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**A NEW LOOK AT SEDIMENTATION
IN TURBULENT STREAMS**

by

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Preliminary Remarks

The field of sedimentation in all its facets has been a most important one in scientific exploration through the years due to its significant role in many modern technologies and in all measures of human interference in natural flow processes. On the one hand we seek to lower the cost of bulk transportation by conveying materials such as coal, ore, grain, sand, gravel and silt in overland pipelines over short or very long distances. On the other hand we dislocate large sediment masses by agricultural practices, by deepening or realigning river channels and estuaries, by mining and exploiting sediment deposits, by denying natural deposition areas to our streams when floods are controlled by reservoirs, by dikes and land reclamation in flood plains. Many engineering plans often hastily conceived and executed did not anticipate nature's reaction in the form of erosion and deposition of large masses of sediment and consequently did not result in the hoped-for benefits.

To this day our understanding of the natural dynamic processes in sediment movement remains essentially one of empirical trial and error with regard to quantitative predictions, although the last fifty years have seen the development of many qualitative criteria through laboratory research and experimentation as well as through extensive field studies. Gross production of sediment from the watersheds of the United States alone exceeds 4 billion tons per year from all natural causes such as surface erosion, stream bank cutting, channel bed degradation and landslides. As yet in this overall figure, man's contribution through mining, industrial and domestic wastes, roads, housing and land clearing generally is relatively small, but nevertheless important and costly in his various development schemes. Individual projects in various countries have been seriously affected by the changes in sedimentation patterns produced by engineering measures with major side effects on the adjacent environment.

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As an example from the most recent months the completion of the giant Aswan Dam in Egypt may be cited. For thousands of years the Nile delta people depended on the annual floods for their agricultural production. The deposition of the Nile's sediment in Lake Nasser will now disturb the equilibrium in the downstream portion of the river and cause a degradation in its bed by erosion, lowering gradually the water surface of the stream and in the adjacent underground. Similar experiences were reported on the Colorado River after the completion of Hoover Dam. Changes in the water quality are also incurred and Egyptian planners are now faced with new environmental challenges as the result of Aswan Dam.

While it is tempting to evaluate these consequences for the environment for which major changes in the sedimentation processes in river valleys are responsible, the task for this lecture must of necessity be more restricted. It will be more narrowly concerned with some of the mechanical features of sediment transport and, as much as feasible, dwell on some of the more recent research results in this difficult field. A word of warning is in order here, however. The literature on the various aspects of sedimentation is so vast and the purpose of the investigations digress so greatly depending on professional motivations that it is virtually impossible to relate a summary such as this to more than a specific theme. This restriction is also indicated by the complexity of the basic processes involved and it seems more worthwhile to deal with one of these more thoroughly than to resort to a superficial overview. In any case, excellent summaries have been published before and particular reference may be made in this connection to the various reports issued by the Committee on Sedimentation of the Hydraulics Division of the A.S.C.E. [1] and to the recent book by W. H. Graf [2].

Fundamentals of Bed Load Transport

Only the most important parameters for sediment transport as they have been established by analysis may be mentioned here. Amongst these the fall velocity w of a sediment grain of a given diameter in still water ranks prominently. It appears mostly in relation to the so-called shear velocity U_* which is defined through the bottom shear stress τ_0 by $U_*^2 = \tau_0 / \rho = g y_0 S_0$ in which ρ is the density, g is the acceleration of gravity, y_0 is the total depth, and S_0 the bottom gradient or energy gradient of a stream under steady uniform flow conditions.

Next, the layer of flow near the boundary which is governed by viscous forces primarily and is therefore mostly referred to as the laminar sublayer δ' has become an important concept in relation to the mean particle diameter d . The ratio d/δ' can then be stated also in the form:

$$\alpha \frac{d}{\delta'} = \frac{U_* d}{\nu} \quad (1)$$

which combines the stream characteristic U_* , the kinematic viscosity ν with the particle diameter d into a useful Reynolds parameter expressing physically the relative magnitude of d versus δ' . The factor α is often given the value of 11.6 but may have other values after proper definition. It is assumed then that as long as d is smaller than δ' the turbulence of the stream will not affect the flat bed of the stream composed of particles of diameter d . If d becomes much larger than δ' it will affect the value of the shearstress τ_0 itself and determine the hydraulically effective roughness of the boundary.

