{u\} + a[B^W]\{v''\} \ldots\ldots\ldots\ldots (10)$$

Introducing Eqs. 2a, 2b into Eq. 1o, rearranging and then introducing the abbreviations

$$ab[A^W] - ab[B^W] = [K_2^W] \ldots\ldots\ldots\ldots (11a)$$

$$b^2[A^W] + a^2[B^W] = [L_2^W] \ldots\ldots\ldots\ldots (11b)$$

$$bc[A^W] - ad[B^W] = [M_2^W] \ldots\ldots\ldots\ldots (11c)$$

Eq. 10 becomes

$$\left\{\frac{\partial U_T^W}{\partial v}\right\} = [K_2^W]\{u\} + [L_2^W]\{v\} + [M_2^W]\{\phi\} \ldots\ldots\ldots (12)$$

By analogous procedures

$$\left\{\frac{\partial U_T^W}{\partial \phi}\right\} = [K_3^W]\{u\} + [L_3^W]\{v\} + [M_3^W]\{\phi\} \ldots\ldots\ldots (13)$$

in which

$$ca[A^W] + bd[B^W] = [K_3^W] \ldots\ldots\ldots\ldots (14a)$$

$$bc[A^W] - ad[B^W] = [L_3^W] \ldots\ldots\ldots\ldots (14b)$$

$$c^2[A^W] + d^2[B^W] + [C^W] = [M_3^W] \ldots\ldots\ldots\ldots (14c)$$

Eqs. 9, 12 and 13 have been derived for only one shear wall; they can be, however, considered as including all shear walls of the building, if the combined stiffness matrices [K], [L], [M] are extended to include stiffness matrices for all shear walls.

5 Considerations for Frames

A. Kinematic Considerations

The displacement of frame f1 shown in Fig. 3 is considered in detail. Firstly the frame centroid at a floor level is rotated about point C, moving from position 0 to position 1 as shown in Fig. 7. This movement will cause rotational and translational displacements of the frame centroid. The frame is then translated from position 1 to position 2 in accordance with the translations of the rigid floor diaphragm shown in Fig. 4. Position 2 of the frame is shown in Fig. 8, in which also are shown the displacements u″ and v″ pertaining to longitudinal and transverse axes of the frame. Consideration of the total movement of the frame would indicate that the frame undergoes two types of displacement: (a) a displacement in which the frame moves in the direction of its horizontal longitudinal axis, (b) a displacement in which each column of the frame moves in the direction perpendicular to the frame horizontal longitudinal axis.

The first type of displacement being similar to that encountered in the previous section, is termed frame action displacement.

In the second type of displacement the columns behave as individual elements coupled to each other by means of spandrel beams. If the torsional stiffness of the spandrel beams is negligible, the columns can be treated in a manner analogous to that used for individual shear walls. This type of displacement is termed cantilever action displacement.

B. *Frame Action Energy Relationships*

The component of displacement parallel to the frame longitudinal axis is given by Eq. 2a and is designated by u″ in Fig. 8. Energy relationships for the frame can be established from the previously established relationships for the shear wall. Thus the following relationships can be set up analogous to Eq. 9 by setting $[B^W]' = 0$

$$\left\{\frac{\partial U_T^f}{\partial v}\right\} = [K_1^f]\left\{u\right\} + [L_1^f]\left\{v\right\} + [M_1^f]\left\{\phi\right\} \qquad \text{(15)}$$

