DISPERSION PHENOMENA IN COASTAL ENVIRONMENTS

by KLAS CEDERWALL *

(1970 John R. Freeman Memorial Lecture presented before the Boston Society of Civil Engineers and the Hydraulics Section on January 28, 1970)

Introduction

The disposal to the sea of man-produced wastes — sewage, heat, etc. — gives rise to many questions of a scientific nature. Complete removal of all the pollutants from the waste water is seldom practical because of high costs and lack of adequate technology. Hence, the physical aspects of water pollution problems are of great significance due to the fact that knowledge of the processes by which the waste water is mixed and dispersed in the receiving water is required for evaluation of pollutant levels. Knowing the natural processes of transportation and dilution of the disposed waste products, we have a basis for controlling hygienic and aesthetic nuisances as well as ecological disturbances. Studies of the hydrology and hydrodynamics of lakes, rivers and coastal environments exposed to waste discharge are therefore a fundamental feature of water resources planning and water quality management. Let us consider two common pollution problems.

A complex engineering problem in the field of water pollution control is to select the best site for a future outlet of sewage water. Several alternative sites usually have to be investigated as to their efficiency for dispersion of effluent. A qualitative judgment must be based on a statistical presentation of the result of the study. A statistical description of the problem can generally not be obtained without comprehensive hydrological surveys to establish the dispersive properties of the environment. Adequate information is usually required not only on the physical but also on the chemical and biological characteristics of the affected area for which water quality standards have to be established. Hence, such field studies turn out to be most costly and time consuming. This necessitates a systematic approach not only for the planning of the investigation but also for the future water quality control management.

Let us turn to another problem of current interest. For the successful operation of a thermal power plant located on the coast, an essential aspect concerns the effect on the sea caused by the discharge of warmed condenser

---

* W. M. Keck Laboratory of Hydraulics and Water Resources, California Institute of Technology.

water. Thermal pollution of the sea and recirculation of warmed condenser water are affected by several factors, for instance, distance between intake and outlet, the heat budget of the sea, and finally to a substantial degree also by the spreading behaviour of the warmed condenser water. Much attention has been given in Sweden, as in many other countries, to studies on the potential of the coastline for siting of planned thermal power plants, and various approaches for analyses of spread characteristics of the discharged cooling water have been considered. This would ultimately lead to design criteria for intake and discharge structures.

Scope and Objectives

In this paper we will discuss some approaches for studying dispersion phenomena in coastal environments. The complexity of the problem necessitates a review of applicable methodology. Because of the multitude of factors that usually have to be considered, a systematic approach to the control of water pollution is required. Since mathematical models are widely used in hydrology for prototype simulation, an objective of this paper is to present some principles for mathematical modeling of dispersion phenomena. Some advances in field measuring techniques are reviewed as they apply to water pollution studies.

It should be emphasized that the conditions for marine disposal of waste water in the Scandinavian waters, with which this study is most concerned, are very often quite different from conditions in other parts of the world. This is due to the fact that these water areas exhibit some unique hydrographical as well as topographical features. The fjord and the archipelago, for instance, are significant elements of our coast, and the Baltic Sea is an important feature of the salt water and fresh water balance for the whole area.

Until recently, only limited experience from rational waste disposal practice in Scandinavian waters has been gained. Worth mentioning is a permanent interstate committee which was founded in 1960 for planning and execution of a scientific program for investigations in the sound between Denmark and Sweden. The Norwegian Institute for Water Research has carried out extensive investigations in the Oslofjord, and monitoring schemes for conceivable major expansions of marine waste disposal systems are established for the municipalities of Stockholm and Gothenburg. Of special oceanographic interest is the Baltic Sea and concentrated effort is devoted to field studies and surveys in order to gain a better understanding of the general hydrography of this huge body of brackish water.

Mode of Analysis

It goes without saying that a complete physiographical description of a lake or a coastal water area is not possible to obtain. This is strictly true also for a more well defined process such as, for instance, the spread of a pollutant from a waste outfall. Every approach to study the characteristics of a receiving water area has to introduce some simplifications. The degree to which a model of the prototype — analytical or experimental — represents the true conditions is dependent on the assumptions introduced concerning prototype behaviour.

There are few areas of engineering activity which involve as many variables as are found in applied hydrology, and notably in water pollution studies. The computerized mathematical models, as well as the use of automatic data processing systems, are therefore becoming extremely valuable. It is, however, essential that the investigator be aware of the limitations inherent in his choice of approach to a study of the problem, and how they affect the ultimate result. Sometimes even the applied numerical procedure for solving the governing equations introduces errors which can destroy the physical significance of certain important parameters, e.g. pseudo-dispersion.

There are several alternative methods for the prediction of the dispersion pattern in the receiving water area. Four main approaches may be distinguished.

a) A purely theoretical analysis supported by general experience on the diffusive properties, circulation and exchange conditions of the water area in question.

b) The same as under a), but with supplementary field surveys to establish characteristic prototype behaviour.

c) Tracer technique for direct in-situ simulation of transport and mixing of the waste effluent.

d) Scale tests by means of a hydraulic model.

In the future, when the next generation of electronic computers are available, we will probably use advanced theoretical flow models for computer simulation of dispersion phenomena.

Sometimes the possibilities to use a natural simulation technique are limited because adequate tracer field tests are not possible. For instance, in the case of the design of a cooling water system for a planned thermal power plant it is virtually impossible to accomplish a true simulation of this hydraulic flow pattern in the prototype, i. e. the coastline without the power station. This suggests an analytical analysis of the problem and scale model studies as the appropriate support for the engineering design of the cooling water arrangement. Hydraulic models have been employed for many years as a valuable tool in the solution of waste disposal problems. The technique and practice applied depend on the water area to be reproduced, the general nature of the problem to be studied, and many other considerations.

For the present study we will confine ourselves to discussion of the principle of prototype simulation by mathematical models.

Mathematical Models

The development of a mathematical model follows three distinct phases: conceptual, functional and computational. The conceptual model of the problem analyzes the fundamental physical elements to be incorporated in the model. As a second stage, we have to convert the proposed physical model into mathematical terms. The computational solution is the final step in the development of the mathematical model. The predictive capability of the proposed model must be tested in the laboratory or in the field by comparison to observed data.

The ultimate objective of a dispersion model is the effective control of concentrations of contaminants released within the area. Hence, this model is an integral part of the water quality planning within the control regime. The pattern of spreading of discharged water is dependent on a great number of parameters several of which may have a significant effect on the pollution of the environment. Transportation and distribution of the pollutants result not only from the details of movement and mixing within the receiving area, but depend also very often to a substantial degree on the external circulation, i.e., exchange with adjacent larger water bodies. Hence, considerable effort has to be made to investigate dispersion mechanisms from well-defined sources of pollution as well as to expand the knowledge of the physiography of the coastal environment which will be exposed to waste discharge.

Most fjord and nearshore problems are so much related to such local factors as topography, stratification, flow-through, and meteorlogical and tidal activity, that a mathematical treatment is very difficult to achieve. Unfortunately, it is in these cases that the characteristics of the receiving water are of primary interest, as they ultimately determine the concentration levels and the residence time distribution in the water quality region considered. We will discuss approaches of mathematical modeling with special reference to this particular problem.

A properly designed dispersion model should be able to predict pollutant levels at any location in the receiving area, if the source emission and the environmental factors are known. Using terminology of control theory, the non-manipulative input variables are the environmental factors, and the control variables of the system which can be selected to meet some water quality requirements are the sources of pollutants and their locations, outlet arrangements and finally, of course, the source strengths. Once such a model of high predictive capability has been constructed, various control problems can be posed.

The analytical procedure of the mathematical modeling is based on either a finite or an infinitesimal approach. The differential control-volume analysis describes the flow characteristics and mass balance at a fixed point in space. Consequently, the basis for this method is the general equations of motion and appropriate equations for the conservation of mass and volume. Oceanic circulation, for example, has been successfully studied by means of infinitesimal calculus. The predictive capabilities of these ocean models are fairly good due to the fact that even drastic simplifications of the flow properties do not significantly affect the overall pattern of circulation.

Theoretically, the spatial and time history of discharged pollutants could be obtained by solving the general equation of diffusion and the equations of motion. The lack of detailed knowledge of all the flow properties of the water area is one of the reasons that an exact solution to the diffusion problem is not available.

The finite control-volume analysis is perhaps the most widely used technique for studies of flow problems. The water body is divided into segments of finite size and mean values are assigned to each element according to the physical properties of interest. The classical method for estimating flushing rates in estuaries due to tidal action is the tidal-prism analysis. Using finite control volumes we will never gain a thorough understanding of the flow mechanisms which can only be reached by means of a particle approach analysis.

Statistical correlation approaches have proved to be successful tools for analysing dispersion phenomena, being alternatives to the differential analysis using coefficients of diffusion. The classical theory was developed by Taylor who considered "diffusion by continuous movements" from a fixed point in an isotropic, turbulent flow field. In recent years several techniques have been proposed for simulation of turbulent diffusion by means of stochastic models.

Statistical models are frequently used in operational hydrology and may be a likely analytical procedure for water quality management when large quantities of data are available. The "data bank" is then the historical observations of flow characteristics, quality parameters and environmental conditions gathered during a certain period of time. The problem is to extract from the data bank enough information to construct a statistical model which demonstrates properly the interaction between significant system parameters.

Condenser Water Discharge

For predictions of the process of thermal diffusion produced by condenser water discharge, we usually have to rely on a combination of analytical reasoning, scale modeling and field surveys. It is a complex problem involving aspects of stratified flow as well as thermodynamics. The objectives of a hydraulic study of a cooling water arrangement are two-fold: determination of the thermal effects on the receiving water area and the degree of recirculation of heated condenser water.

Several experimental studies have contributed to our understanding of the diffusion phenomena inherent in the surface discharge of warm water jets. Jen, et.al. (1966)[21] considered the reduction of temperature excess due to turbulent jet mixing. Hayashi and Shuto (1967)[18] performed similar studies. The results of these experiments can be compared to a model for the initial flow zone proposed by Cederwall and Sjöberg (1969)[8] using the jet diffusion theory as will be outlined.