Whether or not movement of bed particles results for given flow conditions, is influenced by a second dimensionless ratio which relates the shearstress τ_0 to the buoyant specific weight of the particles $(\gamma_s - \gamma)$ and to the mean diameter d . With $\gamma_s/\gamma = S$ the specific gravity, and with the density of the ambient fluid as $\rho = \gamma/g$, this parameter may be expressed as:

$$\frac{\tau_0}{(\gamma_s - \gamma)d} = \frac{\tau_0/\rho}{g(S-1)d} = \frac{U_*^2}{(g(S-1)d)}$$

Shields [3] first combined the two ratios just defined into a general functional relation:

$$\frac{U_*^2}{gd(S-1)} = \phi \left(\frac{U_* d}{\nu} \right) \quad (2)$$

He showed by extensive experimental evidence that unique values of the parameters in equation (2) determined the initiation of motion for the sediment grains on the bed. His entrainment function ϕ given graphically in the literature has become generally accepted in research and practice although somewhat different results have been obtained in more recent studies.

In summary then three parameters have been found to have general significance for sedimentation studies:

$$\left(\frac{U_*}{w} \right), \left(\frac{U_*}{gd(S-1)} \right), \left(\frac{U_* d}{\nu} \right)$$

A theory for sediment transport of universal application has so far however been impossible to formulate. In explanation of this state of affairs a short descriptive resume may be given as follows:

After the grains along a flat bed start moving in considerable quantity, the type of transport is referred to as bed-load movement. As the volume concentration in the layers adjacent to the bed increases, the motion consists of sliding, rolling, agitated intermingling and saltation. The flow exerts drag and lift

forces on the particles, unknown interaction of the particles in high concentration takes place, and the highly turbulent zone adjacent of the boundary exerts random impulses on the fluid-sediment mixtures. Statistical approaches seemed indicated and have been attempted, prominently so by H. A. Einstein [2]. But in essence the parameters discussed above were again the building blocks of the various functions derived for the rate of bed-load transport usually given as q_s in cu. ft./sec. ft. Experimental data and field data have seldom shown general correlation, even though certain equations and certain sets of data proved consistent for the particular flow conditions and sediment properties employed. Most sediment transport formulae for bed-load have therefore the general formulation:

$$S \cdot q_s = \frac{U_*^2}{g} f \left(\frac{U_* d}{\nu}, \frac{U_*}{w}, \frac{U_*^2}{gd(S-1)} \right) \quad (3)$$

The application of more specific empirical relationships to practical design is still at best an educated guess and requires considerable experience with specific field conditions. This is not to say that at various times there have not been very satisfactory solutions.

Other factors entering into a more detailed analysis are the complications introduced by mixtures of sediments which are sorted by their variable response to the dynamics of the flow near the boundary, so that all kinds of movement coexist, such as intermittent rolling, sliding, saltation and suspension. In turn the interaction of the moving sediment with the fluid flow modifies the state of turbulence, the boundary shear and the state of the mean flow itself.

The surface shape of the bed itself deforms in alluvial streams. For small values of the Froude Number $F = U/\sqrt{gy_0}$ an initially flat bed will fairly rapidly change to a rippled surface and at a later stage dunes will appear. As the flow rate and thus the Froude number is increased further towards a critical value $F = 1$ the dunes will be washed away and a flat bed reappears. When the flow is near critical however, standing waves are formed on the surface and so-called sandwaves appear at the bottom. These seem to grow with the surface waves until the latter break with considerable turbulence generation similar to the hydraulic jump, and the sandwaves are washed away also. These relatively slow processes are then repeated at another favorable location. In contrast to ripples measured in inches, sandwaves may reach a height of many feet. In supercritical flow, finally large dunes of long length in terms of depth are formed which move slowly in the upstream direction. It is clear that the rate of sediment transport varies greatly under all of these bed roughness conditions as the shearstress becomes non-uniform both in the direction of flow and laterally.

These phenomena are mentioned here in passing only to call attention to the difficulty of formulating a general theory of sediment transport. As beds

may deform at different stages of river flow, thus modifying materially the stage-discharge relationships, the same discharge may occur for considerable periods of time over beds of different roughness. As floods pass, bed forms do not revert immediately to equilibrium conditions if equilibrium conditions do indeed exist except in the laboratory. Nevertheless reasonable progress has been achieved in recent years also in this difficult area on various specific streams by field research. In the theoretical analysis of certain characteristics of dune and anti-dune behaviour, the basic work of J. F. Kennedy [4] may be mentioned which has stimulated others to get involved. In view of the complexities cited, the field of bed-load movement has many challenges confronting it in future research.