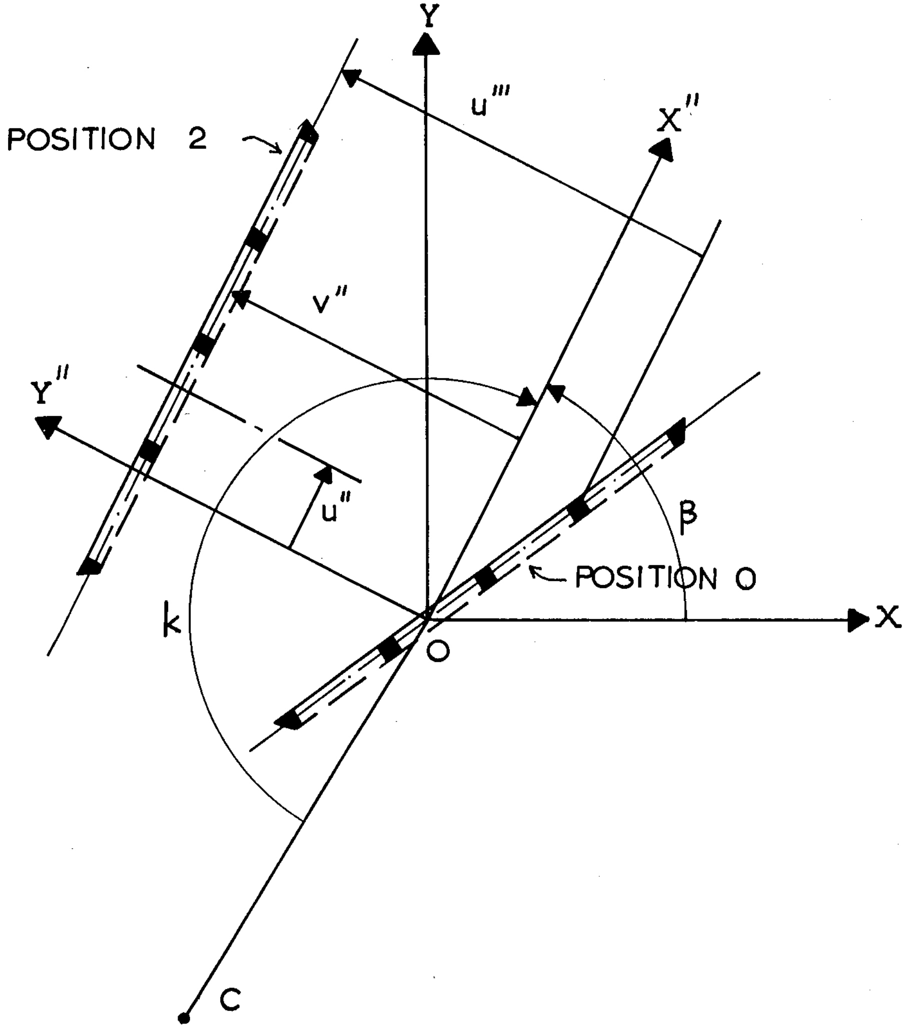

Fig. 7 — Frame Displacements Due to Rotation

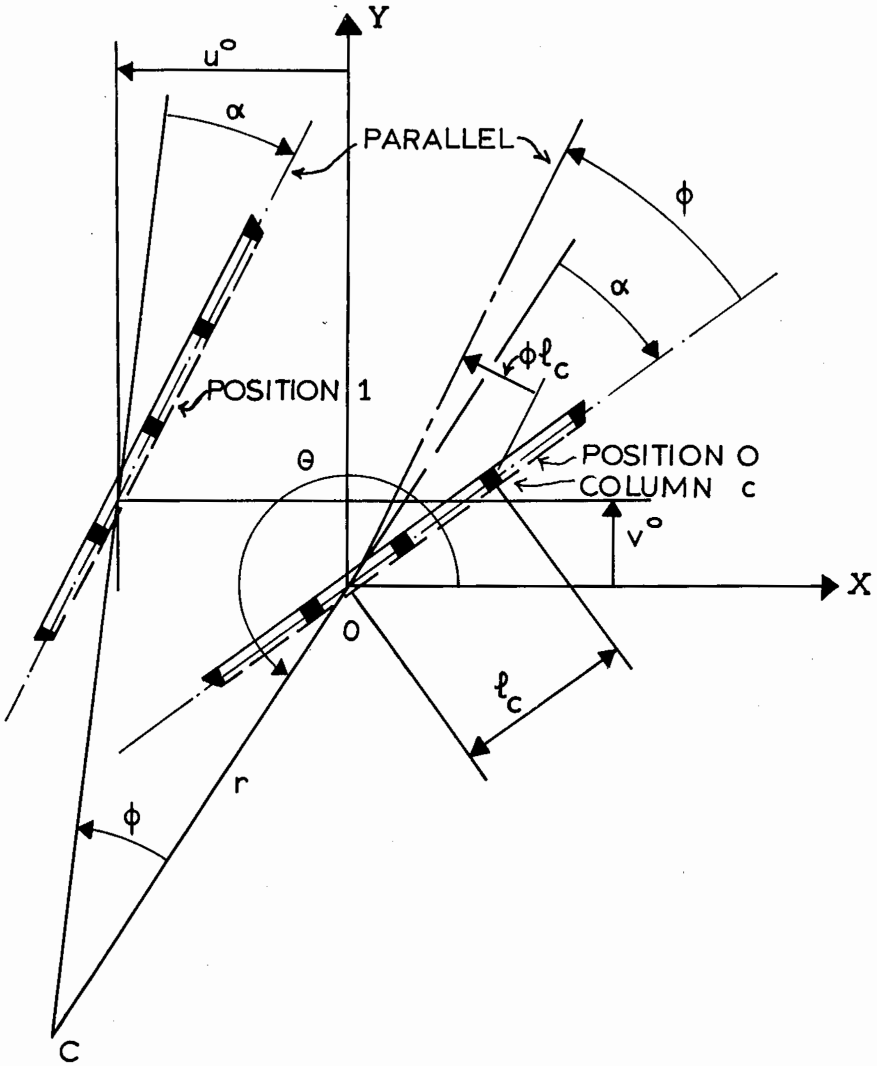

Fig. 8 — Frame Coordinate Transformation

Similarly from Eq. 12 by setting $[B^W] = 0$

$$\left\{\frac{\partial U_T^f}{\partial v}\right\} = [K_2^f]\left\{u\right\} + [L_2^f]\left\{v\right\} + [M_2^f]\left\{\phi\right\} \qquad (16)$$

And from Eq. 13 by setting $[\overset{w}{B}] = [\overset{w}{C}] = 0.$

$$\left\{\frac{\partial U_T^f}{\partial \phi}\right\} = [K_3^f]\left\{u\right\} + [L_3^f]\left\{v\right\} + [M_3^f]\left\{\phi\right\} \qquad (17)$$

Eqs. 15, 16, and 17 have been derived for only one frame; they can however be considered as including all frames of the building, if the combined stiffness matrices [K], [L], [M] are extended to include stiffness matrices for all frames.

C. *Cantilever Action Energy Relationships*

The strain energy stored in the column is due to translational displacements in the direction of the transverse frame axis shown in position 2 in Fig. 8. The strain energy due to column torsion is neglected in the present analysis.

Column c shown in Fig. 7 is chosen in exemplifying the energy relationships. By reference to Figs. 7 and 8, and by analogy to Eq. 2a, the displacement in the direction of the transverse frame axis is given by

$$u^{III} = (\sin\beta)u + (\cos\beta)v - r(\cos k)\phi + \ell_c\phi \qquad (18)$$

in which ℓ_c is the distance from the frame centroid to the column centroid. The above relationship is of the form

$$u^{III} = au + bv + c\phi \qquad (19)$$

Since the total strain energy in the column is due to uniaxial translations, energy relationships can be obtained from the corresponding relationships for a shear wall by setting $[B] = [C] = 0$.

Thus, by analogy to Eqs. 9, 12 and 13

$$\left\{\frac{\partial U_T^c}{\partial u}\right\} = [K_1^c]\left\{u\right\} + [L_1^c]\left\{v\right\} + [M_1^c]\left\{\phi\right\} \qquad (20)$$

$$\left\{\frac{\partial U_T^c}{\partial v}\right\} = [K_2^c]\left\{u\right\} + [L_2^c]\left\{v\right\} + [M_2^c]\left\{\phi\right\} \quad \ldots\ldots\ldots\ldots \quad (21)$$

$$\left\{\frac{\partial U_T^c}{\partial \phi}\right\} = [K_3^c]\left\{u\right\} + [L_3^c]\left\{v\right\} + [M_3^c]\left\{\phi\right\} \quad \ldots\ldots\ldots\ldots \quad (22)$$

in which $\left\{U_T^c\right\}$ denotes the total strain energy stored in the column. Eqs. 20, 21 and 22 have been derived for only one column; they can however be considered as including all columns of the building if the combined stiffness matrices [K], [L], [M] are extended to include stiffness matrices for all columns.

6 Potential of External Loads

Load $F_{1.n}$ acting on face 1 at the n^{th} floor of the building, as shown in Fig. 3, is chosen to exemplify the pertinent relationships. For the above particular building face and floor level, the potential is given by

$$V_{1.n} = -u''_{1.n} F_{1.n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (23)$$

in which $u''_{1.n}$ is the displacement of the point of application of the load in the direction of the load. Since the displacement $u''_{1.n}$ can be expressed as a function of the type of Eq. 2a, Eq. 23 becomes

$$V_{1.n} = -(a_{1.n} u_n + b_{1.n} v_n + c_{1.n} \phi_n) F_{1.n} \quad \ldots\ldots\ldots \quad (24)$$

in which the constants $a_{1.n}$, $b_{1.n}$, $c_{1.n}$ refer to the coordinate transformation XOY X″OY″ such that the positive direction of the OX″ axis coincides with the direction of the load as shown in Fig. 9.

