AN ANALOGY TO THE STRUCTURAL BEHAVIOR OF SHEAR-WALL SYSTEMS

by

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Abstract

An analogy is presented to illustrate the structural behavior of shearwall-frame systems. It is shown that the rigid frame, subjected to horizontal loading deforms similarly to a tensioned cable under transverse loading. A shearwall behaves as a cantilever beam fixed at its base. A system composed of shearwalls and frames exhibits the same deformation as a cantilevered beam, subjected simultaneously to transverse loading and axial tension.

This analogy in structural behavior lends itself to a simple application in a physical model by which the relative distribution of the horizontal loading to the shearwall and frames can be directly established. Variation in stiffness of the components of the system and loading on it will exhibit in its analogous model the corresponding patterns of deformation and load distribution.

Keywords: Structural behavior, *Shearwall-Frame Systems, Analogies* (structural), Structural engineering, Tall buildings, Models.

Introduction

The interaction between various structural elements and their respective participation in the resistance to horizontal loadings in multistory buildings has been for the past few decades the subject of numerous technical papers (1)+ to (5). Invariably these dwell on the intricacies of the analysis which tries to establish the distribution of the external loads among the individual members, some by approximate methods others by supposedly "exact" ones; supposedly, since the exactness of any such solution is highly questionable in view of a multitude of assumptions, not the least of which is the stiffness of the sections. However, the major problem of design, as opposed

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+Numbers in parentheses refer to the references given at the end of the paper.

to analysis, remains most often unsolved, namely where to position these structural members and how to proportion them. No analysis can even get started without a preliminary design. Whereas the design approaches are commonly of an iterative nature, in which preliminary — fairly arbitrary — assumptions are successively corrected, none of them can really validate a basically wrong design layout.

It is, therefore, not surprising that most of the approximate approaches are particularly aimed at establishing a quick and simple procedure by which a first intelligent guess can be checked. Such a guess, however, would mainly depend on the designer's experience and adequate understanding of the structural behavior. To deepen this understanding and to lessen the complete dependence on intuition, regardless of its infinite merits, is the main task of proper design tools and training. An important means to accomplish this is the use of simple analogies, by which the behavior of a structural element can be simulated by another phenomenon which can either be readily visualized or easily tested or more simply analyzed than the original problem. The soap film analogy, for instance, has been an eyeopener as to the behavior of any section under torsion, far beyond what purely analytical results could ever accomplish. Similarly, the column analogy has served most appropriately for clarification and easier understanding of the computational procedure for the determination of moments and physical properties of frames and members with varying cross-section.

The present paper proposes an analogy by which the behavior of a rigid frame in a multistory building, subjected to transverse loading, can be visualized. The main emphasis is placed on the analogy as an educational tool, to develop a better conceptual understanding of the interaction of rigid frames with shearwalls in a multistory building. This discussion is therefore of a descriptive nature, presenting the basic concept and illustrating it by an appropriate model. The analytical application of this analogy will lead to a design procedure for establishing the distribution of horizontal loading between frames and shearwalls in multistory buildings. That subject will be presented in a separate report.

The Analogy

We shall now try to show that the structural behavior of a system composed of rigid frames and shearwalls under transverse loading is analogous to that of a cantilever beam subjected simultaneously to transverse loads and to axial tension. Basically, judging from the deflection pattern, we note that the rigid frame under transverse loads deforms similarly to a cable under tension (Figs. 1a, b), that the shearwall deforms similarly to a canti-

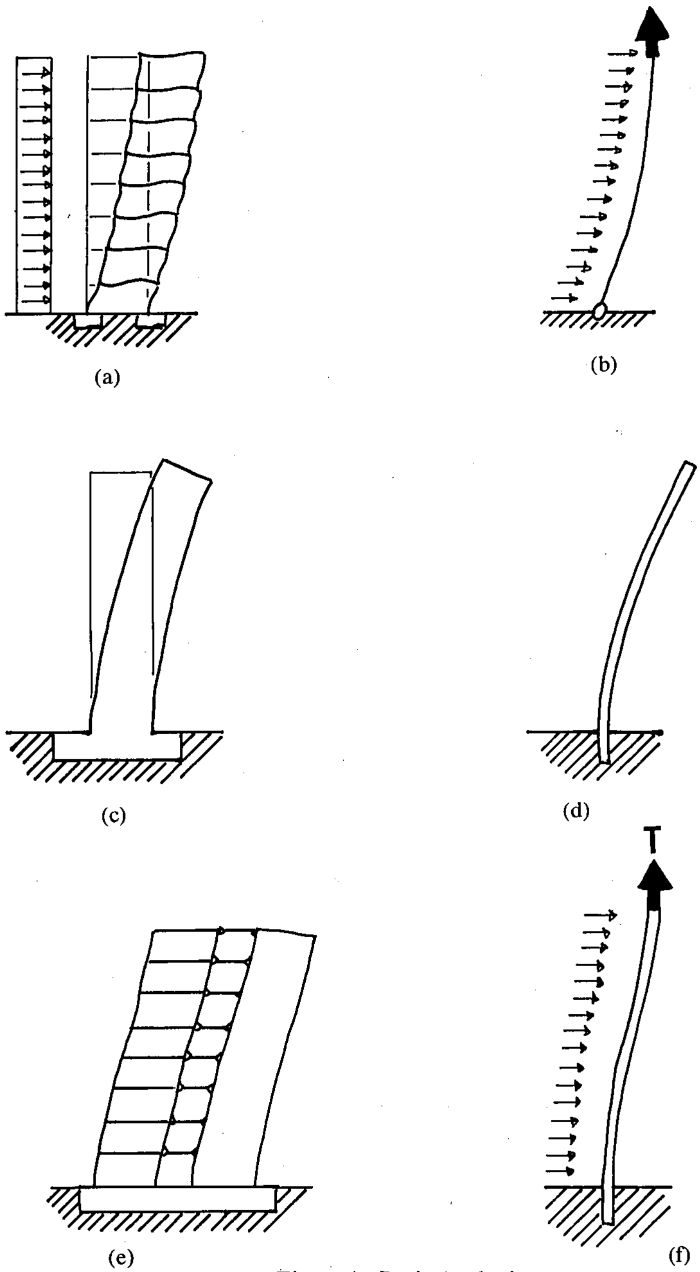


Figure 1 - Basic Analogies

lever beam (Figs. 1c, d), and that a system composed of rigid frames and shearwalls will deform similarly to a cantilever beam, subjected simultaneously to transverse loading and axial tension (Figs. 1e, f).

The frame under transverse loading may be regarded as a beam elastically restrained along its length from freely rotating. The rigidity of this restraint varies with the stiffness of the frame components. Let us call such a beam a R.R. Beam, short for rotationally restrained beam. Although the frame is subjected to rotational restraints at discrete points only (the column to girder joints), its behavior can be simulated by a flexible rod on continuous elastic rotational supports (Fig. 2). This will enable us to set up

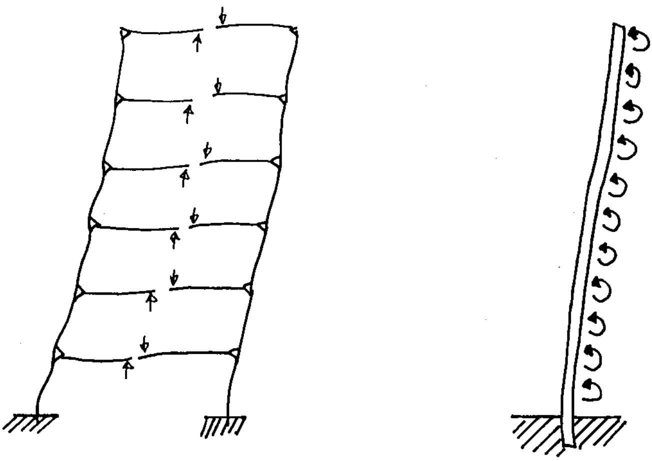


Figure 2 - Rigid Frame - A rotationally restrained beam

the basic differential equation for this case. It will then become evident that this basic differential equation is of the same form as that of a transversely loaded cable under tension, and the analogy between the frame and the cable can then be established by correlating the various parameters in these equations.

