JOURNAL *of the*

BOSTON SOCIETY OF CIVIL ENGINEERS

1848
BOSTON SOCIETY OF CIVIL ENGINEERS

122 YEARS
1848 - 1970

JANUARY — 1970

VOLUME 57 NUMBER 1

JOURNAL OF THE

BOSTON SOCIETY OF CIVIL ENGINEERS

Volume 57 JANUARY 1970 Number 1

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Journal of Boston Society of Civil Engineers is indexed regularly by Engineering Index, Inc.

Second-Class postage paid at Boston, Massachusetts

Published four times a year, January, April, July and October, by the Society
47 Winter Street, Boston, Massachusetts 02108

Subscription Price $7.00 a Year (4 Copies)
$1.75 a Copy
All orders must be accompanied by check

BOSTON SOCIETY OF CIVIL ENGINEERS
OFFICERS, 1969 - 1970

JOURNAL OF THE
BOSTON SOCIETY OF CIVIL ENGINEERS

Volume 57 JANUARY 1970 Number 1

BACK BAY BOSTON
PART 1

By

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(Presidential address presented at the Annual Meeting of the Boston Society of Civil Engineers held on March 24, 1969)

SYNOPSIS

This paper concerns the Back Bay, a former tidal estuary in Boston which was filled a century ago to create land for an expanding population. In Part I of the paper, the geology of the Back Bay and subsurface soil conditions are described. Topographic development of the area is traced and early foundation practice in the Back Bay is discussed. In Part II, the design and construction of sewers and subways are included, insofar as they provide data on soil conditions and affect ground water levels in the Back Bay. Finally, the soil mechanics and foundations aspects of building design and construction are summarized.

The author hopes that the paper will provide engineers and contractors with a useful and interesting reference for information relative to soil conditions, ground water levels, existing underground facilities and foundation practice in the area, from the earliest days of development in the Back Bay, to the deep foundations supporting the New Boston.

INTRODUCTION

The shore line of Boston today bears little resemblance to the shore line when Boston was settled in the seventeenth century. The original high water line, superimposed on a map of present day Boston in Figure 1,

* Principal, Haley & Aldrich, Inc., Cambridge, Massachusetts.

shows this relationship vividly. Over a period of two centuries, tidal areas adjacent to the land were filled by cutting down the hills and hauling materials from land outside the City.

Back Bay as defined herein extends from Boston Common to Massachusetts Avenue and from the Charles River to Washington Street, a residential and commercial area of approximately 600 acres, Figure 2. The Back Bay Fens, located west of Massachusetts Avenue, is not included in the discussion.

GEOLOGY AND SUBSURFACE SOIL CONDITIONS

GENERAL

Soil conditions in Back Bay and indeed the topography of seventeenth century Boston, owe their origin primarily to events which took place during the Pleistocene. During this period, there were successive advances and retreats of glacial ice from the region, followed by extreme variations in climate and sea level relative to the land, all of which influenced the sediments and their engineering properties.

Typical soil and rock profiles in Back Bay are shown in Figure 3. Although it is the overburden soils that are primarily of interest to the civil engineer practicing in the area, the underlying bedrock has become increasingly important with the construction of major high-rise buildings on deep foundations. Thus, we begin with a description of the rock and progress upward through the more recent sediments.

BEDROCK

The best account of the bedrock geology in the Boston area is given by LaForge (12)*. Bedrock in the Boston Basin belongs to the Boston Bay Group which includes two formations, a lower one called the Roxbury Conglomerate and an upper one named the Cambridge Slate. The Cambridge Slate underlies the Back Bay and indeed most of the Boston peninsula, Cambridge, Watertown and Somerville as well as parts of Medford, Everett, Chelsea, East Boston, North Quincy and Hingham. It is believed to be at least 2000 to 4000 ft. in thickness.

The Cambridge Slate consists dominently of fine-grained clayey rocks which are slaty in places. The slaty cleavage is frequently absent and "argillite" is a better and more common name for the rock.

The parent sediments were deposited in a period which Kaye (11) believes is probably Carboniferous in age. The formation was subjected to tec-

*Numbers in parenthesis refer to References listed at end of paper.

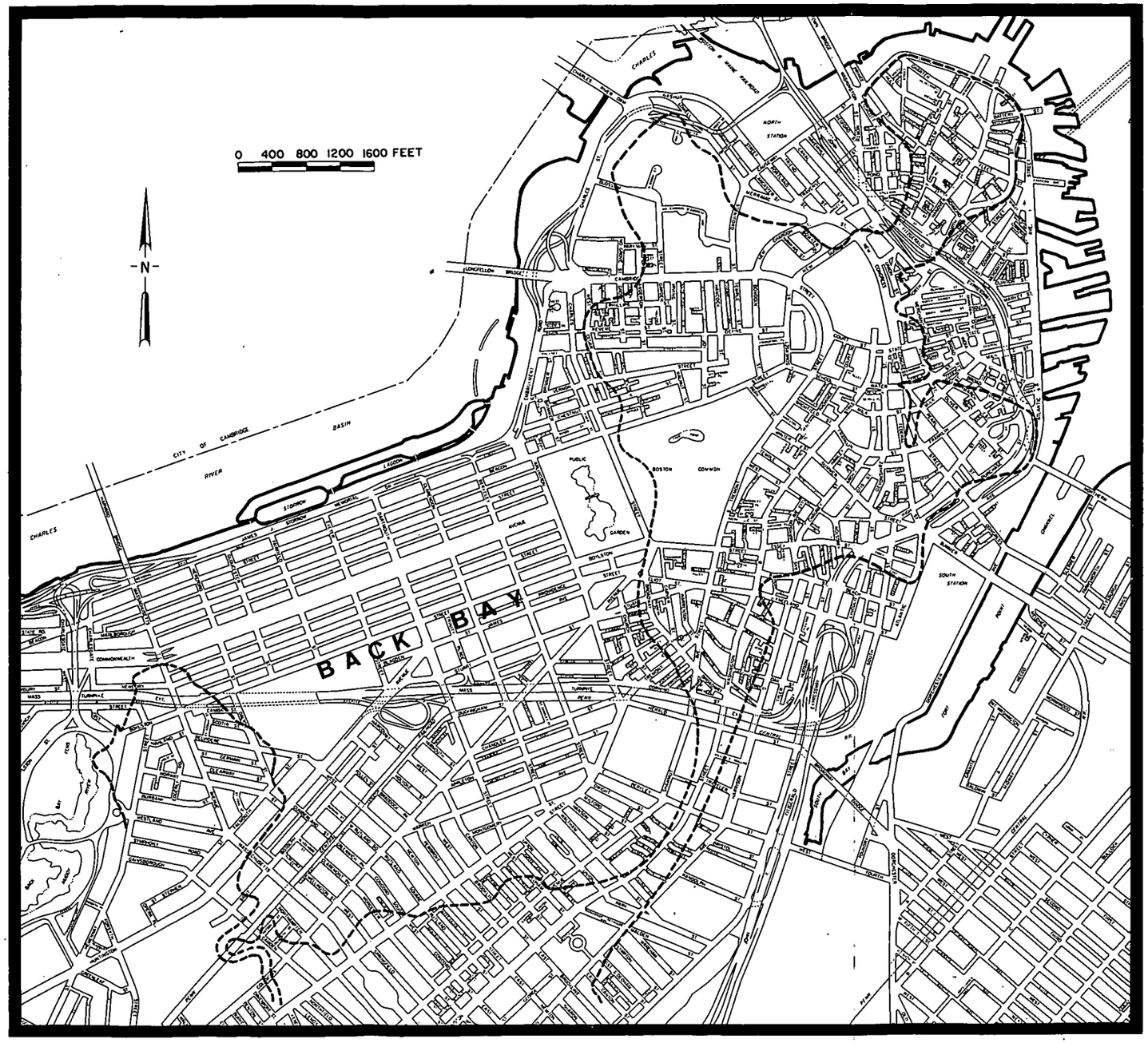

Figure 1 - Colonial Shoreline Superimposed on a Modern Map.

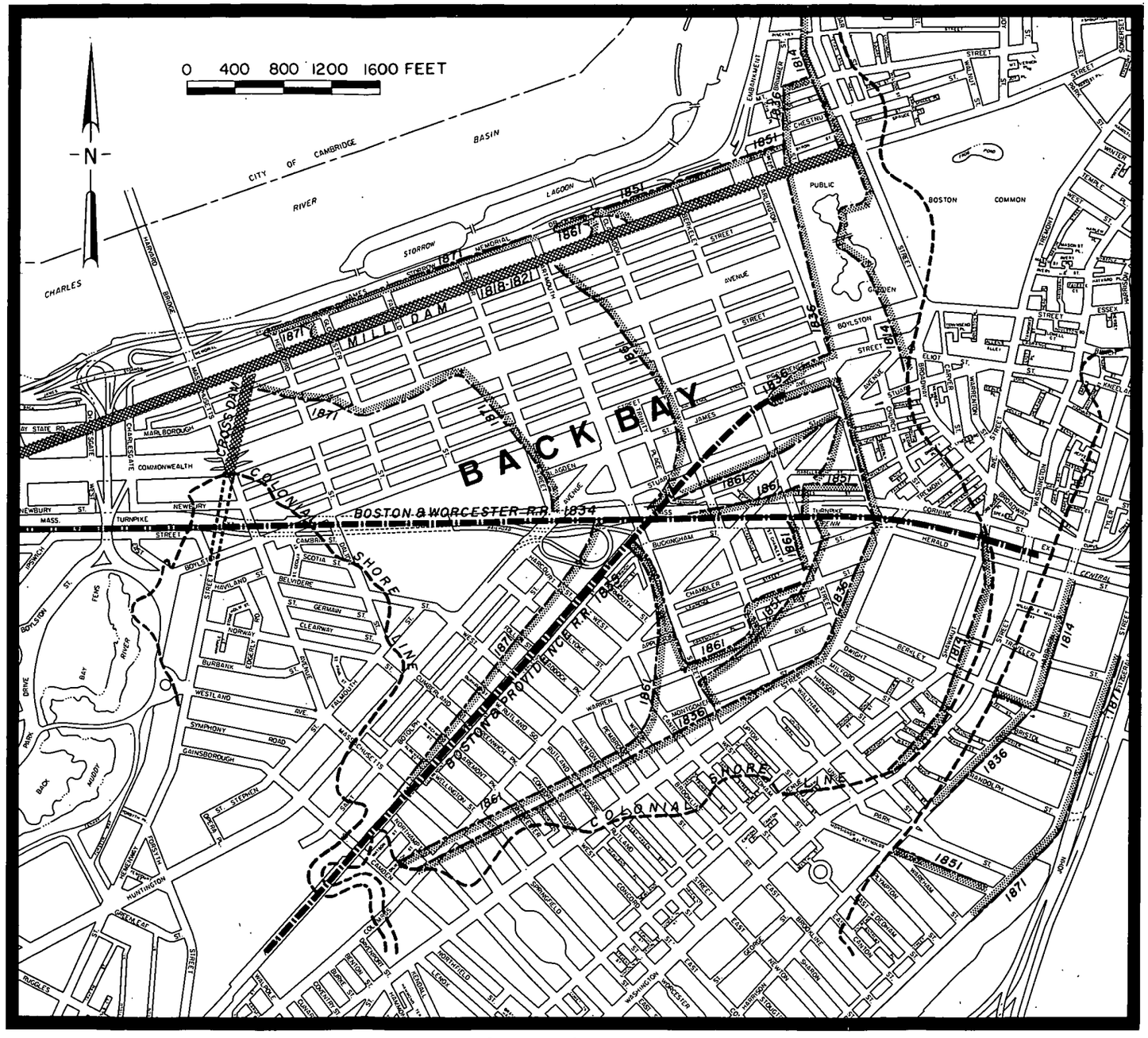

Figure 2 - Sequence of Filling in Back Bay.

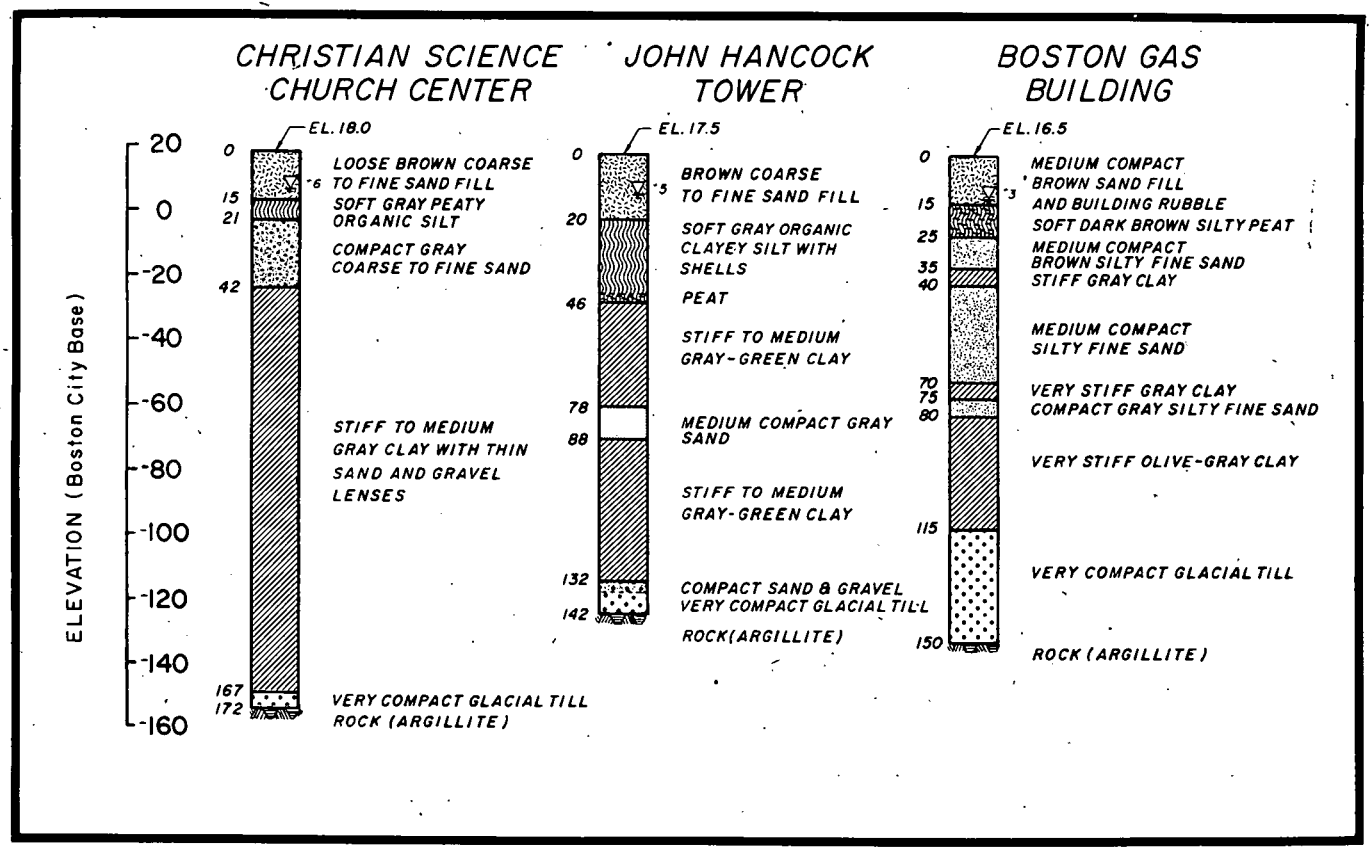

Figure 3 - Typical Soil Profiles in Back Bay.

tonic stresses which produced several broad folds and a number of lesser ones in the Boston Basin. The major fold axes are aligned roughly east-west and plunge toward the east. Faults are fairly numerous and dikes and sills cut through the formation, the most common intrusive rock being diabase. Other fine-grained igneous rocks and volcanic tuff have also been encountered in core borings.

The argillite is derived from siltstone, claystone or shale and is generally bluish-gray or brownish-gray in color. It is well-stratified with a dip commonly from 50 to 60 degrees but varying from 30 degrees to near vertical. Kaye (11) describes the rock locally as "----having a fairly well developed slaty cleavage. Typically, these rocks are thin bedded or banded, and consist of alternating light- and dark-gray strata ranging from 1 to 100* centimeters in thickness. Bedding parting is absent or poorly developed, and fissility is lacking." Descriptions of the rock encountered in tunnels driven below the greater Boston area are included in papers by others (1), (2), (13) and (14).

Deep borings in the Boston area have shown that the argillite has been altered or weathered at some locations to a soft light-gray clayey material which is predominently kaolin. Intensive alteration has occurred in some areas to depths of 300 ft. or more.

In 1914, in his report on the new Cambridge site for M.I.T., Professor W. O. Crosby (18,p.225) noted "----the slate to be extensively and deeply decomposed. In fact, the slate is, in large part, rotted to a whitish and more or less plastic clay; and close observation is necessary to determine the line between the drift and the bedrock."

Kaye (11) attributes the kaolinized zones most probably to the roots of an extensive lateritic regolith that mantled southern New England in the Tertiary, but recognizes the possibility that the alteration is hydrothermal in origin.

Within the Back Bay, altered argillite has been encountered in test borings at Beacon and Clarendon Streets, below the Boston Common Garage, at Castle Square, and for the Boston Gas building at Park Square. It was not found during drilling for the Prudential Center or for the new John Hancock building and parking garage.

To the foundation engineer, the presence of the altered argillite and the occurrence of clay seams within an otherwise relatively hard indurated rock, present an important condition to be explored for any major building project.

*Kaye now believes that 0.1 to 10 centimeters is more typical although much thicker strata do occur, for example at the Christian Science Church Center development.

Before and during the Pleistocene glaciation, the surface of the rock was eroded to form deep valleys. The Back Bay is located on the eastern edge of one such valley. As a result, rock is relatively deep, generally from 100 to 200 ft. below ground surface.

Subsequently, the valleys were filled with thick deposits of sediments of glacial origin.

OVERBURDEN SOILS

General: During the past 65 years, the Boston Society of Civil Engineers has made a significant contribution to our knowledge of the distribution of overburden soils in the Boston area by publishing the logs of test borings, and maps showing boring locations. In 1903, J. R. Worcester (17) contributed a paper to the Society which included logs of the earliest borings. He supplemented this information in a paper on "Boston Foundations" (18) published in the first volume of the Journal in 1914. Subsequently, the Subsoils of Boston Committees extended the work, collecting data and publishing boring logs. Information on the Back Bay is contained in Journal issues of September 1931 and October 1949. The most recent contribution, published in the July-October 1969 issue of the Journal, includes logs of many deep borings made during the last 15 years.

A detailed description of Back Bay sediments is given by Judson (6,p.7-48) in his contribution to "The Boylston Street Fishweir II", a fascinating series of papers of the Robert S. Peabody Foundation for Archaelogy.

Soils which overlie rock in the Back Bay include glacial till, a marine clay, sand and gravel outwash and organic soils. Finally, a century ago, sand and gravel fill was transported into the Back Bay to cover these natural deposits. Typical soil conditions at three locations within the Back Bay are shown on Figure 3. The stratigraphy around the easterly fringe is very complex.

Glacial Till: The first deposit of any significance to cover the bedrock during the Pleistocene was the glacial till or hardpan, deposited by the overriding glacier. Throughout the Boston area, the till commonly mantles the bedrock, varying in thickness from a few feet to over 100 ft. Thick deposits of glacial till form numerous islands in Boston Harbor (Deer Island) and other distinct hills on the mainland (Orient Heights) which are known as drumlins.

Originally, all of the major hills on the Boston peninsula including Copp's Hill in the North End and Fort Hill in the South End were thought to be drumlins, Crosby (4,p.345), underlain by shallow bedrock. However, recent test borings and geological investigations have suggested that the

"Trimountain," which included Mount Vernon Hill, Beacon Hill, and Pemberton (Cotton) Hill, is a far more complex geologic feature which includes deep deposits of overthrust sediments of all types which were bulldozed up and over the underlying till and outwash materials by a secondary advance of glacial ice.

Glacial till is an unsorted, generally non-stratified mixture of rock fragments and minerals of all sizes, varying from cobbles and boulders to silt and clay-size particles. The unweathered till is generally blue-gray in color, but weathering has oxidized the material in the topographic highs to a rusty buff color. The till is very compact and generally difficult to excavate. In the Back Bay, the unweathered till is relatively thin, varying from a few feet to perhaps 30 ft. in thickness.

Occurring with the till in many places in the Back Bay is a relatively pervious stratum of sand and gravel, probably an outwash deposit. The continuity of this stratum was demonstrated during extended dewatering for deep caisson foundations to support a building located on Harrison Avenue between Herald and Traveler Streets. Deep observation wells located at the Prudential Center, approximately one mile away, dropped as much as 30 ft. and piezometers installed below the clay across the Charles River at the M.I.T. Hayden Memorial Library were lowered by 1 to 2 ft.

Clay: The most famous of the local sediments is known as Boston blue clay, actually a silty clay of medium plasticity which is blue-gray to a drab olive-green in color. Silt and clay-sized particles, sorted from the till by glacial streams, settled out in a relatively quiet marine environment in bays around Boston, primarily from Boston to Lynn. Generally, the clay occupies the topographic lows between the predominently glacial till highs.

In the Back Bay, the clay is typically from 50 to 125 ft. in thickness, but clay to a depth of 180 ft. was encountered in borings for an apartment building located at the corner of Beacon and Fairfield Streets. Clay underlies all of the Back Bay. The stratum contains many lenses of fine sand, local strata and pockets of granular soils and occasional boulders.

At the time the clay was deposited, the sea stood 30 ft. or more higher than its present level. Subsequently, sea level fell relative to the land to expose the clay surface to weathering and erosion. At that time, when the sea level was perhaps 70 or 80 ft. below that at present, the surface of the clay at the higher elevations dried to form a stiff to hard weathered crust, commonly called yellow clay. Drying had less effect with increasing depth below the surface and the clay commonly becomes medium to soft in consistency toward the bottom. The stiff crust of the clay stratum plays an important part in supporting structures within the Back Bay area.

Sand and Gravel Outwash: Following a readvance of glacial ice perhaps twelve to fourteen thousand years ago, termed the Lexington Substage by Judson (6,p.23), well-stratified sand and fine gravel outwash materials were deposited over parts of the surface of the eroded and weathered Boston clay. In the Back Bay area, the sand and gravel is well-developed and generally continuous west of Copley Square. Beginning around Dartmouth and Exeter Streets, it increases in thickness westerly toward Massachusetts Avenue on the colonial peninsula in the Back Bay called, appropriately, Gravelly Point. At the Christian Science Church Center, the coarse to fine sand is approximately 20 ft. thick, Figure 3. East of Copley Square, the outwash occurs irregularly. It is absent at the John Hancock site, Figure 3.

The outwash is generally a medium compact to compact gray well-graded gravelly sand, deposited by rapidly moving streams of glacial melt waters. The outwash is very pervious and, in contrast to the glacial till, it can be excavated easily since it contains little binding silt and clay-size particles.

Organic Soils: In recent times following the glacial age, organic deposits formed throughout the Back Bay. Three distinct types of organic soil have been encountered: (1) fresh water peat, formed in areas having sluggish drainage; (2) organic silt with shells, deposited in salt water by tidal action; and (3) salt marsh peat, which accumulated along the shore line of a slowly rising sea.

An ancient fresh water swamp, in which peat formed and trees grew, occurred in the central to easterly section of the Back Bay. According to Judson (6,p.29), good surface drainage, which had undoubtedly been established by erosion when the clay stratum was exposed to drying, was probably blocked by the irregular outwash sand deposits. The peat which accumulated is relatively thin, generally less than 5 ft. in thickness.