Because it is difficult to solve the dynamic and thermal equations simultaneously, the problem is studied assuming the temperature to be a conservative property. Turbulent diffusion normally accounts for the most efficient reduction of excess temperature, although heat loss to the atmosphere sometimes is far from negligible. However, if we want to design a discharge structure for the purpose of achieving a high degree of dilution, (this is usually the most appropriate way of meeting water quality standards and reducing recirculation) then the assumption of a conservative temperature is a justified approximation. In such cases the heat loss may be estimated through a stepwise correction of the heat flux following the calculated path of the condenser water flow.

To describe the initial flow zone of a surface discharged warm water jet we start studying the basic flow properties of a submerged three-dimensional momentum jet in the zone of established flow, Albertson, et. al. (1950)[3], see Fig. 1 for definitions.

$$\frac{U_m}{U_o} = 6.2 \frac{D}{s} \qquad (1)$$

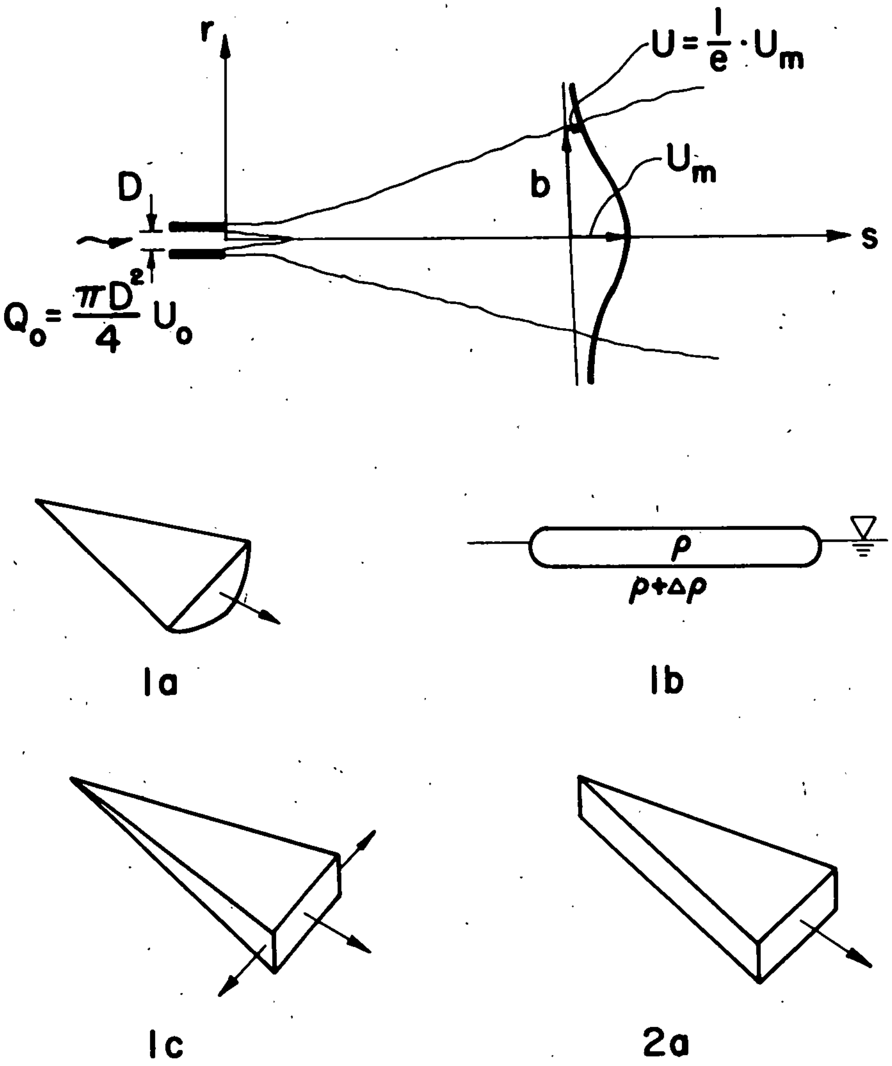

Fig. 1. — Velocity field of submerged three-dimensional jet and sketch of jet diffusion models.

$$\frac{U}{U_m} = e - \frac{r^2}{b^2} = e - \frac{r^2}{(0.114s)^2} \tag{2}$$

$$\frac{Q}{Q_o} = 0.32 \frac{s}{D} \tag{3}$$

In the case of a submerged jet in a large reservoir, the absence of external forces requires the flux of momentum, m, to be the same at all successive sections.

$$m(s) = m_o = \rho Q_o U_o \tag{4}$$

The growth of the jet flow is characterized by the following equation:

$$\frac{dQ}{ds} = a \cdot 2\pi b U_m \tag{5}$$

where a is the coefficient of entrainment which, from dimensional considerations, has to be a constant, $a = 0.057$ for this particular case.

Introducing the kinematic flux of momentum, $jV = Q_o U_o$, we can write Eq. (1).

$$U_m = 7.0\ J_o^{1/2} \cdot \frac{1}{s} = 7.0\ j^{1/2} \cdot \frac{1}{s} \tag{6}$$

For the growth of the jet flow we may write

$$Q = 0.28\ J^{1/2} \cdot s \tag{7}$$

or by means of Eq. (5)

$$dQ = 1.41\ a \left[\frac{J}{\pi b^2}\right]^{1/2} \cdot 2\pi b ds \tag{8}$$

where $J/\pi b^2$ is a characteristic local velocity, and $2\pi b ds$ "the local area of entrainment". Similar expressions hold for the submerged, two-dimensional momentum jet.

Now, returning to the surface discharged warm water jet we have to take into consideration the following conditions:

a. The water surface and the bottom are characterized by zero rate of entrainment.

b. The flux of momentum is reduced due to turbulent shear at the fixed boundaries.

c. Increased lateral spread is caused by the density deficit of the heated water.

We assume that the density deficit $\Delta\rho$ is a linear function of the temperature excess ΔT. The following situations are modelled, see Fig. 1:

1. Subsurface discharge in a deep and homogeneous stagnant environment.

1a. $\Delta\rho_o = 0;\ m_o > 0$ Non-buoyancy case

According to Eqs. (6) to (8) we get

$$U_m = \sqrt{2} \cdot 7.0\ J^{1/2} \cdot \frac{1}{s} = 9.9\ J^{1/2}\ \frac{1}{s} \qquad (9)$$

or

$$\frac{U_m}{U_o} = 8.8\ \frac{D}{s} \qquad (10)$$

$$Q = \frac{1}{\sqrt{2}} \cdot 0.28\ J^{1/2} \cdot s = 0.20\ J^{1/2} \cdot s \qquad (11)$$

The turbulent diffusion of mass and heat is similar to the diffusion of momentum and in the reference case of a three-dimensional submerged simple jet the distribution of concentration, C, of the discharged effluent can be written:

$$\frac{C_m}{C_o} = 5.6\ \frac{D}{s} \qquad (12)$$

$$\frac{C}{C_m} = e^{-\frac{r^2}{(\lambda b)^2}} \qquad (13)$$

where λ, the turbulent Schmidt number, is close to unity.

We can modify the distribution of concentration in the same way as the velocity field to yield the case of a subsurface non-buoyant jet. Hence, Eq. (12) takes the following form:

$$\frac{C_m}{C_o} = \sqrt{2}\ 5.6\ \frac{D}{s} = 7.9\ \frac{D}{s} \qquad (14)$$

Eqs. (2) and (13) expressing the lateral spread of momentum and mass remain as they are.

Eq. (10) is in good agreement with experimental results of Horikawa (1958) as reported by Jen, et. al. (1966)[21] and is a better representation of the velocity field than Eq. (1) which they proposed. Furthermore, the lateral profile of velocity of the subsurface jet was not found to differ significantly from that of a submerged jet. Laboratory experiments performed at the hydraulics laboratory, Chalmers Institute of Technology, show that Eq. (11) is a valid description of the growth of the jet flow.

Measurements of the distribution of temperature from experiments ranging in Froude number, $F = U_o / \sqrt{\frac{\Delta \varrho}{\varrho} g D}$, from 18 to 180 (that is when the effect of buoyancy on the spreading is small) was reported by Jen, et. al. (1966).[21] The experimental data of Hayashi and Shuto (1967)[18] range in Froude number from 1 to 16. Tamai, et. al. (1969)[31] describe some experiments performed at relatively small values of the Froude number. For $F > 3$ the effect of buoyancy is not very pronounced in the initial flow zone. Hence, Eq. (14) should be applicable which also is confirmed by the experiments.

1b. $\Delta \rho_o > 0; m_o \approx 0$ Buoyancy case.

The spread is then induced essentially by the density deficit. Theoretical and experimental studies indicate that the front velocity, U_Δ, may be described roughly by Abbot (1961).[1]

$$U_\Delta = \left(\frac{\Delta \rho}{\rho} gh \right)^{1/2} \qquad (15)$$

where h is a characteristic thickness of the flow field at the edge.

1c. $\Delta \rho_o > 0; m_o > 0$ Intermediate case.

This is a subsurface jet characterized by initial momentum and induced gravitational spread. To model this flow situation we make the following assumptions:

a. The overall rate of entrainment is not affected by the buoyancy. Hence, F must not be too small.

b. The principle of superposition is assumed to yield the mechanisms causing the jet expansion.

The expansion of the jet flow due to diffusion may be evaluated from case 1a. The flow pattern of the non-buoyant subsurface jet is generalized to have a rectangular cross-section with a width-depth ratio of 2 and a constant longitudinal velocity equal to $k \cdot U_m$. Furthermore, we assume the velocity and concentration profiles to be identical. From continuity we get

$$Q = \tfrac{1}{2}\,\pi U_m b^2 = k^2 U_m \cdot \frac{B^2}{2} \qquad (16)$$

$$Q_o = \tfrac{1}{2}\,\pi U_m \cdot C_m \frac{b^2}{2} = k^2 U_m \cdot C_m \cdot \frac{B^2}{2} \qquad (17)$$

where B is the width of the generalized subsurface jet.