2.a. *The analogy between the rotationally restrained beam and beams under tension.*

For any rigid member, subjected to loading P(x) transversely to its longitudinal axis, the relationship between bending moment M_x^p and loading is expressed by equ. (1) as:

$$\frac{d^2 M_x^p}{dx^2} = -P(x) \qquad (1)$$

In the R.R. Beam (the rotationally restrained beam), the additionally acting continuous moment m_x^r, exerted by the rotationally elastic restraints is:

$$m_x^r = -\phi \cdot K \tag{2}$$

where ϕ is the angle of rotation at the point at which m_x^r acts, and K is the spring constant of the elastic rotational restraint. Therefore, the increment of the bending moment M_x^r in the beam per unit length due to its rotational restraint is:

$$\frac{dM_x^r}{dx} = m_x^r = -\phi \cdot K = \frac{dy}{dx} \cdot K \tag{3}$$

Expressing the angle of rotation as the derivative of y, equation (3) becomes

$$\frac{dM_x^r}{dx} = -\frac{dy}{dx} \cdot K \tag{4}$$

Now, when the two effects — of transverse loading, equ. (1), and of rotational restraints, equ. (4) — are superimposed, we get for the total bending moment:

$$\frac{d^2Mx}{dx^2} = \frac{d^2(M_x^p + M_x^r)}{dx^2} = -p(x) - K\frac{d^2y}{dx^2} \tag{5}$$

For the case of a flexible rod (EI = 0) the moment at any point along it vanishes, so that equ. (5) becomes:

$$\frac{d^2y}{dx^2} = -\frac{P(x)}{K} \tag{6}$$

Now, for the basic equation of cable theory, relating the displacements to transverse loading and to the tension force T in the cable, we have (Fig. 3).

$$\frac{d^2y}{dx^2} = -\frac{P(x)}{T} \tag{7}$$

Thus the analogy between the completely flexible R.R. Beam (EI = 0) and the cable under tension is clearly noted from the identical form of equations (6) and (7). It thus becomes obvious that the stiffness K of the rotational restraint in an R.R. Beam is analogous to the tension force T in the cable. If the transverse loading, p(x), is the same in such a beam and the analogous cable, both structures will exhibit the same pattern of deformation. The cable under tension can, therefore, serve as analog to the flexible R.R. Beam.

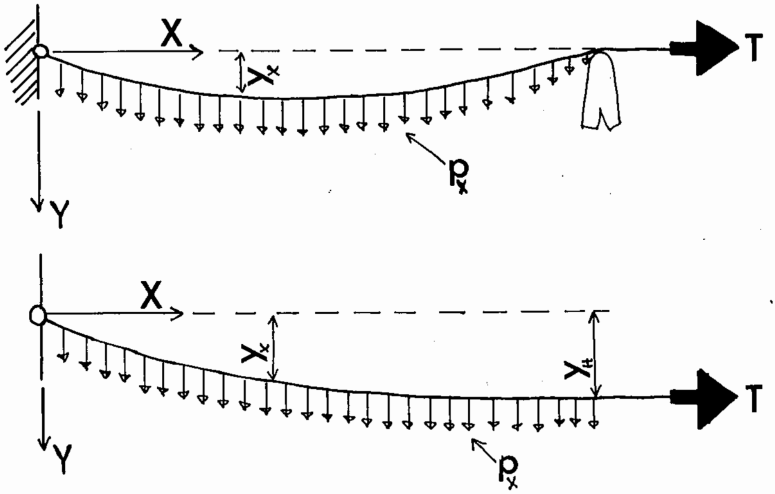


Figure 3 - Deformation of a transversly loaded tensioned cable

So far we have discussed only the case of a completely flexible rod for which EI = 0. Equ. (5) can now be used to cover the general case of a rigid R.R. Beam with a finite but constant value of EI. When substituting in equu. (5) for $\frac{d^2Mx}{dx^2}$, the value

$$\frac{d^2Mx}{dx^2} = -EI\frac{d^4y}{dx^4} \tag{8}$$

equ. (5) becomes

$$EI\frac{d^4y}{dx^4} - K\frac{d^2y}{dx^2} = p(x) \tag{9}$$

The differential equation for a rigid beam with constant EI, which is simultaneously subjected to transverse loading and axial tension is given by (6)

$$EI\frac{d^4y}{dx^4} - T\cdot\frac{d^2y}{dx^2} = p(x) \tag{10}$$

where T is the applied tensile force acting on the beam. Thus, again, equations (9) and (10) are of analogous form; the rigid R.R. Beam is therefore analogous to a beam under tension.

In conclusion of this part the following analogies are thus established:

Flexible (EI = 0) rotationally restrained beam — cable under tension.

Rigid (EI $\neq$ 0) rotationally restrained beam — beam under tension.

2.b. *Frame-cable analogy.*

The previous section considered the case of an R.R. Beam, a beam on which continuous rotational restraint is exerted. A frame is not substantially different from such a beam. A rigid frame might be conceived of as such a beam, except that the rotational restraints are acting at discrete points. It will now be shown that such a frame will be displaced at its nodal points analogously to a cable subjected to concentrated transverse loads.

This analogy clearly identifies the deformation of the frame due to external shear force. In fact, the displacement pattern of a frame results basically from two components: the effect of bending of the frame members, and the effect of their axial deformation. While the first is associated with a displacement pattern that is governed by the magnitude of the horizontal shear at any level (Fig. 1.a.), the latter is predicated by the external moments at any level. That causes an elongation of the columns on one side of the frame and a shortening of the columns on the opposite side, with a deformation resembling that of Fig. 1.d.

The analogy between the frame and the transversely loaded cable under tension, presented below, reflects only the shearforce pattern of the deformation (Fig 1.a.). The other component, which generally is of considerably smaller order of magnitude than the former, reflects the same behavior as that of the shearwall and is accounted for with it.

For a frame with girders of infinite rigidity ($EI^g = \infty$), the increment of horizontal displacements between two adjacent nodal points is given by (Fig. 4.a):

$$\Delta^{col}_i Y = \frac{S_i \cdot h_i^3}{12E\sum I^c} = \frac{S_i}{\frac{12E}{h_i^2} \cdot \sum I^c} \cdot h_i \qquad (11)$$

For a portion of a vertical cable under tension, subjected to transverse concentrated loads at points which are a distance h_i apart, corresponding to the nodal points of a frame, we note from Fig. 4.b that

$$\Delta^{c}_i Y = \frac{S_i}{T} \cdot h_i \qquad (12)$$

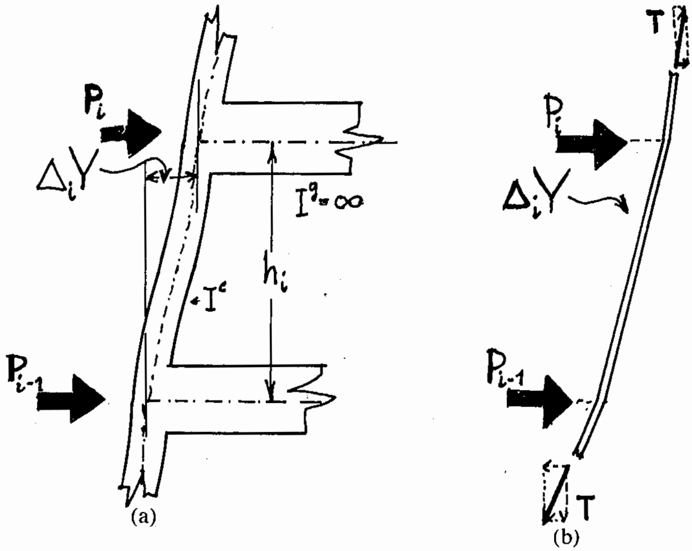


Figure 4 - Frame-cable displacement

Again, the expression of equ. (11) for the frame is analogous to equ. (12) for the polygonally deformed cable, where the applied tensile force on the cable, T, corresponds to a modified column stiffness $\left[F \right]_{Ig=\infty} = \frac{12E \cdot \Sigma I^c}{h_i^2}$ in the frame.

Equations (11) and (12) give the exact value for the increment of lateral displacement in the frame and the cable respectively, but the first is confined only to the idealized case of infinitely rigid girders. If, however, the girders of the frame, as is always the case, have some finite rigidity, EI^g, the frame stiffness must be defined by a more appropriate expression. To arrive at its value, the lateral displacement between two adjacent nodal points of the frame, one story, i.e., h_i apart, must be established.