Differentiating Eq. 24 with respect to u_n

$$\frac{\partial V_{1.n}}{\partial u_n} = -a_{1.n} F_{1.n} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (25)$$

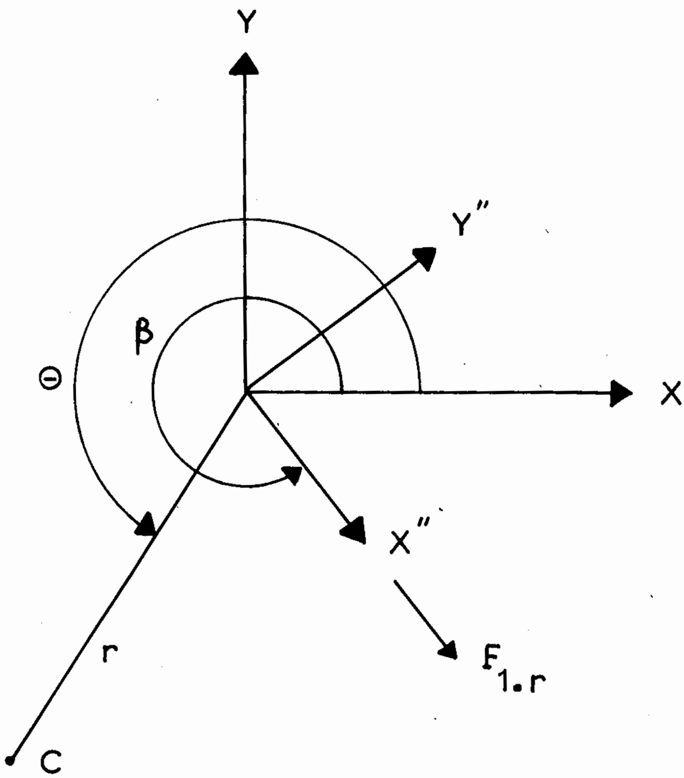

Fig. 9 — Force Coordinate Transformation

Considering expressions similar to Eq. 25 for all floors of the building at face 1

$$\left\{\begin{matrix} \frac{\partial V_{1.1}}{\partial u_1} \\ \frac{\partial V_{1.2}}{\partial u_2} \\ \cdot \\ \cdot \\ \frac{\partial V_{1.p}}{\partial u_p} \end{matrix}\right\} = - \left\{\begin{matrix} a_{1.1} F_{1.1} \\ a_{1.2} F_{1.2} \\ \cdot \\ \cdot \\ a_{1.p} F_{1.p} \end{matrix}\right\} \qquad \ldots\ldots\ldots\ldots\ldots \quad (26)$$

Setting Eq. 26 in matrix form

$$\left\{\frac{\partial V_{1.}}{\partial u}\right\} = - \left\{a_{1.} F_{1.}\right\} \ldots\ldots\ldots\ldots\ldots (27)$$

Considering potential relationships similar to Eq. 27 for all faces, 1 to 7, of the building, grouping together and replacing the matrix sums by equivalent matrices

$$\left\{\frac{\partial V_T}{\partial u}\right\} = - \left\{aF\right\} \qquad \ldots\ldots\ldots\ldots\ldots \quad (28)$$

in which V_T is the potential due to all loads acting on the building. Establishing relationships similar to Eq. 28 for displacements v and ϕ

$$\left\{\frac{\partial V_T}{\partial v}\right\} = - \left\{bF\right\} \qquad \ldots\ldots\ldots\ldots\ldots \quad (29)$$

$$\left\{\frac{\partial V_T}{\partial \phi}\right\} = - \left\{cF\right\} \qquad \ldots\ldots\ldots\ldots\ldots (30)$$

7 Application of Minimum Total Potential Theorem

According to the minimum total potential theorem, at equilibrium

$$\left\{\frac{\partial U_T}{\partial u}\right\} + \left\{\frac{\partial V_T}{\partial u}\right\} = 0 \quad \text{(31a)}$$

$$\left\{\frac{\partial U_T}{\partial v}\right\} + \left\{\frac{\partial V_T}{\partial v}\right\} = 0 \quad \text{(31b)}$$

$$\left\{\frac{\partial U_T}{\partial \phi}\right\} + \left\{\frac{\partial V_T}{\partial \phi}\right\} = 0 \quad \text{(31c)}$$

Since U_T and V_T in the above equations refer to the entire sustem, the following relationships exist

$$\left\{\frac{\partial U_T}{\partial u}\right\} = \left\{\frac{\partial U_T^w}{\partial u}\right\} + \left\{\frac{\partial U_T^f}{\partial u}\right\} + \left\{\frac{\partial U_T^c}{\partial u}\right\} \quad \text{(32a)}$$

$$\left\{\frac{\partial U_T}{\partial v}\right\} = \left\{\frac{\partial U_T^w}{\partial v}\right\} + \left\{\frac{\partial U_T^f}{\partial v}\right\} + \left\{\frac{\partial U_T^c}{\partial v}\right\} \quad \text{(32b)}$$

$$\left\{\frac{\partial U_T}{\partial \phi}\right\} = \left\{\frac{\partial U_T^w}{\partial \phi}\right\} + \left\{\frac{\partial U_T^f}{\partial \phi}\right\} + \left\{\frac{\partial U_T^c}{\partial \phi}\right\} \quad \text{(32c)}$$

Combining Eqs. 9, 15, 20, 28, 31a and 32a

$$([K_1^w] + [K_1^f] + [K_1^c])\left\{u\right\} + ([L_1^w] + [L_1^f] + [L_1^c])\left\{v\right\} +$$

$$([M_1^w] + [M_1^f] + [M_1^c])\left\{\phi\right\} = \left\{aF\right\} \quad \text{(33)}$$

Abbreviating the matrix sums in Eq. 33

$$[K_1^T]\left\{u\right\} + [L_1^T]\left\{v\right\} + [M_1^T]\left\{\phi\right\} = \left\{aF\right\} \quad \ldots\ldots\ldots\ldots (34)$$

By an analogous process combining Eqs. 12, 16, 31b, 32b, 29 and 21

$$[K_2^T]\left\{u\right\} + [L_2^T]\left\{v\right\} + [M_2^T]\left\{\phi\right\} = \left\{bF\right\} \quad \ldots\ldots\ldots\ldots (35)$$

Similarly, combining Eqs. 13, 17, 22, 30, 31c, 32c and 22

$$[K_3^T]\left\{u\right\} + [L_3^T]\left\{v\right\} + [M_3^T]\left\{\phi\right\} = \left\{cF\right\} \quad \ldots\ldots\ldots\ldots (36)$$

Solution of the system of Eqs. 34, 35 and 36 will yield values for the displacements u, v and ϕ.

The forces necessary to keep the structural systems of the building in the known deflected configuration can then be computed from the pertinent stiffness equations. Stress resultants can then be determined by conventional means from either the known displacements or computed forces.