Hence, $k = 0.5$

$$\frac{dB}{ds} = \sqrt{2\pi} \cdot 0.114 = 0.29 \qquad (18)$$

$$U_d = \frac{0.29 U_m}{2 \cdot 2} = 0.072\ U_m$$

where U_d is the lateral spread velocity of the jet due to diffusion. If the gravitational effect is superposed we arrive at the following expression:

$$\frac{dB}{ds} = 4\ \frac{U_d + U_\Delta}{U_m} \qquad (19)$$

and the equation characterizing the jet expansion is then

$$\frac{dB}{ds} = 0.29 + \frac{0.20}{F}\ \frac{S^{3/2}}{DB^{1/2}} \qquad (20)$$

The depth may be found from an equation of continuity.

Eq. (20) is in good agreement with experimental results reported by Hayashi and Shuto (1967)[18] corresponding to steady-state conditions in Froude numbers exceeding say 2.5, and seems to be a better representation of the spreading pattern than the linear relationships proposed by Jen, et. al (1966)[21] and Wood and Wilkinson (1967).[33] For very small Froude numbers there is, in these scale tests, a pronounced buoyancy effect which hampers the mixing. Hence Eq. (20) cannot hold.

2. Subsurface discharge in shallow, homogeneous and stagnant environment.

2a. $\Delta\rho_o = 0;\ m_o > 0$ Non-buoyancy case.

For shallow water the jet tends to penetrate the whole depth. Hence, there is only lateral entrainment of ambient water. Assume the slope of the bottom to be S. Then the depth of the flow is given by:

$$h = h_o + S \cdot s \qquad (21)$$

This case is represented by a plane jet and we arrive at the following characteristic flow equations using formulas for submerged two-dimensional jets, see Albertson, et al. (1950).[3]

$$U_m = 2.28 \left[\frac{J}{(h_o + S \cdot s)\, s}\right]^{1/2} \tag{22}$$

$$\frac{dQ}{ds} = 0.31 \; (J/s)^{1/2} \; (h_o + S \cdot s)^{1/2} \tag{23}$$

$$J = J_o - \int_{-\infty}^{s} \int_{-\infty}^{\infty} \frac{\tau}{\rho_o} \, dy \, ds \tag{24}$$

where y is the lateral coordinate and, τ, the shear at the bottom expressed by

$$\tau = \frac{f}{8} \rho_o \, U_m^2 \cdot e^{-2 \frac{y^2}{b^2}} \tag{25}$$

f is the friction factor and b stands for a local characteristic length of the plane jet. Hence,

$$\frac{J}{J_o} = \left[1 + S \frac{s}{h_o}\right]^{-\frac{f}{8S}} ; \qquad \text{for } S \neq 0. \tag{26}$$

and

$$\frac{J}{J_o} = e^{-\frac{fs}{8h_o}} \qquad \text{for } S = 0. \tag{27}$$

This approach is supported by laboratory experiments carried out at Chalmers Institute of Technology.

2b. $\Delta\rho_o > 0$; $m_o \approx 0$. Buoyancy case, see 1b.

2c. $\Delta\rho_o > 0$; $m_o > 0$. Intermediate case.

If the entrainment is mainly in lateral direction, a generalized flow model is preferably based on a plane jet. Assuming that the lateral entrainment is not significantly affected by the buoyancy we may define a mean velocity in the longitudinal direction and a lateral velocity of expansion similar to case 1c. These velocities are evaluated to be 0.71 U_m and 0.124 U_m respectively. The combined effect of diffusion and gravitational spread leads to the following expression of the expansion of the flow field.

$$\frac{dB}{ds} = 0.35 + \frac{3.0}{F}\left[\frac{U_o^3}{U_m^3}\ \frac{D}{B}\right]^{1/2} \tag{28}$$

Similar to Eq. (20), this relation requires that the Froude number is not too small. It should also be mentioned that the hampering of vertical mixing due to the density difference is more pronounced at larger distances from the source.

The depth of the jet flow is obtained from an equation of continuity.

If the receiving water is density stratified, the vertical mass exchange is considerably reduced. This suggests that a subsurface, plane jet of initial momentum and induced gravitational spread -2c- best represents the flow field. The two models proposed, however, define extreme situations as to the overall rate of entrainment and the true flow pattern is likely to fall somewhere between.

The two-dimensional wall jet is well-known from literature. If we apply the proposed jet diffusion model to this case, and neglect the shear at the wall, we arrive at the following equations.

$$U_m = 3.2\left(\frac{J}{hs}\right)^{1/2} \tag{29}$$

$$Q = 0.44\ sh\left(\frac{J}{hs}\right)^{1/2} \tag{30}$$

Analyzing experimental data, see Rajaratnam and Subramanya (1968),[28] we find the following expressions for the wall jet:

$$U_m = 3.4\left[\frac{J}{hs}\right]^{1/2} \tag{31}$$

$$Q = 0.44\ sh\left[\frac{J}{hs}\right]^{1/2}\left(0.52 + 4.1\ \frac{B_o}{s}\right) \tag{32}$$

Eqs. (31) and (32) deviate slightly from Eqs. (29) and (30) which is due to the fact that the shear at the wall affects the flux of momentum as well as the velocity distribution across the jet.

Coastal Currents

Currents in the receiving water area affect the dispsrsion pattern of the discharged condenser water and have to be accounted for when estimating the rate of recirculation. For shallow waters the jet tends to penetrate the whole depth and the jet trajectory is bent over towards the coast if coastal currents are prevailing. This may create a zone of local recirculation where a portion of the flow is continuously fed back to the jet. This mechanism of feed-back is less pronounced in deep water where the jet flow does not penetrate the full depth. Experiments have shown that such curved, two-dimensional jets have local concentration and velocity profiles similar to free jets, see Sawyer (1963).[29] The total rate of entrainment of the curved jet is almost identical to that of a free plane jet — the increased rate of entrainment along the outer edge ($\approx 2/3$) being balanced by a reduced rate of entrainment along the inner edge ($\approx 1/3$) . Hence, as an engineering approximation it is justified to use the formulas for the growth of flow as a function of s, previously deduced for the stagnant environment case.

To determine the curvature of the jet path we have to consider the interaction between the flux of momentum of the ambient water and the jet flow. Hence, the rate of change of momentum flux may be set equal to the rate of entrainment of momentum flux plus a gross drag force acting on the jet, Fan (1967.)[11] However, several other models can be tested, Häggström (1969).[19]

Laboratory experiments are presently being carried out at Chalmers Institute of Technology to study the behavior of warm water jets discharged into a flowing environment.

Example

Proposed models of the dispersion of condenser water discharge are applied to the planned nuclear power plant, Barsebäck, located at the sound between Denmark and Sweden. The cooling water discharge is 150 m^3/s initially issued at a speed of 2 m/s. The temperature excess represents an initial density deficit of $\Delta\rho_0 = 1.0$ kg/m^3. Figs. 2 and 3 show the result for stagnant ambient water as well as for the case of a coastal current of 0.5 knot.

SUBMARINE DISPOSAL OF SEWAGE

The dispersion mechanism following a marine waste water discharge includes essentially two stages: first, the initial mixing of the waste water in the immediate proximity of the outfall; and second, the subsequent transport and dispersion of the disposed effluent. When sewage is discharged into

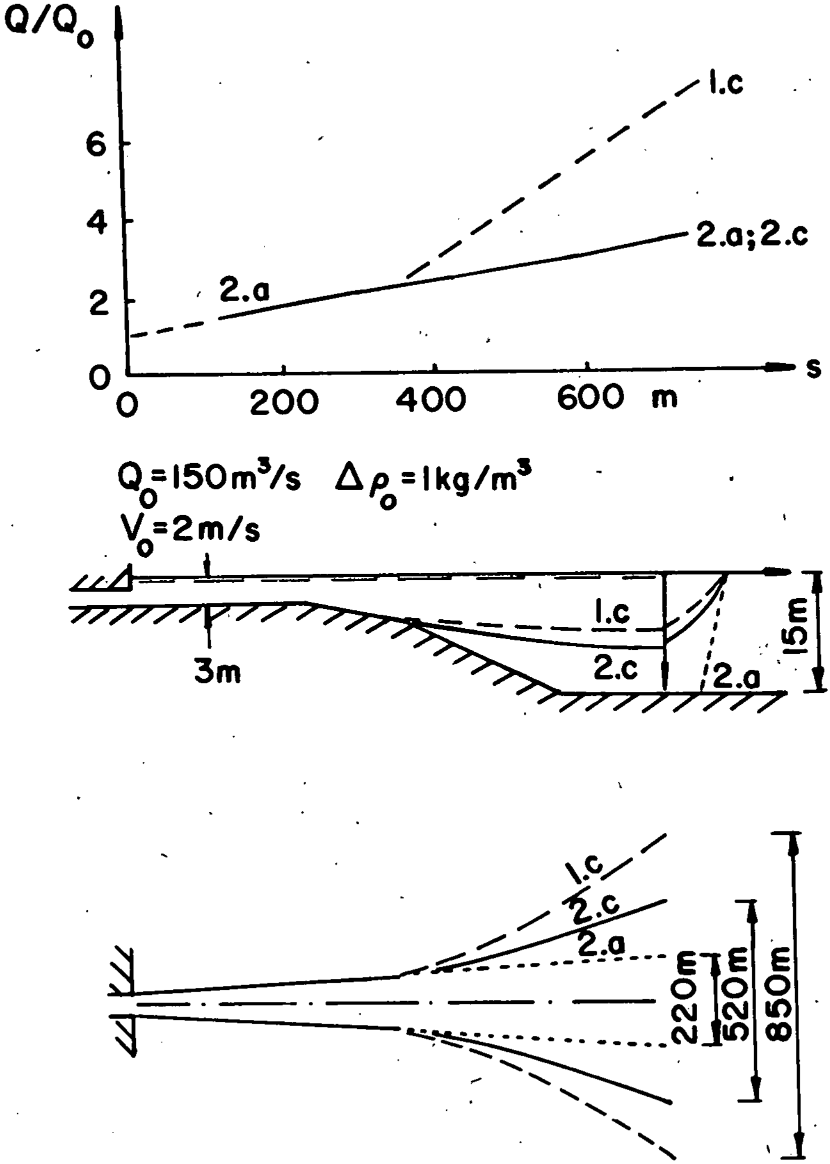

Fig. 2. — Flow characteristics for Barsebäck power plant. Applying models 2a (0 < s < 330m) and 1c and 2c (s > 330m) in the case of a stagnant receiving water.