The increment of horizontal displacement at any story is a function of shearforces and of the rigidity of frame members. However, the predominant factors determining the magnitude of this increment at any given story are the shearforce acting at that story, and the rigidities of all the frame members at that level. Based on that consideration, Wilbur (7) has developed approximate expressions for the increment of such lateral displacement for frames with variable moments of inertia of their members. If Wilbur's expressions are adapted to the case of Fig. 5, the lateral displacement between two adjacent nodal points, h_i, apart, will become:

$$\Delta^{F}_{i} Y = S_i \cdot \left[\frac{h_i^{3}}{12E \cdot \sum I^c} + \frac{h_i^{2}}{12E \cdot \sum \frac{I^g}{L}} \right] = \frac{S_i}{F_i}\, h_i \qquad (13)$$

where
$$F_i = \frac{12E}{hi \cdot \left[\frac{1}{\sum \frac{I^c}{h_i}} + \frac{1}{\sum \frac{I^g}{L}} \right]} + \frac{12E}{hi \cdot \left[\frac{1}{\sum K^c} + \frac{1}{\sum K^g} \right]} \qquad (14)$$

In equations, (11), (13), and (14) the sum of the rigidities of columns $\sum K^c$ and girders $\sum K^g$ ought to be that of all the respective members at the story considered. F_i is defined as the modified stiffness of the frame. Any other valid expression, or an experimentally established value for F, could be used instead of that given by equ. (14), if any greater accuracy was required.

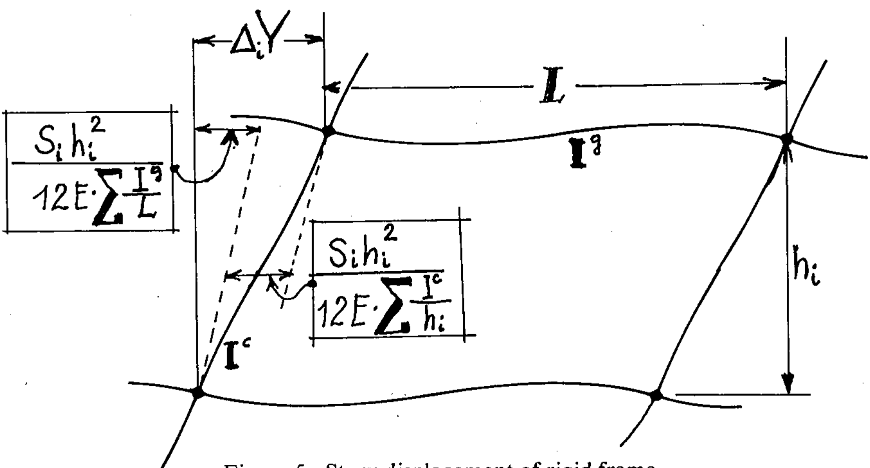


Figure 5 - Story displacement of rigid frame

In comparing equ. (13) with equ. (12) it is again noted that the modified frame stiffness, F, is analogous to the tension force T, applied on the transversely loaded cable.

2.c. *Frame-wall systems — tensioned beam analogy.*

So far the frame alone has been considered, and its analogy with the similarly loaded cable under tension has been established (Fig. 1 a,b). We shall now see that the interaction of frames and shear walls in a building can be simulated by the behavior of a rigid beam under simultaneously acting transverse loading and longitudinal tension.

The system composed of frames and walls is diagrammatically shown in Fig. 6a, which is the former case, Fig. la, with the addition of a shear wall. In this analogy, since the shear wall may be considered as a cantilever

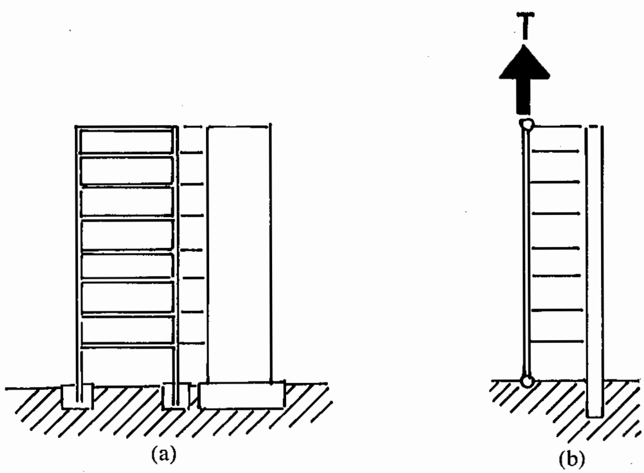


Figure 6 - Frame-shearwall system

beam, we will have to attach to the cable of Fig. 1b a cantilever beam of corresponding rigidity (Fig. 6b). It thus follows, that the frame-shearwall system is analogous to the cantilever beam linked to a cable under tension, which in turn is simply the case of a rigid beam subjected simultaneously to transverse loading and longitudinal tension.

The modified stiffness F of the frame, equ. (14), thus represents the general form of the concept of continuous rotational restraint of stiffness K. Equ. (9) can now be used, with F replacing K;

$$EI \frac{d^4y}{dx^4} - F \frac{d^2y}{dx^2} = p(x) \qquad (15)$$

Equation (15) may be considered as the basic differential equation governing the interaction of shearwalls of a constant bending stiffness EI, with rigid frames of a constant modified stiffness F. It may be readily applied to the case of variable stiffness, both of walls and frames. In such cases equ. (15) will have to be solved for each range over which those stiffnesses are constant.

Conceptual Application of the Analogy

A clear visual illustration of the behavior of the frame-shearwall system, based on the analogy presented above, can be obtained from the following consideration. For a rigid cantilever, subjected simultaneously to transverse loading and longitudinal tension, the bending moment can be directly expressed as (Fig. 7):

$$M_X^{EI} = -EI \frac{d^2y}{dx^2} = M_X^o - T(Y_X - Y_H) \tag{16}$$

where: M_X^o, is the moment due to the external transverse loading only, and T is the applied tension force.

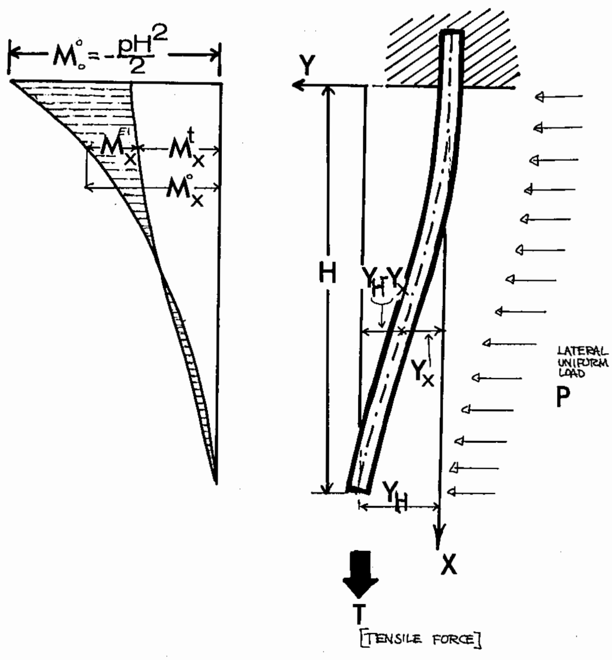


Figure 7 - A cantilever subjected to axial tension and transverse loading

M_x^w, the bending moment in a shearwall, I^w denoting the moment of inertia of its cross-section, is similarly expressed on the basis of the analogy by:

$$M_x^w = -EI^w \frac{d^2y}{dx^2} = M_x^o - F(Y_x - Y_H) \qquad (17)$$

where M_x^o is again the moment due to the external transverse loading and F is the modified stiffness of the frame, as derived by equ. (14). The restraining effect of the frame on the distribution of the bending moments in the shearwall is clearly brought to light by the second term of the right side of equ. (17). The magnitude of this restraint considerably varies with the height of the building and the ratio between the stiffness of the frame and that of the shearwall. For a low building, with relatively small lateral displacement, the second term of equ. (17) is small compared with the first; thus horizontal loads are mainly resisted by the action of the shearwall alone. The taller the building, the more pronounced becomes the restraining effect of the frames.