As the sea level rose relative to the land, beginning some eight to ten thousand years ago, the fresh water peat bogs were eventually flooded, and marine silts and peats formed in the new salt water environment. The organic silt which accumulated in the sluggish tidal currents is generally gray in color and varies from a non-plastic silty fine sand to a plastic peaty clayey silt with shells.

The silt overlies the lower fresh water peat, or where the peat is absent, the silt was deposited directly on the outwash sands or the surface of the clay stratum. Around the fringes of the Back Bay in particular, the organic silt is overlain by salt marsh peat which began to accumulate to keep pace with a slowly rising sea.

Organic soils blanket the Back Bay area continuously and vary in thickness from 5 to 25 ft. Where the thickness is greatest in the central section of the Back Bay, the top surface of organic soil generally occurs near or below El. 0, Boston City Base*. Originally, the surface was much higher, but considerable compression has occurred under the weight of man-made fills. At the fringes of the Back Bay and on Gravelly Point (Massachusetts Avenue), the top surface of organic soils occurs up to El. +9, see Kaye (8).

SEA LEVEL CHANGES AND CRUSTAL RISE

Positive evidence that sea level was considerably lower relative to the land than at present, is the occurrence of a thin layer of fresh water peat overlying the clay as much as 20 to 30 ft. below present mean sea level. The well-preserved stump of a pine or cedar tree with roots was found at El. -15 Boston City Base, a depth of 30 ft. below street level, during construction of the Boylston Street Subway just west of Church Street in 1913. This discovery is reported by Manley (18,p.406). In one corner of the excavation for the Berkeley Street John Hancock building in 1946, oak and maple stumps were found at El. -20. In addition, sharpened stakes and wattles, remains of ancient Indian fishweirs, were found at El. -12 to -20 in the excavations for the New England Mutual and John Hancock buildings. A fascinating description of these discoveries is provided by Judson (6,p.7) and Barghoorn (6,p.49). At fishweir time, perhaps 4000 to 5000 years ago, water level was at least 15 ft. below the present sea level.

A sample of fresh water peat, recovered from a caisson excavated to support the I.B.M. building located at the corner of Clarendon and Boylston Streets was radiocarbon-dated to be approximately 5,600 years old, Kaye and Barghoorn (9). At this location, the peat occurred at El. -20 and was approximately 1 ft. in thickness.

Sea level change and crustal rise in the Boston area are described in detail by Kaye and Barghoorn (9). They conclude that sea level at Boston reached to within 2 ft. of its present level about 2800 years ago. Furthermore, Kaye (10) reports that while sea level was about at today's elevation 116 years ago, it was approximately 0.5 ft. lower at the turn of the century.

It is interesting to note that half a century ago, many engineers believed that settlement and perhaps displacement of the clay were responsible for the presence of peat, fishweirs and tree stumps substantially below sea

*All elevations used herein are referenced to Boston City Base where El. 0.0 is 5.65 ft. below USCGS Mean Sea Level.

level. For example, in his 1914 paper, Worcester (18,p.3) writes the following interesting account "----under a section of Cambridgeport and a part of the Back Bay the material (clay) is extremely soft, so soft, in fact, that it apparently is quite free to flow from heavily loaded areas towards places where the load is less. It is not definitely determined, so far as the writer knows, whether such a flowing takes place, or the clay is gradually being compressed. It is certain, however, that widely-spread settlements have occurred, in some instances to a very marked extent. A section of Cambridgeport covering about one-half square mile, centering roughly on Massachusetts Avenue and Albany Street, has settled to a maximum amount of about 2 ft.* In Boylston Street, between Berkeley and Clarendon Streets, the Transit Commission found the well-preserved remains of a weir or fence at about grade -18. It does not seem possible that this could have been constructed below low tide level or grade 0. Near Church Street was found a well-preserved stump of a tree with roots, at about grade -15. Another instance of subsidence is found in the depth at which peat is encountered. This material must have been formed above water, but is now found, overlaid with silt, far below grade 0. On Tremont Street, above Dover, it was found at about grade -12, and on Boylston Street it has been found at grade -19. This tendency to settle will have to be taken into consideration in locating heavy structures in the future. It is not enough to gain the necessary support in piles which may rest in a gravel crust, but the settlement of the crust may seriously injure important structures, as it is believed to have already done in the case of the Public Library and the New Old South Church."

In commenting on the tree stump found during subway construction, L. B. Manley (18,p.406) reasoned that "----its presence at this depth indicates a settlement of the surface of at least 25 ft."

In his discussion of the Worcester Paper, Charles R. Gow (18,p.191) relates the presence of peat below the organic silt to a rather rapid subsidence: "Thus, when we find peat deposits at great depths below the marsh level, we may assume that such settlements as their presence indicates may reasonably have occurred during a comparatively short period of subsidence such as the one we are now discussing. This assumption is strengthened by the known fact that the peat deposits are usually covered with a deposit of silt, proving that the vegetation was suddenly stopped by a rapid subsidence of the marsh level below the surface of the water. Had the subsidence been as gradual as that which we now assume it to be in general, there seems to be no good reason why the peat should not be continuous to the surface."

*This settlement was later attributed to compression of organic soil below recently filled land.

Henry F. Bryant (18,p.205) was not convinced, however, and had some rather astute comments on the subject: "Mr. Worcester suggests that peat at considerable depth indicates land subsidence. I accept that statement with some hesitation. In the case of fresh water peat, that is certainly not the case, as we find it to depths of forty, seventy and even one hundred feet, completely filling old glacial pot holes. I have in mind one or two instances of tidal marshes where the subsidence would of necessity be quite irregular had the bottom of the peat ever been at or near the surface. I think that the evidence is favorable for Mr. Worcester's theory, but I do not think it is by any means proven."

TOPOGRAPHIC DEVELOPMENT

GENERAL

The last remaining element in the soil formation throughout the Back Bay area is the man-made fill placed during the last 175 years. Historically, it is of some interest to recount the topographic development in the area, for the filling of this great tidal basin was to be the most drastic single alteration in the history of Boston's changing topography. A summary of the sequence of filling is shown on Figure 2 and a detailed account of the topographic development is given by Whitehill (16). Further information can be found in Bunting (3). A number of maps prepared by the Coast and Geodetic Survey and the engineering firm of Fuller and Whitney, Figures 4 through 13, provide an interesting chronology of the Back Bay filling and building development.

MILL DAM

The earliest encroachment on the Back Bay tide flats occurred in 1794 when the town granted the marshy flats at the foot of Boston Common to be filled for the building of five ropewalks (long sheds for the manufacture of rope) to replace those which burned in the fire of that year.

The first significant filling in the Back Bay took place when a mill dam was constructed from Charles Street at the foot of the Common, westerly to Sewall's Point in Brookline, near the present Kenmore Square. The Mill Dam ran along what is now Beacon Street, at that time called Western Avenue.

To complete the tidal power project, a cross dam was built from Gravelly Point in Roxbury to intersect the main dam along a line just east of the present Massachusetts Avenue. At high tide, water was admitted into the "full basin" located in the Fens west of the cross dam. It powered machinery in mills located along the cross dam on Gravelly Point, discharging

into the easterly "receiving basin". At low tide, water was sluiced back into the Charles River through the main dam near the present Exeter Street.

Uriah Cotting began construction of the Mill Dam for the Boston and Roxbury Mill Corporation, chartered in 1814. Mr. Cotting died in 1819 and the work was finished under Colonel Laommi Baldwin. The dam, which carried a toll road, was opened for travel on July 2, 1821. In 1880, Mr. E. W. Howe (7,p.87) described the design and construction of the Mill Dam as follows:

> As an example of an engineering structure of sixty years ago, perhaps a description of this sea-wall may be of some interest. The "Mill Dam" as it is called, was built for the purpose of utilizing the rise and fall of the tide as a source of power, but has been chiefly used as a public highway. Its construction was begun about the year 1818, and completed in 1821. It is about a mile and one half in length, and consists of two parallel walls about 50 feet apart between their outer faces. In excavating through them for the construction of the new sluices at the outlet of the lake in the Back Bay Park, the construction of the old dam was found to be as follows: For the northerly wall starting from a grade of 1.75 feet below low water, there was first laid a course of 12″ × 12″ timbers, four in number, running lengthwise of the wall, the four occupying a width of 6 feet; on these was laid a course of 9″ × 9″ timber crossways of the wall and about 9 inches apart; next there was another course of five 12″ × 12″ timbers laid lengthwise. The timber was white pine, and the courses were treenailed together with oak treenails 1¾″ square; one treenail in every other bearing. The southerly wall has only two courses of timber, the lower course of 12″ × 12″ laid lengthwise, and the upper of 9″ × 9″ laid crosswise. Otherwise the two walls are alike. The walls are of rubble masonry, 6 feet wide at the bottom and 3 feet wide at the top of Roxbury pudding-stone, laid dry and very loosely. The wall is ballasted with small stones from the bottom to the top of the masonry; the ballast having a width of 8 feet at the bottom and nothing at the top. The back-filling is of mud to a height of 8.5 feet above the timber work, then 5 feet of sand, and then from 1.5 to 2 feet of road material. The whole height of the masonry is 15 feet. The wall has evidently settled somewhat and is somewhat out of a straight line, but not so much so as to cause any fear of its destruction. The wall is all afloat, so to speak, on the mud; there being from six to eight feet of mud underneath it,

with no piling or other foundation other than the timber work before described; while the average thickness of the wall is but three tenths of the height.

With low water maintained in the Back Bay receiving basin, the tide flats dried up and clouds of fine dust blew in every direction. For a time, then, before sluice-ways were built to keep the flats covered with water, the organic silts and peat at higher levels were subject to dessication.

PUBLIC GARDEN

In 1819, the ropewalks at the foot of the Common burned in their turn, and subsequently in 1824 the City of Boston bought back the land for about $50,000 and voted that it be "forever after kept open and free of buildings of any kind for the use of the citizens," (16,p.98). During the period 1824 to 1836, most of the remainder of what is now the Public Garden was filled.

There were many attempts following acquisition of the land by the City to develop the area for commercial purposes, especially during the period 1840 to 1850. They were always defeated, and finally in 1859 the land was officially voted the Public Garden by an act of the Legislature.

RAILROADS

In 1831, both the Boston & Worcester and the Boston & Providence Railroads were chartered. Embankment construction across the Back Bay was immediately begun to bring the rail lines into Boston.

The Boston & Worcester line was opened for travel as far as Needham in 1834. The tracks crossed the Back Bay on an embankment at the location of the present Boston & Albany tracks. The following year the Boston & Providence line was opened. It crossed the Back Bay in a southwest-northeast alignment from Roxbury to a station at Park Square. The two lines intersected near the present Back Bay Station, at the site for the new John Hancock Garage under construction in air rights over the Massachusetts Turnpike.

The railroads influenced the growth of the Back Bay in two important ways. First, they greatly interferred with the flow of water, hence reducing the usefulness of the area as a power project, increasing its undesirable aspects and hastening the day of its filling. Second, they influenced materially the ultimate layout of streets in the Back Bay, which factor had a tremendous impact on its physical and sociological development.

MISCELLANEOUS EARLY FILLING

In addition to the railroad embankments, a certain amount of piece-

meal filling took place during the 35 years following the construction of the Mill Dam. By 1836, the shoreline ran south from Beacon Street roughly along Arlington Street to Tremont, thence southwest along Tremont to approximately Dover Street, then west to about where Massachusetts Avenue and Columbus Avenue now join, Figure 2. A little filling also took place during this period north of Beacon Street and west of Charles Street up to Cambridge Street, and west on Beacon Street to Embankment Road.

The principal change occurring in the Back Bay during these years was that the erstwhile tidal basin had become an offensive open sewer and Boston residents demanded that it all be filled.

MAJOR BACK BAY FILLING

In 1856, after several years of wrangling, a tripartite indenture was completed among owners of the Back Bay land and water; the Commonwealth, the City and various private parties. The Boston and Roxbury Mill Corporation was given the tide flats north of their Beacon Street Dam (later called the "water side of Beacon"). The Commonwealth was given the area bounded roughly by Beacon, Arlington, Boylston and an irregular line between Exeter and Fairchild Streets. The Boston Water-Power Company was given the remainder of the Back Bay. The City, "uncooperative throughout, and rapacious in its demands", (16,p.151), was left out. It did, however, build Arlington Street jointly with the State.

The indenture was confirmed in 1857 and the Commissioners were authorized to fill and sell the Commonwealth's land. The Commonwealth let a contract in 1858 to Norman Munson and George Goss, partners in a contracting office at 22 Congress Street. A year later, a separate contract between Munson and Goss and the Boston Water-Power Company was signed to fill the Power Company's land north of Beacon Street.

Sand and gravel fill was brought by rail from a farm in Needham belonging to the Charles River Railroad Company. This farm was located near the present day Route 128 at Needham Avenue. The operation involved 145 cars, 80 men and two of the earliest steam shovels. Three 35-car trains were continually on the road, one arriving at the Back Bay every 45 minutes. When the trains arrived at the borrow pit, they were divided in half and each half was fed by one 25-horsepower steam shovel. Two shovelfulls filled one car and the 35-car train could be loaded in ten minutes. Some of the sand hills leveled were 50 ft. high and in the first year about twelve acres were leveled, fourteen having been created. The rate of filling was approximately 2500 cu.yd. per day. Generally, fill was placed to about El. 12 but streets were built up to approximately El. 18. As fill was placed

north of Beacon Street, a granite sea wall was constructed on the north side of the new Back Street.

The rate of filling can be traced by a series of maps prepared at ten year intervals by Fuller & Whitney, Figures 7 through 13. The extent of filling shown on these maps is summarized on Figure 2. By 1861 the shoreline was just west of Clarendon Street, in 1871 it was an irregular line between Exeter and Hereford Streets, and by 1882 filling had been completed to approximately Massachusetts Avenue. In the following ten years, all of the Back Bay Fens was filled, ending up with the layout of the Fenway and adjacent areas.

RECENT FILLING

Subsequent events in the topographic development of the Back Bay area include the first Esplanade filling, a 100 ft. promenade along the south shore of the Charles River adjacent to Back Street and the Beacon Street houses. In 1910, the tidal dam was constructed, controlling water in the Charles River Basin to El. 8. In 1929-31, the Storrow embankment and ponds were constructed and in 1951, Storrow Drive was built.

BUILDING FOUNDATION PRACTICE BEFORE WORLD WAR I

GENERAL

Construction of buildings followed closely behind the Back Bay filling. One of the first major buildings was the Arlington Street Church, constructed at the corner of Arlington and Boylston Streets in 1859. Shortly thereafter, the Museum of Natural History (now Bonwit Teller's) and the M.I.T. Rogers Building, both designed by W. T. Preston, were constructed. Within 50 years, private homes, hotels, churches, schools, a public library and many other buildings were to occupy the former tidal basin.

Members of the Boston Society of Civil Engineers contributed significantly to the evolution of foundation design and construction in the area. The 1914 J. R. Worcester paper (18) provoked voluminous discussion which reflects the practice of the times. From this work and other records and publications, we can reconstruct the important features of this early foundation practice.

SOIL BEARING PRESSURES

In 1903, Worcester (17) had recommended safe bearing pressures for soils found in Boston which varied from 2.5 tons per sq. ft. for soft clay to 4.5 tons for hard compact materials. By 1914 (18,p.19), he had found the

upper limit to be conservative. Based on his experience and results of load tests on one ft. square plates, he suggested the following tentative safe soil bearing pressures:

Soil Type	Safe Bearing Pressure (tons per sq. ft.)
Dry, hard, yellow clay, "Bowlder clay", dry sand or gravel	6.0
Compact, damp sand, hard sandy clay, hard blue clay	5.0
Medium blue clay, whether or not mixed with fine sand	3.5
Soft clay, running sand (confined)	2.5

By comparison, our present code defines eight materials and expands the range from 1 to 10 tons per sq. ft., using 1 ton for soft clay. Worcester's recommended safe bearing pressure for soft clay was later found to be too high, especially for large loaded areas.

The importance of footing size and the overlapping effects of stresses from adjacent footings had been discussed earlier and Worcester acknowledged the danger in extrapolating from load tests run on small plates. However, he reasoned (18,p.10), somewhat incorrectly, that "There is also present in every test a condition having exactly the opposite tendency, which renders them unreliable. This is, that when a limited area is loaded, the soil has a chance to flow out in every direction and, as the area of the load increases, the opportunity for flow is relatively decreased. The first error which would be liable to give too high capacities, is important in the case of a harder ground over a softer. The second, which may give too low results, is more likely to be found in the case of a soft plastic material, like clay, immediately under the loaded point." The importance of long term settlement from consolidation had not yet been recognized, although significant settlement of major structures, many founded on closely spaced friction piles, had already been observed.

Charles R. Gow (18,p.181) believed that the soil pressures suggested by Worcester were conservative, except for soft clays, and he offered that "----he has at times adopted values as high as eight tons for the cemented clays and gravels with no unsatisfactory results." In addition, Henry F. Bryant (18,p.208) frequently used eight tons for the "bowlder clay in the Boston drumlins." Others, including Charles T. Main (18,p.216) felt that the soil bearing pressures were too high, in particular that for the soft clay. Main stated that "Because of the effect of vibration and the observance of

what happened in a weaving mill in the earlier part of my experience, I have been very conservative regarding the loads on soils, and many years ago decided on the following:

Soft clay	1 ton per sq. ft.
Compact sand	1 to 2 tons per sq. ft.
and gravel	2 to 3 tons per sq. ft.
Hardpan	(under favorable conditions,4)"

In his later experience, he increased loads on all but the soft clay by about 50 percent. His assessment of the soft clay proved to be correct, although others were using from 1.5 to 2.5 tons per sq. ft.

Engineers of the time generally agreed that all parts of the structure should be supported on a stratum of soil below the organic silt and peat. They further concurred in the importance of taking borings to determine subsurface soil conditions.

WOOD PILES AND PILE CUTOFF

In the Back Bay area, buildings were commonly supported by wood piles driven through fill and organic soils. As Professor W. O. Crosby (18,p.226) of M.I.T. put it: "This formation (blue clay), reinforced by piles, has been the main reliance for deep foundations, or the foundations of important structures, throughout a large part of the lowland areas of the Metropolitan District."

A safe load of ten tons was commonly used on spruce piles having a tip diameter of approximately 6 inches. In the Back Bay, piles were driven to bear in the sand and gravel outwash or the hard clay crust where these materials offered point resistance. Elsewhere, piles were driven into the medium to soft clay to act as friction piles. Drop hammers were used, having weights commonly from 1800 to 2300 lb. which were dropped from 10 to 25 ft.

Most specifications required that piles be driven in accordance with the Engineering News formula: ($P = \frac{2WH}{p + 1}$) and the applicability of this formula was widely discussed. Following an evaluation of several pile load tests, Worcester (18,p.19) concluded that the Engineering News formula could be modified to allow a 50 percent higher load, i.e., by allowing 3WH, although he recognized that the Engineering News formula "----does not appear always to have a factor of safety of 6, as it is supposed to have." Most other engineers were more conservative and thought the Engineering News formula should be used. Harry E. Sawtell (18,p.246), a structural engineer

with Charles T. Main, cited five good reasons why he would regret to see Worcester's modification made: "First, that it would result in greater settlements under working loads; second, from long observation it is believed that a large part of the piles driven are, unlike test piles, seldom given the penetration required which now results in doing what Mr. Worcester would do by changing the formula; third, that an unknown percentage of spruce piles driven under the present conditions are unreliable, due to brooming and breakage; fourth, that as this construction is out of sight, a greater factor of safety should be obtained than for construction in sight which can be inspected; fifth, that the factor of safety obtained by the Engineering News formula is now relatively low when based on a reasonable settlement of the pile itself."

Although inspection procedures have improved and over-driving is less common, these arguments are still sound.

From some fourteen load tests performed on friction piles, Worcester (18,p.18) found that the average skin friction over the embedded length of the pile was 628 lb. per sq. ft. for a deflection of ¼ in. For design, he suggested using 300 lb. per sq. ft. to give a factor of safety equivalent to the Engineering News formula. With great wisdom he noted, however, that "--- it is not always safe to take into account the portion of the pile which is embedded in an inferior material, and the objection to the use of this method is the uncertainty as to how much length to consider."

Henry Bryant (18,p.208) replied emphatically that embedment in inferior material should never be considered. "In fact, I think that with a layer of soft material underlying a considerable depth of hard filling, the latter should be considered as negative since it is likely to seize the pile and, in settling, to push it down. This has occurred several times in my observation." For 20 years, Bryant had designed for a skin friction equal to 1,000 lb. per sq. ft. for the area of pile embedded in the supporting soil. For an average pile diameter of 8 inches, this is approximately 1 ton per ft. of length. This was considered to be 50 percent of the actual skin friction. "From this I would deduct a similar amount (1 ton per ft.) for penetration in filling underlaid by any considerable depth of peat or silt." Again, for comparison, our present building code allows a skin friction in inorganic clay equal to 500 lb. per sq. ft. and requires that effects of downward friction forces from subsiding fill be considered.

On the matter of design skin friction, H. S. Adams (18,p.211) used a skin friction equal to one-third of the safe bearing pressure. "For example, if the clay is good for 3 tons per sq. ft. for foundation, it is good for about 1

ton per sq. ft. in grip upon the pile." For wood piles driven in the medium to soft Boston clay, the one-third rule can be accepted today, although we recognize that the allowable bearing capacity is less than 3 tons per sq. ft.

In addition, both negative skin friction from subsiding fill and false driving resistance were also recognized and discussed in 1914. In his discussion of pile driving and testing for the new M.I.T. buildings in Cambridge, Charles T. Main (18,p.217) stated "These piles generally pass through a fairly hard fill of blue-black silty mud and shells before reaching the harder sand stratum in which they get most of their support. This fill gives considerable resistance to driving, and soon after the pile reaches the sand, it would generally appear, by the small penetrations under the hammer, that a theoretically satisfactory bearing power had been reached. This is not practically acceptable, however, as the fill is unreliable and subject to large future settlement owing to decomposition, etc., and should not be depended upon for permanent support, even if it appears to give temporary support, therefore the piles are driven into the hard sand stratum to a depth that will give a satisfactory support to them from that material alone."

Wood piles were commonly spaced 2.5 ft. on centers but a spacing of 2 ft. was not uncommon where heavy loads were to be supported. While his explanation was a little strange, Henry S. Adams (18,p.211) recognized the danger in driving piles too close together in clay. "If they are driven closer than that (3 ft.) in clumps, the material between the piles is so compressed that it looses its grip, and does not hold the interior piles to the extent that it should." There was ample evidence to support his concern. Below the Trinity Church tower at Copley Square, there are over 2,000 friction piles in an area 90 ft. square, an average spacing of about 2 ft. The Church had settled nearly a foot. Closely spaced wood friction piles also support the Old South Church on Boylston Street. The average load under the base of the tower is 3.18 tons per sq. ft. By 1914, the tower leaned 2.5 ft. toward Boylston Street as a result of differential settlement. (It was later dismantled and reassembled on a level foundation.

It was common practice in Boston to cut off wood piles at the average tide level, El. 5 Boston City Base, with entire safety. After the Back Bay was filled and through the remainder of the nineteenth century, the ground water level in the Back Bay was approximately El. 8 and as a result many buildings were constructed on piling cut off *above* El. 5.

Although there was ample evidence by 1914 that sewers and drains in the Back Bay were affecting the ground water table, Worcester (18,p.6) felt that El. 5 was too low and suggested a cutoff as high as El. 8. Wisely, most engineers at the time disagreed with him, believing that El. 5 or 6 should be

maintained. Frederick P. Stearns (18,p.201) reasoned that with the presence of an increasing number of floor drains and decreased infiltration of surface water as the land was built upon, ". . . piles to support important structures should be cut off below rather than above grade 5." Charles T. Main (18,p.397) indicated that piles for the M.I.T. building would be cut off no higher than grade 13, Cambridge base (El. 8 Boston City Base).