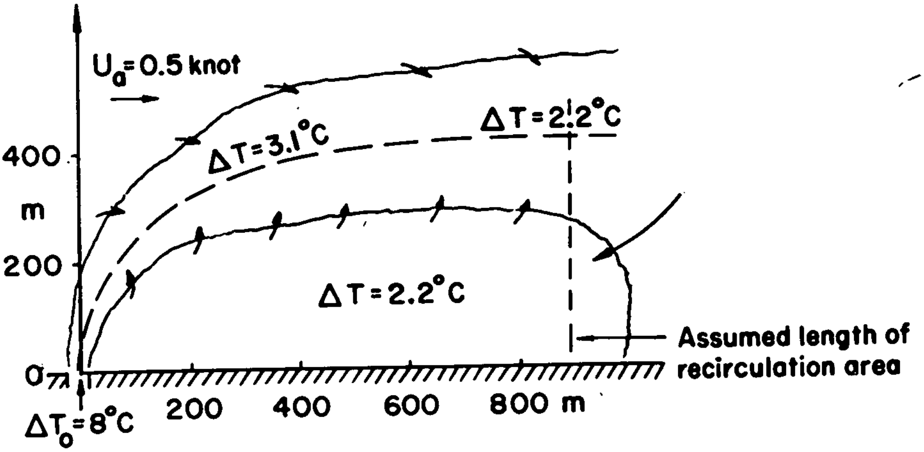

Fig. 3. — The spread of the cooling water from Barsebäck power plant discharged into a coastal current of 0.5 knot reproduced by a plane jet and ignoring density differences.

the sea it is subjected to a buoyant force proportional to the density difference between the sea water and the waste water. Hence, the discharged sewage rises towards the sea surface while mixed with ambient water. After the excess jet energy has been dissipated and a sewage field is established at some neutrally buoyant level, mixing due to natural turbulent diffusion becomes dominant.

In a stratified receiving environment there is a possibility of submerged sewage fields forming above a submarine outfall. The amount of pollutants that is brought up to the surface is then considerably reduced. This is usually looked upon as a desirable result, considering the increased recreational value of the receiving water area. However, other basic interests, primarily fishing, may prefer dispersion of the disposed waste water in surface layers where transportation and associated dilution are more efficient than in the deep waters.

The initial mixing of the discharged waste water is basically a problem of jet diffusion, whereas sea dispersion is a more complex phenomenon due to the multitude of significant parameters involved. It has, from initial mixing considerations, been the usual practice to provide multiple-port diffusers with horizontal ports. Hence, studies on horizontal jets discharged into environments of various hydrographical complexity is of great engineering interest.

A definition sketch for horizontal jet diffusion is given in Fig. 4. The S-shaped form of the jet is observed in flowing ambient water or when the jet effluent is discharged at large values of the Froude number in a stratified environment. Several solutions to horizontal jet diffusion in stagnant homogeneous water have been proposed. Abraham (1963)[2] introduces empirical functions for the spread of momentum and mass across the jet section where the rate of spread is a function of the local angle of inclination. Fan and Brooks (1966)[12] use the principle of entrainment which they advocate as being more logical from a physical standpoint. The two theoretical solutions give practically the same result when adequate values of coefficients are chosen. For cases of linear density profiles in the environment, a variety of numerical solutions to jet diffusion problems have been presented by Fan and Brooks (1969).[13] A numerical step-by-step procedure has been suggested by Cederwall (1966)[5] to obtain a solution for the case of a horizontal jet issued into a stably but otherwise arbitrarily stratified environment. A similar program has been worked out by Ditmars (1969).[9] These upproaches estimate the levels between which the sewage field will be established. Furthermore, the Danish Isotope Centre has worked out a set of computer programs which are concerned with the trapping effect in stratified water, and the conversion of all the single results to statistical expressions, Hansen (1967).[15]

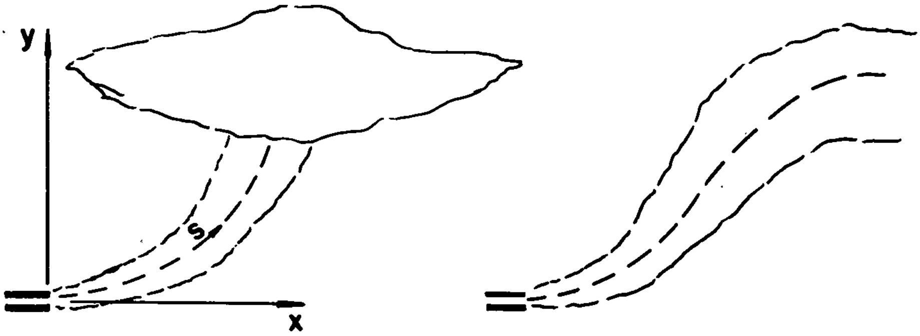

Fig. 4. — Definition sketch for horizontal jet diffusion. The S-shaped profile is observed in flowing ambient water or when the jet is discharged at a high Froude number in stratified water.

When the sewage jet reaches the surface or some trapping level below the surface, there is a transition to horizontal spreading. It is difficult to assign any special flow pattern which gives a general description of the transition zone because this mechanism depends very much on the hydrographical situation at the outfall site, and no detailed study has been focused on this problem. If fairly strong currents are prevailing in the disposal area, the spreading feature of the sewage field — submerged or established at the water surface — may be evaluated. However, when the currents are weak, the transport of the sewage away from the disposal area is a more complex phenomenon. If the sewage field is established at the water surface, then the propagation of the front of the expanding field is induced by density differences and is likely to resemble the spreading of oil on water, Abbot (1961).[1] For this case the front speed is given by

$$U_\Delta = \sqrt{\frac{\Delta\rho}{\rho} gh} \tag{33}$$

where $\frac{\Delta\rho}{\rho}$ is the relative density difference between the ambient water and the sewage-seawater mixture, and h is the thickness of the field. A solution based on Eq. (33) was given by Larsen and Sörensen (1967)[23] for the case of a sewage jet reaching the surface of a uniform flow. Sharp (1969)[30] studied the surface spread following a horizontal jet discharge in a stagnant homogeneous environment and provided an experimental solution for the rate of spread.

Eq. (33) expresses the front propagation of a gravity current in a homogeneous environment. The front abvances as an "instability front" with a tendency to have a vertical front (the dam burst analogy), see Benjamin (1968).[4] Assume for the moment that a similar mechanism of gravitation spread is applicable for the case of a trapped sewage field. The stratification of the receiving water is assumed to be linear between the levels occupied by the sewage field. Analogous to Eq. (33) we then get for the front velocity:

$$U_\Delta = \sqrt{\frac{\Delta\rho}{\rho} g \frac{h}{6}} \tag{34}$$

However, in stratified surroundings the front shape is more wedgelike suppressed by the stratification, and the front advance does not follow Eq. (34). This is observed from experimental modeling of the collapse of a mixing region in a stratified environment developing a thin triangular front with a low velocity of propagation, Wu (1969),[34] see Fig. 5. As a result there is a pronounced tendency of accumulation of sewage above the outfall for the case of trapped jets in a stagnant environment.

It is well-known that the directional variation of the horizontal velocity with depth-variation of the mean value as well as time fluctuations, is a

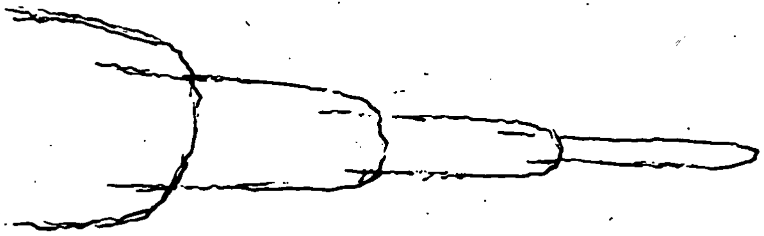

Fig. 5. — A mixed region collapsing successively in stratified water, Wu (1969).

most efficient mechanism for spreading a tracer cloud apart, see Fig. 6. Suppose that the environment is stratified thermally or due to existing differences in salinity. An increase of the initial vertical expansion of the sewage field favorable to subsequent dispersion is obtained when using diffusers with a slight but systematic variation of the port directions, or by a variation of port diameters. This suggests that sometimes, and specifically in stratified waters, the appropriate design of the outfall arrangement has to be carefully analyzed as to its effect on the dispersion mechanism following the initial dilution.

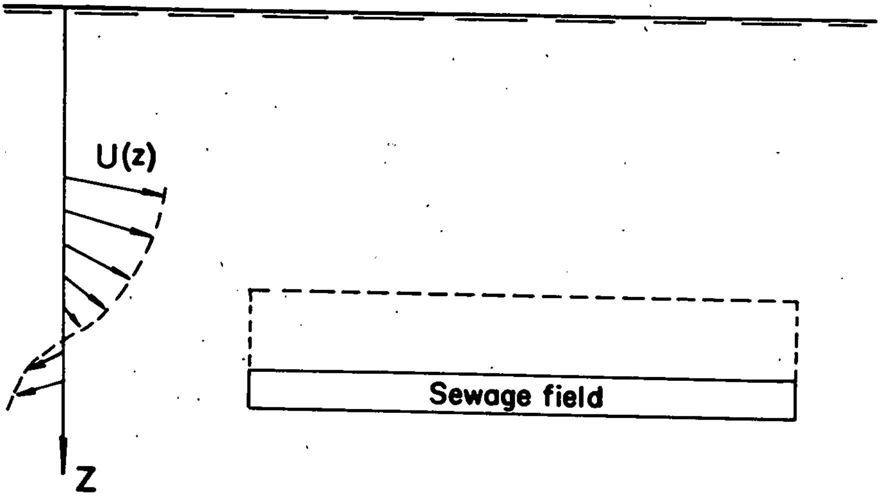

Fig. 6. — Increasing the initial depth of a submerged sewage field improves the lateral dispersion if the velocity vector of the ocean varies significantly with depth.