To arrive in general terms at the expressions for the respective internal forces acting in the frame and the shearwalls, let us solve first the differential equation (10) or (16) for an analogous tensioned cantilever. The equ. (16), as can be seen when comparing with Fig. (7), may be rewritten as:

$$M_x^o = M_x^{EI} + M_x^T \qquad (18)$$

which expresses the total moment due to external loading, M_x^o, as the sum of the bending moment M_x^{EI} in the tensioned beam and M_x^T, the moment caused by the axial tension force T. The first term on the right side of equ. (18) was given in equ. (16) as

$$M_x^{EI} = -EI \cdot \frac{d^2y}{dx^2} \qquad (19)$$

and the second term as

$$M_x^T = T(Y_x - Y_H) \qquad (20)$$

Assuming the external moment M_x^o to be caused by a uniformly distributed horizontal loading p, we get for it

$$M_x^o = -\frac{p(H-x)^2}{2} \tag{21}$$

With these expressions, the solution of the aforementioned differential equations yields (after recognizing the boundary conditions that are evident from Fig. 7):

$$Yx = \frac{p}{Tk^2 \,\mathrm{Cosh}\,(kH)} \left[K^2 \,(Hx - \frac{x^2}{2}) \;\mathrm{Cosh}\,(kH) + \mathrm{Cosh}\,(kx) + kH\mathrm{Sinh}\left[k\,(H-x)\right] - kH\mathrm{Sinh}\,(kH) - 1 \right] \tag{22}$$

where $$K^2 = \frac{T}{EI} \tag{23}$$

With the solution for the horizontal displacements of the tensioned beam given by equ. (22), the bending moment in the beam caused by the transverse uniform loading is obtained as:

$$M_x^{EI} = -EI\,\frac{d^2y}{dx^2} = -\frac{p}{k^2\,\mathrm{Cosh}\,(kH)} \left[kH\mathrm{Sinh}\,\left[k\,(H-x)\right] + \mathrm{Cosh}\,(kx) - \mathrm{Cosh}\,(kH) \right] \tag{24}$$

The moment caused by the axial tension T is given by

$$M_x^T = T\,(y_x - y_H) = \frac{p}{k^2\,\mathrm{Cosh}\,(kH)} \left[kH\mathrm{Sinh}\,\left[k\,(H-x)\right] - \mathrm{Cosh}\,(kH) + \mathrm{Cosh}\,(kx) - 1/2k^2\,(H-x)^2\,\mathrm{Cosh}\,(kH) \right] \tag{25}$$

When adding equs. (24) and (25), as a check, equ. (18) is indeed satisfied, resulting in the expression for the external moment as given by equ. (21).

The two component parts of the shearforce are S_x^{EI} and S_x^T, the latter representing the component of the internal axial tensile force in the direction perpendicular to the initial axis of the beam.

$$S_x^{EI} = \frac{dM_x^{EI}}{dx} = -EI\,\frac{d^3y}{dx^3} = -\frac{p}{k\mathrm{Cosh}\,(kH)} \left[\mathrm{Sinh}\,(kx) - kH\mathrm{Cosh}\left[k\,(H-x)\right] \right] \tag{26}$$

$$S_x^T = T\frac{dy}{dx} = \frac{p}{k\text{Cosh}(kH)}\left[k(H-x)\,\text{Cosh}(kH) + \text{Sinh}(kx) - kH\text{Cosh}\left[k(H-x)\right]\right] \quad (27)$$

It is readily seen again that when these two parts of the shearforces are added, the total shear is obtained, namely:

$$S_x^{EI} + S_x^T = p(H-x) \quad (28)$$

Equs. (24) to (27) thus give the components of moments and shears in the tensioned cantilever beam.

The analogy, that was presented here, stated that the shearwall-frame system behaves exactly as does the tensioned cantilever beam; with the modified frame stiffness, F, of the former being analogous to the cable tension T in the latter. Similarly, the bending stiffness of the shearwall EI^W is analogous to that of the cantilever beam EI.

With these analogous substitutions introduced into equ. (24), the bending moment in the shearwall is obtained. Similarly equ. (25) will represent the total bending moment in the rigid frames; equ. (26) the shearforce in the wall, and equ. (27) the total shearforce in the frame. All these expressions are comparatively summarized in Table I.

A Physical Model of the Analogy

The first models that simulate the behavior of shearwall-frame systems, according to the analogy presented above, were built at the Graduate School of Design of Harvard University and at the Department of Architecture of the Massachusetts Institute of Technology (Figs. 8 and 9). The models consist of vertical strips of cast thermoplastic acrylic resin (Plexiglass) which can be subjected simultaneously to horizontal loading and vertical longitudinal tension. The horizontal loading represents the scaled load acting on the prototype shearwall-frame system; the tensile force represents the scaled rigidity of the frames. The bending stiffness of the Plexiglass strips is the scaled value for the stiffness of the shearwalls.

To facilitate the application of loads and the testing, the model is placed in its supporting frame in an upsidedown position, its support at the top representing the foundation of the prototype structure. The bottom of the model, at which the tensile force is applied, represents the uppermost level at the top of the real structure. The vertical strips of the model are joined by horizontal connectors to which horizontal and vertical loading can be

TABLE 1: BASIC ANALOGY

Tensioned Cantilever Beam		Shearwall - Frame System	
	Stiffness: EI		EI_W
k	T (axial tension) $\sqrt{\frac{T}{EI}}$	$\bar{k}$	F (modified frame stiffness $\sqrt{\frac{F}{EI_W}}$
M_x^{EI}	$-\frac{p}{k^2\cosh(kH)}\left[kH\sinh k(H-x) + \cosh(kx) - \cosh(kH)\right]$ (transverse bending moment in beam)	M_x^W	$-\frac{p}{\bar{k}^2\cosh(\bar{k}H)}\left[\bar{k}H\sinh \bar{k}(H-x) + \cosh(\bar{k}x) - \cosh(\bar{k}H)\right]$ (bending moment in shearwall)
S_x^{EI}	$-\frac{p}{k\cosh(kH)}\left[\sinh(kx) - kH\cosh k(H-x)\right]$ (shear force due to transverse loading)	S_x^W	$-\frac{p}{\bar{k}\cosh(\bar{k}H)}\left[\sinh(\bar{k}x) - \bar{k}H\cosh \bar{k}(H-x)\right]$ (shear force in wall)
M_x^T	$\frac{p}{k^2\cosh(kH)}\left[kH\sinh k(H-x) + \cosh(kx) - \cosh(kH) - \frac{1}{2}k^2(H-x)^2\cosh(kH)\right]$ (moment caused by tension force)	M_x^F	$\frac{p}{\bar{k}^2\cosh(\bar{k}H)}\left[\bar{k}H\sinh \bar{k}(H-x) + \cosh(\bar{k}x) - \cosh(\bar{k}H) - \frac{1}{2}\bar{k}^2(H-x)^2\cosh(\bar{k}H)\right]$ (total moments in all frames)
S_x^T	$\frac{p}{k\cosh(kH)}\left[k(H-x)\cosh(kH) - kH\cosh k(H-x) + \sinh(kx)\right]$ (transverse shear due to tension force)	S_x^F	$\frac{p}{\bar{k}\cosh(\bar{k}H)}\left[\bar{k}(H-x)\cosh(\bar{k}H) - \bar{k}H\cosh \bar{k}(H-x) + \sinh(\bar{k}x)\right]$ (total shear force in all frames)

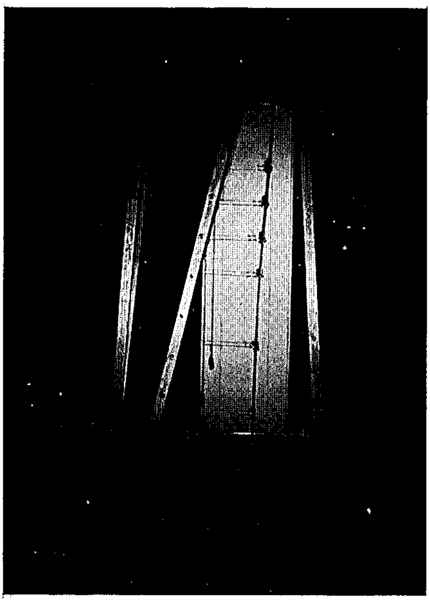


Figure 8 - Model of the analogy

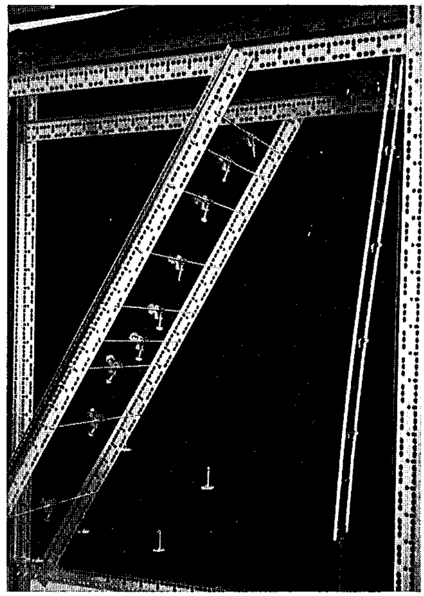


Figure 9 - Loading of the model

applied. Thus the vertical strips have unobstructed surfaces, for measurements and for attaching to them additional vertical strips, so as to simulate any possible variation in the rigidity of the prototype shearwalls. Application of an appropriate vertical tensile force at any one of these connectors would correspond to a change of stiffness of the frames at that level in the prototype structure. At the top support of the model a clamping device permits rotating of the axis of the model, thus simulating any elastic restraint of the prototype shearwalls in their foundation.