In final discussions, Worcester (18,p.415) challenged his fellow engineers to cite a case where rotted piles had been found below El. 8. Although no examples were forthcoming, he changed his recommendation to El. 6.

In 1931, following the discovery of rotted wood piles below the Boston Public Library, the BSCE Committee on Boston Subsoils (14,p.244) was of the opinion that untreated wood piles should be cut off not higher than El. 3 in the Back Bay.

A further discussion of ground water levels throughout the Back Bay is presented in Part II of this paper.

PILE CAPS AND CONCRETE

Piles were commonly topped with a granite capping stone or a series of stones upon which the stone and brick foundation walls were constructed. After the turn of the century, concrete almost completely took the place of stone for foundations and also drove out the use of steel beams and girders in grillages and cantilevers. Prior to the time concrete was used to cap foundations, it was placed around the heads of piles to prevent the lateral motion of the piles, and to some extent connect them together. At Trinity Church, for example, two feet of dry concrete were placed around the wood pile heads in four layers, each 6-inch layer being thoroughly compacted. The upper surface of concrete was kept 1 inch below the heads of the piles so that every granite stone could be firmly rested on the piles.

OTHER PILES AND GOW CAISSONS

Toward the latter part of the century, concrete was being used in foundation construction. The Simplex pile, the Raymond standard taper pile and Gow caissons were introduced just after the turn of the century. Composite piles had been used since the turn of the century to overcome the cost of cutting off wood piles at a low grade. For the concrete extensions, headless barrels were used for forms, stacking one above the other. A history of the use of concrete piling is given by Gow (5).

HYDROSTATIC PRESSURE

It was a further requirement of the Boston Building Department that basement floors be placed at or above El. 12. However, with "waterproofed" construction, some basements were placed below this grade. For

determination of uplift pressure on floors, Worcester (18,p.19) initially assumed El. 12 as the highest water level on the basin side of the City, but later changed his recommendation to El. 11 (18,p.415). It is interesting to note his design assumption relative to uplift pressure (18,p.8). When a structural floor supported by piles was used, he assumed full water pressure acting over the entire area of the floor because the earth was likely to settle away. On the other hand, when the floor rested directly on the ground, he reasoned that ". . . obviously less than the whole area is exposed to water, for part must bear on the soil. Experiments reported by J. C. Meem would indicate that with a sandy soil not over 50 percent is so exposed. The writer has been in the habit of making this assumption in Boston." In subsequent discussions of his paper, no one questioned his assumption. Perhaps no uplift failure occurred because El. 12 was a conservative assumption for water level.

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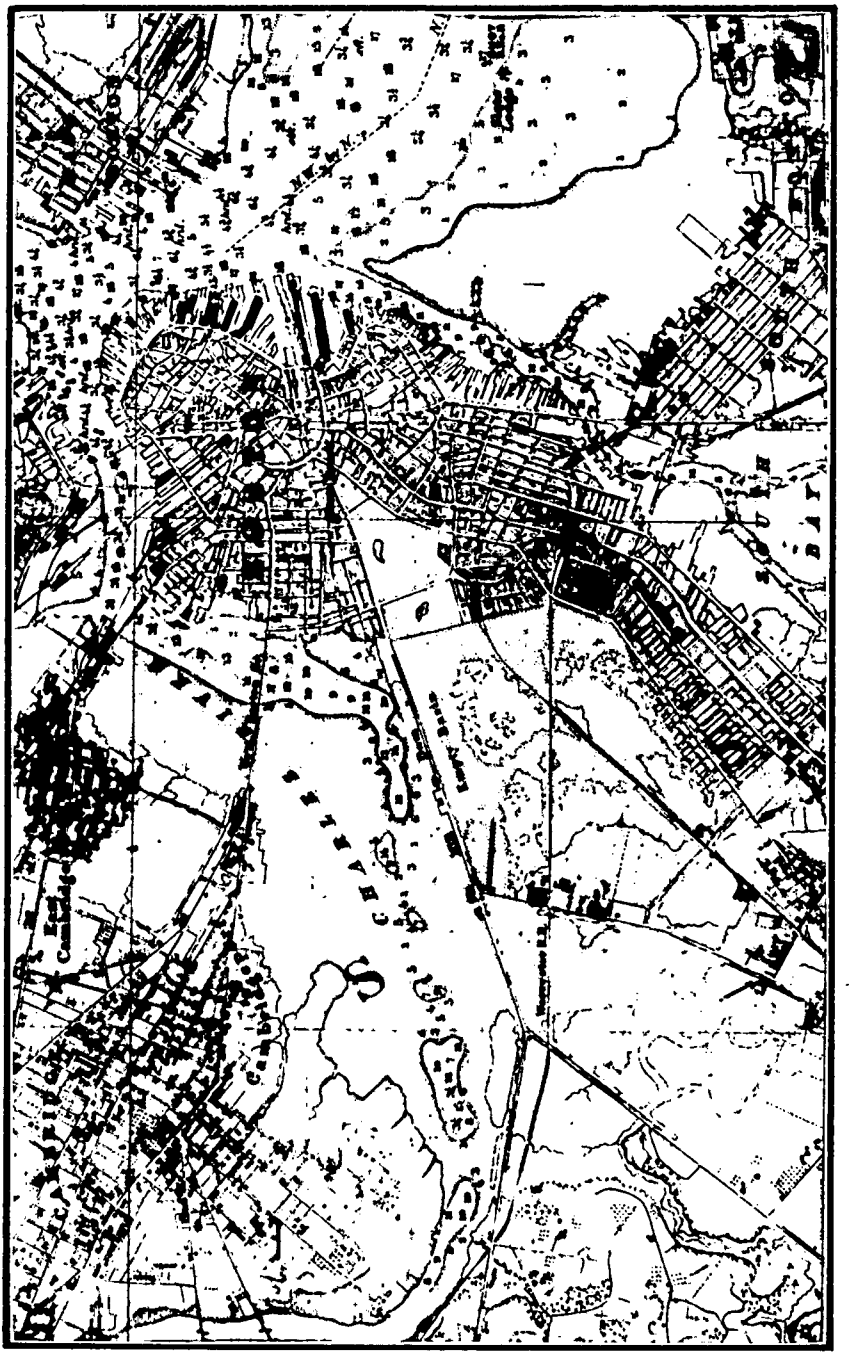

Figure 4 - Boston, About 1848, from a Coast and Geodetic Survey Map Dated 1857.

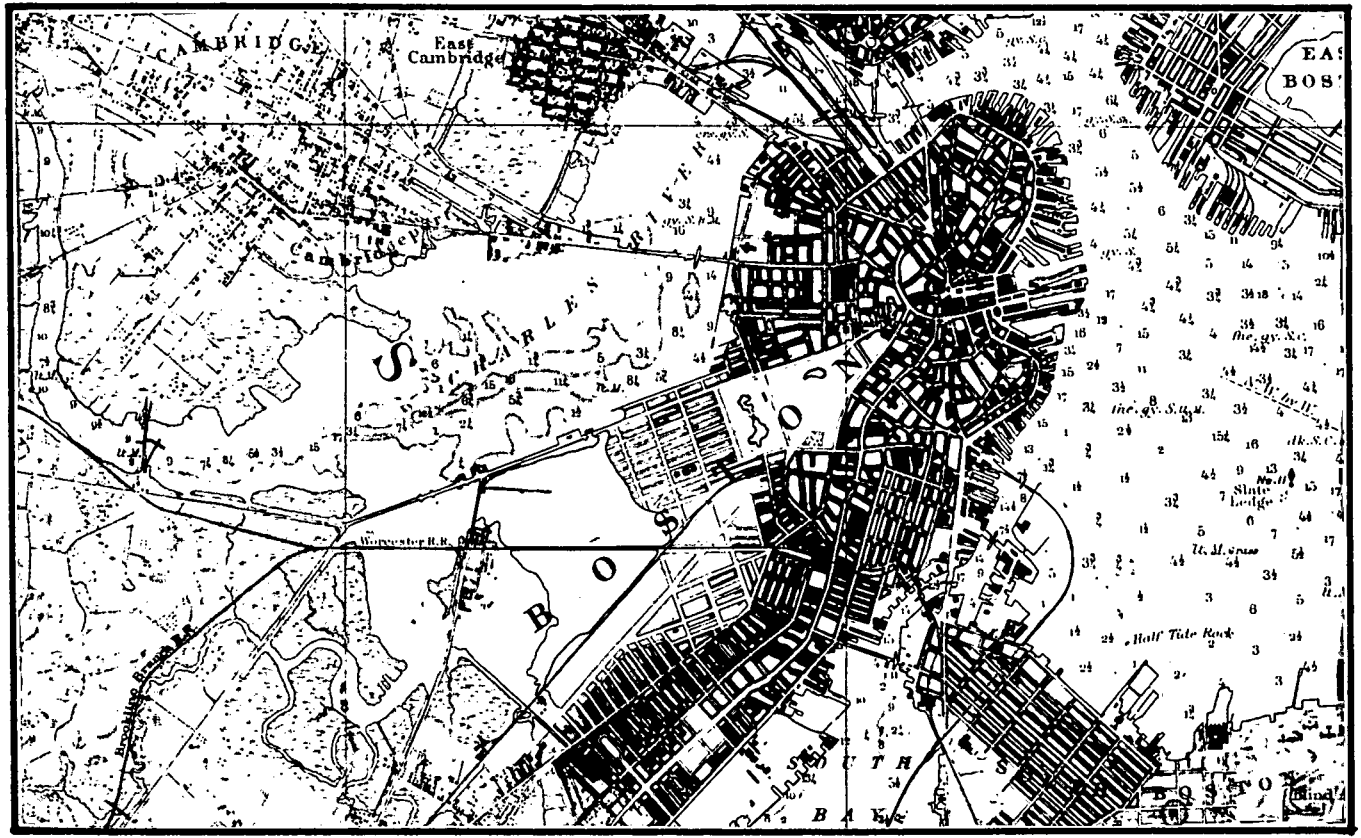

Figure 5 - Boston, About 1863, from a Coast and Geodetic Survey Map Dated 1872.

Figure 6 - Boston, About 1895, from a Coast and Geodetic Survey Map Dated 1901.

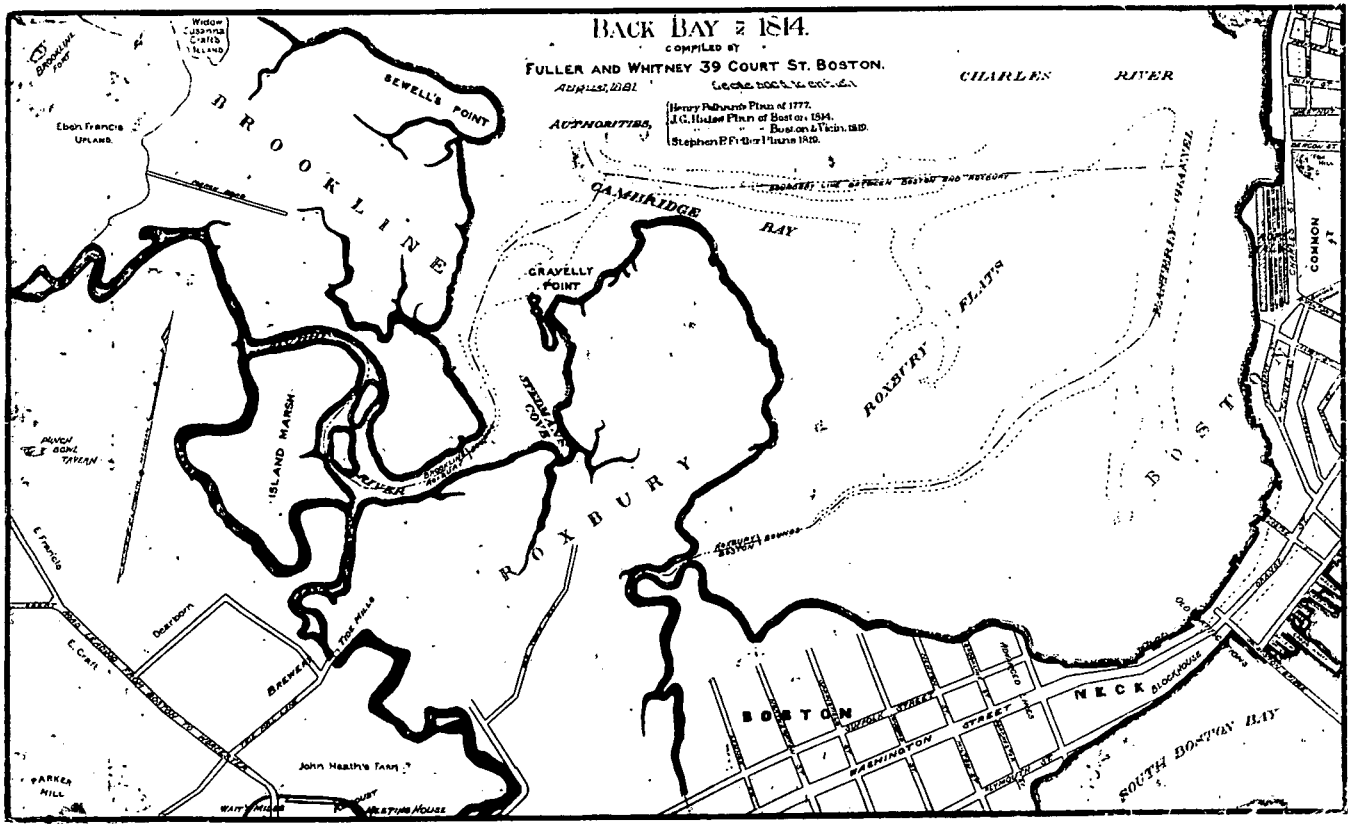

Figure 7 - Back Bay in 1814.

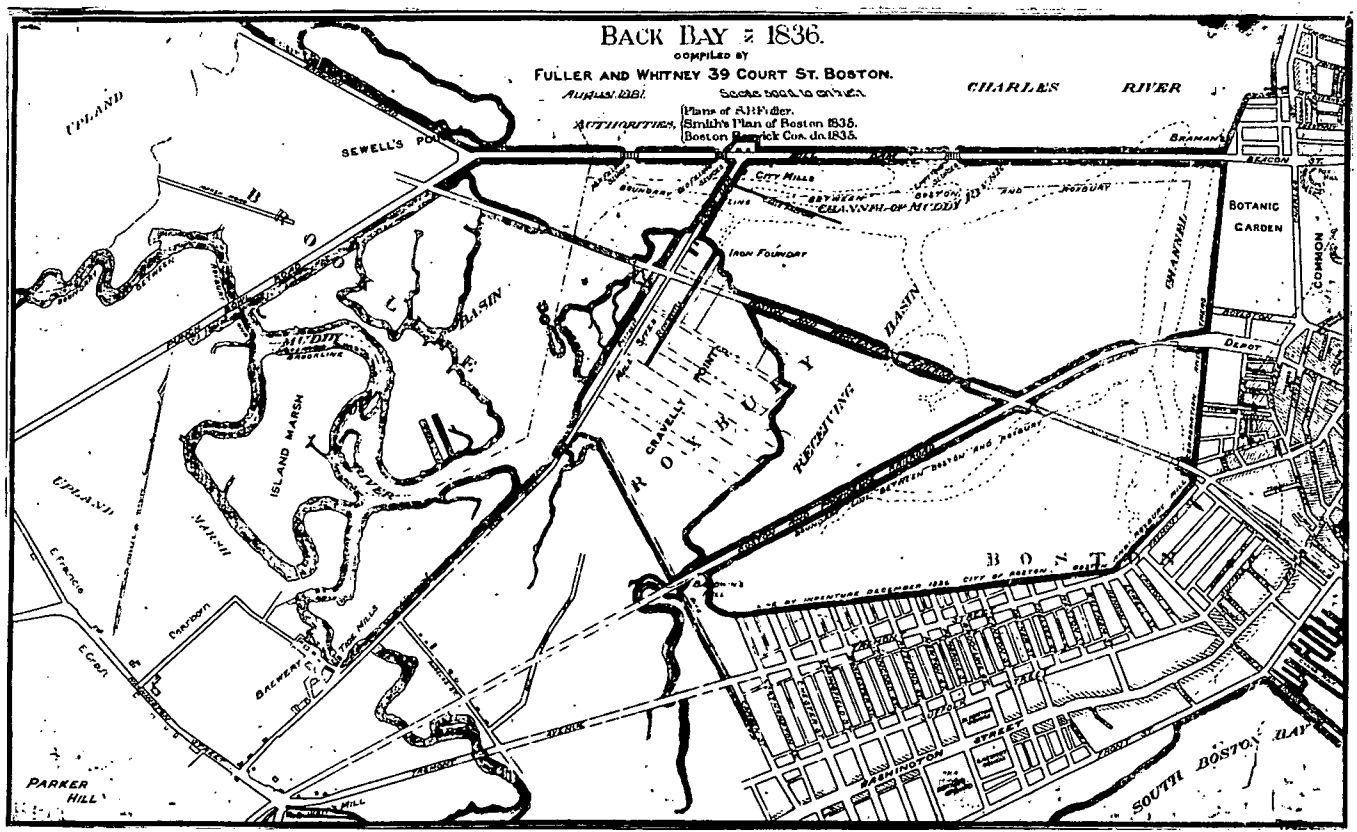

Figure 8 - Back Bay in 1836.

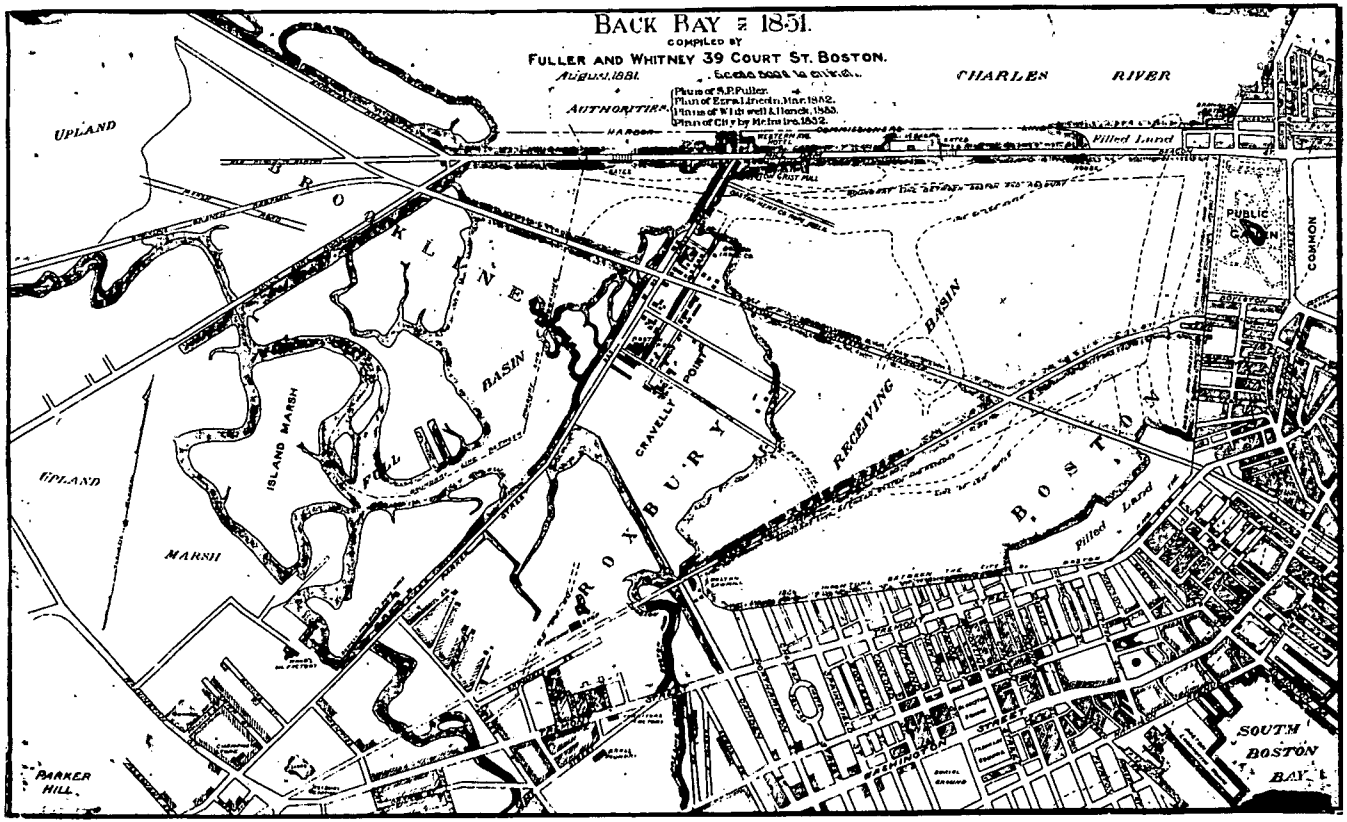

Figure 9 - Back Bay in 1851.

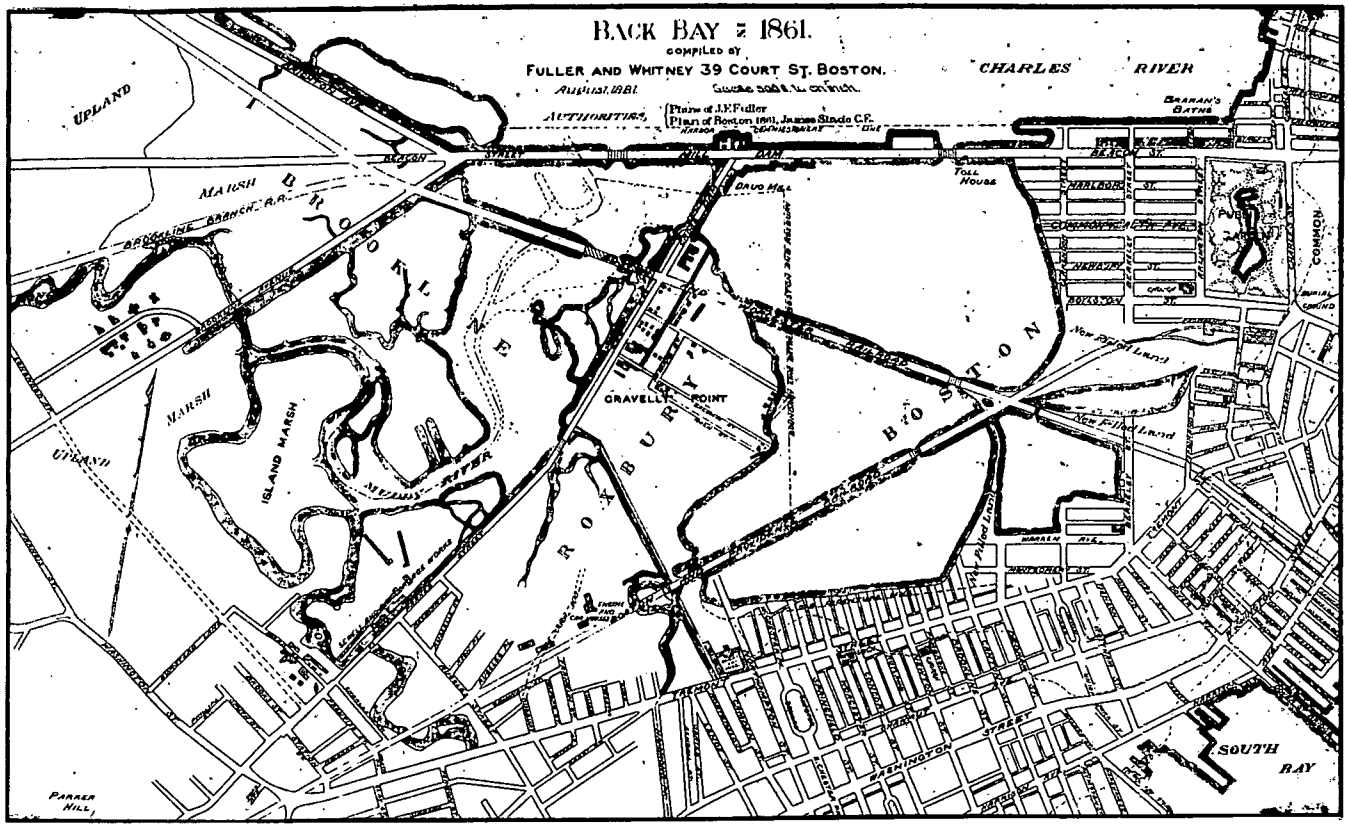

Figure 10 - Back Bay in 1861.