Several mathematical models have been proposed for ocean diffusion. Unfortunately, there are but few studies on dispersion in natural waters systematically relating the pattern of dispersion to environmental parameters. Hence, there is still a considerable lack of knowledge on many features of sea dispersion despite the efforts which have been devoted to the problem, see for instance Foxworthy (1968),[14] Okubo (1968)[25] and Kullenberg (1968).[22] Processes of transportation and mixing in coastal waters and in the open ocean are so complex that a single mathematical model that can explain the entire pattern of dispersion seems far away. Hence, we are very much dependent on empirical data obtained from field tracer investigations. This emphasizes the need for rational methods and sensitive instrumentations to reduce the high costs of performing tracer simulation studies on waste dispersion.

Sometimes the dispersion pattern following a continuous discharge of waste water is of less significance to the pollution problem than the overall water circulation of the receiving area. This is usually the case for confined water bodies with reduced communication with the open sea, such as fjords and many types of archipelagos. For such environments the background pollution level built up by the waste water discharge may be the factor which limits the receiving capacity of the water body. Hence, considerable emphasis must be given to studies on the external as well as the internal circulation of confined water bodies exposed to waste discharge. This particular problem will be discussed in the next section.

CIRCULATION IN CONFINED WATER BODIES

The circulation in confined bodies of water, as in fjords and embayments, is usually very complex and built up by various types of basic currents. It is essential when dealing with water quality problems to understand these transport mechanisms and how the water body considered communicates with the open sea. A classical example of such large scale circulation is the outflow of high salinity water from the Mediterranean through the Straits of Gibraltar. The heavy underflow is compensated for by a surface flow of less saline water entering from the Atlantic. A similar process is observed for the Baltic Sea; however, in this case precipitation exceeds evaporation and a saline underflow enters the Baltic Sea. The brackish outflow of water through the sound between Denmark and Sweden — the Baltic Current — sweeps along the Swedish west coast due to the Coriolis force. This current has been of considerable interest when major future waste disposal projects in these areas have been discussed.

For the purpose of this study let us look at the circulation of a confined water body which communicates with the open sea through a narrow strait, see Fig. 7. Fresh water is discharged into the interior of the water area. The result is a density stratified system with a typical circulation pattern — outflowing brackish surface water, the loss of salt in the confined water area being compensated for by a subsurface inflow of salt water. This exchange mechanism is affected not only by the flow-through of fresh water, but also significantly by tidal activity. Furthermore, the circulation of the closed basin is driven by heating from the atmosphere and considerably by the wind stress. We will find that the vertical mixing between the brackish surface water and the salt bottom water is a process which contributes efficiently to the mechanism of external exchange of water. A characteristic feature of turbulent shear flow is the transport of energy from mean motion to turbulent motion. For the case of a homogeneous flow, this energy is dissipated by viscosity only, whereas in the stably stratified case, part of the turbulence energy is converted into potential energy by means of vertical diffusion. To maintain the turbulent structure of the flow, the rate of energy input must exceed the rate at which potential work is done by turbulent diffusion. In terms of the local Richardson number, R_i, characterizing the dynamic stability of stratified flow, we then have:

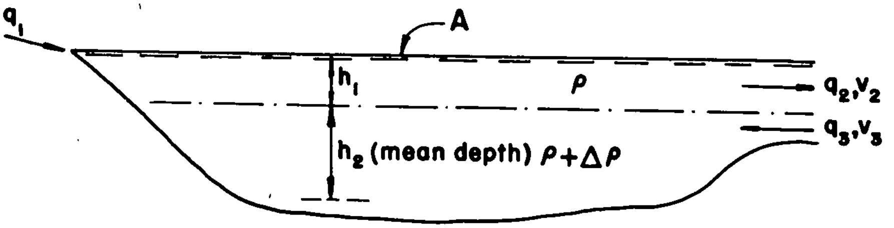

Fig. 7. — Section through a confined water body communicating with the open sea.

$$R_i = \frac{g}{\rho}\frac{\partial\rho}{\partial z}\Big/\left(\frac{\partial u}{\partial z}\right)^2 < \frac{\epsilon_{mz}}{\epsilon_{sz}} \qquad (35)$$

where ϵ_{mz} and ϵ_{sz} are momentum and mass diffusivities respectively. Small values of R_i indicate increase of turbulence whereas a large value indicates suppression of vertical diffusion and high dynamic stability.

From experimental studies we know that vertical mixing occurs in a two-layered system at a certain velocity difference between the layers. If the parameter θ, first proposed by Keulegan, see Ippen (1966),[20] exceeds a certain critical value the interface will be stable.

$$\theta = \frac{\nu \frac{\Delta\rho}{\rho} g}{U^3} \tag{36}$$

ν is the kinematic viscosity, and $\Delta\rho$ and U are a characteristic density difference and velocity of the interface. θ is essentially a Richardson number. If there is a pronounced interface between the two layers, the mass exchange depends on the tendency of small disturbances at the interface to be damped or amplified. This is theoretically deduced as well as verified in the laboratory. As a matter of fact, most of our knowledge on stratified flow has been gained from experiments in small laboratory flumes. There are two reasons why it is so difficult to apply laboratory test results to ocean conditions. First, we cannot be quite sure that empirical interfacial data of stability and mixing do not suffer from scale effects. Second, the bulk hydrodynamic parameters which are easily controlled in the laboratory are not as readily evaluated in the field. Furthermore, few field experiments on vertical diffusion have related the data to the complete set of relevant environmental conditions such as hydrographical parameters and wind and wave characteristics. Hence, a proper prediction of vertical mass exchange in a receiving water area, essential in many water pollution studies, is a very difficult task.

A major input of energy into a coastal water area is due to wind action. The rate at which energy is transferred from the wind to the water body depends on both normal and tangential stresses. It is convenient to introduce an apparent shear stress, τ_a , at the water surface:

$$\tau_a = \epsilon_a \rho_a \frac{\partial W}{\partial z} \tag{37}$$

where ρ_a is the density of the air, ϵ_a is the eddy viscosity of the air and W is the wind velocity. Assuming a rough water surface — waves are developing — we get the following well-known velocity profile:

$$\frac{W}{w_*} = 2.5 \ln \frac{30z}{k} = 2.5 \ln \frac{z}{z_o} \tag{38}$$

k is an equivalent roughness and z_o a roughness parameter related to the average wave height. For moderate and strong winds, z_o is assumed to have a constant value of about 0.6 cm. w* is the shear velocity $\sqrt{\frac{\tau_a}{\rho_a}}$.

If W_{10} is the mean wind speed at the 10 m level and ζ is a resistance coefficient, we can write:

$$\tau_a = \zeta \cdot \rho_a \cdot W^2{}_{10} \tag{39}$$

Combining Eqs. (38) and (39) we get $\zeta = 2.9 \cdot 10^{-3}$. A number of writers have reported values close to $2.6 \cdot 10^{-3}$. If u_* is the shear velocity at the water surface defined by $\sqrt{\frac{\tau_w}{\rho_w}}$ where ρ_w is the density of the water and τ_w the shear we arrive at:

$$\tau_a = w_*^2\, \rho_a = u_*^2 \cdot \rho_w = \tau_w \tag{40}$$

$$\frac{u_*}{w_*} = \left(\frac{\rho_a}{\rho_w}\right)^{1/2} \tag{41}$$

and if we put $\rho_a = 1.3$ kg/m³ we find that $\frac{u_*}{w_*}$ is approximately 3.6%. This value is somewhat higher but fairly close to observed relations between wind velocity and induced wind drift at the water surface. The rate at which energy is transferred to the water body per unit surface area is then

$$P_w = \tau_a \cdot W \cdot \frac{u_*}{w_*} = 2.9 \cdot 10^{-3}\, W^3 \frac{u_*}{w_*} \tag{42}$$

The generation of wind induced currents is closely related to the mechanism of wave generation and a considerable fraction of the energy input by wind action is diverted to wave motion. In deep water the waves produce very little turbulence except when breaking. In shallow waters, however, and, specifically in the breaking zone close to the shore, there is a considerabls production of turbulence. It is, however, the turbulence that penetrates into deeper waters that causes vertical mass transport and hence it is the wind-induced currents and not the wave action that is the main factor contributing to vertical diffusion in the sea.

Let us take an example. Referring to Fig. 7, the following values of the variables represent the situation of the inner archipelago of Stockholm.[1]

$A = 110$ Mm²

$h_1 = 10$ m

$h_2 = 7$ m

$\Delta\rho = 2.5$ kg/m³

$q_1 = 170$ m³/s

$q_2 = 370$ m³/s

$q_3 = 200$ m³/s

$v_2 = 0.10$ m/s

$v_3 = 0.15$ m/s

---

[1] From an investigation by Vattenbyggnadsbyron (VBB), Stockholm, for a planned atomic power plant at Värtan.

The required vertical mass transfer to maintain the circulation of this system corresponds to the following potential work per time unit

$$P_p = g\Delta\rho q_3 \cdot h_1/2 \qquad (43)$$

For this particular case P_p amounts to 24,500 Nm/s. Assuming an efficient wind velocity of 4 m/s, P_w is the order of 10^6 Nm/s. The inflows q_1 and q_3 may be neglected as to their contribution of energy being just a fraction of the required rate P_p. Hence, it is the wind induced energy which maintains the circulation of the water body.

Measurements in this archipelago have confirmed that the upper layer 'erodes' the lower layer. The mass transport is essentially a one-way transport — saline bottom water is brought up to the more turbulent surface layer while the salinity of the bottom water remains fairly constant throughout the depth. A planned atomic power station to be located here will take bottom water — 10 m³/s — for cooling purposes and discharge the heated condenser water in such a way that it will be submerged below the interface. Hence, recirculation of cooling water can be avoided as long as the mechanism of vertical transport just outlined can be maintained. During some winter months ice covers the water surface and there is no input of energy from wind action. The vertical transport of salt water into the surface layer is then considerably reduced and recirculation of heated condenser water is difficult to avoid. This indicates that an alternative intake arrangement for surface water is favourable during winter time.