METHOD OF ANALYSIS FOR UNIAXIAL DISPLACEMENTS

A structure is considered that consists of planar arrays of shear walls and frames as shown in Fig. 10 and incorporating the following characteristics: (a) interconnecting links may be omitted from several floor levels, (b) external loading may be applied on each shear wall and frame individually. Frames are characterized by f1, f2 etc., and shear walls by w1, w2 etc. Floors are numbered as follows:

The floor levels of the first system proceeding from left to right are assigned Arabic numerals increasing from bottom to top, and all floor levels of system 1 are included in this operation. Then, system 2 is inspected starting from the lowest floor and proceeding upward. The floor levels of system 2 that are interconnected with system 1 are assigned the same numerals as those for system 1 while each encountered level which is not interconnected to system 1 is assigned the next numeral in the series. System 3 is treated relative to system 2 in the same manner that system 2 was treated relative to system 1.

Externally applied forces are then assigned the superscript pertaining to the system, and the subscript pertaining to the floor level. Floor level displacements are assigned the subscript pertaining to the level in question. The complete column matrices of externally applied forces and displacements for the system combination $1 + 2 + 3$ are given by

$$\{\bar{F}\} = \begin{Bmatrix} F_1^{f1} \\ F_2^{f1} + F_2^{w1} + F_2^{f2} \\ F_3^{f1} + F_3^{w1} + F_3^{f2} \\ F_4^{f1} \\ F_5^{f1} \\ F_6^{w1} + F_6^{f2} \\ F_7^{w1} + F_7^{f2} \end{Bmatrix} \qquad (37a)$$

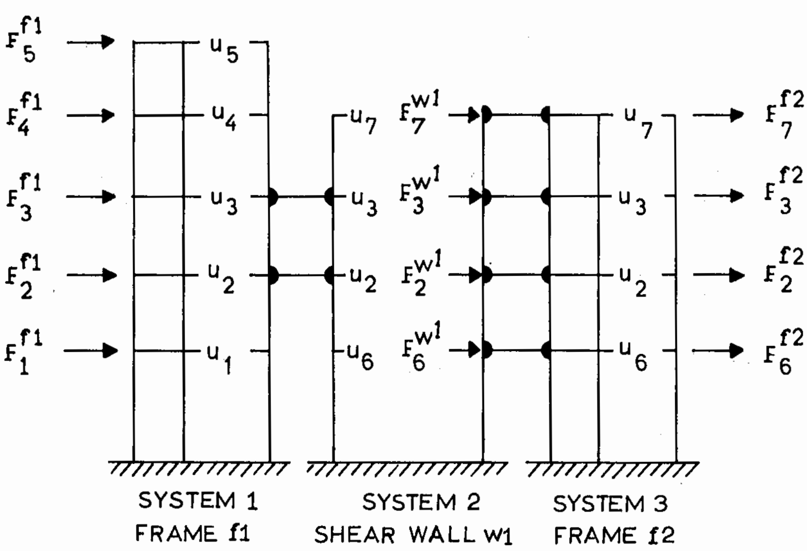

Fig. 10 — Generalized Planar Arrays

$$\left\{ \bar{u} \right\} = \left\{ \begin{matrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ u_7 \end{matrix} \right\} \quad \ldots\ldots\ldots\ldots \quad (37b)$$

Establishing an equation for frame f1 analogous to Eq. 5

$$\left\{ \frac{\partial U^{f1}}{\partial u^{f1}} \right\} = [A^{f1}] \left\{ u^{f1} \right\} \quad \ldots\ldots\ldots\ldots \quad (38)$$

in which $\left\{ u^{f1} \right\}$ is the column matrix of displacements for frame f1 only. Eq. 38 can be modified to

$$\left\{ \frac{\partial U^{f1}}{\partial \bar{u}} \right\} = [\bar{A}^{f1}] \left\{ \bar{u} \right\} \quad \ldots\ldots\ldots\ldots \quad (39)$$

in which $[\bar{A}^{f1}]$, resulting from $[A^{f1}]$ by the introduction of appropriate zero elements, corresponds to the complete displacement column matrix $\left\{ \bar{u} \right\}$. Establishing expressions similar to Eq. 39 for shear wall w1 and frame f2 and combining them with Eq. 39 gives

$$\left\{ \frac{\partial U}{\partial \bar{u}} \right\} = [\bar{A}] \left\{ \bar{u} \right\} \quad \ldots\ldots\ldots\ldots \quad (40)$$

in which U is the total strain energy for the combined system 1 + 2 + 3 and

$$[\bar{A}] = [\bar{A}^{f1}] + [\bar{A}^{w1}] + [\bar{A}^{f2}]$$

Applying Eq. 27 to frame f1

$$\left\{\frac{\partial V^{f1}}{\partial u^{f1}}\right\} = -\left\{F^{f1}\right\} \quad \text{.............................} \quad (41)$$

Eq. 41 can be modified as

$$\left\{\frac{\partial V^{f1}}{\partial \bar{u}}\right\} = -\left\{\bar{F}^{f1}\right\} \quad \text{.............................} \quad (42)$$

in which $\left\{\bar{F}^{f1}\right\}$, resulting from $\left\{F^{f1}\right\}$ by the introduction of appropriate zero elements, corresponds to the complete displacement column matrix given by Eq. 37b. Establishing expressions similar to Eq. 42 for shear wall w1 and frame f2 and combining them with Eq. 42

$$\left\{\frac{\partial V}{\partial \bar{u}}\right\} = -\left\{\bar{F}\right\} \quad \text{.............................} \quad (43)$$

in which V is the potential for the combined system 1+2+3, and $\left\{\bar{F}\right\}$ is given by Eq. 37a.

Applying the theorem of minimum total potential to Eqs. 40 and 43

$$[\bar{A}]\left\{\bar{u}\right\} = \left\{\bar{F}\right\} \quad \text{.............................} \quad (44)$$

Solution of Eq. 44 will give the complete set of horizontal displacements, from which the forces necessary to keep the system in the known deflected configuration and stress resultants can be computed. The method can be applied to any structure regardless of number of systems and floors.

The effect of foundation rotation rotation may be treated by the introduction of a member of equivalent stiffness, as is outlined in Reference 18.

The effect of plastic hinge formation may be taken into account by resolving the structure into two or more constituent systems. In the first, the structure is assumed to consist of the original material behaving in a linearly elastic manner up to the point of the first plastic hinge formation. In the second constituent system, the plastic hinge is replaced by an equivalent hypothetical linearly elastic material. Determination of the stiffness matrix of both systems must be carried out. Resultant action of the original structure can be obtained by superposition of the results of the two constituent systems.

Formation of more than one plastic hinge can be treated in a similar manner.