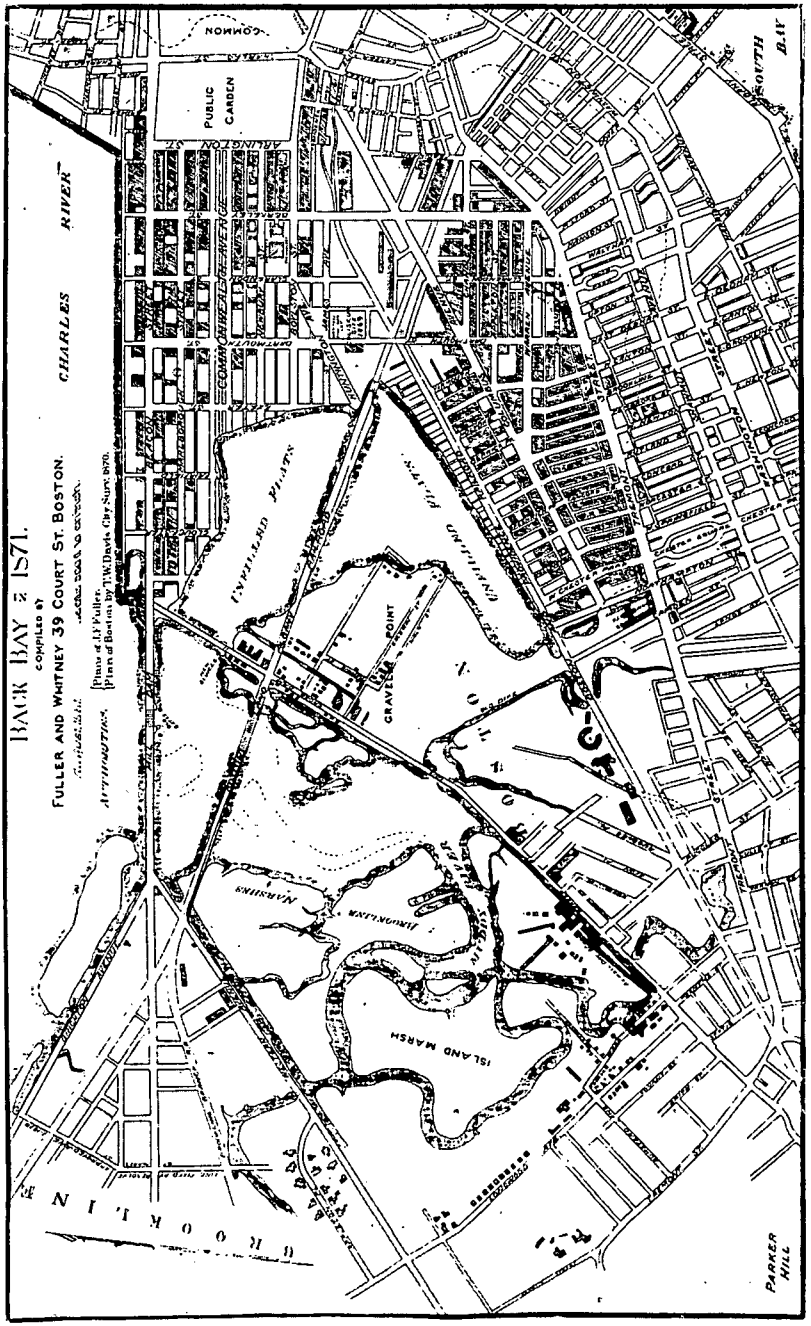

Figure 11 - Back Bay in 1871.

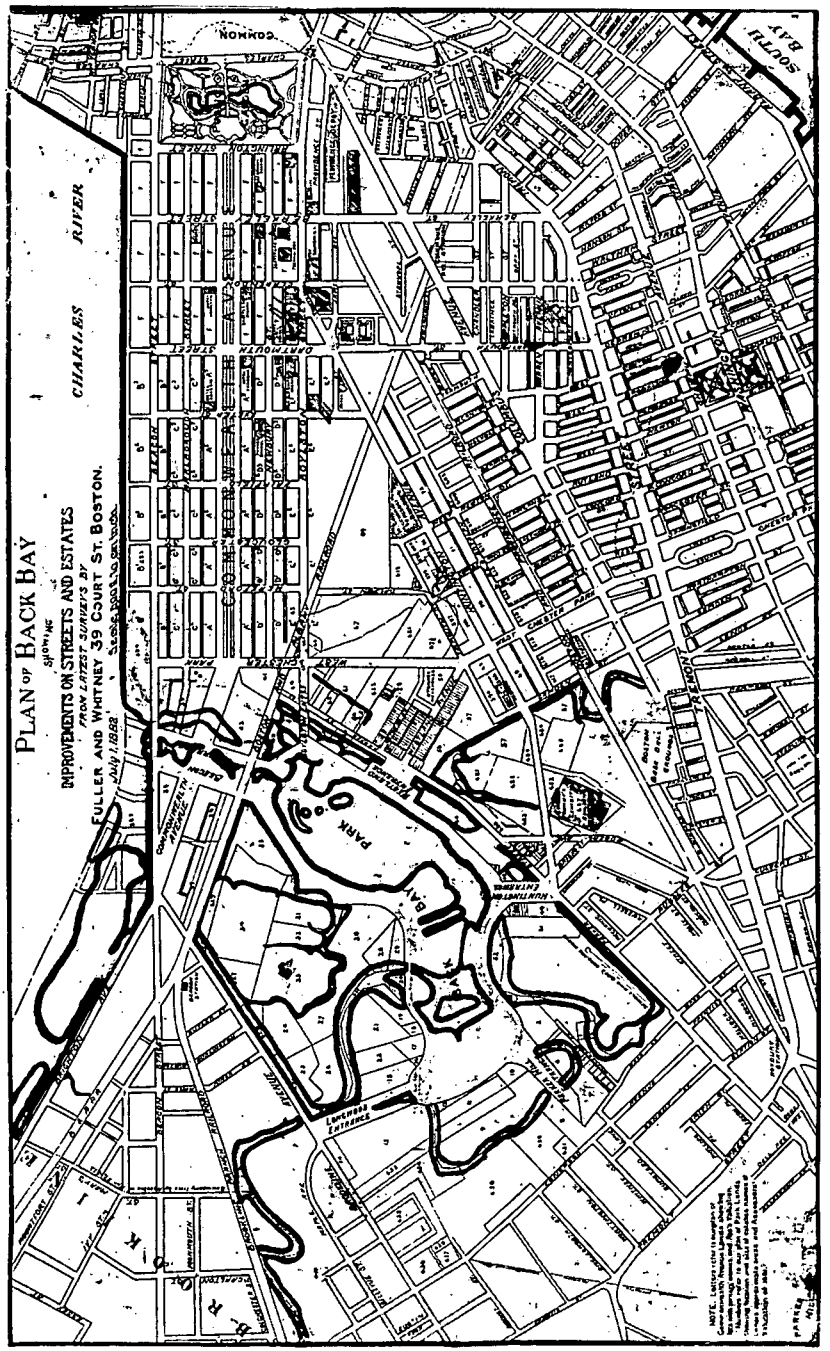

Figure 12 - Back Bay in 1882.

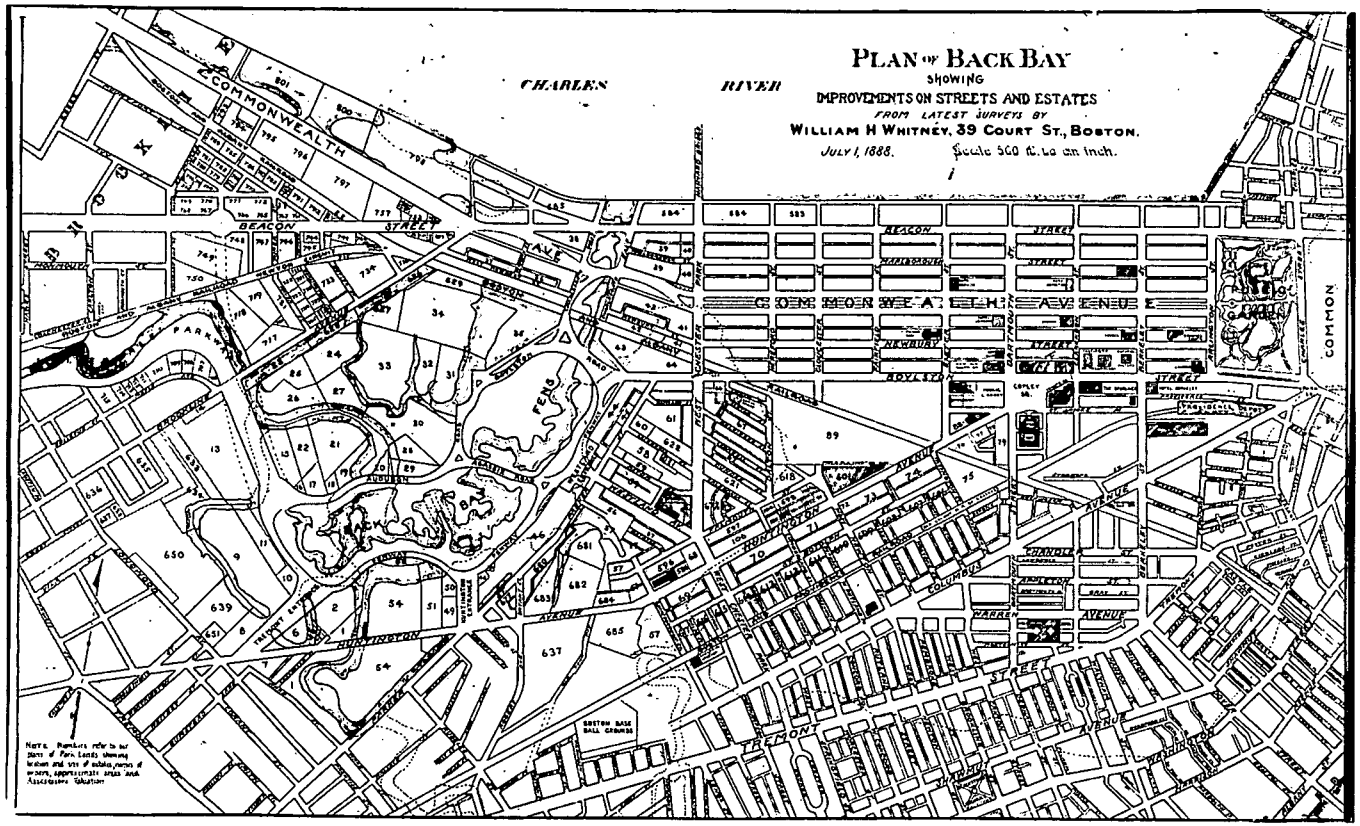

Figure 13 - Back Bay in 1888.

DISPERSION PHENOMENA IN COASTAL ENVIRONMENTS

by KLAS CEDERWALL *

(1970 John R. Freeman Memorial Lecture presented before the Boston Society of Civil Engineers and the Hydraulics Section on January 28, 1970)

Introduction

The disposal to the sea of man-produced wastes — sewage, heat, etc. — gives rise to many questions of a scientific nature. Complete removal of all the pollutants from the waste water is seldom practical because of high costs and lack of adequate technology. Hence, the physical aspects of water pollution problems are of great significance due to the fact that knowledge of the processes by which the waste water is mixed and dispersed in the receiving water is required for evaluation of pollutant levels. Knowing the natural processes of transportation and dilution of the disposed waste products, we have a basis for controlling hygienic and aesthetic nuisances as well as ecological disturbances. Studies of the hydrology and hydrodynamics of lakes, rivers and coastal environments exposed to waste discharge are therefore a fundamental feature of water resources planning and water quality management. Let us consider two common pollution problems.

A complex engineering problem in the field of water pollution control is to select the best site for a future outlet of sewage water. Several alternative sites usually have to be investigated as to their efficiency for dispersion of effluent. A qualitative judgment must be based on a statistical presentation of the result of the study. A statistical description of the problem can generally not be obtained without comprehensive hydrological surveys to establish the dispersive properties of the environment. Adequate information is usually required not only on the physical but also on the chemical and biological characteristics of the affected area for which water quality standards have to be established. Hence, such field studies turn out to be most costly and time consuming. This necessitates a systematic approach not only for the planning of the investigation but also for the future water quality control management.

Let us turn to another problem of current interest. For the successful operation of a thermal power plant located on the coast, an essential aspect concerns the effect on the sea caused by the discharge of warmed condenser

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water. Thermal pollution of the sea and recirculation of warmed condenser water are affected by several factors, for instance, distance between intake and outlet, the heat budget of the sea, and finally to a substantial degree also by the spreading behaviour of the warmed condenser water. Much attention has been given in Sweden, as in many other countries, to studies on the potential of the coastline for siting of planned thermal power plants, and various approaches for analyses of spread characteristics of the discharged cooling water have been considered. This would ultimately lead to design criteria for intake and discharge structures.

Scope and Objectives

In this paper we will discuss some approaches for studying dispersion phenomena in coastal environments. The complexity of the problem necessitates a review of applicable methodology. Because of the multitude of factors that usually have to be considered, a systematic approach to the control of water pollution is required. Since mathematical models are widely used in hydrology for prototype simulation, an objective of this paper is to present some principles for mathematical modeling of dispersion phenomena. Some advances in field measuring techniques are reviewed as they apply to water pollution studies.

It should be emphasized that the conditions for marine disposal of waste water in the Scandinavian waters, with which this study is most concerned, are very often quite different from conditions in other parts of the world. This is due to the fact that these water areas exhibit some unique hydrographical as well as topographical features. The fjord and the archipelago, for instance, are significant elements of our coast, and the Baltic Sea is an important feature of the salt water and fresh water balance for the whole area.

Until recently, only limited experience from rational waste disposal practice in Scandinavian waters has been gained. Worth mentioning is a permanent interstate committee which was founded in 1960 for planning and execution of a scientific program for investigations in the sound between Denmark and Sweden. The Norwegian Institute for Water Research has carried out extensive investigations in the Oslofjord, and monitoring schemes for conceivable major expansions of marine waste disposal systems are established for the municipalities of Stockholm and Gothenburg. Of special oceanographic interest is the Baltic Sea and concentrated effort is devoted to field studies and surveys in order to gain a better understanding of the general hydrography of this huge body of brackish water.

Mode of Analysis

It goes without saying that a complete physiographical description of a lake or a coastal water area is not possible to obtain. This is strictly true also for a more well defined process such as, for instance, the spread of a pollutant from a waste outfall. Every approach to study the characteristics of a receiving water area has to introduce some simplifications. The degree to which a model of the prototype — analytical or experimental — represents the true conditions is dependent on the assumptions introduced concerning prototype behaviour.

There are few areas of engineering activity which involve as many variables as are found in applied hydrology, and notably in water pollution studies. The computerized mathematical models, as well as the use of automatic data processing systems, are therefore becoming extremely valuable. It is, however, essential that the investigator be aware of the limitations inherent in his choice of approach to a study of the problem, and how they affect the ultimate result. Sometimes even the applied numerical procedure for solving the governing equations introduces errors which can destroy the physical significance of certain important parameters, e.g. pseudo-dispersion.

There are several alternative methods for the prediction of the dispersion pattern in the receiving water area. Four main approaches may be distinguished.

a) A purely theoretical analysis supported by general experience on the diffusive properties, circulation and exchange conditions of the water area in question.
b) The same as under a), but with supplementary field surveys to establish characteristic prototype behaviour.
c) Tracer technique for direct in-situ simulation of transport and mixing of the waste effluent.
d) Scale tests by means of a hydraulic model.

In the future, when the next generation of electronic computers are available, we will probably use advanced theoretical flow models for computer simulation of dispersion phenomena.

Sometimes the possibilities to use a natural simulation technique are limited because adequate tracer field tests are not possible. For instance, in the case of the design of a cooling water system for a planned thermal power plant it is virtually impossible to accomplish a true simulation of this hydraulic flow pattern in the prototype, i. e. the coastline without the power station. This suggests an analytical analysis of the problem and scale model studies as the appropriate support for the engineering design of the cooling

water arrangement. Hydraulic models have been employed for many years as a valuable tool in the solution of waste disposal problems. The technique and practice applied depend on the water area to be reproduced, the general nature of the problem to be studied, and many other considerations.

For the present study we will confine ourselves to discussion of the principle of prototype simulation by mathematical models.

Mathematical Models

The development of a mathematical model follows three distinct phases: conceptual, functional and computational. The conceptual model of the problem analyzes the fundamental physical elements to be incorporated in the model. As a second stage, we have to convert the proposed physical model into mathematical terms. The computational solution is the final step in the development of the mathematical model. The predictive capability of the proposed model must be tested in the laboratory or in the field by comparison to observed data.

The ultimate objective of a dispersion model is the effective control of concentrations of contaminants released within the area. Hence, this model is an integral part of the water quality planning within the control regime. The pattern of spreading of discharged water is dependent on a great number of parameters several of which may have a significant effect on the pollution of the environment. Transportation and distribution of the pollutants result not only from the details of movement and mixing within the receiving area, but depend also very often to a substantial degree on the external circulation, i.e., exchange with adjacent larger water bodies. Hence, considerable effort has to be made to investigate dispersion mechanisms from well-defined sources of pollution as well as to expand the knowledge of the physiography of the coastal environment which will be exposed to waste discharge.

Most fjord and nearshore problems are so much related to such local factors as topography, stratification, flow-through, and meteorlogical and tidal activity, that a mathematical treatment is very difficult to achieve. Unfortunately, it is in these cases that the characteristics of the receiving water are of primary interest, as they ultimately determine the concentration levels and the residence time distribution in the water quality region considered. We will discuss approaches of mathematical modeling with special reference to this particular problem.

A properly designed dispersion model should be able to predict pollutant levels at any location in the receiving area, if the source emission and the

environmental factors are known. Using terminology of control theory, the non-manipulative input variables are the environmental factors, and the control variables of the system which can be selected to meet some water quality requirements are the sources of pollutants and their locations, outlet arrangements and finally, of course, the source strengths. Once such a model of high predictive capability has been constructed, various control problems can be posed.

The analytical procedure of the mathematical modeling is based on either a finite or an infinitesimal approach. The differential control-volume analysis describes the flow characteristics and mass balance at a fixed point in space. Consequently, the basis for this method is the general equations of motion and appropriate equations for the conservation of mass and volume. Oceanic circulation, for example, has been successfully studied by means of infinitesimal calculus. The predictive capabilities of these ocean models are fairly good due to the fact that even drastic simplifications of the flow properties do not significantly affect the overall pattern of circulation.

Theoretically, the spatial and time history of discharged pollutants could be obtained by solving the general equation of diffusion and the equations of motion. The lack of detailed knowledge of all the flow properties of the water area is one of the reasons that an exact solution to the diffusion problem is not available.

The finite control-volume analysis is perhaps the most widely used technique for studies of flow problems. The water body is divided into segments of finite size and mean values are assigned to each element according to the physical properties of interest. The classical method for estimating flushing rates in estuaries due to tidal action is the tidal-prism analysis. Using finite control volumes we will never gain a thorough understanding of the flow mechanisms which can only be reached by means of a particle approach analysis.

Statistical correlation approaches have proved to be successful tools for analysing dispersion phenomena, being alternatives to the differential analysis using coefficients of diffusion. The classical theory was developed by Taylor who considered "diffusion by continuous movements" from a fixed point in an isotropic, turbulent flow field. In recent years several techniques have been proposed for simulation of turbulent diffusion by means of stochastic models.

Statistical models are frequently used in operational hydrology and may be a likely analytical procedure for water quality management when large quantities of data are available. The "data bank" is then the historical observations of flow characteristics, quality parameters and environmental

conditions gathered during a certain period of time. The problem is to extract from the data bank enough information to construct a statistical model which demonstrates properly the interaction between significant system parameters.

Condenser Water Discharge

For predictions of the process of thermal diffusion produced by condenser water discharge, we usually have to rely on a combination of analytical reasoning, scale modeling and field surveys. It is a complex problem involving aspects of stratified flow as well as thermodynamics. The objectives of a hydraulic study of a cooling water arrangement are two-fold: determination of the thermal effects on the receiving water area and the degree of recirculation of heated condenser water.

Several experimental studies have contributed to our understanding of the diffusion phenomena inherent in the surface discharge of warm water jets. Jen, et.al. (1966)[21] considered the reduction of temperature excess due to turbulent jet mixing. Hayashi and Shuto (1967)[18] performed similar studies. The results of these experiments can be compared to a model for the initial flow zone proposed by Cederwall and Sjöberg (1969)[8] using the jet diffusion theory as will be outlined.

Because it is difficult to solve the dynamic and thermal equations simultaneously, the problem is studied assuming the temperature to be a conservative property. Turbulent diffusion normally accounts for the most efficient reduction of excess temperature, although heat loss to the atmosphere sometimes is far from negligible. However, if we want to design a discharge structure for the purpose of achieving a high degree of dilution, (this is usually the most appropriate way of meeting water quality standards and reducing recirculation) then the assumption of a conservative temperature is a justified approximation. In such cases the heat loss may be estimated through a stepwise correction of the heat flux following the calculated path of the condenser water flow.