For water pollution control of a confined water area exposed to waste disposal, we have to rely on some model of the water circulation. Most theories proposed for estuarine mixing are based on the one-dimensional concept. They assume a well-mixed region and do not consider the detailed mechanism of dispersion within the system, see Pritchard (1952),[26] Waldichuk (1964).[32]

The flushing mechanism of a partially mixed estuary may be analysed by means of a two-dimensional model, if the circulation and mixing is assumed to be governed by advection and vertical mixing between the surface layer and the bottom layer, but not significantly by longitudinal dispersion, Pritchard (1969).[27]

Let us consider the case of a three-dimensional, partially stratified, semi-enclosed coastal area. Suppose that there is an influx of fresh water to this water body causing pronounced salinity differences. Let Q_f denote the fresh water flow. If C_o is the salinity of the sea water outside the bay, and C is the salinity at any point within the confined water area found from a survey of salinity measurements, we can express the fresh water content by

$$C_f = \frac{C_o - C}{C_o} \tag{44}$$

Hence, at that particular time the total volume of fresh water, V_f, present in the body of water considered, V, is found to be:

$$V_f = \int_V C_f \, dV \tag{45}$$

The mean residence time, τ, related to the flow-through of fresh water is given by:

$$\tau = \frac{V_f}{Q_F} \tag{46}$$

If a pollutant is introduced into the system in the same way as the fresh water, then τ is a good estimate of the detention time for this particular waste discharge. This situation, however, is not always the case, and we must have other methods to evaluate the dispersion within the region and the external water exchange. Let us first assume that there is a significant and well-defined flow through the free connection with the open sea exhibiting a periodical response to tidal activity. This condition is satisfied if the passage is relatively narrow as in the case of a fjord. Considering the characteristics of the alternating flow pattern through the passage, we may estimate an overall flushing rate of the confined water body which is not restricted to the fresh water.

The flow entering the water area has initial spread characteristics similar to a plane jet. The distance to which the incoming water will reach increases with some defined flushing time t as $\alpha t^{2/3}$ where α is a coefficient related primarily to the flow conditions at the passage. The water leaving the embayment, on the other hand, is flowing towards a sink and consequently the expansion of the withdrawal zone follows $\beta t^{1/2}$ where β is a coefficient which can be derived from initial flow conditions similar to α.

An exchange factor r′ may be defined as:

$$r' = \left(1 - \frac{\beta}{\alpha} t^{-1/6}\right) \frac{\Delta V}{V} \tag{47}$$

where $\frac{\Delta V}{V}$ is the nominal exchange ratio generally used in flushing theories relating the change of volume due to tidal activity to the efficient flushing volume of the water body. The same reasoning applies to the flow entering and leaving the open sea and, similar to Eq. (47), we may define exchange factors r″.

Now, assume that flushing of the surface layer in a particular water body is due not only to tidal activity but also significantly to erratic meteorological surges of the system, see Fig. 8. The net water exchange following a series of water level fluctuations is then described statistically as a first order model by

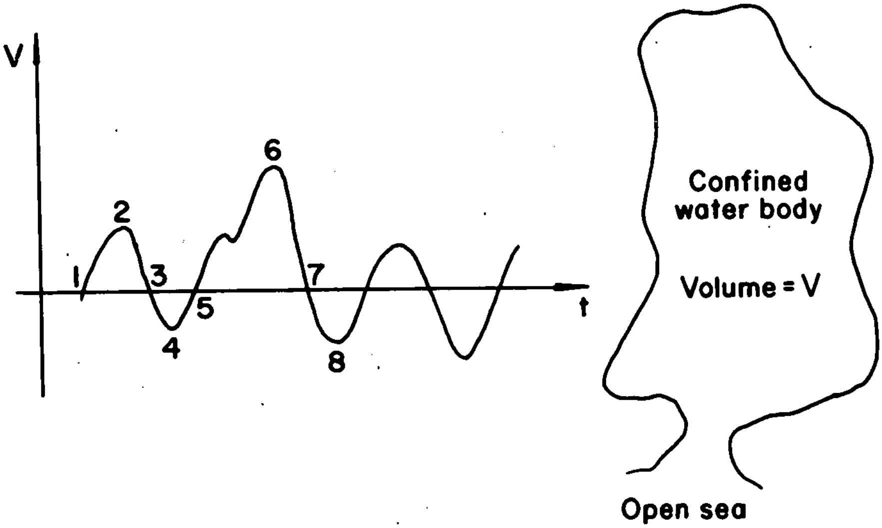

Fig. 8. — Flushing model of the confined water body related to the flow pattern of the free connection with the open sea, Eqs. (41)-(42).

$$V \cdot \left[r'_{1\text{-}3} + r'_{3\text{-}5} + r'_{5\text{-}7} + \ldots\ldots \right. \tag{48}$$

assuming that flushing is the only significant mechanism contributing to water exchange. We may write the exchange factors as:

$$\begin{aligned} r'_{1\text{-}3} &= \left[1 - \frac{a}{(t_3 - t_1)^n} \right] \frac{\Delta V_2}{V} \\ r'_{3\text{-}5} &= \left[1 - \frac{b}{(t_5 - t_3)^m} \right] \frac{\Delta V_4}{V} \end{aligned} \tag{49}$$

and so on[2]

---

[2] Appropriate values of the assumed constants-a, b, n and m may be obtained from tracer field experiments, preferably by use of drouges or driftcards released at the cross-section of the passage.

Let us now consider an embayment with a broad communication with the open sea. In this case, the flushing effect probably does not contribute more to water circulation and exchange than the wind. Furthermore, the dispersive characteristics may differ considerably from site to site within the embayment. A straight-forward approach for study of the water circulation of the surface layer would be in-situ tracer experiments in order to simulate the processes of dispersion. Let us however, discuss the feasibility of a mathematical model for predicting the dispersive properties.

In recent years several methods have been developed for tidal computation in coastal waters, Dronkers (1969)[10] and Leendertse (1967).[24] These numerical schemes account for the effect of bottom friction but neglect viscosity effects in the lateral direction which unfortunately very often is a significant feature of two-dimensional dispersion (separation effects). Let us assume that the dispersion of the surface layer may be properly studied by means of a two-dimensional model. We may then write the equation for conservation of mass as follows:

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}(uc) + \frac{\partial}{\partial y}(vc) = \frac{\partial}{\partial x}\left(\epsilon_x \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(\epsilon_y \frac{\partial c}{\partial y}\right) \qquad (50)$$

where c is the concentration of a conservative tracer. In finite-difference form we get:

$$\frac{c_{ij}(t_o+\Delta t) - c_{ij}(t_o)}{\Delta t} + \frac{(uc)_{i+1,j} - (uc)_{i-1,j}}{2\,\Delta x} + \frac{(vc)_{i,j+1} - (vc)_{i,j-1}}{2\,\Delta y}$$

$$= \epsilon_{xij}\,\frac{c_{i+1,j} - 2c_{ij} + c_{i-1,j}}{\Delta x^2} + \epsilon_{yij}\,\frac{c_{i,j+1} - 2c_{i,j} + c_{i,j-1}}{\Delta y^2} \qquad (51)$$

or in matrix form

$$\left[a_{ij}(t_o)\right]\;\left[c_{ij}(t_o)\right] + \left[b_{ij}(t_o)\right]\;\left[c_{ij}(t_o)\right]^{*}\longrightarrow c_{ij}(t_o+\Delta t) \qquad (52)$$

The matrices $\left[a_{ij}\right]$ and $\left[b_{ij}\right]$ account for the combined effect of advection and diffusion in x- and y-direction respectively. The step from Eq. (50) to (52) means that we have laid out on the region considered a rectangular array of spatial points and specified the values of the variables at the points of the rectangular grid. Furthermore, we have to denote proper boundary conditions and, above all, we have to consider the three dimensional nature of the circulation. In most coastal areas of the kind we are discussing here, we find that the bottom layer has a fairly constant salinity. The mean depth of the surface layer does not vary much, although internal seiches give rise to temporary displacements of the interface. Hence, in those cases the two-dimensional model may be accepted as an engineering approximation. If the salinity is chosen as the tracer material — which is very convenient, provided that there are pronounced variations of salinity within the area — then we have to account for the vertical transport of salt water. Assume a one-way transport from the lower layer of salinity c_o to the upper layer, we then get:

$$\begin{aligned} &\left[a_{ij}(t_o)\right] \left[c_{ij}(t_o)\right] + \left[b_{ij}(t_o)\right] \left[c_{ij}(t_o)\right]^* \\ &\qquad + c_o \left[d_{ij}(t_o)\right] \longrightarrow c_{ij}(t_o + \Delta t) \end{aligned} \tag{53}$$

Furthermore, we assume that there are a certain number of environmental conditions that from a statistical point of view are relevant for describing the circulation of the system. Hence, we are searching for a set of solutions controlled by the values of the environmental parameters — wind, fresh water outflow, etc. Our principal interest is in the steady state solutions because they are unique and not affected by initial conditions such as salinity distributions which implies an uncontrolled dynamic effect. Hence, to represent a particular grouping of environmental parameters we may write:

$$\left[a_{ij}\right] \left[c_{ij}\right] + \left[b_{ij}\right] \left[c_{ij}\right]^* + c_o \left[d_{ij}\right] \longrightarrow c_{ij} \longrightarrow \left[c_{ij}\right] \tag{54}$$

If the velocity field and the tracer distribution are measured, Eq. (54) may be solved for the unknown vertical mass transfer velocities expressed by matrix $\left[d_{ij}\right]$ provided that appropriate values of the eddy diffusivities are selected. Hence, the full set of matrices $\left[a_{ij}\right]$, $\left[b_{ij}\right]$ and $\left[d_{ij}\right]$ characterizing the dispersion of the surface water can be calculated. To summarize, we have proposed an analytical dispersion model overlying an empirical flow model. We could also have included as a third step a reaction model which delays substances, generates reactions between substances, and simulates sources and sinks. It is obvious, however, that an analytical simulation technique as just outlined leads to extensive work even for a modern computer, and the field work implied is less significant.

TRACER SIMULATION OF DISPERSION

The analytical approach for modeling of dispersion phenomena in coastal waters discussed in the previous section, meets with the difficulties arising from the general complexity of the circulation. Hence, tracer measurements performed in the field represent an important engineering technique to determine the dispersion properties of a receiving water area for the conditions that will prevail when, for instance, an outfall has been built.

During the last twenty years the use of tracers has been very common. This is due to the development of very sensitive instrumentation for both laboratory and field investigations and furthermore, to the fact that many new tracers have appeared so that for each case one should be able to find the "ideal tracer".