Both models were constructed of two 1″ wide, 1/4″ thick, and 5′-0″ long Plexiglass strips (Fig. 10). As is evident from Table I, the distribution of shearforces and moments among the frames and walls is a function of the dimensionless parameter $kH = \frac{F}{EI^W} H^P$. Its magnitude determines the characteristic pattern of deformation of the entire structure. The analogous parameter to this in the axially-tensioned cantilever beam has the dimensionless value $kH = \frac{T}{EI} H^M$. The model, which forms such an axially-tensioned cantilever beam must, therefore, have its parameter $(kH)_M$ equal to that of the prototype (frame-wall) structure $(\bar{k}H)_P$:

$$(kH)_M = (kH)_P \tag{29}$$

For the prototype structure, from Table I

$$(\bar{k}H)_P = \frac{F}{EI^W} H^P \tag{30}$$

where F is the modified frame stiffness, as determined from equ. (14), I^W the moment of inertia of the shearwalls, and H^P the height of the building.

For the model

$$(kH)_M = \sqrt{\frac{T}{E^M I^M}}\, H^M \tag{31}$$

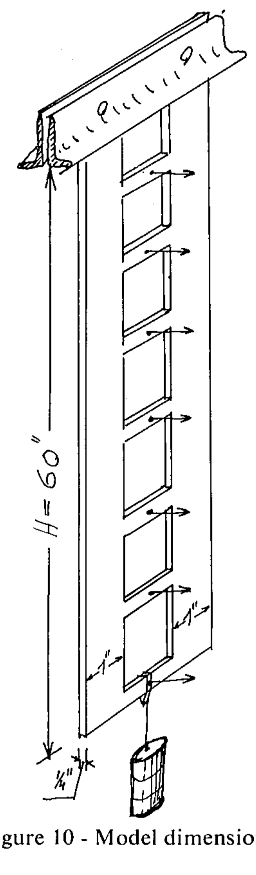


Figure 10 - Model dimensions

where T is the applied axial tension force, E^M — the modulus of elasticity of the model material, and I^M the moment of inertia about the axis perpendicular to the plane of bending. H^M is the height of the model. Thus the required tensile force, T, to be applied in the model, is established from equation (31) as a function of the parameter kH.

For the models built at the Harvard and M.I.T. workshops we have:

$$E^M = 450{,}000 \text{ psi}$$

$$I^M = 2 \times \frac{1.0 \times 0.25^3}{12}$$

$$E^M I^M = 1172 \text{ lbs-in}^2$$

$$H^M = 60 \text{ in.}$$

Introducing this into equ. (31), we get

$$(kH)_M = \sqrt{\frac{T}{1172}}\, 60 \qquad (32)$$

or

$$T = \frac{(kH)_M^2}{3.08} \quad [\text{lbs.}] \qquad (33)$$

which becomes, considering equ. (29),

$$T = \frac{(kH)_P^2}{3.08} \quad [\text{lbs.}] \qquad (34)$$

Thus equ. (34) gives for any value of the parameter $(kH)_P$ of the prototype structure, the required tensile force T to be applied on the model described above.

From the deformation of the model, subjected simultaneously to the longitudinal tension and a horizontal loading which represents the scaled value of wind or earthquake loads on the prototype, its structural behavior can be studied. A deformation curve which resembles that of Fig. 1.d., pure wall action, will indicate the fact that most of the external moment is resisted by the shearwalls. A curve that shows substantial deviation from that of Fig.

1d, and approximates that of Fig. 1.b., indicates the preponderance of frame action.

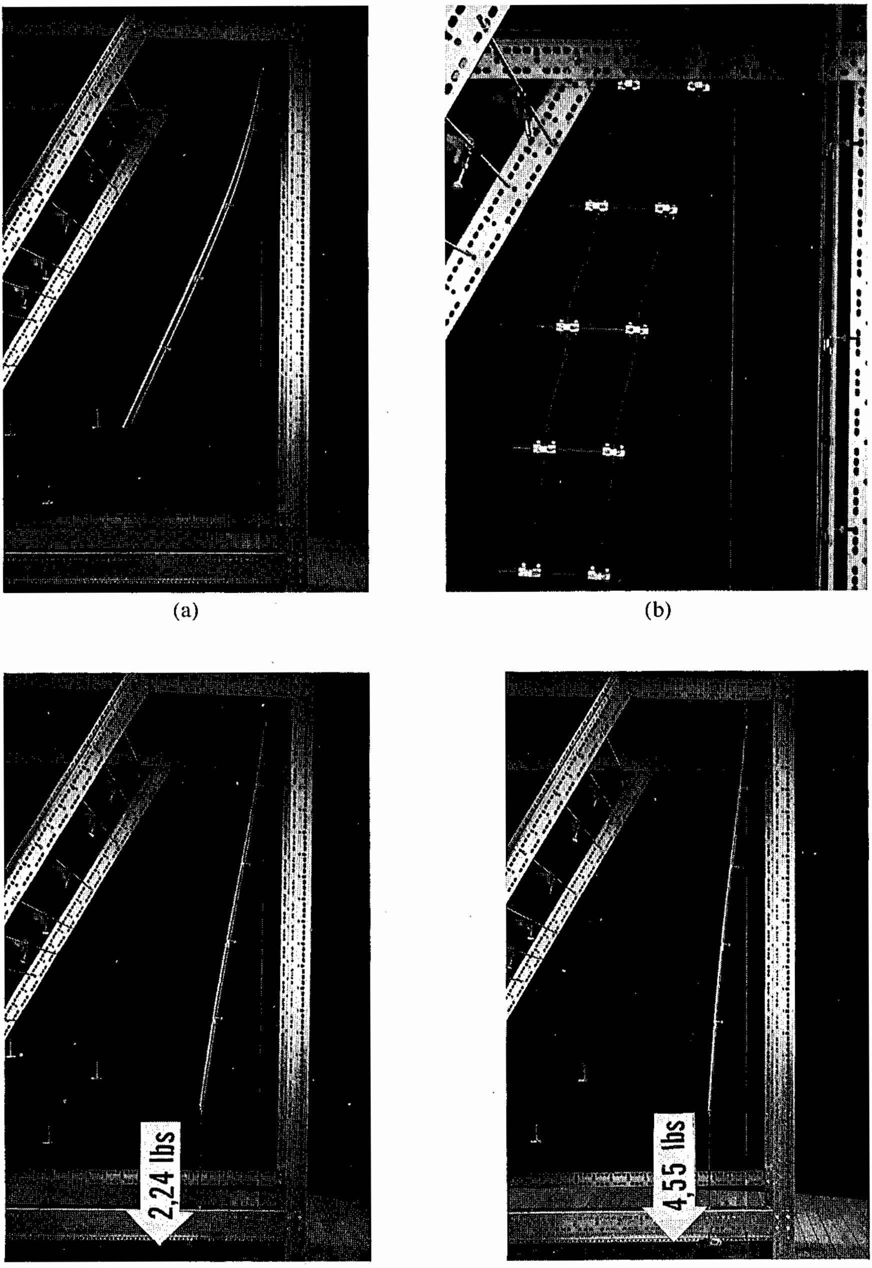
A glance at the deformation of the model immediately reveals its structural behavior. (Fig. 11) shows a series of photos of the Harvard-M.I.T. model under different relative frame-shearwall-height proportions. Fig. 11a shows the horizontal displacement simulating the action of a shearwall alone, a convex curve when viewed from the right (center of curvature to left of curve). Fig. 11b shows that of a rigid frame alone, a concave curve when viewed from the right (center of curvature to right of curve). Figs. 11c to 11f show the simulated combined effect of interacting frames and shearwalls. Each of these figures represents a progressively larger kH value. Corresponding floor plans and building sections are shown in the Appendix, in which the analogous tensile loadings on the model also are computed. It becomes apparent that, the larger the applied tension force, the closer the curve becomes to a concave one, when viewed from the right. It is, however, of interest to note that in all the cases shown in Figs. 11c to 11f, there is a point of inflection on the curve. This indicates that the lower part of the building, close to the foundation, in all cases resists the horizontal loads by shearwall-action, even though the upper part might exhibit predominant frame-action.

For comparison with these experimental results it is of interest to investigate two extreme cases of structural behavior: that of predominant wall action, i.e. larger kH values. Fig. 12a gives a graph for the displacement y of a structure for which $kH = 1$. The moments and shearforces, computed by the analogy presented here for a tensioned beam with the expressions summarized in Table I, are also shown in the same figure.