To describe the initial flow zone of a surface discharged warm water jet we start studying the basic flow properties of a submerged three-dimensional momentum jet in the zone of established flow, Albertson, et. al. (1950)[3], see Fig. 1 for definitions.

$$\frac{U_m}{U_o} = 6.2 \frac{D}{s} \tag{1}$$

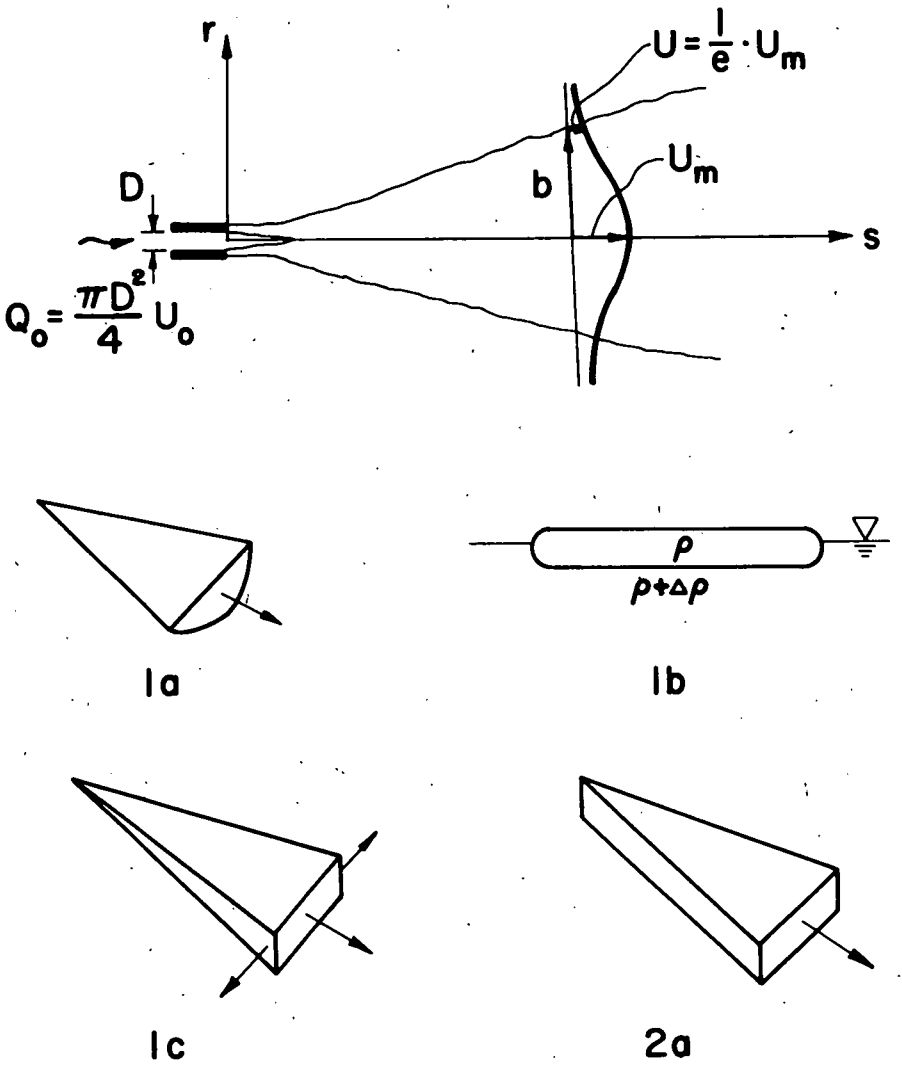

Fig. 1. — Velocity field of submerged three-dimensional jet and sketch of jet diffusion models.

$$\frac{U}{U_m} = e - \frac{r^2}{b^2} = e - \frac{r^2}{(0.114s)^2} \tag{2}$$

$$\frac{Q}{Q_o} = 0.32 \frac{s}{D} \tag{3}$$

In the case of a submerged jet in a large reservoir, the absence of external forces requires the flux of momentum, m, to be the same at all successive sections.

$$m(s) = m_o = \rho Q_o U_o \tag{4}$$

The growth of the jet flow is characterized by the following equation:

$$\frac{dQ}{ds} = a \cdot 2\pi b U_m \tag{5}$$

where a is the coefficient of entrainment which, from dimensional considerations, has to be a constant, $a = 0.057$ for this particular case.

Introducing the kinematic flux of momentum, $JV = Q_o U_o$, we can write Eq. (1).

$$U_m = 7.0\ J_o^{1/2} \cdot \frac{1}{s} = 7.0\ J^{1/2} \cdot \frac{1}{s} \tag{6}$$

For the growth of the jet flow we may write

$$Q = 0.28\ J^{1/2} \cdot s \tag{7}$$

or by means of Eq. (5)

$$dQ = 1.41\ a \left[\frac{J}{\pi b^2}\right]^{1/2} \cdot 2\pi b ds \tag{8}$$

where $J/\pi b^2$ is a characteristic local velocity, and $2\pi b ds$ "the local area of entrainment". Similar expressions hold for the submerged, two-dimensional momentum jet.

Now, returning to the surface discharged warm water jet we have to take into consideration the following conditions:

a. The water surface and the bottom are characterized by zero rate of entrainment.

b. The flux of momentum is reduced due to turbulent shear at the fixed boundaries.

c. Increased lateral spread is caused by the density deficit of the heated water.

We assume that the density deficit $\Delta\rho$ is a linear function of the temperature excess ΔT . The following situations are modelled, see Fig. 1:

1. Subsurface discharge in a deep and homogeneous stagnant environment.

1a. $\Delta\rho_o = 0;\ m_o > 0$ Non-buoyancy case

According to Eqs. (6) to (8) we get

$$U_m = \sqrt{2} \cdot 7.0\ J^{1/2} \cdot \frac{1}{s} = 9.9\ J^{1/2}\ \frac{1}{s} \tag{9}$$

or

$$\frac{U_m}{U_o} = 8.8\ \frac{D}{s} \tag{10}$$

$$Q = \frac{1}{\sqrt{2}} \cdot 0.28\ J^{1/2} \cdot s = 0.20\ J^{1/2} \cdot s \tag{11}$$

The turbulent diffusion of mass and heat is similar to the diffusion of momentum and in the reference case of a three-dimensional submerged simple jet the distribution of concentration, C, of the discharged effluent can be written:

$$\frac{C_m}{C_o} = 5.6\ \frac{D}{s} \tag{12}$$

$$\frac{C}{C_m} = e^{-\frac{r^2}{(\lambda b)^2}} \tag{13}$$

where λ, the turbulent Schmidt number, is close to unity.

We can modify the distribution of concentration in the same way as the velocity field to yield the case of a subsurface non-buoyant jet. Hence, Eq. (12) takes the following form:

$$\frac{C_m}{C_o} = \sqrt{2}\ 5.6\ \frac{D}{s} = 7.9\ \frac{D}{s} \tag{14}$$

Eqs. (2) and (13) expressing the lateral spread of momentum and mass remain as they are.

Eq. (10) is in good agreement with experimental results of Horikawa (1958) as reported by Jen, et. al. (1966)[21] and is a better representation of the velocity field than Eq. (1) which they proposed. Furthermore, the lateral profile of velocity of the subsurface jet was not found to differ significantly from that of a submerged jet. Laboratory experiments performed at the hydraulics laboratory, Chalmers Institute of Technology, show that Eq. (11) is a valid description of the growth of the jet flow.

Measurements of the distribution of temperature from experiments ranging in Froude number, $F = U_o \Big/ \sqrt{\frac{\Delta\varrho}{\varrho} g D}$, from 18 to 180 (that is when the effect of buoyancy on the spreading is small) was reported by Jen, et. al. (1966).[21] The experimental data of Hayashi and Shuto (1967)[18] range in Froude number from 1 to 16. Tamai, et. al. (1969)[31] describe some experiments performed at relatively small values of the Froude number. For $F > 3$ the effect of buoyancy is not very pronounced in the initial flow zone. Hence, Eq. (14) should be applicable which also is confirmed by the experiments.

1b. $\Delta\rho_o > 0$; $m_o \approx 0$ Buoyancy case.

The spread is then induced essentially by the density deficit. Theoretical and experimental studies indicate that the front velocity, U_Δ, may be described roughly by Abbot (1961).[1]

$$U_\Delta = \left(\frac{\Delta\rho}{\rho} gh\right)^{1/2} \tag{15}$$

where h is a characteristic thickness of the flow field at the edge.

1c. $\Delta\rho_o > 0$; $m_o > 0$ Intermediate case.

This is a subsurface jet characterized by initial momentum and induced gravitational spread. To model this flow situation we make the following assumptions:

a. The overall rate of entrainment is not affected by the buoyancy. Hence, F must not be too small.
b. The principle of superposition is assumed to yield the mechanisms causing the jet expansion.

The expansion of the jet flow due to diffusion may be evaluated from case 1a. The flow pattern of the non-buoyant subsurface jet is generalized to have a rectangular cross-section with a width-depth ratio of 2 and a constant longitudinal velocity equal to $k \cdot U_m$. Furthermore, we assume the

velocity and concentration profiles to be identical. From continuity we get

$$Q = \tfrac{1}{2}\,\pi U_m b^2 = k^2 U_m \cdot \frac{B^2}{2} \tag{16}$$

$$Q_o = \tfrac{1}{2}\,\pi U_m \cdot C_m \frac{b^2}{2} = k^2 U_m \cdot C_m \cdot \frac{B^2}{2} \tag{17}$$

where B is the width of the generalized subsurface jet.

Hence, $k = 0.5$

$$\frac{dB}{ds} = \sqrt{2\pi} \cdot 0.114 = 0.29 \tag{18}$$

$$U_d = \frac{0.29 U_m}{2 \cdot 2} = 0.072\ U_m$$

where U_d is the lateral spread velocity of the jet due to diffusion. If the gravitational effect is superposed we arrive at the following expression:

$$\frac{dB}{ds} = 4\ \frac{U_d + U_\Delta}{U_m} \tag{19}$$

and the equation characterizing the jet expansion is then

$$\frac{dB}{ds} = 0.29 + \frac{0.20}{F}\ \frac{S^{3/2}}{DB^{1/2}} \tag{20}$$

The depth may be found from an equation of continuity.

Eq. (20) is in good agreement with experimental results reported by Hayashi and Shuto (1967)[18] corresponding to steady-state conditions in Froude numbers exceeding say 2.5, and seems to be a better representation of the spreading pattern than the linear relationships proposed by Jen, et. al (1966)[21] and Wood and Wilkinson (1967).[33] For very small Froude numbers there is, in these scale tests, a pronounced buoyancy effect which hampers the mixing. Hence Eq. (20) cannot hold.

2. Subsurface discharge in shallow, homogeneous and stagnant environment.

2a. $\Delta\rho_o = 0;\ m_o > 0$ Non-buoyancy case.

For shallow water the jet tends to penetrate the whole depth. Hence, there is only lateral entrainment of ambient water. Assume the slope of the bottom to be S. Then the depth of the flow is given by:

$$h = h_o + S \cdot s \tag{21}$$

This case is represented by a plane jet and we arrive at the following characteristic flow equations using formulas for submerged two-dimensional jets, see Albertson, et al. (1950).[3]

$$U_m = 2.28 \left[\frac{J}{(h_o + S \cdot s)\, s} \right]^{1/2} \tag{22}$$

$$\frac{dQ}{ds} = 0.31\ (J/s)^{1/2}\ (h_o + S \cdot s)^{1/2} \tag{23}$$

$$J = J_o - \int_{-\infty}^{s} \int_{-\infty}^{\infty} \frac{\tau}{\rho_o}\ dy\ ds \tag{24}$$

where y is the lateral coordinate and, τ, the shear at the bottom expressed by

$$\tau = \frac{f}{8}\ \rho_o\ U_m^2 \cdot e^{-2\frac{y^2}{b^2}} \tag{25}$$

f is the friction factor and b stands for a local characteristic length of the plane jet. Hence,

$$\frac{J}{J_o} = \left[1 + S\frac{s}{h_o} \right]^{-\frac{f}{8S}}; \qquad \text{for } S \neq 0. \tag{26}$$

and

$$\frac{J}{J_o} = e^{-\frac{fs}{8h_o}} \qquad \text{for } S = 0. \tag{27}$$

This approach is supported by laboratory experiments carried out at Chalmers Institute of Technology.

2b. $\Delta\rho_o > 0$; $m_o \approx 0$. Buoyancy case, see 1b.

2c. $\Delta\rho_o > 0$; $m_o > 0$. Intermediate case.

If the entrainment is mainly in lateral direction, a generalized flow model is preferably based on a plane jet. Assuming that the lateral entrainment is not significantly affected by the buoyancy we may define a mean velocity in the longitudinal direction and a lateral velocity of expansion similar to case 1c. These velocities are evaluated to be 0.71 U_m and 0.124 U_m respective-

ly. The combined effect of diffusion and gravitational spread leads to the following expression of the expansion of the flow field.

$$\frac{dB}{ds} = 0.35 + \frac{3.0}{F}\left[\frac{U_o^3}{U_m^3}\ \frac{D}{B}\right]^{1/2} \tag{28}$$

Similar to Eq. (20), this relation requires that the Froude number is not too small. It should also be mentioned that the hampering of vertical mixing due to the density difference is more pronounced at larger distances from the source.

The depth of the jet flow is obtained from an equation of continuity.

If the receiving water is density stratified, the vertical mass exchange is considerably reduced. This suggests that a subsurface, plane jet of initial momentum and induced gravitational spread -2c- best represents the flow field. The two models proposed, however, define extreme situations as to the overall rate of entrainment and the true flow pattern is likely to fall somewhere between.

The two-dimensional wall jet is well-known from literature. If we apply the proposed jet diffusion model to this case, and neglect the shear at the wall, we arrive at the following equations.

$$U_m = 3.2\left(\frac{J}{hs}\right)^{1/2} \tag{29}$$

$$Q = 0.44\ sh\left(\frac{J}{hs}\right)^{1/2} \tag{30}$$

Analyzing experimental data, see Rajaratnam and Subramanya (1968),[28] we find the following expressions for the wall jet:

$$U_m = 3.4\left[\frac{J}{hs}\right]^{1/2} \tag{31}$$

$$Q = 0.44\ sh\left[\frac{J}{hs}\right]^{1/2}\left(0.52 + 4.1\ \frac{B_o}{s}\right) \tag{32}$$

Eqs. (31) and (32) deviate slightly from Eqs. (29) and (30) which is due to the fact that the shear at the wall affects the flux of momentum as well as the velocity distribution across the jet.

Coastal Currents

Currents in the receiving water area affect the dispsrsion pattern of the discharged condenser water and have to be accounted for when estimating the rate of recirculation. For shallow waters the jet tends to penetrate the whole depth and the jet trajectory is bent over towards the coast if coastal currents are prevailing. This may create a zone of local recirculation where a portion of the flow is continuously fed back to the jet. This mechanism of feed-back is less pronounced in deep water where the jet flow does not penetrate the full depth. Experiments have shown that such curved, two-dimensional jets have local concentration and velocity profiles similar to free jets, see Sawyer (1963).[29] The total rate of entrainment of the curved jet is almost identical to that of a free plane jet — the increased rate of entrainment along the outer edge $(\approx 2/3)$ being balanced by a reduced rate of entrainment along the inner edge $(\approx 1/3)$. Hence, as an engineering approximation it is justified to use the formulas for the growth of flow as a function of s, previously deduced for the stagnant environment case.

To determine the curvature of the jet path we have to consider the interaction between the flux of momentum of the ambient water and the jet flow. Hence, the rate of change of momentum flux may be set equal to the rate of entrainment of momentum flux plus a gross drag force acting on the jet, Fan (1967.)[11] However, several other models can be tested, Häggström (1969).[19]

Laboratory experiments are presently being carried out at Chalmers Institute of Technology to study the behavior of warm water jets discharged into a flowing environment.

Example

Proposed models of the dispersion of condenser water discharge are applied to the planned nuclear power plant, Barsebäck, located at the sound between Denmark and Sweden. The cooling water discharge is 150 m^3/s initially issued at a speed of 2 m/s. The temperature excess represents an initial density deficit of $\Delta\rho_0 = 1.0$ kg/m^3. Figs. 2 and 3 show the result for stagnant ambient water as well as for the case of a coastal current of 0.5 knot.

SUBMARINE DISPOSAL OF SEWAGE

The dispersion mechanism following a marine waste water discharge includes essentially two stages: first, the initial mixing of the waste water in the immediate proximity of the outfall; and second, the subsequent transport and dispersion of the disposed effluent. When sewage is discharged into

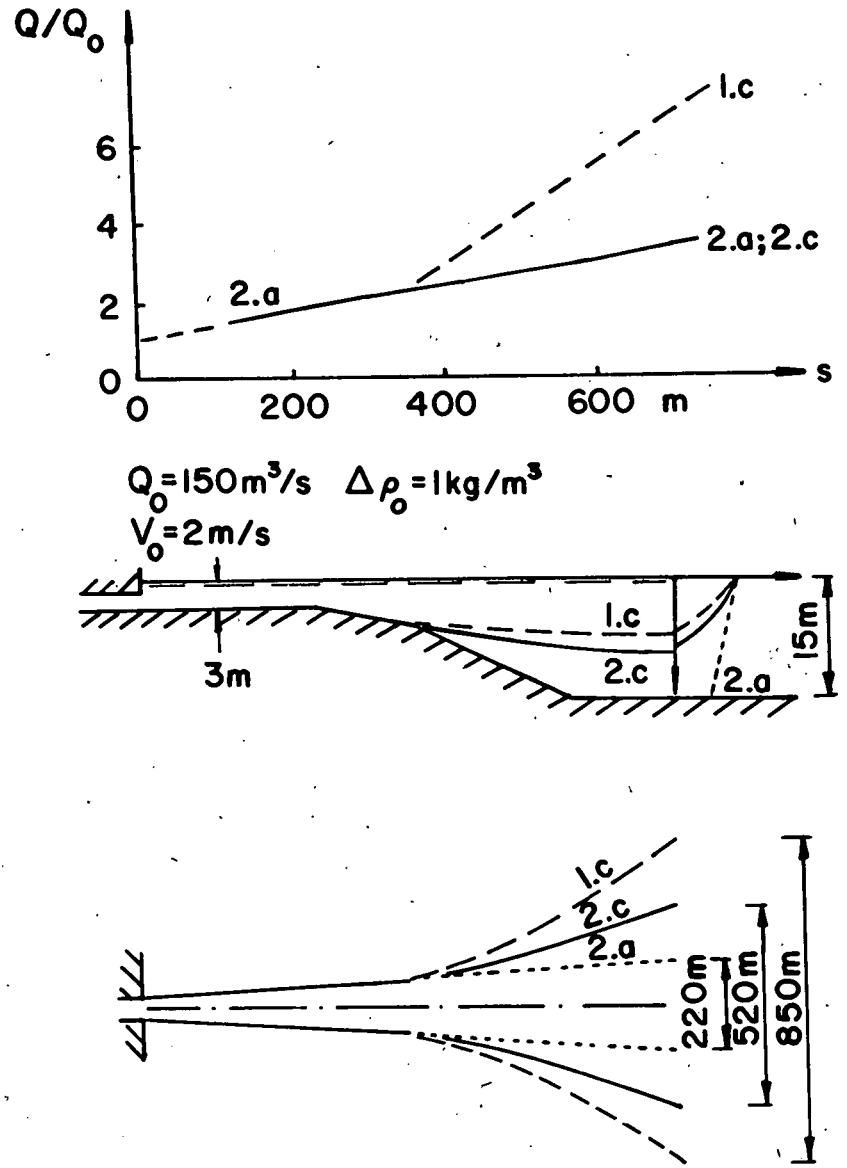

Fig. 2. — Flow characteristics for Barsebäck power plant. Applying models 2a (0 < s < 330m) and 1c and 2c (s > 330m) in the case of a stagnant receiving water.

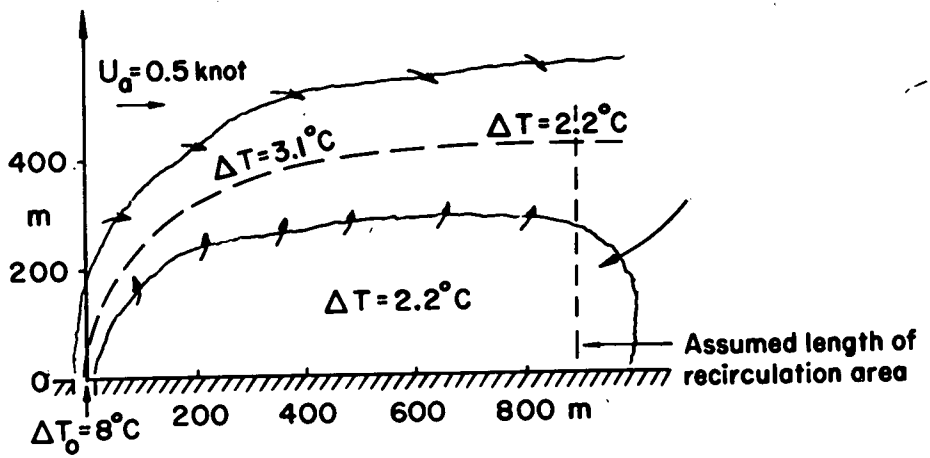

Fig. 3. — The spread of the cooling water from Barsebäck power plant discharged into a coastal current of 0.5 knot reproduced by a plane jet and ignoring density differences.

the sea it is subjected to a buoyant force proportional to the density difference between the sea water and the waste water. Hence, the discharged sewage rises towards the sea surface while mixed with ambient water. After the excess jet energy has been dissipated and a sewage field is established at some neutrally buoyant level, mixing due to natural turbulent diffusion becomes dominant.

In a stratified receiving environment there is a possibility of submerged sewage fields forming above a submarine outfall. The amount of pollutants that is brought up to the surface is then considerably reduced. This is usually looked upon as a desirable result, considering the increased recreational value of the receiving water area. However, other basic interests, primarily fishing, may prefer dispersion of the disposed waste water in surface layers where transportation and associated dilution are more efficient than in the deep waters.

The initial mixing of the discharged waste water is basically a problem of jet diffusion, whereas sea dispersion is a more complex phenomenon due to the multitude of significant parameters involved. It has, from initial mixing considerations, been the usual practice to provide multiple-port diffusers with horizontal ports. Hence, studies on horizontal jets discharged into environments of various hydrographical complexity is of great engineering interest.

A definition sketch for horizontal jet diffusion is given in Fig. 4. The S-shaped form of the jet is observed in flowing ambient water or when the jet effluent is discharged at large values of the Froude number in a stratified environment. Several solutions to horizontal jet diffusion in stagnant homogeneous water have been proposed. Abraham (1963)[2] introduces empirical functions for the spread of momentum and mass across the jet section where

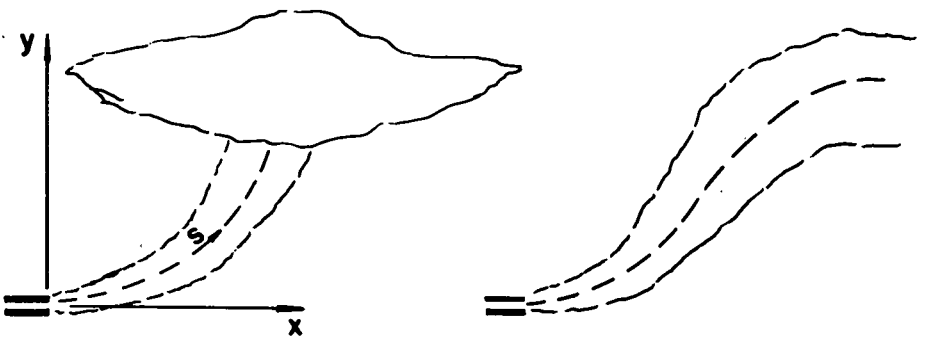

Fig. 4. — Definition sketch for horizontal jet diffusion. The S-shaped profile is observed in flowing ambient water or when the jet is discharged at a high Froude number in stratified water.

the rate of spread is a function of the local angle of inclination. Fan and Brooks (1966)[12] use the principle of entrainment which they advocate as being more logical from a physical standpoint. The two theoretical solutions give practically the same result when adequate values of coefficients are chosen. For cases of linear density profiles in the environment, a variety of numerical solutions to jet diffusion problems have been presented by Fan and Brooks (1969).[13] A numerical step-by-step procedure has been suggested by Cederwall (1966)[5] to obtain a solution for the case of a horizontal jet issued into a stably but otherwise arbitrarily stratified environment. A similar program has been worked out by Ditmars (1969).[9] These upproaches estimate the levels between which the sewage field will be established. Furthermore, the Danish Isotope Centre has worked out a set of computer programs which are concerned with the trapping effect in stratified water, and the conversion of all the single results to statistical expressions, Hansen (1967).[15]

When the sewage jet reaches the surface or some trapping level below the surface, there is a transition to horizontal spreading. It is difficult to assign any special flow pattern which gives a general description of the transition

zone because this mechanism depends very much on the hydrographical situation at the outfall site, and no detailed study has been focused on this problem. If fairly strong currents are prevailing in the disposal area, the spreading feature of the sewage field — submerged or established at the water surface — may be evaluated. However, when the currents are weak, the transport of the sewage away from the disposal area is a more complex phenomenon. If the sewage field is established at the water surface, then the propagation of the front of the expanding field is induced by density differences and is likely to resemble the spreading of oil on water, Abbot (1961).[1] For this case the front speed is given by

$$U_\Delta = \sqrt{\frac{\Delta\rho}{\rho} gh} \tag{33}$$

where $\frac{\Delta\rho}{\rho}$ is the relative density difference between the ambient water and the sewage-seawater mixture, and h is the thickness of the field. A solution based on Eq. (33) was given by Larsen and Sörensen (1967)[23] for the case of a sewage jet reaching the surface of a uniform flow. Sharp (1969)[30] studied the surface spread following a horizontal jet discharge in a stagnant homogeneous environment and provided an experimental solution for the rate of spread.