Definition of a tracer is scarcely possible without taking into account the actual purpose of its use. Almost every substance could in one connection or another be regarded as a useful tracer. For more conventional use, however, we can choose between radioactive isotopes or fluorescent dyes. In most cases it is convenient to distinguish only between conservative and nonconservative tracers. When the amount of tracer decreases with time, the tracer is non-conservative or decaying. Very often, rapidly decaying tracers are preferred to more conservative ones. For instance, the presence of the tracer may be wanted only for a short period of time. Use of a slowly decaying tracer then could give rise to undesirable effects of accumulation.

The decay of tracers can normally be described by an exponential decay function e^{-kt} where k is the decay parameter and t the time. The half-time $t_{0.5}$ then equals:

$$t_{0.5} = \frac{\ell n2}{k} \tag{55}$$

The decay parameter may be a constant characterizing the tracer in question, or it may be a function of light conditions, acidity and turbidity, etc. of the medium. In each case the decay parameter is well-defined only when these conditions are fully specified.

For the study of dispersion in a receiving water, two properties are often of special interest: dilution, and residence time of a discharged pollutant. Residence time should be understood as the time after the discharge of the waste water element in question. These two properties are of great interest with reference to later consideration of e.g. ecological and hygienic consequences of a planned discharge of waste water.

The purpose of the field tracer simulation, regardless of the method used, is to reach a statistical description of the dispersion pattern. For reliable prediction we must be able to reproduce or account for the initial stage of mixing as well as the effect of gravitational spread, provided that this phenomenon is of significance. There are two major approaches for tracer release — continuous injection during a certain period of time; and repeated, instantaneous injections covering the full range of environmental conditions. The most adequate simulation of a continuous waste discharge is certainly attained by applying a continuous tracer release technique. Assume that the waste product which is to be traced may be characterized by a decay parameter, k, corresponding to the most critical or most conservative component of the waste. Then a continuous injection of a tracer material with a decay parameter equal or close to k is the most straight-forward and most reliable approach. This situation is, however, very seldom possible. In order to simulate the continuous discharge of a conservative contaminant by means of a decaying tracer, the injection must be steadily reduced at a rate equal to the tracer decay. This restricts severely the length of the period of injection, especially when rapidly decaying radioactive isotopes are used, due to the fact that both the initial and final rates of tracer injection, for safety and practical reasons respectively, have to be neither too high nor too low. To overcome this difficulty a dual tracer technique may be applied.

The method of parallel injection of tracers have been described elsewhere, Cederwall (1968),[7] Cederwall and Hansen, (1968),[6] and just the principle will be outlined. To be used for a continuous, parallel injection two tracers are used which fulfill the following conditions:

a) The two decay parameters must be well-defined.

b) The two tracer substances must differ in at least one property, detectable in low concentrations of the substance.

Consider a decaying tracer with the decay parameter k continuously injected at a constant rate into a receiving water area. The concentration C(T) recorded at a certain point in the receiving water area at a time, T, after the start of the injection, can be considered as a sum of contributions from tracer elements released in succession at the point of tracer discharge. Then the concentration can be expressed by means of a frequency function, $f(T-t)$, i.e. an age distribution with reference to time of release:

$$C(T) = \int_{0}^{T} f(T-t) \cdot e^{-k(T-t)} dt \doteq \int_{0}^{T} F(s) \cdot e^{-ks} ds \qquad (56)$$

Thus, for a steady-state situation of the water circulation, f(t) is the impulse function recorded from a δ-input at time t = 0. The frequency function is characterized by its moments, μ_n , around the averaged value $\bar{t}$ — the mean residence time.

$$\mu_n = \int_0^T (t-\bar{t})^n f(t)\,dt \tag{57}$$

where f(t) is assumed to be normalized, that is $\mu_0 = 1$, $\mu_1 = 0$ and $\mu_2 = \sigma^2$. A residence time, τ , is defined by the following equation:

$$C(T) = e^{-k\tau} \int_0^T f(T-t)\,dt = e^{-k\tau} \tag{58}$$

τ is related to the frequency function f(t) and its moments, μ_n . For an arbitrary function f(t) we can write C(T):

$$\begin{aligned} C(T) &= e^{-k\bar{t}} \int_0^T f(s) \cdot e^{-k(s-\bar{t})} ds = \\ &= e^{-kt} \int_0^T f(s) \sum_{n=0}^{\infty} \frac{(-k)^n (s-\bar{t})^n}{n!}\, ds = \\ &= e^{-k\bar{t}} \sum_{n=0}^{\infty} \frac{(-k)^n}{n!} \int_0^T (s-\bar{t})^n f(s)\,ds = e^{-k\bar{t}} \left[1 + \sum_{n=2}^{\infty} \frac{(-k)^n \mu_n}{n!} \right] \end{aligned} \tag{59}$$

Hence,

$$\tau = \bar{t} - 1/2\, \sigma^2 k + \frac{1}{6} \mu_3 k^2 - \frac{1}{24} (\mu_4 - 3\sigma^4) k^3 + O(k^4) \tag{60}$$

For commonly used tracers and normal dispersive properties of the receiving water τ is very close to $\bar{t}$, see Cederwall (1968).[7]

Assume that the two tracers, 1 and 2, with decay parameters k and k_2, are simultaneously and continuously released at a constant rate. A time parameter, τ, is now defined by:

$$C_1 = C_1' \cdot e^{-k_1 \tau}$$

$$C_2 = C_2' \cdot e^{-k_2 \tau} \qquad (61)$$

where C_1 and C_2 are the tracer concentrations at the measuring site, and C_1' and C_2' corresponding concentrations of the tracers assumed conservative. $c_1 \cdot q_1$ and $c_2 \cdot q_2$ are the rates of tracer injections. Then the following equation holds:

$$\frac{C_1'}{c_1 q_1} = \frac{C_2'}{c_2 q_2} \qquad (62)$$

and,

$$\tau = (k_1 - k_2)^{-1} \cdot \ell n \frac{C_2\, c_1\, q_1}{C_1\, c_2\, q_2} \qquad \frac{\ell n \left[R \frac{C_2}{C_1} \right]}{a} \qquad (63)$$

where R and a have constant values.

$$\tau \approx \overline{t} - 1/2\ \sigma^2 (k_1 + k_2) \approx \overline{t} \qquad (64)$$

Consequently, a τ value may be determined for each site in the receiving water where tracers are found in measurable concentrations. Thus a dual tracer technique makes possible the simultaneous registration of concentration distributions and residence time in a receiving water area. This method is useful not only for dispersion studies but also for determination of water exchange, since the residence time is evaluated and the time of injection is not limited. A case report from and investigation in Byfjorden on the west coast of Sweden using the radioactive tracer Bromine-82 and the fluorescent tracer Pontacyl Brilliant Pink has been reported by Cederwall and Hansen (1968).[6]

Instead of using a continuous release method, it is often possible to carry out spread tests by means of instantaneous injections of the tracer. Provided that the pattern of circulation is steady, the dispersion from a continuous source can be calculated by integration of the results obtained from a single injection. Each tracing then gives a quantitative determination of the spreading from the source for the environmental conditions that prevailed during that particular tracing. An adequate statistical description of pollution from a future outfall must be based on the investigation of a sufficient number of situations representing those parameter combinations that are significant to the spreading pattern. If it is difficult to find a reasonable number of such situations which, from a statistical point of view, represent the full picture of tracer dispersion, then the only realistic approach is long-term tests with continuous injection of a single tracer or, alternatively, a dual tracer injection. The theoretical background to the instantaneous injection technique as well as case reports have been described by Harremoës (1964,[16] 1966[17]).

Measurements of radioactive tracer concentrations in surface waters are usually carried out by means of scintillation detectors connected to a counting system. The standard field procedure is to use submerged units measuring the in-situ concentration, although the impulse is averaged over a small volume surrounding the detector. The standard instrument for fluorescence measurements is the fluorometer. For field application the Turner Fluorometer, Type III, provided with a continuous flow cuvette has been extensively used. This measuring technique does not, however, provide instantaneous recording of the concentration, and furthermore, during the passage of the tube, the concentration is affected by dispersion. Hence, sharp tracer boundaries and gradients of the receiving water are not properly recorded. To eliminate this difficulty Kullenberg (1968)[22] developed an in-situ measuring fluorometer. The instrument, see Fig. 9, consists of two photo-multipliers, a depth sensing unit, and a lamphouse with a mercury lamp which irradiates horizontally the water below one of the photomultipliers which therefore senses both the daylight and the fluorescence of the dye. The other photo-multiplier is unaffected by the fluorescence of the dye and only senses

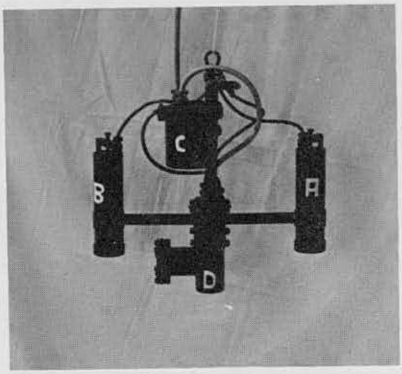

Fig. 9. — In-situ measuring fluorometer.

A, B	*photo-multiplier units*
D	*mercury lamp house*
C	*depth indicator unit*

the daylight. Thus, the effect of the daylight may be eliminated. The instrument has proved reliable during measurements.

The injection technique is a very important element of a tracer investigation. To improve standard methods of tracer release, an instrumentation system has been developed by the Danish Isotope Centre in collaboration with the author, designed primarily for the dual tracer technique but providing for a number of facilities for work in this field.