Figure 12a illustrates the relative contribution of the frame and the shearwall to the deflection pattern, by examining the two extreme cases, namely, that of $F = 0$, i.e., no frame, and that of $EI^W = 0$, i.e. no shearwall. It is noted that in this case the system deforms predominantly as a cantilever; in other words, the frame takes only a very minor part of the applied load.

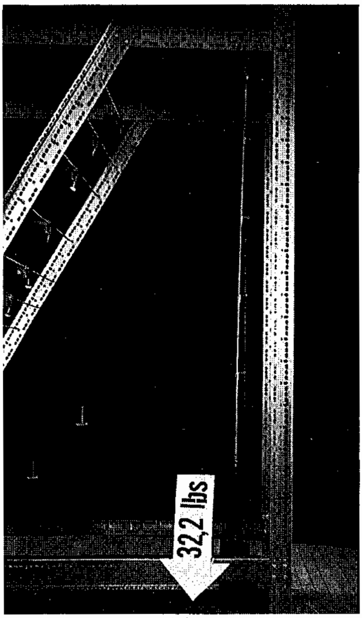
On the other hand, for a fairly tall building, say $kH = 10$ as shown in Fig. 12b, it is readily seen that the deflection pattern is considerably affected by frame action and resembles that of a transversely loaded cable. Again, the deflection pattern for the two extreme cases, that of no frame and that of no shear wall respectively, makes this apparent and shows the predominant effect of frame action in tall buildings.

The model described above, can be used for a detailed determination of the distribution of lateral loads among frames and walls of a real building

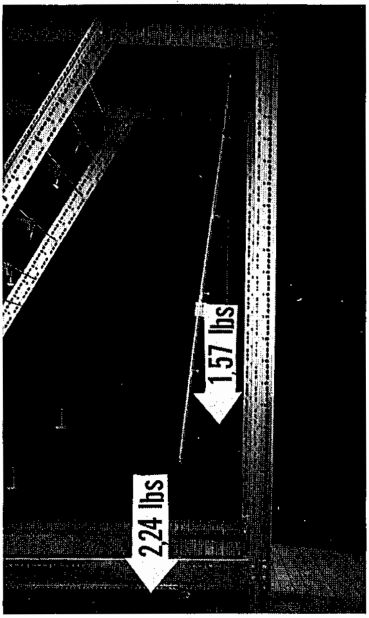


(a) (b)

(c) (d)



(e)



(f)

Figure 11 - Model deformation under loading

represented by it. To do this, its deformation must be measured with accuracy. One method for accomplishing this would be to measure, by strain gauges or other deformometers, the curvatures of the model. This should first be established for the model subjected to the scaled horizontal loading only (Fig. 13a). The curvature $(1/R)^o$ measured would correspond to the external bending moment M^o on the real structure (Fig. 13b). After the model is subjected, in addition to the horizontal loading, also to the vertical tension, simulating the frame action, the curvatures of the model $(1/R)^w$ will be measured again, Fig. (13a). The bending moments M^w in the shear wall of a real structure are then established in proportion to the measured curvatures, as shown in Fig. (13b).

$$M^w = M^o \frac{(1/R)^w}{(1/R)^o} \qquad (35)$$

Another method for applying the experimental results to design calculations of the interaction of frames and walls in a building is the measuring of the horizontal displacements of the model from its original vertical position

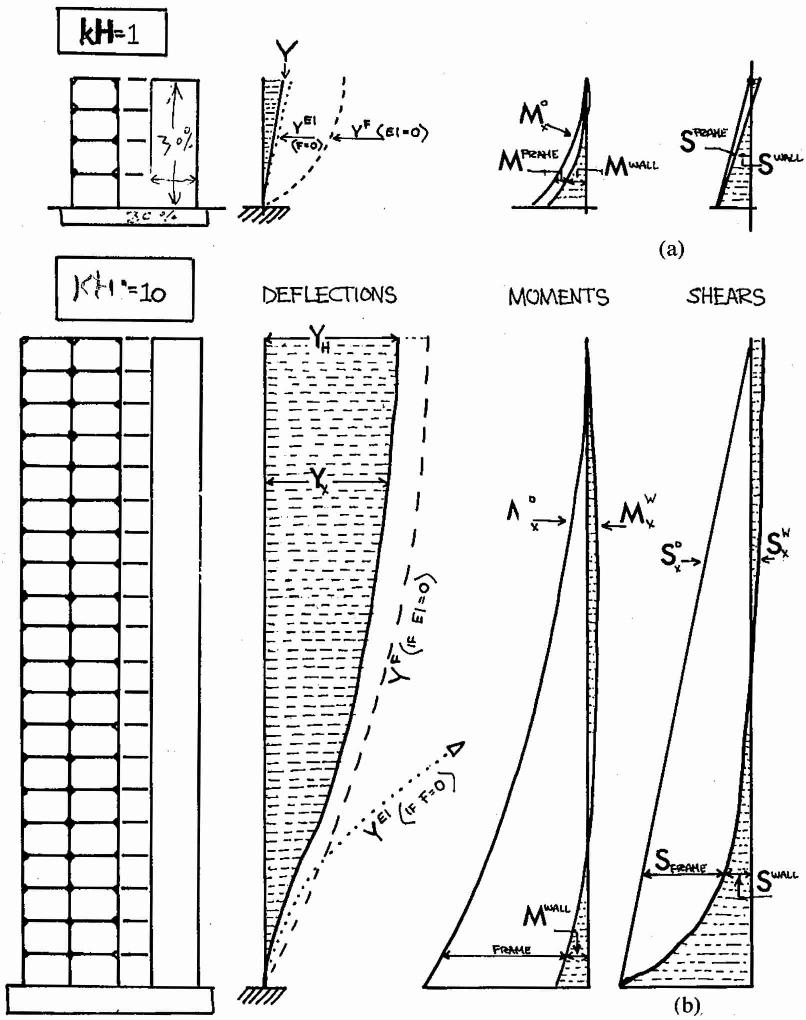


Figure 12 - Deformation, Moments and Shears of Shearwall-Frame Systems

(Fig. 7.). The bending moment MF resisted by frames of the prototype structure is then established from equ. (20), adjusted for its scale factor.

$$M^F = T \cdot (Y_H - Y_x) \cdot \left(\frac{H^P}{H^M}\right)^2 \cdot \frac{p^P}{p^M} \qquad (36)$$

Such measurements have not yet been taken on the relatively crude models described; their results will be reported when available.

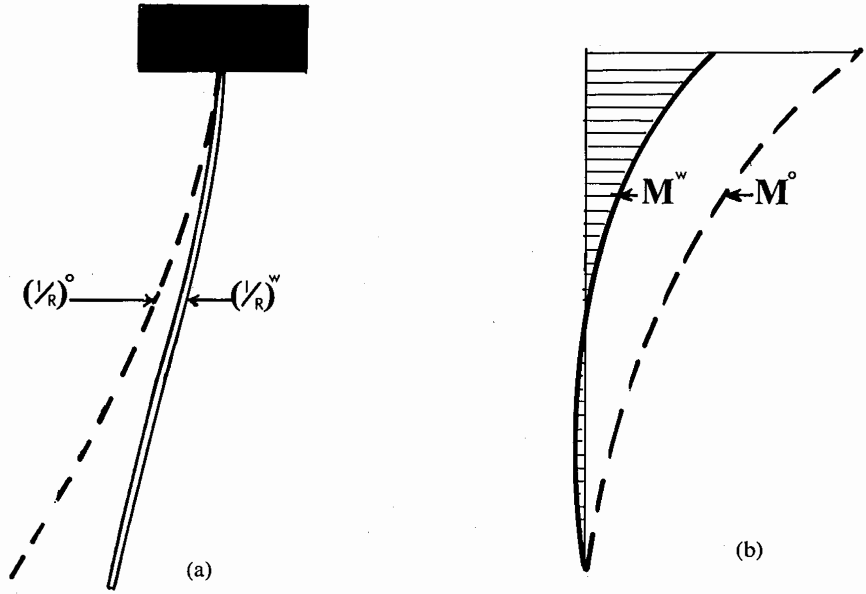


Figure 13 - Model curvature and related bending moments

In Conclusion

All existing methods for determining the distribution of horizontal forces among shearwalls and frames require lengthy numerical or computer substitution of the physical constants into equations, of which the right part of Table 1 demonstrates only a special case. The particular advantage of the method presented here is that it allows the development of a simple experimental procedure which can yield the results directly. From the measurements on the model, the shearforces and moments can be readily derived; and these yield, with the appropriate similitude factor, the corresponding values for the prototype structure.