Eq. (33) expresses the front propagation of a gravity current in a homogeneous environment. The front abvances as an "instability front" with a tendency to have a vertical front (the dam burst analogy), see Benjamin (1968).[4] Assume for the moment that a similar mechanism of gravitation spread is applicable for the case of a trapped sewage field. The stratification of the receiving water is assumed to be linear between the levels occupied by the sewage field. Analogous to Eq. (33) we then get for the front velocity:

$$U_\Delta = \sqrt{\frac{\Delta\rho}{\rho} g \frac{h}{6}} \tag{34}$$

However, in stratified surroundings the front shape is more wedgelike suppressed by the stratification, and the front advance does not follow Eq. (34). This is observed from experimental modeling of the collapse of a mixing region in a stratified environment developing a thin triangular front with a low velocity of propagation, Wu (1969),[34] see Fig. 5. As a result there is a pronounced tendency of accumulation of sewage above the outfall for the case of trapped jets in a stagnant environment.

It is well-known that the directional variation of the horizontal velocity with depth-variation of the mean value as well as time fluctuations, is a

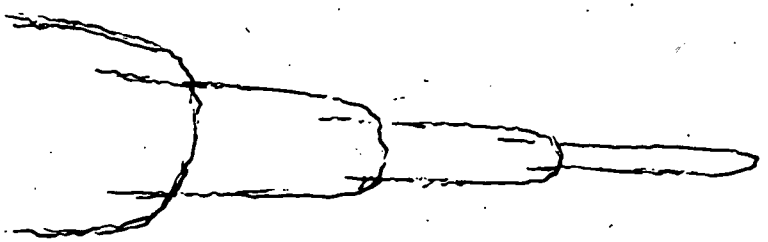

Fig. 5. — A mixed region collapsing successively in stratified water, Wu (1969).

most efficient mechanism for spreading a tracer cloud apart, see Fig. 6. Suppose that the environment is stratified thermally or due to existing differences in salinity. An increase of the initial vertical expansion of the sewage field favorable to subsequent dispersion is obtained when using diffusers with a slight but systematic variation of the port directions, or by a variation of port diameters. This suggests that sometimes, and specifically in stratified waters, the appropriate design of the outfall arrangement has to be carefully analyzed as to its effect on the dispersion mechanism following the initial dilution.

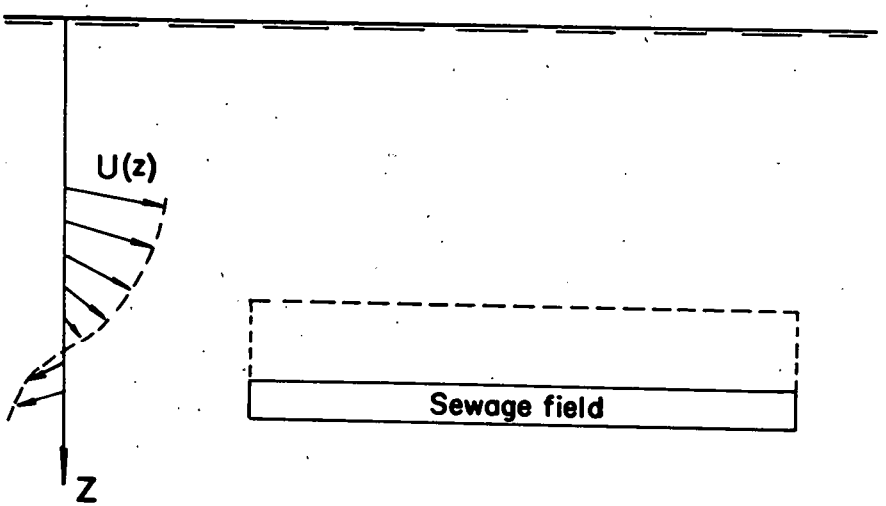

Fig. 6. — Increasing the initial depth of a submerged sewage field improves the lateral dispersion if the velocity vector of the ocean varies significantly with depth.

Several mathematical models have been proposed for ocean diffusion. Unfortunately, there are but few studies on dispersion in natural waters systematically relating the pattern of dispersion to environmental parameters. Hence, there is still a considerable lack of knowledge on many features of sea dispersion despite the efforts which have been devoted to the problem, see for instance Foxworthy (1968),[14] Okubo (1968)[25] and Kullenberg (1968).[22] Processes of transportation and mixing in coastal waters and in the open ocean are so complex that a single mathematical model that can explain the entire pattern of dispersion seems far away. Hence, we are very much dependent on empirical data obtained from field tracer investigations. This emphasizes the need for rational methods and sensitive instrumentations to reduce the high costs of performing tracer simulation studies on waste dispersion.

Sometimes the dispersion pattern following a continuous discharge of waste water is of less significance to the pollution problem than the overall water circulation of the receiving area. This is usually the case for confined water bodies with reduced communication with the open sea, such as fjords and many types of archipelagos. For such environments the background pollution level built up by the waste water discharge may be the factor which limits the receiving capacity of the water body. Hence, considerable emphasis must be given to studies on the external as well as the internal circulation of confined water bodies exposed to waste discharge. This particular problem will be discussed in the next section.

CIRCULATION IN CONFINED WATER BODIES

The circulation in confined bodies of water, as in fjords and embayments, is usually very complex and built up by various types of basic currents. It is essential when dealing with water quality problems to understand these transport mechanisms and how the water body considered communicates with the open sea. A classical example of such large scale circulation is the outflow of high salinity water from the Mediterranean through the Straits of Gibraltar. The heavy underflow is compensated for by a surface flow of less saline water entering from the Atlantic. A similar process is observed for the Baltic Sea; however, in this case precipitation exceeds evaporation and a saline underflow enters the Baltic Sea. The brackish outflow of water through the sound between Denmark and Sweden — the Baltic Current — sweeps along the Swedish west coast due to the Coriolis force. This current has been of considerable interest when major future waste disposal projects in these areas have been discussed.

For the purpose of this study let us look at the circulation of a confined water body which communicates with the open sea through a narrow strait, see Fig. 7. Fresh water is discharged into the interior of the water area. The

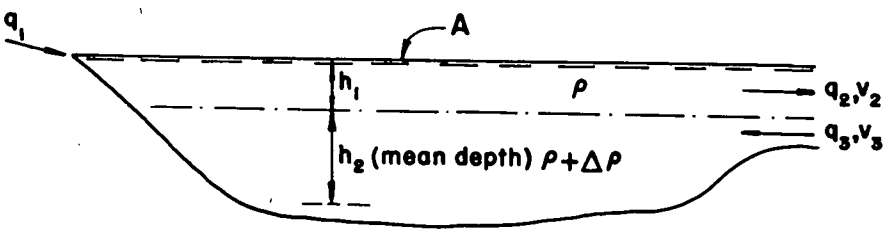

Fig. 7. — Section through a confined water body communicating with the open sea.

result is a density stratified system with a typical circulation pattern — outflowing brackish surface water, the loss of salt in the confined water area being compensated for by a subsurface inflow of salt water. This exchange mechanism is affected not only by the flow-through of fresh water, but also significantly by tidal activity. Furthermore, the circulation of the closed basin is driven by heating from the atmosphere and considerably by the wind stress. We will find that the vertical mixing between the brackish surface water and the salt bottom water is a process which contributes efficiently to the mechanism of external exchange of water. A characteristic feature of turbulent shear flow is the transport of energy from mean motion to turbulent motion. For the case of a homogeneous flow, this energy is dissipated by viscosity only, whereas in the stably stratified case, part of the turbulence energy is converted into potential energy by means of vertical diffusion. To maintain the turbulent structure of the flow, the rate of energy input must exceed the rate at which potential work is done by turbulent diffusion. In terms of the local Richardson number, R_i, characterizing the dynamic stability of stratified flow, we then have:

$$R_i = \frac{g}{\rho}\frac{\partial \rho}{\partial z}\Big/\left(\frac{\partial u}{\partial z}\right)^2 < \frac{\epsilon_{mz}}{\epsilon_{sz}} \tag{35}$$

where ϵ_{mz} and ϵ_{sz} are momentum and mass diffusivities respectively. Small values of R_i indicate increase of turbulence whereas a large value indicates suppression of vertical diffusion and high dynamic stability.

From experimental studies we know that vertical mixing occurs in a two-layered system at a certain velocity difference between the layers. If the

parameter θ, first proposed by Keulegan, see Ippen (1966),[20] exceeds a certain critical value the interface will be stable.

$$\theta = \frac{\nu \frac{\Delta\rho}{\rho} g}{U^3} \tag{36}$$

ν is the kinematic viscosity, and $\Delta\rho$ and U are a characteristic density difference and velocity of the interface. θ is essentially a Richardson number. If there is a pronounced interface between the two layers, the mass exchange depends on the tendency of small disturbances at the interface to be damped or amplified. This is theoretically deduced as well as verified in the laboratory. As a matter of fact, most of our knowledge on stratified flow has been gained from experiments in small laboratory flumes. There are two reasons why it is so difficult to apply laboratory test results to ocean conditions. First, we cannot be quite sure that empirical interfacial data of stability and mixing do not suffer from scale effects. Second, the bulk hydrodynamic parameters which are easily controlled in the laboratory are not as readily evaluated in the field. Furthermore, few field experiments on vertical diffusion have related the data to the complete set of relevant environmental conditions such as hydrographical parameters and wind and wave characteristics. Hence, a proper prediction of vertical mass exchange in a receiving water area, essential in many water pollution studies, is a very difficult task.

A major input of energy into a coastal water area is due to wind action. The rate at which energy is transferred from the wind to the water body depends on both normal and tangential stresses. It is convenient to introduce an apparent shear stress, τ_a, at the water surface:

$$\tau_a = \epsilon_a \rho_a \frac{\partial W}{\partial z} \tag{37}$$

where ρ_a is the density of the air, ϵ_a is the eddy viscosity of the air and W is the wind velocity. Assuming a rough water surface — waves are developing — we get the following well-known velocity profile:

$$\frac{W}{w_*} = 2.5 \ln \frac{30z}{k} = 2.5 \ln \frac{z}{z_o} \tag{38}$$

k is an equivalent roughness and z_o a roughness parameter related to the average wave height. For moderate and strong winds, z_o is assumed to have a constant value of about 0.6 cm. w* is the shear velocity $\sqrt{\frac{\tau_a}{\rho_a}}$.

If W_{10} is the mean wind speed at the 10 m level and ζ is a resistance coefficient, we can write:

$$\tau_a = \zeta \cdot \rho_a \cdot W^2_{10} \tag{39}$$

Combining Eqs. (38) and (39) we get $\zeta = 2.9 \cdot 10^{-3}$. A number of writers have reported values close to $2.6 \cdot 10^{-3}$. If u_* is the shear velocity at the water surface defined by $\sqrt{\frac{\tau_w}{\rho_w}}$ where ρ_w is the density of the water and τ_w the shear we arrive at:

$$\tau_a = w_*^2 \rho_a = u_*^2 \cdot \rho_w = \tau_w \tag{40}$$

$$\frac{u_*}{w_*} = \left(\frac{\rho_a}{\rho_w}\right)^{1/2} \tag{41}$$

and if we put $\rho_a = 1.3$ kg/m³ we find that $\frac{u_*}{w_*}$ is approximately 3.6%. This value is somewhat higher but fairly close to observed relations between wind velocity and induced wind drift at the water surface. The rate at which energy is transferred to the water body per unit surface area is then

$$P_w = \tau_a \cdot W \cdot \frac{u_*}{w_*} = 2.9 \cdot 10^{-3}\ W^3 \frac{u_*}{w_*} \tag{42}$$

The generation of wind induced currents is closely related to the mechanism of wave generation and a considerable fraction of the energy input by wind action is diverted to wave motion. In deep water the waves produce very little turbulence except when breaking. In shallow waters, however, and, specifically in the breaking zone close to the shore, there is a considerabls production of turbulence. It is, however, the turbulence that penetrates into deeper waters that causes vertical mass transport and hence it is the wind-induced currents and not the wave action that is the main factor contributing to vertical diffusion in the sea.

Let us take an example. Referring to Fig. 7, the following values of the variables represent the situation of the inner archipelago of Stockholm.[1]

$A = 110$ Mm²	$q_1 = 170$ m³/s
$h_1 = 10$ m	$q_2 = 370$ m³/s
$h_2 = 7$ m	$q_3 = 200$ m³/s
$\Delta\rho = 2.5$ kg/m³	$v_2 = 0.10$ m/s
	$v_3 = 0.15$ m/s

[1]From an investigation by Vattenbyggnadsbyron (VBB), Stockholm, for a planned atomic power plant at Värtan.

The required vertical mass transfer to maintain the circulation of this system corresponds to the following potential work per time unit

$$P_p = g\Delta\rho q_3 \cdot h_{1/2} \tag{43}$$

For this particular case P_p amounts to 24,500 Nm/s. Assuming an efficient wind velocity of 4 m/s, P_w is the order of 10^6 Nm/s. The inflows q_1 and q_3 may be neglected as to their contribution of energy being just a fraction of the required rate P_p. Hence, it is the wind induced energy which maintains the circulation of the water body.

Measurements in this archipelago have confirmed that the upper layer 'erodes' the lower layer. The mass transport is essentially a one-way transport — saline bottom water is brought up to the more turbulent surface layer while the salinity of the bottom water remains fairly constant throughout the depth. A planned atomic power station to be located here will take bottom water — 10 m^3/s — for cooling purposes and discharge the heated condenser water in such a way that it will be submerged below the interface. Hence, recirculation of cooling water can be avoided as long as the mechanism of vertical transport just outlined can be maintained. During some winter months ice covers the water surface and there is no input of energy from wind action. The vertical transport of salt water into the surface layer is then considerably reduced and recirculation of heated condenser water is difficult to avoid. This indicates that an alternative intake arrangement for surface water is favourable during winter time.

For water pollution control of a confined water area exposed to waste disposal, we have to rely on some model of the water circulation. Most theories proposed for estuarine mixing are based on the one-dimensional concept. They assume a well-mixed region and do not consider the detailed mechanism of dispersion within the system, see Pritchard (1952),[26] Waldichuk (1964).[32]

The flushing mechanism of a partially mixed estuary may be analysed by means of a two-dimensional model, if the circulation and mixing is assumed to be governed by advection and vertical mixing between the surface layer and the bottom layer, but not significantly by longitudinal dispersion, Pritchard (1969).[27]

Let us consider the case of a three-dimensional, partially stratified, semi-enclosed coastal area. Suppose that there is an influx of fresh water to this water body causing pronounced salinity differences. Let Q_f denote the fresh water flow. If C_o is the salinity of the sea water outside the bay, and C is the salinity at any point within the confined water area found from a survey of salinity measurements, we can express the fresh water content by

$$C_f = \frac{C_o - C}{C_o} \tag{44}$$

Hence, at that particular time the total volume of fresh water, V_f, present in the body of water considered, V, is found to be:

$$V_f = \int_V C_f \, dV \tag{45}$$

The mean residence time, τ, related to the flow-through of fresh water is given by:

$$\tau = \frac{V_f}{Q_F} \tag{46}$$

If a pollutant is introduced into the system in the same way as the fresh water, then τ is a good estimate of the detention time for this particular waste discharge. This situation, however, is not always the case, and we must have other methods to evaluate the dispersion within the region and the external water exchange. Let us first assume that there is a significant and well-defined flow through the free connection with the open sea exhibiting a periodical response to tidal activity. This condition is satisfied if the passage is relatively narrow as in the case of a fjord. Considering the characteristics of the alternating flow pattern through the passage, we may estimate an overall flushing rate of the confined water body which is not restricted to the fresh water.

The flow entering the water area has initial spread characteristics similar to a plane jet. The distance to which the incoming water will reach increases with some defined flushing time t as $\alpha t^{2/3}$ where α is a coefficient related primarily to the flow conditions at the passage. The water leaving the embayment, on the other hand, is flowing towards a sink and consequently the expansion of the withdrawal zone follows $\beta t^{1/2}$ where β is a coefficient which can be derived from initial flow conditions similar to α.

An exchange factor r′ may be defined as:

$$r' = \left(1 - \frac{\beta}{\alpha} t^{-1/6}\right) \frac{\Delta V}{V} \tag{47}$$

where $\frac{\Delta V}{V}$ is the nominal exchange ratio generally used in flushing theories relating the change of volume due to tidal activity to the efficient flushing volume of the water body. The same reasoning applies to the flow enter-

ing and leaving the open sea and, similar to Eq. (47), we may define exchange factors r''.

Now, assume that flushing of the surface layer in a particular water body is due not only to tidal activity but also significantly to erratic meteorological surges of the system, see Fig. 8. The net water exchange following a se-

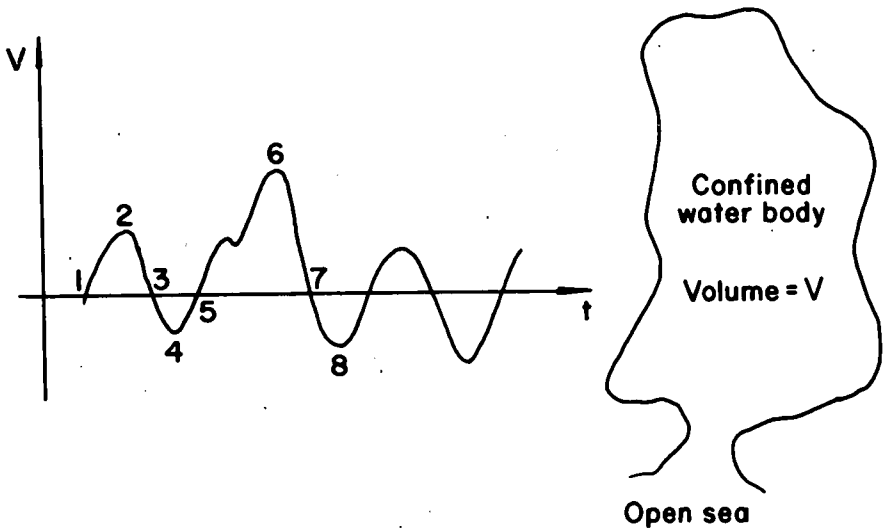

Fig. 8. — Flushing model of the confined water body related to the flow pattern of the free connection with the open sea, Eqs. (41)-(42).

ries of water level fluctuations is then described statistically as a first order model by

$$V \cdot \left[r'_{1\text{-}3} + r'_{3\text{-}5} + r'_{5\text{-}7} + \ldots\ldots \right. \tag{48}$$

assuming that flushing is the only significant mechanism contributing to water exchange. We may write the exchange factors as:

$$r'_{1\text{-}3} = \left[1 - \frac{a}{(t_3 - t_1)^n} \right] \frac{\Delta V_2}{V}$$

$$r'_{3\text{-}5} = \left[1 - \frac{b}{(t_5 - t_3)^m} \right] \frac{\Delta V_4}{V} \quad \text{and so on}^2 \tag{49}$$

[2]Appropriate values of the assumed constants a, b, n and m may be obtained from tracer field experiments, preferably by use of drouges or driftcards released at the cross-section of the passage.

Let us now consider an embayment with a broad communication with the open sea. In this case, the flushing effect probably does not contribute more to water circulation and exchange than the wind. Furthermore, the dispersive characteristics may differ considerably from site to site within the embayment. A straight-forward approach for study of the water circulation of the surface layer would be in-situ tracer experiments in order to simulate the processes of dispersion. Let us however, discuss the feasibility of a mathematical model for predicting the dispersive properties.

In recent years several methods have been developed for tidal computation in coastal waters, Dronkers (1969)[10] and Leendertse (1967).[24] These numerical schemes account for the effect of bottom friction but neglect viscosity effects in the lateral direction which unfortunately very often is a significant feature of two-dimensional dispersion (separation effects). Let us assume that the dispersion of the surface layer may be properly studied by means of a two-dimensional model. We may then write the equation for conservation of mass as follows:

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}(uc) + \frac{\partial}{\partial y}(vc) = \frac{\partial}{\partial x}\left(\epsilon_x \frac{\partial c}{\partial x}\right) + \frac{\partial}{\partial y}\left(\epsilon_y \frac{\partial c}{\partial y}\right) \qquad (50)$$

where c is the concentration of a conservative tracer. In finite-difference form we get:

$$\frac{c_{ij}(t_o + \Delta t) - c_{ij}(t_o)}{\Delta t} + \frac{(uc)_{i+1,j} - (uc)_{i-1,j}}{2\,\Delta x} + \frac{(vc)_{i,j+1} - (vc)_{i,j-1}}{2\,\Delta y}$$

$$= \epsilon_{xij} \frac{c_{i+1,j} - 2c_{ij} + c_{i-1,j}}{\Delta x^2} + \epsilon_{yij} \frac{c_{i,j+1} - 2c_{i,j} + c_{i,j-1}}{\Delta y^2} \qquad (51)$$

or in matrix form

$$\left[a_{ij}(t_o)\right] \left[c_{ij}(t_o)\right] + \left[b_{ij}(t_o)\right] \left[c_{ij}(t_o)\right]^* \longrightarrow c_{ij}(t_o + \Delta t) \qquad (52)$$

The matrices $\left[a_{ij}\right]$ and $\left[b_{ij}\right]$ account for the combined effect of advection and diffusion in x- and y-direction respectively. The step from Eq. (50) to (52) means that we have laid out on the region considered a rectangular array of spatial points and specified the values of the variables at the points of the rectangular grid. Furthermore, we have to denote proper

boundary conditions and, above all, we have to consider the three dimensional nature of the circulation. In most coastal areas of the kind we are discussing here, we find that the bottom layer has a fairly constant salinity. The mean depth of the surface layer does not vary much, although internal seiches give rise to temporary displacements of the interface. Hence, in those cases the two-dimensional model may be accepted as an engineering approximation. If the salinity is chosen as the tracer material — which is very convenient, provided that there are pronounced variations of salinity within the area — then we have to account for the vertical transport of salt water. Assume a one-way transport from the lower layer of salinity c_o to the upper layer, we then get:

$$\left[a_{ij}(t_o)\right]\left[c_{ij}(t_o)\right] + \left[b_{ij}(t_o)\right]\left[c_{ij}(t_o)\right]^* + c_o\left[d_{ij}(t_o)\right] \longrightarrow c_{ij}(t_o + \Delta t) \tag{53}$$

Furthermore, we assume that there are a certain number of environmental conditions that from a statistical point of view are relevant for describing the circulation of the system. Hence, we are searching for a set of solutions controlled by the values of the environmental parameters — wind, fresh water outflow, etc. Our principal interest is in the steady state solutions because they are unique and not affected by initial conditions such as salinity distributions which implies an uncontrolled dynamic effect. Hence, to represent a particular grouping of environmental parameters we may write:

$$\left[a_{ij}\right]\left[c_{ij}\right] + \left[b_{ij}\right]\left[c_{ij}\right]^* + c_o\left[d_{ij}\right] \longrightarrow c_{ij} \longrightarrow \left[c_{ij}\right] \tag{54}$$

If the velocity field and the tracer distribution are measured, Eq. (54) may be solved for the unknown vertical mass transfer velocities expressed by matrix $\left[d_{ij}\right]$ provided that appropriate values of the eddy diffusivities are selected. Hence, the full set of matrices $\left[a_{ij}\right]$, $\left[b_{ij}\right]$ and $\left[d_{ij}\right]$ characterizing the dispersion of the surface water can be calculated. To summarize, we have proposed an analytical dispersion model overlying an empirical flow model. We could also have included as a third step a reaction model which delays substances, generates reactions between substances, and simulates sources and sinks. It is obvious, however, that an analytical simulation technique as just outlined leads to extensive work even for a modern computer, and the field work implied is less significant.