The "DIC Injector System", see Fig. 10, consists of a pump unit and a control unit. The pump unit contains two separate pump systems. One of the pumps, the feed pump, is running at a constant speed and thus provides an injector for a non-decaying tracer. The other pump, the activity pump, is running at a speed which is variable and governed by the control unit. Its speed is measured by means of a tachometer generator. Both pumps are peristaltic pumps with separate inlets but a common outlet. In the pump

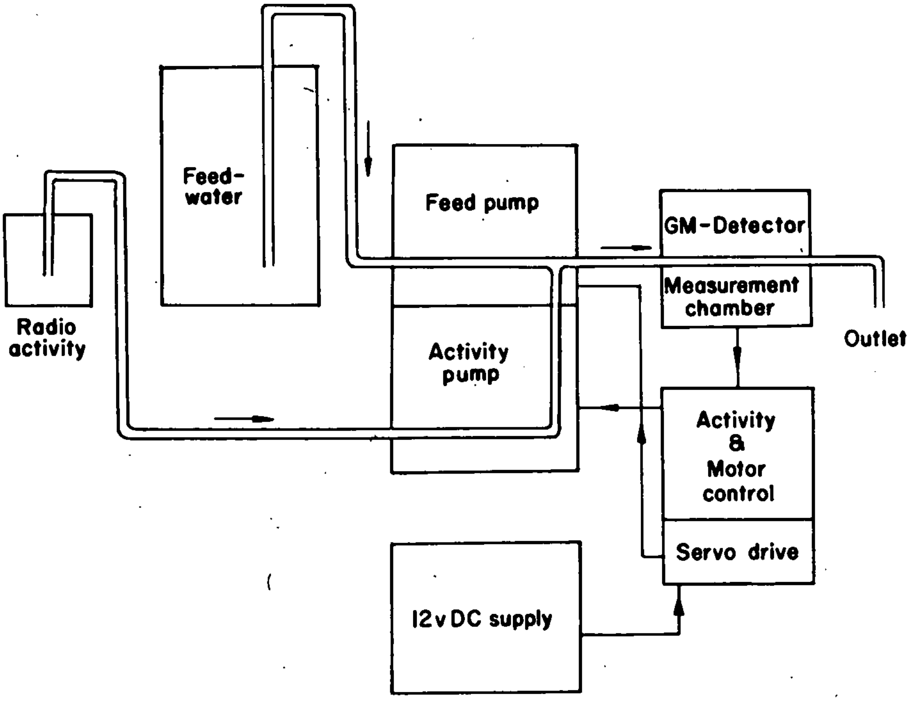

Fig. 10. — DIC injector system schematic.

unit, the radioactive concentration of the outlet is measured by means of a GM-detector. As the pump capacity of the feed pump is larger compared to that of the activity pump, the outlet flow will remain constant with time, and the signal from the GM-detector is thus a measure of the activity flow rate, e.g millicuries per hour of the radioactive tracer. The inlet and outlet of each pump are accessible, so that each pump can be operated separately. The control unit is a fully transistorized servo-unit, which ensures proper operation of the pump systems. Either the activity pump speed or the activity flow rate can be selected at the start of an investigation, and the control unit will insure that the chosen operation conditions does not change during the investigation.

The system is capable of injecting constant activity in the range:

4 mc/hour - 400 mc/hour of Bromine-82 (Br-82).

Manually operated the system can control activity flow rates down to 0.4 mc/hour of Br-82.

SUMMARY

Some approaches have been outlined for prediction of waste dispersion in coastal environments associated with sewage disposal and heat emission. It should be emphasized that a water pollution study represents an integral part of a broad engineering analysis of a particular environmental problem. Like most hydrological questions, water quality planning is very much interdisciplinary in nature.

The dispersive properties of the receiving water area are studied primarily to control hygienic and aesthetic harm to the environment, but also to control possible ecological disturbances and modifications. An important part of the overall analysis is the economical feasibility tests of various alternative projects which include benefit-cost studies of the water area considered. The complexity of the problem stresses the need for highly systematized methods of evaluation throughout the analysis.

The electronic computer has had a profound impact on water resources planning and it is now, and will be still more in the future, an indispensable tool for the engineer working in this field. However, the ability to make extensive calculations must correspond to an equivalent capacity in forming useful models and hypotheses. In this development towards an operational type of water quality management, we will find both purely theoretical and semi-empirical approaches for modeling of flow phenomena extremely valuable.

REFERENCES

1. Abbot, M.B., "On the Spreading of One Fluid Over Another," *La Houille Blanche,* Vol. 16, No. 6, 1961, pp. 827-846.
2. Abraham G., "Jet Diffusion in Stagnant Ambient Fluid," Delft Hydraulics Laboratory 1963, Publ. No. 29, July 1963.
3. Albertson, M.L., et. al., "Diffusion of Submerged Jets," *Trans. ASCE,* Vol. 115, 1950, pp. 639-664.
4. Benjamin, T.B., "Gravity Currents and Related Phenomena," *Jour. Fluid Mech.,* Vol. 31, No. 2, Jan. 1968, pp. 209-248.
5. Cederwall, K., "Rational Judgment on Ocean Outfalls," *Vatten-hygien,* No. 2, 1966, pp. 45-54.
6. Cederwall, K. and Hansen, J., "Tracer Studies on Dilution and Residence Time Distribution in Receiving Waters," *Water Research,* IAWPR, Vol. 2, No. 4, June 1968, pp. 297-310.
7. Cederwall, K., "Hydraulics of Marine Waste Water Disposal," Hydraulics Div., Chalmers Institute of Technology, Report No. 42, 1968.
8. Cederwall, K. and Sjöberg, A., "Discharge of Cooling Water from Thermal Power Plants, *"Särtryckur Väg-och vattenbyggaren,* No. 9, 1969.
9. Ditmars, J.D., "Computer Program for Round Buoyant Jets Into Stratified Ambient Environments," W.M. Keck Laboratory, California Institute of Technology, Tech. Memo. 69-1, 1969.
10. Dronkers, J.J., "Tidal Computations for Rivers, Coastal Areas and Seas," *Jour. of Hyd. Div., ASCE,* Vol. 95, HY 1, Jan. 1969, pp. 29-77.
11. Fan, L.-N., "Turbulent Buoyant Jets Into Stratified or Flowing Ambient Fluids," W.M. Keck Laboratory, California Institute of Technology, Report No. KH-R-15, July 1967.
12. Fan, L.-N. and Brooks, N.H., Discussion of "Horizontal Jets in Stagnant Fluid of Other Density," by Gerrit Abraham, *Jour. of Hyd. Div., ASCE,* Vol. 92, HY 2, March 1966, pp. 423-429.
13. Fan, L.-N. and Brooks, N.H., "Numerical Solution of Turbulent Buoyant Jet Problems," W.M. Keck Laboratory, California Institute of Technology, Report No. KH-R-18, Jan. 1969.
14. Foxworthy, J.E., "Eddy Diffusivity and the Four-Thirds Law in Near-Shore Coastal Waters," University of Southern California, A. Hancock Foundation Report 68-1, 1968.
15. Hansen, J., "Computer Calculations on Jet Diffusion and Dispersion in Stratified Receiving Water," The Danish Isotope Centre, Copenhagen, 1967.
16. Harremöes, P., "Prediction of Pollution from Planned Sewage Outlets," *Tekn. Kem. Aikak,* Vol. 21, No. 6, 1964, pp. 251-260.
17. Harremöes, P., "Tracer Studies on Jet Diffusion and Stratified Dispersion," *Advances on Water Pollution Research,* Vol. 3, Munich 1966, pp. 65-86.
18. Hayashi, T. and Shuto, N., "Diffusion of Warm Water Jets Discharged Horizontally at the Water Surface," I.A.H.R. Colorado, Vol. 4, June 1967, pp. 47-59.
19. Häggström, S., "Cooling Water Discharge Into a Coastal Current," Hydraulics Div., Chalmers Institute of Technology, Progress Report, 1969.
20. Ippen, A.T., (Ed.), *Estuary and Coastline Hydrodynamics.* McGraw-Hill, 1966.
21. Jen, Y., et. al., "Surface Discharge of Horizontal Warm-Water Jet," *Jour. of Power Div., ASCE,* Vol. 92, PO 2, 1966, pp. 1-30.
22. Kullenberg, G., "Measurements of Horizontal and Vertical Diffusion in Coastal Waters," University of Copenhagen, Institute of Physical Oceanography, Rep. No. 3, 1968.
23. Larsen, I. and Sorensen, T., Theory on Ref. 7, 1967, (unpublished).
24. Leendertse, J.J., "Aspects of the Computational Model for Long-Period Water-Wave Propagation," The Rand Corp., Mem. RM-5294-PR, 1967.

25. Okubo, A., "A New Set of Oceanic Diffusion Diagrams," Chesapeake Bay Inst., John Hopkins University, Techn. Rep. 38, June 1968.
26. Pritchard, D.W., "Estuarine Hydrography," *Advances in Geophysics,* Vol. 1, Academic Press, Inc., New York, 1952, pp. 243-280.
27. Pritchard, D.W., "Dispersion and Flushing of Pollutants in Estuaries, *Jour. of Hyd. Div., ASCE,* Vol. 95, HY 1, Jan. 1969, pp. 115-124.
28. Rajaratnam, N. and Subramanya, K., "Plane Turbulent Reattached Wall Jets," *Jour. of Hyd. Div., ASCE,* Vol. 94, HY 1, Jan. 1968, pp. 95-112.
29. Sawyer, R.A., "Two-Dimensional Reattaching Jet Flows Including the Effects of Curvature on Entrainment," *Jour. of Fluid Mech.,* No. 17, Dec. 1963, pp. 481-498.
30. Sharp, J.J., "Spread of Buoyant Jets at Free Surface," *Jour. of Hyd. Div., ASCE,* Vol. 95, HY 3, May 1969, pp. 811-825.
31. Tamai, N., et al., "Horizontal Surface Discharge of Warm Water Jets," *Jour. of Power Div., ASCE,* Vol. 95, PO 2, Oct. 1969, pp. 253-276.
32. Waldichuk, M., "Estimation of Flushing Rates from Tide Height and Current Data In An Inshore Marine Channel of the Canadian Pacific Coast," *Advances in Water Pollution Research,* Vol. 3, Tokyo 1964, pp. 133-166.
33. Wood, I.R. and Wilkinson, D.R., Discussion of "Surface Discharge of Horizontal Warm Water Jet," by Jen, Y., et al, *Jour. of Power Div., ASCE,* Vol. 93, PO 1, 1967, pp. 149-151.
34. Wu, J., "Mixed Region Collapse with Internal Wave Generation in a Density-Stratified Medium," *Jour. of Fluid Mech.,* Vol. 35, No. 3, Feb. 1969, pp. 531-544.