PART 3
NUMERICAL APPLICATION

1. System Characteristics

A four story building, having a plan configuration as shown in Fig. 11, was analyzed for earthquake loading acting in the OX direction. The number of stories was chosen deliberately small in order to keep the computa-

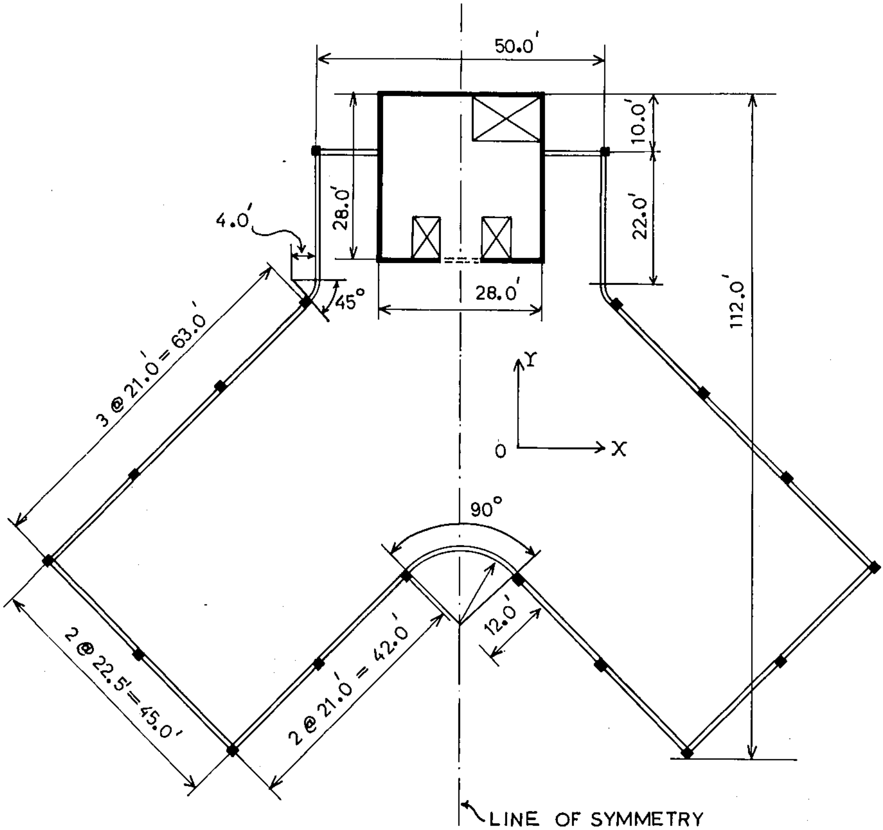

Fig. 11 — Example Building — Floor Plan

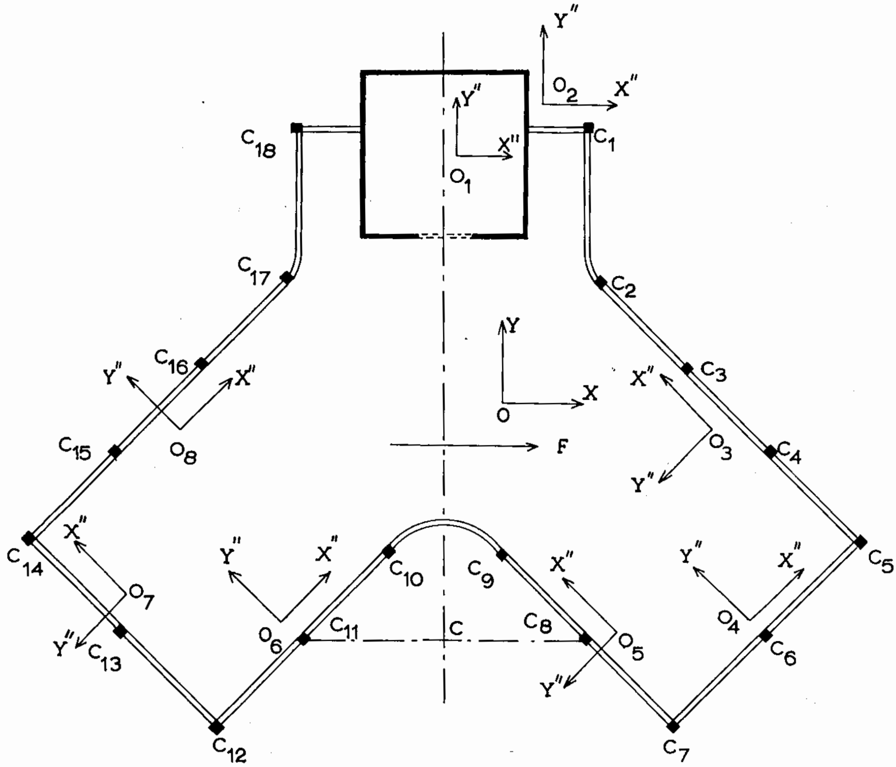

Fig. 12 — Example Building — Reference Systems

tions in a manageable space. The building is of reinforced concrete, waffle slab construction. The core is 10 in. thick and the columns 15 in. square.

The perimetric spandrel supporting the floor is 6.0 ft. deep and 12 in. wide, yielding (I/L) spandrel/ (I/L) column $\cong$ 25, and therefore justifying the assumption of discrete spring type systems for the perimetric frames. The first story is 13.3 ft. high and the remaining stories are 11.3 ft. high. Core and perimetric frames are resting on a rigid diaphragm at the first floor level.

The total lateral force was calculated as 560 kips, which is 0.057 of the total building weight. In applying the lateral loading, the center of stiffness was arbitrarily assumed at the center of the core and the center of mass was located by calculations at 50.0 feet from it. This is the point at which lateral forces F are applied as shown in Fig. 12. This apparent eccentricity thus exceeds 25% of the 112.0 ft. lateral building dimension; the effects of torsion were therefore doubled (according to recommendation of the current National Building Code of Canada) by doubling the total lateral force to 1,120 kips. The individual lateral forces were calculated according to a formula of the above Code and are shown in Table 2 below.

The flexural rigidity EI of the core and columns was calculated as $1.5(10^9)$ kip-ft.^{2} and $1.3(10^5)$ kip-ft.^{2} respectively.

The following assumptions were made in the analysis:

1. Floor plates are extensionally rigid in their own plane, but flexurally or torsionally flexible about horizontal axes lying in their plane.
2. Lateral forces are horizontal and are applied at floor levels.
3. The torsional rigidity of the core and the torsional and transverse rigidities of the columns are negligible. All axial deformations are negligible.
4. Shear deformations are effective for the core but negligible for the columns.
5. The effective height for flexure in the columns is the story height minus the spandrel depth.

The structural system is sectionalized into eight stiffness elements consisting of the core and column groups. Each element is given a characteristic number, as shown in Table 1, which is used as an identifying subscript in subsequent development. Individual coordinate axes systems $X'' O_n Y''$ (subscript n denotes the element characteristic number) are assigned one to a stiffness element as shown in Fig. 12 in which also is shown the center of rotation C for each floor plate.