TRACER SIMULATION OF DISPERSION

The analytical approach for modeling of dispersion phenomena in coastal waters discussed in the previous section, meets with the difficulties arising from the general complexity of the circulation. Hence, tracer measurements performed in the field represent an important engineering technique to determine the dispersion properties of a receiving water area for the conditions that will prevail when, for instance, an outfall has been built.

During the last twenty years the use of tracers has been very common. This is due to the development of very sensitive instrumentation for both laboratory and field investigations and furthermore, to the fact that many new tracers have appeared so that for each case one should be able to find the "ideal tracer".

Definition of a tracer is scarcely possible without taking into account the actual purpose of its use. Almost every substance could in one connection or another be regarded as a useful tracer. For more conventional use, however, we can choose between radioactive isotopes or fluorescent dyes. In most cases it is convenient to distinguish only between conservative and nonconservative tracers. When the amount of tracer decreases with time, the tracer is non-conservative or decaying. Very often, rapidly decaying tracers are preferred to more conservative ones. For instance, the presence of the tracer may be wanted only for a short period of time. Use of a slowly decaying tracer then could give rise to undesirable effects of accumulation.

The decay of tracers can normally be described by an exponential decay function e^{-kt} where k is the decay parameter and t the time. The half-time $t_{0.5}$ then equals:

$$t_{0.5} = \frac{\ell n 2}{k} \tag{55}$$

The decay parameter may be a constant characterizing the tracer in question, or it may be a function of light conditions, acidity and turbidity, etc. of the medium. In each case the decay parameter is well-defined only when these conditions are fully specified.

For the study of dispersion in a receiving water, two properties are often of special interest: dilution, and residence time of a discharged pollutant. Residence time should be understood as the time after the discharge of the waste water element in question. These two properties are of great interest with reference to later consideration of e.g. ecological and hygienic consequences of a planned discharge of waste water.

The purpose of the field tracer simulation, regardless of the method used, is to reach a statistical description of the dispersion pattern. For reliable prediction we must be able to reproduce or account for the initial stage of mixing as well as the effect of gravitational spread, provided that this phenomenon is of significance. There are two major approaches for tracer release — continuous injection during a certain period of time; and repeated, instantaneous injections covering the full range of environmental conditions. The most adequate simulation of a continuous waste discharge is certainly attained by applying a continuous tracer release technique. Assume that the waste product which is to be traced may be characterized by a decay parameter, k, corresponding to the most critical or most conservative component of the waste. Then a continuous injection of a tracer material with a decay parameter equal or close to k is the most straight-forward and most reliable approach. This situation is, however, very seldom possible. In order to simulate the continuous discharge of a conservative contaminant by means of a decaying tracer, the injection must be steadily reduced at a rate equal to the tracer decay. This restricts severely the length of the period of injection, especially when rapidly decaying radioactive isotopes are used, due to the fact that both the initial and final rates of tracer injection, for safety and practical reasons respectively, have to be neither too high nor too low. To overcome this difficulty a dual tracer technique may be applied.

The method of parallel injection of tracers have been described elsewhere, Cederwall (1968),[7] Cederwall and Hansen, (1968),[6] and just the principle will be outlined. To be used for a continuous, parallel injection two tracers are used which fulfill the following conditions:

a) The two decay parameters must be well-defined.
b) The two tracer substances must differ in at least one property, detectable in low concentrations of the substance.

Consider a decaying tracer with the decay parameter k continuously injected at a constant rate into a receiving water area. The concentration C(T) recorded at a certain point in the receiving water area at a time, T, after the start of the injection, can be considered as a sum of contributions from tracer elements released in succession at the point of tracer discharge. Then the concentration can be expressed by means of a frequency function, f(T − t), i.e. an age distribution with reference to time of release:

$$C(T) = \int_0^T f(T-t) \cdot e^{-k(T-t)} dt = \int_0^T F(s) \cdot e^{-ks} ds \qquad (56)$$

Thus, for a steady-state situation of the water circulation, f(t) is the impulse function recorded from a δ-input at time $t = 0$. The frequency function is characterized by its moments, μ_n, around the averaged value $\bar{t}$ — the mean residence time.

$$\mu_n = \int_0^T (t-\bar{t})^n f(t)\,dt \tag{57}$$

where f(t) is assumed to be normalized, that is $\mu_0 = 1, \mu_1 = 0$ and $\mu_2 = \sigma^2$. A residence time, τ, is defined by the following equation:

$$C(T) = e^{-k\tau} \int_0^T f(T-t)dt = e^{-k\tau} \tag{58}$$

τ is related to the frequency function f(t) and its moments, μ_n. For an arbitrary function f(t) we can write C(T):

$$C(T) = e^{-k\bar{t}} \int_0^T f(s) \cdot e^{-k(s-\bar{t})} ds =$$

$$= e^{-kt} \int_0^T f(s) \sum_{n=0}^{\infty} \frac{(-k)^n (s-\bar{t})^n}{n!} ds = \tag{59}$$

$$= e^{-k\bar{t}} \sum_{n=0}^{\infty} \frac{(-k)^n}{n!} \int_0^T (s-\bar{t})^n f(s)ds = e^{-k\bar{t}} \left[1 + \sum_{n=2}^{\infty} \frac{(-k)^n \mu_n}{n!}\right]$$

Hence,

$$\tau = \bar{t} - 1/2\, \sigma^2 k + \frac{1}{6} \mu_3 k^2 - \frac{1}{24} (\mu_4 - 3\sigma^4)k^3 + O(k^4) \tag{60}$$

For commonly used tracers and normal dispersive properties of the receiving water τ is very close to $\bar{t}$, see Cederwall (1968).[7]

Assume that the two tracers, 1 and 2, with decay parameters k and k_2, are simultaneously and continuously released at a constant rate. A time parameter, τ, is now defined by:

$$C_1 = C_1' \cdot e^{-k_1 \tau}$$
$$C_2 = C_2' \cdot e^{-k_2 \tau} \tag{61}$$

where C_1 and C_2 are the tracer concentrations at the measuring site, and C_1' and C_2' corresponding concentrations of the tracers assumed conservative. $c_1 \cdot q_1$ and $c_2 \cdot q_2$ are the rates of tracer injections. Then the following equation holds:

$$\frac{C_1'}{c_1 q_1} = \frac{C_2'}{c_2 q_2} \tag{62}$$

and,

$$\tau = (k_1 - k_2)^{-1} \cdot \ell n \frac{C_2 c_1 q_1}{C_1 c_2 q_2} \qquad \frac{\ell n \left[R \frac{C_2}{C_1}\right]}{a} \tag{63}$$

where R and a have constant values.

$$\tau \approx \bar{t} - 1/2 \; \sigma^2 (k_1 + k_2) \approx \bar{t} \tag{64}$$

Consequently, a τ value may be determined for each site in the receiving water where tracers are found in measurable concentrations. Thus a dual tracer technique makes possible the simultaneous registration of concentration distributions and residence time in a receiving water area. This method is useful not only for dispersion studies but also for determination of water exchange, since the residence time is evaluated and the time of injection is not limited. A case report from and investigation in Byfjorden on the west coast of Sweden using the radioactive tracer Bromine-82 and the fluorescent tracer Pontacyl Brilliant Pink has been reported by Cederwall and Hansen (1968).[6]

Instead of using a continuous release method, it is often possible to carry out spread tests by means of instantaneous injections of the tracer. Provided that the pattern of circulation is steady, the dispersion from a continuous source can be calculated by integration of the results obtained from a single injection. Each tracing then gives a quantitative determination of the spreading from the source for the environmental conditions that prevailed during that particular tracing. An adequate statistical description of pollution from a future outfall must be based on the investigation of a sufficient number of situations representing those parameter combinations that are significant to the spreading pattern. If it is difficult to find a reasonable

number of such situations which, from a statistical point of view, represent the full picture of tracer dispersion, then the only realistic approach is long-term tests with continuous injection of a single tracer or, alternatively, a dual tracer injection. The theoretical background to the instantaneous injection technique as well as case reports have been described by Harremoës (1964,[16] 1966[17]).

Measurements of radioactive tracer concentrations in surface waters are usually carried out by means of scintillation detectors connected to a counting system. The standard field procedure is to use submerged units measuring the in-situ concentration, although the impulse is averaged over a small volume surrounding the detector. The standard instrument for fluorescence measurements is the fluorometer. For field application the Turner Fluorometer, Type III, provided with a continuous flow cuvette has been extensively used. This measuring technique does not, however, provide instantaneous recording of the concentration, and furthermore, during the passage of the tube, the concentration is affected by dispersion. Hence, sharp tracer boundaries and gradients of the receiving water are not properly recorded. To eliminate this difficulty Kullenberg (1968)[22] developed an in-situ measuring fluorometer. The instrument, see Fig. 9, consists of two photo-multipliers, a depth sensing unit, and a lamphouse with a mercury lamp which irradiates horizontally the water below one of the photomultipliers which therefore senses both the daylight and the fluorescence of the dye. The other photo-multiplier is unaffected by the fluorescence of the dye and only senses

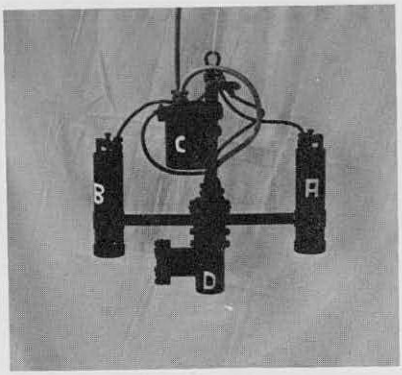

Fig. 9. — In-situ measuring fluorometer.
A, B photo-multiplier units
D mercury lamp house
C depth indicator unit

the daylight. Thus, the effect of the daylight may be eliminated. The instrument has proved reliable during measurements.

The injection technique is a very important element of a tracer investigation. To improve standard methods of tracer release, an instrumentation system has been developed by the Danish Isotope Centre in collaboration with the author, designed primarily for the dual tracer technique but providing for a number of facilities for work in this field.

The "DIC Injector System", see Fig. 10, consists of a pump unit and a control unit. The pump unit contains two separate pump systems. One of the pumps, the feed pump, is running at a constant speed and thus provides an injector for a non-decaying tracer. The other pump, the activity pump, is running at a speed which is variable and governed by the control unit. Its speed is measured by means of a tachometer generator. Both pumps are peristaltic pumps with separate inlets but a common outlet. In the pump

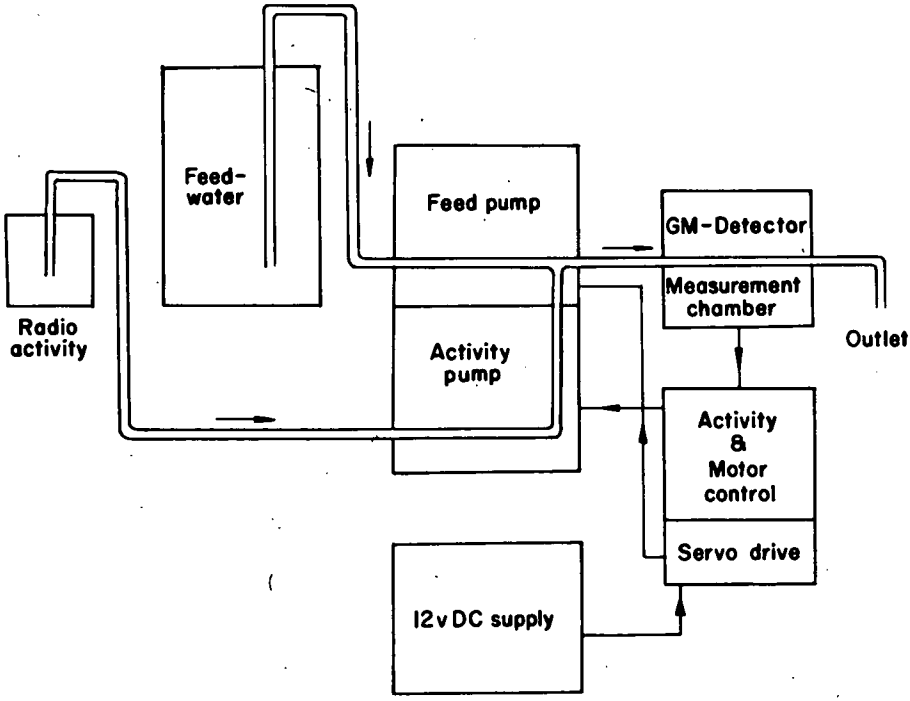

Fig. 10. — DIC injector system schematic.

unit, the radioactive concentration of the outlet is measured by means of a GM-detector. As the pump capacity of the feed pump is larger compared to that of the activity pump, the outlet flow will remain constant with time, and the signal from the GM-detector is thus a measure of the activity flow rate, e.g millicuries per hour of the radioactive tracer. The inlet and outlet of each pump are accessible, so that each pump can be operated separately. The control unit is a fully transistorized servo-unit, which ensures proper operation of the pump systems. Either the activity pump speed or the activity flow rate can be selected at the start of an investigation, and the control unit will insure that the chosen operation conditions does not change during the investigation.

The system is capable of injecting constant activity in the range:

4 mc/hour - 400 mc/hour of Bromine-82 (Br-82).

Manually operated the system can control activity flow rates down to 0.4 mc/hour of Br-82.

SUMMARY

Some approaches have been outlined for prediction of waste dispersion in coastal environments associated with sewage disposal and heat emission. It should be emphasized that a water pollution study represents an integral part of a broad engineering analysis of a particular environmental problem. Like most hydrological questions, water quality planning is very much interdisciplinary in nature.

The dispersive properties of the receiving water area are studied primarily to control hygienic and aesthetic harm to the environment, but also to control possible ecological disturbances and modifications. An important part of the overall analysis is the economical feasibility tests of various alternative projects which include benefit-cost studies of the water area considered. The complexity of the problem stresses the need for highly systematized methods of evaluation throughout the analysis.

The electronic computer has had a profound impact on water resources planning and it is now, and will be still more in the future, an indispensable tool for the engineer working in this field. However, the ability to make extensive calculations must correspond to an equivalent capacity in forming useful models and hypotheses. In this development towards an operational type of water quality management, we will find both purely theoretical and semi-empirical approaches for modeling of flow phenomena extremely valuable.

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A0, 1 ALGORITHM FOR CITY PLANNING

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ABSTRACT

This paper describes a model for the allocation of research funds to a series of urban experiments whose outcomes determine the ultimate disposition and allocation of very much greater amounts of money for urban housing projects. It is shown that the allocation model is a particular application of a technique which can be widely applied in the design of statistical experiments, and the paper describes the algorithm for solving the 0,1 integer programming problem which results from the formulation of the urban model. Central to the working of the model is the derivation of an association matrix which expresses the likelihood that certain experimental procedures will be paired in actual practice.

Key Words: experimental design, housing, city planning, mathematical programming, random sampling, gradient procedure.

Introduction

The operations research literature contains literally dozens of references to solutions of 0,1 programming problems. Quite properly, these many papers focus on the algorithm for extracting the solution and on demonstration of its convergence, uniqueness, and other desirable properties. In most cases the underlying problem — abstracted from the physical, military, management or social sciences — is given short shrift in favor of the more appealing, relevant, and tractable analysis.

This paper reverses the traditional emphasis and concentrates primarily on construction of the objective function for a 0,1 programming problem; passing attention is paid to a new solution algorithm based on a gradient technique. This algorithm does not purport to find the global optimum but rather a series of local optima from which it is possible to decide whether to accept the best available solution and terminate the process, or to run additional trials in the hope of locating a more desirable solution.

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The problem originated from a study of urban housing factors, but is analytically akin to an extension of sampling theory and as such can be modified and rendered generally applicable in that statistical discipline. The original study was undertaken under the auspices of the Department of Housing and Urban Development, and the author acknowledges the assistance provided to him as a consultant to Abt Associates, Inc., which has approved the release and publication of this material; unfortunately, the numerical problem which spawned this solution was not available for publication.

The Problem

Suppose a large sum of money is to be made available for urban communities to spend on housing. The primitive institutional and technological constraints imposed on the housing industry are widely known and seemingly insurmountable; they have led Harvey Brooks to characterize housing as America's "largest cottage industry". The sponsor, whether it be a federal, state or local agency, a foundation, or a private combine, is anxious to overcome as many institutional and technological obstacles as possible, and therefore identifies a number of potential changes in these institutional and technological constraints in the hope that some combination of them might effect major benefits (e.g., cost reduction or quality improvement) in the projected housing development.

The list of such potential changes is very long indeed; a few typical entries are:

1. tax and financing advantages,
2. the use of exotic construction materials,
3. modernization of building codes,
4. modernization of union and restrictive rules, and
5. factory assembly, plumbing, wiring, and drilling of modular components.

The optimal combination of factors is defined as that set of changes which is best for a particular community, and, unhappily, no analysis seems capable of determining this optimum. Experimentation using various combinations in several cities offers some promise, but it is certain to be frightfully expensive and, even worse, doomed to inadequacy because the number of possible combinations is prohibitively large and consequently precludes examination of all but a small fraction of the alternatives. The problem addressed here is whether prior analysis can delineate, for particular cities, certain combinations which are more advantageous than others in that they provide more *information* about the alternatives. Note that this is quite different from attempting to find the optimal combination. The optimal combi-

nation maximizes (or minimizes) some housing-oriented benefit function while here we are concerned with *information* and its maximization by selection of experimental modules. To be sure, the two processes (experimentation and construction of the prototype) are intimately and ultimately related, but formally they are quite distinct.

The following paragraphs describe the decision process in formal, analytical terms. The alternatives are called *experiments* and the several places available for experimentation are called *cities* (this notation is made explicit because the locales may in fact not be cities and the technological and institutional changes may not in fact resemble experiments in the scientific sense).

A set of experiments $\{E_1, E_2, .., E_n\}$ is available, and some sub-set of E must be assigned to a group of cities $\{C_1, C_2, .., C_m\}$ so that the total amount of information derived from performing the experiments at the several cities is maximized. Symbolically, we seek an $n \times m$ matrix called Δ such that the element δ_{ij} of Δ is unity if experiment E_i is performed at city C_j and is zero otherwise. The matrix Δ is a decision matrix, as shown in array A-1:

$$\begin{array}{cc} & \text{Cities} \\ & C_1 \quad C_2 \quad \cdots\cdots \quad C_m \\ \text{Experiments} \begin{array}{c} E_1 \\ E_2 \\ \cdot \\ \cdot \\ \cdot \\ E_n \end{array} & \boxed{\delta_{ij} = \begin{array}{l} 1\text{: perform } E_i \text{ at } C_j \\ 0\text{: do not perform } E_i \text{ at } C_j \end{array}} \end{array} = \Delta \qquad \text{(A-1)}$$

Decision Matrix

Clearly, certain combinations of (0,1) in Δ are more appropriate than others. For example, the cost of performing a certain sub-set of experiments at city C_j may be quite different from performing the same sub-set of experiments at city C_k, so that if the amount of information obtained from the experiments is equal, then clearly the experiments should be done at that city in which the cost is least. It follows that the optimal solution for Δ must consider a matrix of costs; the cost matrix is also of dimension $n \times m$ and the element c_{ij} is the cost of performing E_i at C_j, as shown in array A-2.

		Cities		
	C_1	C_2	.....	C_m
E_1				
E_2				
Experiments		c_{ij} = cost of E_i at C_j		
E_n				

(A-2)

Cost Matrix

We make the fundamental assumption that the information obtained from performing a sub-set of experiments at C_j is precisely the same as that obtained from the same sub-set at C_k. The cities are indistinguishable with respect to results but not with respect to costs. The cost of performing experiment E_i is not independent of the city C_j at which the experiment is performed. Therefore, in all but the most trivial cases, the rows of the cost matrix in array A-2 are not identical so that the cost information cannot be compressed into an n-dimensional vector.

Of course, the political and economic realities encountered in any such experimental enterprise impose a large number of constraints on the specification of the decision matrix Δ. Obedience to geographical distribution,

whether mathematically prudent or not, demands that each city C_j get its fair share of the experimental budget. This paper does not purport to judge the worthiness of any particular distributional requirement, but merely presents a technique whereby the cost of geographical distribution can be measured by the difference in information between the optimal experimental procedure and the actual; from the magnitude of this loss it is possible to impute some economic metric to the political luxury of geographical distribution, and to render an informed judgment on the degree of geographical distribution which the experimental procedure ought to accommodate.

Other constraints enter the decision-making process. For example, certain experiments may be uniquely adaptable to certain cities while it may be quite impossible to perform these same experiments elsewhere. Therefore the analysis must have some way of forcing certain elements δ_{ij} to unity and others to zero.

The experiments cannot be scaled or sub-divided; that is, experiment E_i *is* or *is not* performed at city C_j. It is presumed that the level of experimentation (for any experiment) is uniquely determined at any city and that this level is reflected by the cost c_{ij}. At first blush this would seem to make the problem easier because it eliminates the necessity for determining how intensive each potential experiment should be in each of the cities $\{C_i\}$ and replaces it with a bistable, polar decision represented by the pair (0,1). But in fact the converse is true; solving the (0,1) problem is vastly more difficult than solving the corresponding continous problem in which intermediate levels of experimental intensity can be accommodated.

In any event, a set of decisions must be made to define an experimental design on the grid represented by the intersection of experiments $\{Ei\}$ and cities $\{C_j\{$ A set of 0's and 1's are to be located so as to maximize the total amount of information obtained from the experimental design, all subject to appropriate geographical, institutional, and budgetary constraints. If the number of experiments n is of the order of 20, and the number of cities m is of the order of 10, a solution consists of some 200 binary digits. But this unimpressively small number belies the enormous number of feasible combinations and permutations which are somehow inferior to the optimal solution. Sorting through this enormous number of candidates in not a trivial task!

Failure of Standard Techniques

Consider an experimental design from which it is desired to evaluate two effects, and let these effects be measured by experiments E_1 and E_2. Traditional experimental design calls for four experiments: (i) both E_1 and E_2

absent, (ii) E_1 absent and E_2 present, (iii) E_1 present and E_2 absent, and (iv) both E_1 and E_2 present. From this arrangement it is possible to evaluate the results of each effect alone and in combination, with conclusions usually cushioned by the limits of statistical significance. The number of combinations which must be considered in a complete factorial experiment is 2^n, which is 1024 for the number of experiments $n = 10$. In this study, each city is associated with a unique combination of experiments; that is, one of the 1024 possible arrangements can be tried at each city. With 20 cities, the total number of different assignments is the total number of combinations of 1024 items taken 20 at a time, or

$$_{1024}C_{20} = \binom{1024}{20} = \frac{1024!}{1004!\ 20!} > 10^{60} \quad , \qquad (1)$$

a truly staggering quantity. One of these assignments is best in the sense that it gives more information than any other, and it is our task to find that one.

We are stymied on several accounts. First, with only 20 opportunities (i.e., cities) for experimentation rather than 1024, it is patently impossible to perform a factorial experiment which would (i) uniquely isolate the effect of any factor and (ii) specify interactions between that factor and all other combinations of factors. Second, accepting the constraint of 20 cities, it is clearly impossible to consider the systematic extraction of potential experiments because their number is so formidable. As a corollary, the variable cost structure which represents the fact that c_{ij} might differ substantially from c_{ik} makes impractical a randomized block experimental design. Third, we have not yet come to grips with the essential problem of what it is that constitutes a "good" experiment, having devoted ourselves mainly to the vague notion that good experiments provide lots of information while poor ones do not.