A single physical model suffices to simulate the behavior of any structure consisting of shearwalls and frames. The model, as presently used in the illustrations above, presupposes a symmetrical position of the walls and frames in the building. It can, however, be adjusted so as to simulate the behavior of assymmetrical buildings also. By changing the position and intensity of the tensile load applied on the model, any frame stiffness can be represented. By changing the cross-section of the model material, any wall stiffness or variation in it can be represented.

The value of such an analogy as a visual tool cannot be overemphasized. The loaded model depicts practically at a glance the interaction between frames and walls. The shape of the deflection curve indicates directly whether the horizontal loadings are resisted mainly by wall or by frame action. Measurements on the model can yield values for the moments and shears in the individual elements with an accuracy that is useful for design purposes. The particular advantage of any such visual design tool is a considerably reduced chance for gross design errors.

The method, developed here, is being extended now to serve as a general design tool for the estimate of interaction of walls and frames in any multistory building. It becomes evident that the extension of this analogy to assymmetrical buildings will show additional merits of this method in offering a simple solution to normally complex cases.

NOTATIONS

E	modulus of elasticity
E^M	modulus of elasticity of model
F, F_i	modified stiffness of rigid frame
H, H^p	height of building
H^M	height of model
h_i	height of story i
I	moment of inertia
I^c	moment of inertia of column
I^g	moment of inertia of girder
I^M	moment of inertia of model
I^w	moment of inertia of shear wall
K	Spring constant of rotational restraint
K^c	rigidity of column
K^g	rigidity of girder
kH	dimensionless parameter
$(kH)_p$	dimensionless parameter of a prototype building
$(kH)_M$	dimensionless parameter of model

L	span of a girder in frame
M, M_x, M_x^p	bending moment in a rigid element
M_x^{EI}, M_x^w	bending moment in beam or wall
M_x^T	part of total moment equilibrated by eccentricity of tensile force
M_x^F	part of total moment resisted by frame
M_x^o	total moment due to external loading
M_x^r	bending moment in R.R. beam due to restraints
m_x^r	continuous moment of rotational restraints
P	transverse loading per unit length
P^p	transverse loading per unit length of real structure
P^M	transverse loading per unit length of model
S_i	shear force acting in story i
S_x^{EI}	shear force in wall or tensioned beam
S_x^T	shear force component equilibrated by tensile force
T	tensile force
$\Delta_i^{col}Y$	relative horizontal displacement in rigid frame due to columns only
$\Delta_i^{F}Y$	relative horizontal displacement in rigid frame
$\Delta_i^{c}Y$	relative horizontal displacement in transversely loaded tensioned cable
ϕ	angle of rotation of R.R. Beam

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APPENDIX

EXAMPLES

A building structure, (Figure 14) composed of 8 frames and 2 shear walls is used in the five following examples representing five characteristic cases of application of the analogy presented.

EXAMPLE 1

A 26-story-high building, (Figure 15) with structural layout and R.C. walls and frames having dimensions as shown in Figure 14.

Modulus of elasticity of concrete $E_c = 3 \times 10^6$ psi.

Mom. of inertia of column 16 col.'s C_1 at one level. $C_1;\ I_1^C = \frac{12^4}{12} = 1728\ \text{in}^4;$

Mom. of inertia of column 8 col.'s C_2 at one level. $C_2;\ I_2^C = \frac{12 \cdot 18^3}{12} = 5832\text{in}^4;$

Mom. of inertia of girder 16 girders G at one level. $G;\ I^g = \frac{12 \times 20^3}{12} = 8000\text{in}^4;$

$h = 10' = 120''$; $L = 15' = 180''$;

$$\sum \frac{I^C}{h} = 16 \times \frac{1728}{120} = 230.4\ \text{in}^3$$

+

$$\sum \frac{I^C}{h} = 8 \times \frac{5832}{120} = \underline{\underline{388.8\ \text{in}^3}}$$

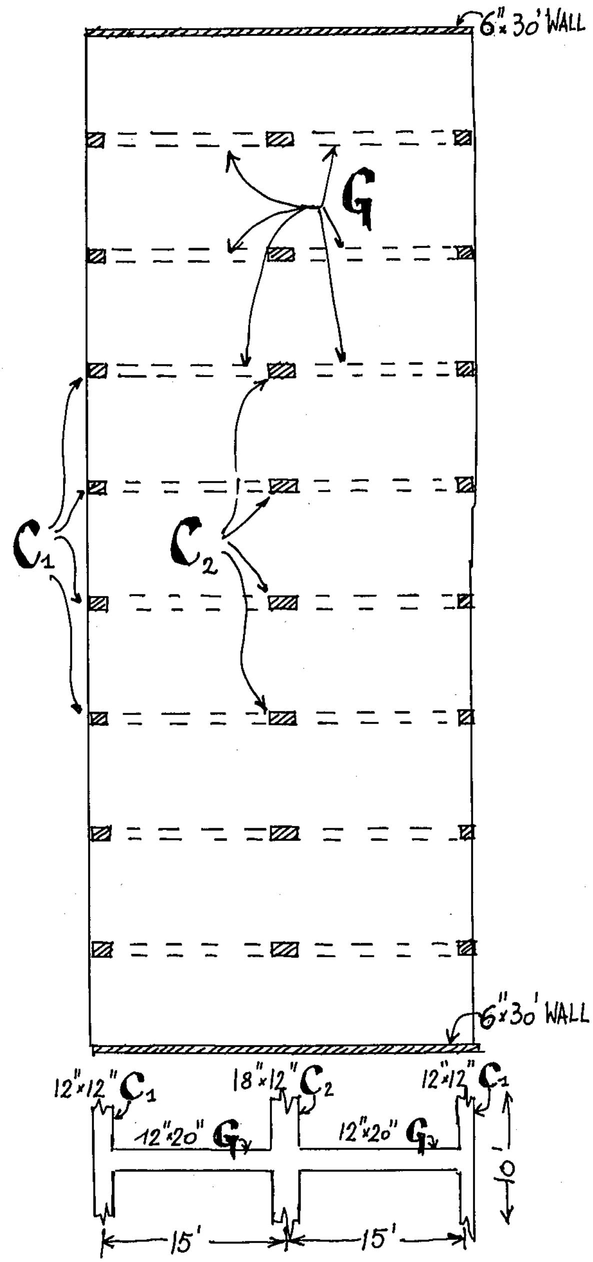

Figure 14

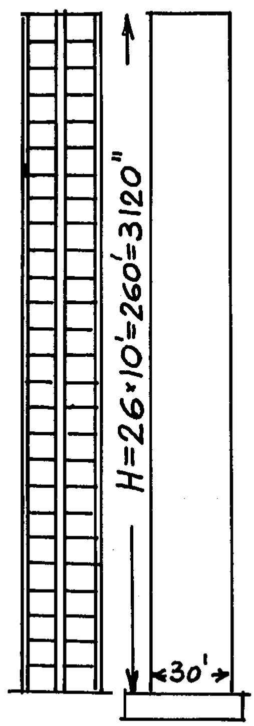

Figure 15

$$\sum K_i^C = \sum \frac{I_i^C}{h} = \underline{619.2 \text{ in}^3}$$

$$\sum K_i^g = \sum \frac{I_i^g}{L} = 16 \times \frac{8000}{180} = \underline{711.1 \text{ in}^3}$$

These values are introduced into formula (14)

$$F = \frac{12E}{h\left[\frac{1}{\Sigma \frac{I^c}{h}} + \frac{1}{\Sigma \frac{I^g}{L}}\right]} = \frac{12E_c}{120\left[\frac{1}{619.2} + \frac{1}{711.1}\right]} = \underline{33.1 \times E_c \text{ [lbs.]}}$$

EI^W – value for two R.C. shear walls:

$$EI^W = E_c \times 2 \, \frac{6 \times (30 \times 12)^3}{12} = \underline{E_c \times 46.6 \times 10^6 \text{ lb} - \text{in}^2}$$

$$H = 26 \times 10' - 260 \text{ ft.} = 3120''$$

$$(kH)^2 = \frac{F}{EI^W} \, H^2 = \frac{33.1 \cdot E_c}{46.6 \times 10^6 E_c} \times 3120^2 = \underline{6.91}$$

According to equ. (34) the tensile load to be applied at the end of the M.I.T.-Harvard models is equal

$$T = \frac{(kH)^2_P}{3.08} = \frac{6.91}{3.08} = \underline{\underline{2.24 \text{ lbs}}}$$

Figure 11.c demonstrates this case.