TABLE 1. IDENTIFICATION OF STIFFNESS ELEMENTS

Stiffness Element	Characteristic Number
Core	1
FCC* C_1, C_{18}	2
FCC C_2, C_3, C_4, C_5	3
FCC C_5, C_6, C_7	4
FCC C_7, C_8, C_9	5
FCC C_{10}, C_{11}, C_{12}	6
FCC C_{12}, C_{13}, C_{14}	7
FCC C_{14}, C_{15}, C_{16}, C_{17}	8

*FCC = Frame comprising columns
**Columns are shown in Fig. 12

Stiffness matrices bearing the stiffness element characteristic number as subscript are shown below. Stiffness matrices for the core incorporate the effect of shear and were computed by the "STRESS" computer program.

$$[A_1] = [B_1] = \begin{bmatrix} 1.588 & -1.253 & 0.161 & 0.090 \\ -1.253 & 2.229 & -1.380 & 0.270 \\ 0.161 & -1.380 & 2.097 & -0.913 \\ 0.090 & 0.271 & -0.913 & 0.532 \end{bmatrix} \quad \ldots (45)$$

$$[A_2] = \begin{bmatrix} 0.023 & -0.020 & 0 & 0 \\ -0.020 & 0.040 & -0.020 & 0 \\ 0 & -0.020 & 0.040 & -0.020 \\ 0 & 0 & -0.020 & 0.020 \end{bmatrix} \quad \ldots (46)$$

$$[A_3] = [A_8] = \begin{bmatrix} 0.047 & -0.-41 & 0 & 0 \\ -0.041 & 0.082 & -0.041 & 0 \\ 0 & -0.041 & 0.082 & -0.041 \\ 0 & 0 & -0.041 & 0.041 \end{bmatrix} \quad \ldots (47)$$

$$\begin{matrix} [A_4] = [A_5] = \\ [A_6] = [A_7] = \end{matrix} \begin{bmatrix} 0.035 & -0.031 & 0 & 0 \\ -0.031 & 0.062 & -0.031 & 0 \\ 0 & -0.031 & 0.062 & -0.031 \\ 0 & 0 & -0.031 & 0.031 \end{bmatrix} \quad \ldots (48)$$

Measured quantities β, r, k and calculated quantities a, b, c, d are shown in Tables 2 and 3.

TABLE 2. CONSTANTS OF FORCE SYSTEM

Floor Level	Force (kips)	β (degrees)	k (degrees)	r (ft.x10^{-6})	a	b	c (ft.x10^{-6})
2	112.0						
3	224.0						
4	336.0	0	270	34.(10^6)	1	0	−34.(10^6)
Roof	448.0						

TABLE 3. CONSTANTS OF STIFFNESS ELEMENTS

Stiffness Element Characteristic Number	ß (degrees)	k (degrees)	r (ft.x10−6	a	b	c (ft.x10⁻⁶)	d (ft.x10⁻⁶)
1	0	270	83.(10^6)	1	0	−83.(10^6)	0
2	0	270	87.(10^6)	1	0	−87.(10^6)	0
3	135	82.5	62.5(10^6)	−0.707	0.707	62.(10^6)	8.(10^6)
4	45	135	54.5(10^6)	0.707	0.707	38.5(10^6)	−38.5(10^6)
5	135	45	24.(10^6)	−0.707	0.707	17.(10^6)	17.(10^6)
6	45	315	24.(10^6)	0.707	0.707	−17.(10^6)	−17.(10^6)
7	135	225	54.5(10^6)	−0.707	0.707	−38.5(10^6)	−38.5(10^6)
8	45	277.5	62.5(10^6)	0.707	0.707	−62.(10^6)	8.(10^6)

2. Calculations

Calculation of the matrices of coefficients of Eqs. 34, 35 and 36 is performed in two stages. In the first stage, equations are developed in terms of symbols. Subscripts appearing in the right hand side of this class of equations denote the stiffness element characteristic number, Symbols [A], [B], a, b, c, d are those pertaining to Eqs. 45 to 48 Table 3. In the second stage of development numerical substitutions are made into the expressions developed in the first stage.

First Stage

$$[K_1^w] = a_1^2 \ [A_1] + b_1^2 \ [B_1] \quad \ldots\ldots \quad (49a)$$

$$\left[K_1^f\right] = \sum_{n=2}^{8} a_n^2 \left[A_n\right] \quad \ldots\ldots \quad (49b)$$

$$\left[L_1^w\right] = a_1 \; b_1 \; \left[A_1\right] - a_1 \; b_1 \; \left[B_1\right] \quad \text{(50a)}$$

$$\left[L_1^f\right] = \sum_{n=2}^{8} a_n \; b_n \; \left[A_n\right] \quad \text{(50b)}$$

$$\left[M_1^w\right] = a_1 \; c_1 \; \left[A_1\right] + b_1 \; d_1 \; \left[B_1\right] \quad \text{(51a)}$$

$$\left[M_1^f\right] = \sum_{n=2}^{8} a_n \; c_n \; \left[A_n\right] \quad \text{(51b)}$$

$$\left[K_2^w\right] = a_1 \; b_1 \; \left[A_1\right] - a_1 \; b_1 \; \left[B_1\right] \quad \text{(52a)}$$

$$\left[K_2^f\right] = \sum_{n=2}^{8} a_n \; b_n \; \left[A_n\right] \quad \text{(52b)}$$

$$\left[L_2^w\right] = b_1^2 \; \left[A_1\right] + a_1^2 \; \left[B_1\right] \quad \text{(53a)}$$

$$\left[L_2^f\right] = \sum_{n=2}^{8} b_n^2 \; \left[A_n\right] \quad \text{(53b)}$$

$$\left[M_2^w\right] = b_1 \; c_1 \; \left[A_1\right] - a_1 \; d_1 \; \left[B_1\right] \quad \text{(54a)}$$

$$\left[M_2^f\right] = \sum_{n=2}^{8} b_n \; c_n \left[A_n\right] \qquad (54b)$$

$$\left[K_3^w\right] = c_1 \; a_1 \left[A_1\right] + b_1 \; d_1 \left[B_1\right] \qquad (55a)$$

$$\left[K_3^f\right] = \sum_{n=2}^{8} a_n \; c_n \left[A_n\right] \qquad (55b)$$

$$\left[L_3^w\right] = b_1 \; c_1 \left[A_1\right] - a_1 \; d_1 \left[B_1\right] \qquad (56a)$$

$$\left[L_3^f\right] = \sum_{n=2}^{8} b_n \; c_n \left[A_n\right] \qquad (56b)$$

$$\left[M_3^w\right] = c_1^2 \left[A_1\right] + d_1^2 \left[B_1\right] \qquad (57a)$$