Modifications of the factorial design include such schemes as Latin Square, Graeco-Latin Squares, randomized blocks, and other techniques which can be studied in any one of many standard references. But these techniques specifically ignore the cost of experimentation at the several alternative locations, the value of information at the several locations, and the difficulties associated with establishing a criterion of performance for the experimental design. Consequently the standard techniques are rejected in this analysis, and it is necessary to consider techniques of mathematical programming.

Formulation of the Experimental Design as a Programming Problem

If the constraints on cities, experiments and combinations can be written as inequalities, mathematical programming offers the preferred solution. The decision variables are the δ_{ij}, the elements of array A-1. There is a constraint on the total budget for the experimental design; this is expressed by the inequality

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \delta_{ij} \; c_i \leq B \; , \tag{2}$$

where B is the total budget for the program. Moreover, there are two constraints on the budgetary allowance for each city C_j, expressed by

$$\sum_{i=1}^{n} \delta_{ij} \; c_{ij} \leq B_j \; , \; \forall_j \; , \tag{3}$$

$$\sum_{i=1}^{n} \delta_{ij} \; c_{ij} \geq B_j^* \; , \; \forall_j \; , \tag{4}$$

where B_j is the maximum budget allocated to city C_j and B_j^* is the minimum. Judicious manipulation of the B_j and B_j^* is tantamount to imposition of geographic distribution, and if all the B_j^* are zero then geographical distribution is not a consideration in the optimal assignment of experiments.

It is also clear that there must be some control exercised over the number of locations at which any experiment is performed. For example, if E_i is performed at every C_j, there is no basis on which to determine its effect because there is no "untreated" city. Conversely, it must be performed someplace, in at least one C_j. These constraints are expressed by

$$\sum_{j=1}^{m} \delta_{ij} \leq N_i, \quad \forall_i \; , \tag{5}$$

$$\sum_{j=1}^{m} \delta_{ij} \geq N_i^* \; , \; \forall_i \; , \tag{6}$$

where N_i and N_i^* are the upper and lower bounds, respectively, on the frequency of experiment E_i.

Finally, to constrain the decision variables δ_{ij} to the values 0,1, the constraint

$$\delta_{ij} = \delta_{ij}^2 \tag{7}$$

is imposed; this is satisfied by the two values $\delta_{ij} = 0$ and $\delta_{ij} = 1$, and further ensures that the decision variables are non-negative and obey the constraint

$$\delta_{ij} \geq 0. \tag{8}$$

The programming problem is to maximize the total information, heretofore undefined but written functionally as

$$I = I(\Delta), \tag{9}$$

subject to the constraints represented by equations (2) through (8). Because of equation (7), it is clear that the problem cannot be cast as a linear programming problem and that one of the more sophisticated relatives of this blessedly simple technique must be employed.

The following sections develop a suitable non-linear objective function corresponding to equation (9), and because of the inherent difficulty of non-linear programming problems, provide an algorithm for approximating the analytical solution.

The Objective Function

Suppose there are three cities and three factors or experiments to be investigated. There are 2^3 combinations in a complete factorial experiment, and all the combinations represented by the complete factorial arrangement cannot be accommodated in the available cities because $2^3 > 3$. Array A-3 shows the eight possible combinations, no more than three of which may be utilized in the experiment:

		Combination								
		1	2	3	4	5	6	7	8	
	E_1	0	0	0	0	1	1	1	1	
Factor	E_2	0	0	1	1	0	0	1	1	(A-3)
	E_3	0	1	0	1	0	1	0	1	

It is necessary to maintain some diversity in the experimental design, so that there would be little benefit in repeating any column or combination in more than one city. In the more general case, for which different cities exert

unique effects, this statement would not be evident *a priori*; however, under the assumption that cities are indistinguishable with respect to effects (but not with respect to costs), replication is not indicated.

Which columns, then, are more appropriate for the limited experimental effort to be undertaken in the several cities? Consider a square matrix A, of dimension n × n, whose elements represent the degree of association which exists between each pair of experiments or factors. For example, it might happen that in housing practice, when the several factors are incorporated into prototype construction projects, certain experiments tend to occur together while others tend to preclude each other. If experiment E_1 is some institutional change which, if implemented, strongly implies that E_2 would be incorporated while E_3 would generally be bypassed, the matrix A has the following general form

$$A \approx \begin{array}{c|ccc|} & E_1 & E_2 & E_3 \\ \hline E_1 & 0 & + & - \\ E_2 & + & 0 & \\ E_3 & - & & 0 \\ \hline \end{array} \qquad \text{(A-4)}$$

It is inappropriate to continue to label the rows and columns as *experiments* E_i because A represents the degree of association encountered in practice, not under the controlled conditions which constitute an experiment. However, for the sake of notational consistency, the symbol E will be used throughout and the context will make its significance abundantly clear. By definition, the elements along the main diagonal of A are equal to zero. Because E_1 and E_2 tend to occur together, the elements a_{12} and a_{21} are positive; similarly, a_{13} and a_{31} are negative. The matrix A should not be thought of as a correlation matrix because there is no implication that E_1 and E_2 force the output of the experiment (whatever that may be) in the same direction, nor conversely for the negatively associated pair $E_1 - E_3$. The elements of A do not specify reinforcement or antagonism in the usual statistical sense, but merely the fact that political and social reality dictate which pairs of experiments are likely to be run together, which are likely to be run individually, and which are independent.

Numerical values are assigned to the elements a_{ij}; the array A-4 merely specifies the signs for several of the constituent pairs. Elements whose absolute values are large reflect strong association or dissociation, and conversely for small values; the proposed solution is independent of an arbitrary

scale factor, so that while it might be convenient to adjust the elements of A to lie within the range $-1 \leq a_{ij} \leq 1$, it is unnecessary to do so.

Suppose a study of the three available experiments suggests

E_1: adoption of a performance-based building code,

E_2: availability of an attractive financing scheme for housing,

E_3: acceptance by trade unions of liberalized restrictive practices.

E_1 and E_2 are strongly associated; E_1 and E_3, for the particular cities involved, are thought to be strongly exclusive; E_2 and E_3 are very nearly independent in that the realization of one does not imply much about the other. The matrix A, or association matrix, is

$$A = \begin{array}{c} \\ E_1 \\ E_2 \\ E_3 \end{array} \begin{array}{c} \begin{array}{ccc} E_1 & E_2 & E_3 \end{array} \\ \boxed{\begin{array}{ccc} 0 & 0.8 & -0.6 \\ 0.8 & 0 & -0.1 \\ -0.6 & -0.1 & 0 \end{array}} \end{array} \qquad \text{(A-5)}$$

(These values are abstracted from the Abt study cited earlier.) Because E_1 and E_2 are strongly associated, combinations 1, 2, 7, and 8 of array A-3 appear to be most promising because in each of these E_1 and E_2 are performed or bypassed jointly. By the same reasoning, combinations 2 and 7 appear to be more suitable than 1 or 8 because E_1 and E_3 are strongly opposed (that is, a_{13}, being negative, suggests that both are unlikely to occur simultaneously). It therefore follows that however the several combinations are ranked, numbers 2 and 7 should fare better than their competitors.

The several combinations are ranked by a simple algorithm. Each score is the weighted sum of elements a_{ij} of the matrix A, with $i \neq j$, and with the weighting factors being positive or negative depending on whether E_i and E_j are run together or not. For combination 1, (0,0,0), all experiments are bypassed so that the score is the sum of elements in the matrix A; for convenience we use only the elements above (or below) the main diagonal, thereby taking advantage of the symmetry of A. Thus for combination 1 the score is $S_1 = 0.8 - 0.6 - 0.1 = 0.1$, as shown in Table 1. Similarly, combination 2, (0,0,1), ignores E_1 and E_2; E_3 is performed, so that the score includes $-a_{13}$; or $+ 0.6$. Finally, because E_2 and E_3 are not jointly performed or bypassed, the sign on a_{13} is negative and the total score is $S_2 = 0.8 + 0.6 + 0.1 = 1.5$, as shown below. The scores do not represent a

physical parameter but rather the degree of "independent return" from the combination k.

Combination, k	1	2	3	4	5	6	7	8
Score, S_k	0.1	1.5	−1.3	−0.3	−0.3	−1.3	1.5	0.1

Table 1 — Scoring System for Experiments

Due to the symmetry of the factorial experiments and the matrix A, the scoring system is indifferent between complementary conbinations. However, if A is not symmetric because of peculiar institutional constraints, the scoring system would necessarily have to consider all elements of A rather than the triangular portion alone.

Table 1 shows the score S_k corresponding to all combinations k, where k runs from 1 to 2^n. If combination 2 were performed at all 3 sites, no information would be derived from the experimental design because there would be no standard against which to compare the information or effects derived from different experimental arrangements.

Suppose city C_i is assigned experimental design or combination number k_i and city C_j is assigned experimental design k_j. The sum of scores for both cities is

$$S_{ij} = S_{k_i} + S_{k_j} , \tag{10}$$

where S_{ij} depends solely on the experimental arrays at each city and not on any measure of replication between them. The contribution to the total score which is due to the pair of cities C_i and C_j is the sum over all experiments

$$\sum_{k=1}^{n} (\delta_{ki} - \delta_{kj})^2 \; S_{ij} \tag{11}$$

which, in effect, assigns a weighting factor of unity to those elements of the experimental design which are different in the two cities and a weighting factor of zero to those elements which are identical. Calculation of the total score is then simply a matter of summing up over all possible pairs of cities in the decision matrix, so that the total amount of information derived from the experimental design may be written

$$I = \sum_{i=1}^{m} \sum_{j=i+1}^{m} \left[\sum_{k=1}^{n} (\delta_{ki} - \delta_{kj})^2 \; S_{ij} \right] , \tag{12}$$

which is to maximized. It is a trivial matter to include an arbitrary weighting factor in each element of the matrix S_{ij}, this factor to represent some *a priori* evaluation of the importance of particular experiments at particular places. If such a factor, say λ_{ijk}, is added, the total information to be maximized is

$$I = \sum_{i=1}^{m} \sum_{j=i+1}^{m} \left[\sum_{k=1}^{n} (\delta_{ki} - \delta_{kj})^2 \lambda_{ijk} S_{ij} \right] . \qquad (13)$$

This completes the formulation of the problem as a non-linear (0,1) integer programming problem subject to linear constraints, but unhappily there is little promise of a solution! The essential feature in formulating the problem is an elaborate structure involving the matrix A; this is necessitated by the fact that there is no specific measure for the benefit accruing to any combination of experiments at a particular city, so the usual notion of economic benefits is replaced by a formulation which measures the absence of replication and, simultaneously, is strongly influenced by the closeness with which experimental arrangements agree with the format and political constraints within which actual construction projects are presumed to operate. Again, it is assumed that the information obtained from any one city is as useful as that obtained from any other; the extent to which this is untrue can be accommodated by assigning a range of values to the parameters λ_{ijk}. For the example cited here, and for the larger problem described above, all values of λ are set at unity so that the program does not distinguish between information obtained at the several cities.

To summarize, the decision variables are the values of δ which appear in equation (13); it is desired to find that set of δ's which maximizes the information defined in equation (13), subject to the several constraints in equations (2) through (8). The next section is devoted to obtaining a numerical approximation to the exact solution of the programming problem.

Numerical Approximation to the Solution

The algorithm developed for this problem is a steepest ascent or gradient technique which starts from a random feasible solution as defined by a decision matrix Δ and proceeds therefrom to a new matrix Δ_1 which is a local optimum in the sense that interchanging any adjacent (0,1) pair produces either an infeasible solution or a lessening of the information $I(\Delta)$. Another random feasible start is then made, and a new local optimum Δ_2 is reached; after several random starts, the most advantageous (or locally) optimal value of $I(\Delta)$ is chosen to approximate the (globally) optimal $I(\Delta)$ and the cor-

responding decision matrix, Δ, is specified as the experimental design to be implemented.

The following steps are executed in the algorithm:

1. Read all control data; the association matrix A; the cost matrix C; the several budgets B,B*, N,N*; and any predetermined combinations which specify that E_I is, or is not, to be performed at C_j.
2. Calculate the score S_k for all experimental combinations, $k = 1,2,\ldots,2^n$. Clearly, for n large, the number of combinations is formidable.
3. Select a pair of rectangularly distributed random sampling integers in the range $1 \leq i \leq n$, $1 \leq j \leq m$, and assign $\delta_{ij} = 1$ unless one or more of the following conditions prevails:
   (a) the intersection δ_{ij} has been precluded by the input data,
   (b) the intersection δ_{ij} has already been established to be unity,
   (c) the budget at C_j is exceeded,
   (d) the limitation on N_i is exceeded, or
   (e) the total budget B is exceeded.

   Continue to put values $\delta_{ij} = 1$ until condition (e) is violated, whereupon a quasi-feasible solution is defined. This solution obeys any constraints on maxima, but not necessarily those on minima. These latter constraints are, for the moment, neglected.
4. Calculate the information $I(\Delta)$.
5. Isolate that pair of experiments E_p, E_q for which a_{pq} is a maximum, and adjust Δ as follows:
   (a) if $a_{pq} > 0$, try to make $\delta_{pk} \neq \delta_{qk}$, starting with that city C_k which minimizes the cost of making the exchange;
   (b) if $a_{pq} < 0$, try to make $\delta_{pk} \neq \delta_{qk}$, again starting with that C_k which minimizes the cost;
   (c) all adjustments are made subject to the budgetary and frequency constraints.
6. Move to a new pair of experiments E_r, E_s for which $|a_{rs}|$ is second-largest, and perform step 5 again. Continue iterating in this way, each time using the largest remaining $|a_{ij}|$. Finally, when all distinct pairs are exhausted (no more than $n(n-1)/2$ pairs are possible, and many of these may have $a_{ij} = 0$), a local optimum is reached.
7. Store the value $I(\Delta)$ corresponding to the decision vector Δ_1.

8. Make a new random start, as described in step 3, and continue until an appropriate number of Δ_i are investigated. The stopping point is defined, in part, by the relative smoothness of the function $I(\Delta)$ and by the execution time required to locate a local optimum.
9. Finally, identify the approximate global optimum and either terminate the solution or start again with new input data, as described in step 1, to determine the sensitivity of the solution to a range of input parameters.

Example

Continuing our numerical example, we assume the following parameters and constraints:

$$B = 2$$

$$B_j = 2, \ \forall j$$

$$B_j^* = 0, \ \forall j$$

$$c_{ij} = 1, \ \forall i,j$$

$$N_i = 2, \ \forall i$$

$$N_i^* = 0, \ \forall i$$

$$\lambda_{ijr} = 1, \ \forall i,j,k$$

and the trial design or decision matrix:

		City 1	2	3
	1	1	0	0
Experiment	2	0	1	0
	3	0	0	0

Thus, from Array A-3, we have $k_1 = 5$, $k_2 = 3$, and $k_3 = 1$. For these 2 cities we have $S_{12} = S_5 + S_3 = -1.6$. If now we sum the products

$(1-0)^2(-1.6) + (0-1)^2(-1.6) + (0-0)(-1.6) = -3.2$ taken down columns 1 and 2 of the decision matrix, equation 11 is evaluated. It is a simple matter to sum down every pair of columns:

1 and 2		-3.2
1 and 3	$-0.2\,[(1-0)^2 + (0-0)^2 + (0-0)^2] =$	-0.2
2 and 3	$-1.2\,[(0-0)^2 + (1-0)^2 + (0-0)^2] =$	-1.2
		$I = -4.6$

where I is the information, equation 12.

Each possible (and feasible) decision matrix is associated with a value of I; for small problems, we could draw an exhaustive list. However, even for this simple problem with $m = n = 3$, it is too demanding to do so by hand. Evaluation by computer shows a total of 28 feasible matrices, with I maximized when the decision matrix is

		City		
		1	2	3
Experiment	1	0	1	0
	2	0	1	0
	3	0	0	0

Inferential Analysis

It is prudent to assume that the response surface representing the information function $I(\Delta)$ has many local optima so that most randomly selected experimental designs will lead to globally non-optimal solutions. The extent of this shortcoming does not depend on the number of local peaks; rather it is a function of the difference between the global solution and the best of the local optima, a quantity which cannot be estimated with any degree of certainty but which does lend itself to certain statistical theorems turning on sampling reliability. For example, the probability that the best of p random and independent trials lies in the upper 100ϕ percent of all possible solutions is $1-(1-\phi)^n$; for example, if $n = 30$ and $\phi = 0.1$, the probability that the best of 30 trials lies in the upper 10 percent of all possible solutions

is $1-(0.9)^{30} = 0.957$. This says nothing about the difference between the best of the sample and the global optimum, but it does provide a lower limit to the reliability of random sampling techniques because by use of gradient methods, the reliability of the result is improved and consequently is better than that which can be ascribed to the unimproved random sampling. This result is independent of the number of decision variables required to characterize any trial design or decision matrix.

The suitability of the best local solution can be estimated by a study of the range of the other local maxima. If the surface appears to be regular and fairly smooth, small values of n are tolerable. If, however, the surface shows abrupt changes of elevation and slope, a more extensive sampling investigation is warranted, provided, of course, that the cost of so doing does not appear to exceed the potential improvement which might be gained in the response.

Conclusion

A computer program to implement the solution algorithm was written in FORTRAN IV for the IBM 7094. It can accommodate up to ten cities and twenty experiments, and has been run successfully on matrices of this size within four to six minutes of computation time. The results have been encouraging, showing major improvements (that is, better response) over the best manual solutions for a wide range of budgetary and geographic constraints. These solutions are being implemented, and it is hoped that a second paper can report on the results of field testing. However, more significant than this numerical achievment is the formalism for casting an urban problem in precise operational terms. It is here, at the interface between mathematics and the social sciences, that the real excitement in modern operational analysis is to be found.

PROCEEDINGS OF THE SOCIETY

Minutes of Meeting

Boston Society of Civil Engineers

November 12, 1969: - A Joint Meeting of the Boston Society of Civil Engineers with the Structural Section of the BSCE was held this evening in the Harvard Room, Purcell's Restaurant, 25 School Street, Boston, Mass.

Dinner was served at 6:30 P.M., 35 members and guests sitting down to the table.

President Robert H. Culver called the meeting to order promptly at 7:30 P.M., and stated that unless there was objection, the minutes of the October 15, 1969 meeting would not be read.

President Culver asked those present to stand and announced the loss to the Society, by death, of the following:-

Charles M Anderson, elected a member November 20, 1929, who died October 14, 1969.

Francis H. Kingsbury, elected a member February 20, 1924, who died August 11, 1969.

George E. Harkness, elected a member November 21, 1906, who died in September, 1969.

Robert W. Mawney, elected a member May 19, 1915, who died July 31, 1969.

President Culver then called upon the Secretary to make any announcements. The Secretary announced that applications for membership had been received from the following:-

Thomas K. Liu, Lexington, Mass.
Mohammed S. Akhtar, Somerville, Mass.
Albert H. Smith, Jr., Quincy, Mass.
Michael G. O'Neill, Jr., Medford, Mass.
Robert W. Fulton, Portland, Maine
Richard J. Forbes, Dedham, Mass.
Reynold A. Hokenson, Chestnut Hill, Mass.

The Secretary also announced that the following had been elected to membership on November 12, 1969:-

Grade of Member - Edward D. Chase, Lindley H. Hall

Grade of Junior - John Mahony, Levin M. Merli, Patrick J. Rowland

Since this was a Joint Meeting with the Structural Section, President Culver turned the meeting over to Richard C. Jasper, Chairman of that Section, to conduct any necessary business. After the business of the Section had been completed, Mr. Jasper introduced the guest speaker of the evening, Mr. Leslie Robertson, Partner of Skillings, Helle, Christansen & Robertson. His subject was "The Design and Construction of High Rise Buildings Including the World Trade Center at New York".

Mr. Robertson gave a very interesting illustrated lecture about the new United States Steel Building in Pittsburgh, Pennsylvania and the World Trade Center Building in New York City. He discussed the general problems associated with ultra-high-rise structures including wind and dynamic loadings. He described in detail, with slides, wind tunnel tests made on both buildings and the results of those tests. During his talk, he integrated the wind loading and thermal distortions problems with the usual problems in structural design. He further described the water-cooled, fire-resistant columns of steel used in the United States Steel Building in Pittsburgh, Pennsylvania.

Forty-eight members and guests were in attendance during the business meeting.

The meeting was adjourned by President Culver at 9:40 P.M.

Respectfully submitted,

Paul A. Dunkerley
Secretary

December 10, 1969. - A Joint Meeting of the Boston Society of Civil Engineers with the Geotechnical Section of the B.S.C.E. was held this evening in the Bristol Bay Room Kevins Wharf, 254 Summer Street, Boston, Mass. Dinner was served at 6:30 P.M., with forty-eight members and guests present.

President Robert H. Culver called the Meeting of the Society to order at 7:30 P.M. He stated that unless there was objection the reading of the minutes of the meeting of the Society held on November 12, 1969 would be waived since those minutes would be published in a forthcoming issue of the Journal.

President Culver then called upon the Secretary for any announcements. The Secretary announced that applications for membership had been received from the following:-

Joel P. Bilodeau, Boston, Mass.
Mrs. Charyl W. Butterworth, Malden, Mass.
Milton M. Cameron, Belmont, Mass.
Richard W. Check, Winchester, Mass.
Max S. Clark, 3rd. Watertown, Mass.
David F. Doyle, North Quincy, Mass.
Wayne T. Fisher, Whitman, Mass.
Melvin W. Morgan, Hampden, Maine
Leslie T. Hatch, Ashland, Mass.
Thomas F.X. Glynn, Derry, New Hampshire
Michael E. Rafferty, Holliston, Mass.
Ching Seng Fang, Watertown, Mass.
Richard G. Sherman, North Reading, Mass.

The Secretary also announced that at the meeting of the Board of Government on this date the following had been elected to membership in the Society:-

Grade of Member - Mohammed S. Aktar, Robert W. Fulton, Reynold A. Hokenson, Thomas K. Liu, Leo F. Peters.

Grade of Junior - Richard J Forbes, Michael G. O'Neill, Jr., Albert H. Smith, Jr.

The Secretary moved, it was seconded, and it was unanimously VOTED "that the Board of Government be authorized to transfer an amount not to exceed $8,000.00 from the Principal of the Permanent Fund to the Current Fund for Current Expenditures."

Since this was a Joint Meeting with the newly formed Geotechnical Section President Culver turned the meeting over to Donald T. Goldberg, Chairman, to conduct any necessary business of the Section. There being no formal business to be transacted, Chairman Goldberg introduced the guest speaker of the evening Mr. William S. Swiger, Consulting Engineer, Stone & Webster Engineering Corporation, who gave a very interesting talk on "Pile Driving Specifications." Mr. Swiger distributed outlined notes on which he elaborated. The speaker emphasized the need for writing tight specifications and for adequate and competent supervision and inspection of pile driving operations. A lively and interesting question and answer period followed the formal presentation of the subject.

Fifty-one members and guests were present during the meeting.

The meeting was adjourned by President Culver at 9:00 P.M.

Respectfully submitted,

Paul A Dunkerley
Secretary

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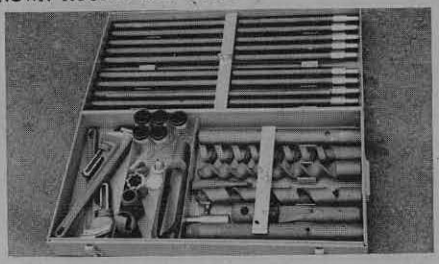

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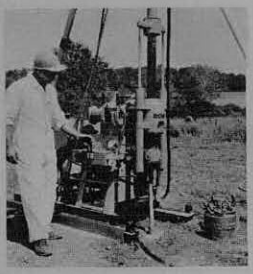

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