EXAMPLE 2

A 37-story-high building, Figure 16, with a typical layout and structure as in Example 1.

Then $\underline{F = 33.1 \times E_c}$

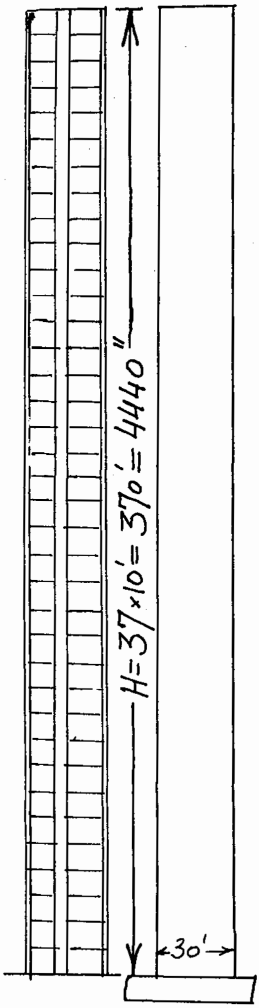

Figure 16

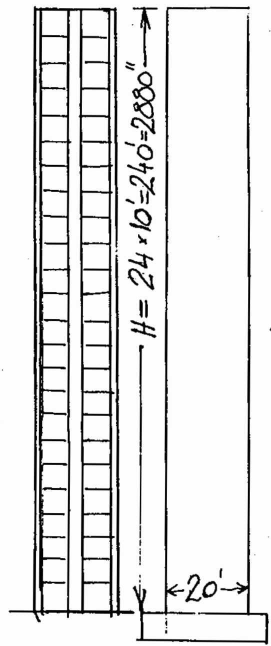

Figure 17

$$\underline{EI^W = 46.6 \times 10^6 E_c}$$

$$H = 37 \times 10' = 370' = 4440''$$

$$(kH)^2 = \frac{F}{EI^W} H^2 = \frac{33.1 E_c}{46.6 \times 10^6 E_c} \times 4440^2 = \underline{14.0}$$

Accordingly the tensile load to be applied on the model is:

$$T = \frac{14.0}{3.08} = \underline{\underline{4.55 \text{ lbs}}}$$

Figure 11.d demonstrates this case.

EXAMPLE 3

A 24-story-high building, Figure 17, with a typical layout, two 6″ thick and 20′ wide R.C. shearwalls and steel frames. Moments of Inertia of all frame members are equal to half the corresponding values for R.C. frames from Examples 1 and 2. Modulus of elasticity of steel; $E_s = 10\ E_c$.

$$\text{Then } F = \frac{1}{2} \cdot 33.1 \times E_s = \frac{1}{2} \times 33.1 \times 10\ E_c = \underline{165.5\ E_c}$$

$$EI^W = E_c \cdot 2\ \frac{6 \times (20 \times 12)^3}{12} = \underline{13824 \times 10^3 \cdot \left[E_c \text{ lb} - \text{in}^2\right]}$$

$$H = 24 \times 10' = 240' = 2880''$$

$$(kH)^2 = \frac{F}{EI^W}\ H^2 = \frac{165.5\ E_c}{13824 \times 10^3\ E_c} \cdot 2880^2 = \underline{\underline{99.5}} \simeq (10)^2. \quad \text{(As 12b)}$$

Accordingly the tensile load to be applied on the model is:

$$T = \frac{99.5}{3.08} = \underline{32.2 \text{ lbs}}$$

Figure 11e demonstrates this case. See also the diagrams Fig. 12b.

EXAMPLE 4

A 30-story-high building, Figure 18a, with a typical layout, showing R.C. frames (as in Example 1), and two 6″ thick R.C. walls — 30 feet wide in lower part of the building 0 to 15 level, and 15 feet wide from 15 level to the top of the building.

$$\text{Then } F = 33.1 \cdot E_c \text{ [lbs]}$$

EI^W value for two shearwalls in the upper part of the building.

$$EI^W_{up} = E_c \times 2\ \frac{6 \times (15 \times 12)^3}{12} = \underline{E_c \times 5.83 \times 10^6 \text{ [lb} - \text{in}^2\text{]}}$$

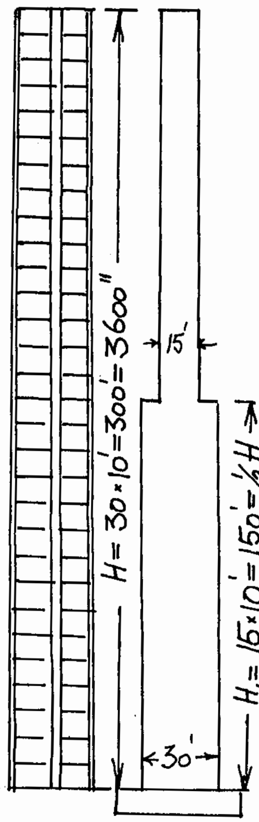

Figure 18(a)

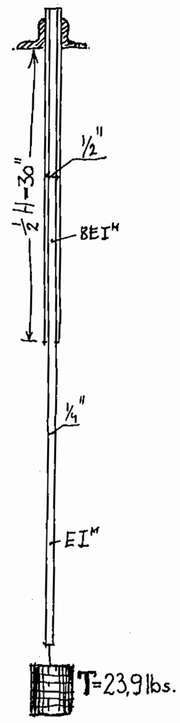

Figure 18(b)

$$H = 30 \times 10' = 300' = 3600''$$

Taking for reference the rigidity parameters in the upper part of the building as corresponding to the properties of the models with their standard dimensions, we obtain:

$$(kH)^2 = \frac{F}{EI^{w}_{up}} H^2 = \frac{33.1\ E_c}{5.83 \times 10^6\ E_c} \cdot 3600^2 = \underline{73.7}$$

Tension in model: $T = \dfrac{73.7}{3.08} = \underline{23.9\ \text{lbs}}$

To represent the variation of the rigidity of the shearwall in the scale model, its upper half must be strengthened accordingly, (Fig. 18b). The tensile force applied on the model is constant, since the rigidity of the frames — to which it is analogous — is constant for the entire height of the building.

EXAMPLE 5

A 26-story-high building, Figure 19, with a typical layout showing R.C. shearwalls (as in Examples 1, 2). The R.C. frames in the upper part of the building, from level 16 to 26, have the same dimensions, and consequently, the same value for F as in Examples 1 and 2.

In the lower part of the building, all members of the frames have rigidities 1.7 times larger than those of the upper standard frames.

Then $F_{up} = 33.1 \times E_c$ (as in Examples 1 and 2)

$$F_{low} = 1.7 \times 33.1 \times E_c = 56.27 \; E_c$$

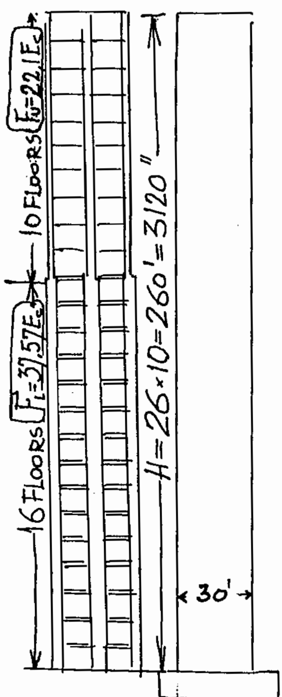

Figure 19

$$EI^{W} = 46.6 \times 10^{6} \ E_c$$

$$H = 26 \times 10' = 260' = 3120''$$

Taking for reference the rigidity parameters in the upper part of the building, the tensioning force in the corresponding lower part of model will be determined as for the case of constant rigidity of frames.

$$(kH)^2 = \frac{F_{up}}{EI^{W}} H^2 = \frac{33.1 \times E_c}{46.6 \times 10^{6} \ E_c} 3120^2 = 6.91$$

The tension to be applied at the bottom of the model:

$$\underline{T_1} = \frac{6.91}{3.08} = \underline{2.24 \text{ lbs}}$$

The tension in the upper part of the model, corresponding to the lower part of the real building, ought to be increased in proportion to the increased rigidity of the real frame in this part of the structure.

Then $T_2 = T_1 \dfrac{F_{low}}{F_{up}} = 2.24 \times 1.7 = \underline{3.81 \text{ lbs}}$

Thus, at the upper part of the model, the end tensile force of $T_1 = 2.24$ lbs must be augmented by

$T = T_2 - T_1 = 3.81 - 2.24 = 1.57$ lbs.

The additional weight of 1.57 lbs has been applied to the model at the level corresponding to that at which the real structure exhibits a change in frame rigidity, Fig. 11.f.