$$\left[M_3^f\right] = \sum_{n=2}^{8} c_n^2 \left[A_n\right] \qquad (57b)$$

Second Stage

$$\left[K_1^t\right] = \left[K^w\right] + \left[K_1^f\right] = \begin{bmatrix} 0.1728 & 0.1376 & 0.0161 & 0.0090 \\ 0.1376 & 0.2476 & 0.1503 & 0.0271 \\ 0.0161 & 0.1503 & 0.2344 & 0.1036 \\ 0.0090 & 0.0271 & 0.1030 & 0.0655 \end{bmatrix} (10) \quad \ldots\ldots\ldots\ldots \quad (58)$$

$$\left[L_1^t\right] = \left[L_1^w\right] + \left[L_1^f\right] = 0 \quad \ldots\ldots\ldots\ldots \quad (59)$$

$$\left[M^t{}_1\right] = \begin{bmatrix} 0.136891 & 0.10841 & 0.013363 & 0.007470 \\ 0.108417 & 0.193843 & 0.118958 & 0.22493 \\ 0.013363 & 0.118958 & 0.182887 & 0.080197 \\ 0.007470 & 0.022493 & 0.080197 & 0.049574 \end{bmatrix} (10^9) \ldots\ldots \quad (60)$$

$$\left[K_2^t\right] = \left[K_2^w\right] + \left[K_2^f\right] = 0 \quad \ldots\ldots\ldots\ldots \quad (61)$$

$$\left[L_2^t\right] = \left[L_2^w\right] + \left[L_2^f\right] = \begin{bmatrix} 0.1705 & 0.1356 & 0.0161 & 0.0090 \\ 0.1356 & 0.2435 & 0.1483 & 0.0270 \\ 0.0161 & 0.1483 & 0.2303 & 0.1016 \\ 0.0090 & 0.0270 & 0.1016 & 0.0635 \end{bmatrix} (10) \quad \ldots\ldots\ldots\ldots \quad (62)$$

$$\left[M_2^t\right] = \left[M_2^w\right] + \left[M_2^f\right] = 0 \quad \ldots\ldots\ldots\ldots \quad (63)$$

$$\left[K_3^t\right] = \left[K_3^w\right] + \left[K_3^f\right] = \begin{bmatrix} 0.136891 & 0.10841 & 0.013363 & 0.007470 \\ 0.108417 & 0.193843 & 0.118958 & 0.022493 \\ 0.013363 & 0.118958 & 0.182887 & 0.080197 \\ 0.007470 & 0.022493 & 0.080197 & 0.049574 \end{bmatrix} (10^9) \ldots\ldots \quad (64)$$

$$\left[L_3^t\right] = \left[L_3^w\right] + \left[L_3^f\right] = 0 \quad \ldots\ldots\ldots\ldots \quad (65)$$

$$\left[M_3^t\right] = \left[M_3^w\right] + \left[M_3^f\right] = \begin{bmatrix} 0.117752 & 0.093472 & 0.011270 & 0.006300 \\ 0.093472 & 0.167554 & 0.102362 & 0.018970 \\ 0.011270 & 0.102362 & 0.158284 & 0.069672 \\ 0.006300 & 0.018970 & 0.069672 & 0.043002 \end{bmatrix} (10^{17}) \ldots\ldots \quad (66)$$

Also with values taken from Table 3

$$\left\{{}_a F\right\} = \begin{Bmatrix} 0.112 \\ 0.224 \\ 0.336 \\ 0.448 \end{Bmatrix} (10^3) \quad \ldots\ldots \quad (67)$$

$$\left\{{}_b F\right\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \quad \ldots\ldots \quad (68)$$

$$\left\{{}_c F\right\} = \begin{Bmatrix} -0.038 \\ -0.076 \\ -0.114 \\ -0.152 \end{Bmatrix} (10^{11}) \quad \ldots\ldots \quad (69)$$

A system of twelve equations with twelve unknown is assembled from the determined values:

$$\begin{bmatrix} \left[K_1^t\right] & \left[L_1^t\right] & \left[M_1^t\right] \\ \left[K_2^t\right] & \left[L_2^t\right] & \left[M_2^t\right] \\ \left[K_3^t\right] & \left[L_3^t\right] & \left[M_3^t\right] \end{bmatrix} \begin{Bmatrix} \{u\} \\ \{v\} \\ \{\phi\} \end{Bmatrix} = \begin{Bmatrix} \{{}_a F\} \\ \{{}_b F\} \\ \{{}_c F\} \end{Bmatrix} \quad \ldots \quad (70)$$

Solution of the above system through a computer program, gives the values shown in Table 4.

TABLE 4. DETERMINED VALUES OF DEFLECTIONS AND ROTATIONS

Floor Level	u (ft.)	v (ft.)	ϕ Radians
Roof	0.0490	0	0.000447
4	0.0453	0	0.000436
3	0.0399	0	0.000405
2	0.0330	0	0.000358

It is seen that v deflections are zero; this is expected since the stiffness elements comprise a system symmetrical about the OY axis and all external forces are perpendicular to that axis. On the basis of the figures of Table 4, the roof plate is shown in the displaced and original position in Fig. 13; displacements are shown magnified 200 times.

In plane displacements u'', for each stiffness element, can be computed from Eq. 2a, and the force system necessary to keep the element in the known deflected configuration may be computed from the stiffness equation incorporating the appropriate stiffness matrix. Stress resultants in the frames can then be computed from the known lateral forces by a conventional method.

CONCLUSIONS

Several methods of analysis of interconnected shear walls and shear wall-frame systems, subjected to horizontal static loading, have been reviewed and summarized. All of the reviewed methods consider structures subjected to the following restrictions: (1) shear walls and frames form rectangular grid plans, (2) the horizontal loading acting on the structure is parallel to the grid, and (3) the structure undergoes uniaxial displacements parallel to the grid.

A method of analysis, free of the above restrictions, and intended to supplement rather than replace the existing methods, has been presented herein. The method, based on the theorem of minimum total potential, is employing stiffness matrices of the constituent stiffness elements of the structure analyzed, as the basic input information. Stiffness matrices may in-

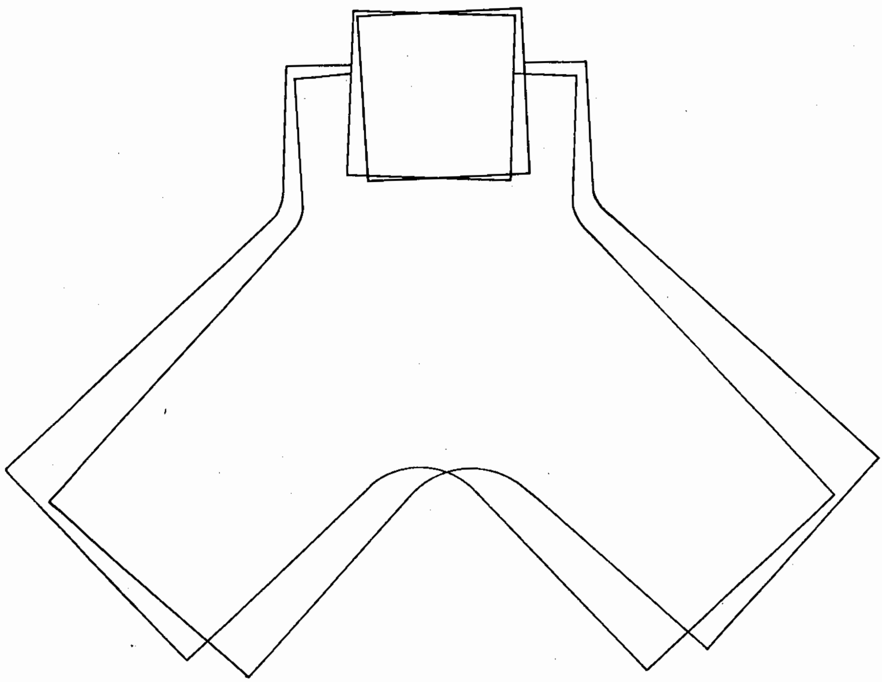

Fig. 13 — Example Building — Roof Displacements

clude the effect of shearing, torsional and axial deformations. The "general case" has been developed first, in which the structure to be analyzed consists of shear walls and frames interconnected through rigid floor diaphragms undergoing biaxial and rotational displacements. Then, the method was applied to the special case of planar arrays of shear walls and frames of arbitrary characteristics, and connected through inextensible links, undergoing uniaxial displacements only. The unknown displacement quantities form algebraic systems of linear equations, which can be solved by means of a digital computer.

In order to exhibit the workability of the method, a numerical example has been presented employing a Y shaped in plan building. In order to keep the numerically developed matrices within a manageable space, the number of stories was limited to four, without, however, impairing the general character of the method. Increasing the number of stories of this example building would have only the effect of increasing the size of computational matrices with all other steps remaining the same.

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APPENDIX I

Determination of Stiffness Coefficients

1. *Shear Walls*

The stiffness coefficients for a shear wall taking into consideration the effect of bending and shear, can be determined directly or indirectly. The direct method would consist of deflecting, by a unit displacement, the cantilever structure, one point at a time, while locking all other points against translational displacements only. Thus, the stiffness coefficient A_{kj} is equal to the reaction required to keep point k locked against displacement, while a load applied at joint j is producing a unit displacement at j.

Reactions can be determined from a moment distribution. Fixed end moments, stiffness and carry over factors can be determined from the relationships in Reference 22, as follows:

$$FEM = \frac{6(EI)}{L^2} \left\{\frac{1}{1-2\beta}\right\} \delta \qquad (47)$$

$$ST = \frac{M_1}{\theta} = \frac{2(EI)}{L} \left\{\frac{2+\beta}{1+2\beta}\right\} \qquad (48)$$

$$COF = \frac{M_2}{M_1} = \frac{1-\beta}{2+\beta} \qquad (49)$$

in which FEM, ST, COF signify fixed end moment, stiffness, and carry over factor respectively; subscripts 1 and 2 refer to the near and far end of the member respectively; θ denotes the angle of rotation of end 1 while end 2 is fixed; β is a parameter given by

$$\beta = \frac{6(EI)}{L^2 (AG)}$$

In the above relationships a constant shear distribution has been assumed through the depth of the member. A parabolic distribution, taking into account the effect of intervening ribs[23], can be effected by a suitable modification of the parameter β.

The indirect method of determining the stiffness coefficients, would involve determining and inverting the flexibility matrix.

In inspecting the stiffness matrix for correctness, it must be borne in mind that it must be symmetric about the main diagonal, and that the sum of the coefficients of any column, including the base shear coefficient if any, must be zero for equilibrium of the system.

2. *Frames*

Stiffness coefficients for beam-column frames due to primary effects (bending plus shear) can be determined according to the direct method outlined previously for shear walls. Axial deformations of columns generated in the deflection operation will induce secondary bending moments which in turn will cause additional deflections and reactions. The combined stiffness coefficients will then be equal to the ratio of the sum of primary and secondary reactions to the sum of primary and secondary deflections.

APPENDIX II — NOTATION

The following symbols have been adopted for use in this paper:

a, b, c = constants;

A = stiffness coefficients;

[A] = square stiffness matrix due to displacements parallel to OX axis;

$[\bar{A}]$ = square stiffness matrix defined by Eq. 39;

AG = shearing rigidity;
[B] = square stiffness matrix due to displacements parallel to OY axis;
[C] = square stiffness matrix due to tortional displacements;
EI = flexural rigidity;
{F} = column matrix of external loads;
$\{\bar{F}\}$ = column matrix of external loads defined by Eq. 42;
[K], [L], [M] = combined stiffness matrices defined by Eqs. 8, 11, 14;
M = bending moment;
OX, OY, OZ = cartesian coordinate system whose OZ axis is parallel to the vertical axis of the building;
OX″, OY″, OZ = cartesian coordinate system whose OX″, OY″ axes are parallel to the principal axes of the shear wall cross section, or frame plan projection;
P = force acting on shear wall or frame at floor level;
{P} = load column matrix;
r = radius of rotation of shear wall or frame centroid;
u, v = displacements of the rigid floor diaphragm with respect to XOY system;

displacements of the shear wall or frame centroid in moving from position 1 to position 2;

uo, vo = displacements of the shear wall or frame centroid (with respect to XOY system) in moving from position 0 to position 1;
u′, v′ = displacements of the shear wall or frame centroid (with respect to XOY system) in moving from position 0 to position 2;
{u} = displacement column matrix;
$\{\bar{u}\}$ = displacement column matrix defined by Eq. 37b;
U = strain energy;
V = potential of external loads;

GREEK SYMBOLS

β = angle between XOY and X″OY″ systems;
θ = angle between axis OX and the line drawn from the center of rotaions C to centroid of shear wall or frame;
ϕ = horizontal rotation of floor diaphragm;

SUPERSCRIPTS

c signifies column;

w denotes shear wall;

f signifies frame;

″ denotes quantities pertaining to X″OY″ system;

SUBSCRIPTS

a signifies quantities pertaining to displacements parallel to OX axis;

b denotes quantities pertaining to displacements parallel to OY axis:

n denotes quantities pertaining to nth floor;

l signifies matrix quantities applicable to 1th face and including all floors;

ℓ.n denotes quantities pertaining to ℓth face and nth floor of the building;

ϕ signifies quantities pertaining to rotational displacements;