The Transport of Suspended Sediment

Historical Remarks

In contrast to bed-load movement which is subject to all the poorly understood processes of the fluid-sediment interaction close to bottom boundary, the analysis of solid suspensions over the depth of the stream has been more successful. This area of study had been of interest to me at the beginning of my professional career over 37 years ago, and then again I became involved in it in more recent years through research conducted in our laboratory at M.I.T. Early in 1934 as a graduate student at the California Institute of Technology I was looking for a topic for a doctoral dissertation and Professor Von Karman suggested that I read a paper by Murrough P. O'Brien in the Transactions of the American Geophysical Union [5]. This paper had appeared the year before and contained the postulation of the equilibrium for the upward dispersion of sedimentary particles in a turbulent stream and their downward settling by gravity in differential form. What was needed as a next step was the introduction of a suitable velocity distribution law to define the distribution over the depth of the turbulent eddy viscosity or momentum transfer coefficient ϵ_m . This would permit the integration of O'Brien's relation which had been derived for suspended particles in liquids. Some years earlier Wilhelm Schmidt had established the corresponding condition for dust particles in the atmosphere [6].

Both authors assumed that at any depth of the stream y , the net downward settling rate of particles w of a given diameter d through a unit horizontal area, was equal to the net upward dispersion by turbulent mass exchange from the higher concentration levels below to the lower concentrations above. Thus they obtained the well know relation:

$$c \cdot w = - \epsilon_s \frac{dc}{dy} \quad (4)$$

in which c = volume concentration
 w = settling velocity in ft./sec.
 y = depth measured from the bottom boundary
 and ϵ_s = kinematic mass exchange coefficient.

The subscript s for this coefficient is to indicate that a difference is assumed between ϵ_s and the momentum transfer coefficient ϵ_m usually termed the eddy viscosity. Both are normally connected by

$$\epsilon_s = \beta \epsilon_m. \quad (5)$$

For the sake of simplicity the factor β is usually assumed to be unity, an assumption justified for small particles by careful analysis of experimental evidence [1b] (pp. 62 and 72). At any rate other necessary assumptions in further developments make the use of $\beta = 1$ quite acceptable.

The simplest integration of equation (4) is accomplished by assuming ϵ_s as constant with respect to the depth y . To produce such uniform turbulence a simple experimental system can be designed consisting of a vertical cylinder with a stirring device agitating the liquid in the column. H. E. Hurst [7] in 1929 performed such experiments first using rotating vanes and fixed baffles. He found the sediment distribution to conform to the one predicted by the equation:

$$\ell_n \frac{c}{c_a} = \frac{aw}{\epsilon_s} \left(1 - \frac{y}{a}\right) \quad (6)$$

in which a indicates a reference depth at which c_a is measured.

A much more extensive study was conducted by H. Rouse [8] a few years later in 1936 with screens oscillating vertically with different frequencies and amplitudes. His definition of the sediment characteristics as well as the extensive scope of his test program revealed many details of the sediment-fluid interactions and encouraged their study in laboratory flumes.

The Sediment Distribution in Turbulent Streams

If the concept of equation (4) is applied to the study of uniform flow in a wide rectangular channel, the following relations must be referred to:

$$\tau = \rho \epsilon_m \frac{du}{dy} \quad (7a)$$

which defines the local shear τ at any depth y in relation to the kinematic eddy viscosity ϵ_m , the density ρ and the velocity gradient. In steady uniform flow in

wide channels, the shear stress τ is varying linearly from zero at the surface to the maximum τ_0 at the bottom. Hence, neglecting the presence of sediment.

$$\tau = \tau_0 \left(1 - \frac{y}{y_0}\right). \quad (7b)$$

Introduction of equations (7a) and (7b) into equation (4) gives the basic expression

$$\frac{dc}{c} = - \frac{w}{\epsilon_s} dy = - \frac{w \cdot du/dy}{\sqrt{\tau_0/\rho} \cdot \beta \sqrt{\tau_0/\rho} \left(1 - \frac{y}{y_0}\right)} \quad (8)$$

With an appropriate function for the velocity distribution, this equation can be readily integrated. This was first done by the writer in 1934 in unpublished notes using the logarithmic velocity distribution law proposed by H. Krey in 1927 [9]. This relation was given by Krey in the form:

$$\frac{u}{U_{\max}} = \frac{\ell n \left(1 + \frac{y}{a}\right)}{\ell n \left(1 + \frac{y_0}{a}\right)} \quad (9)$$

The length a is introduced as a small distance from the bottom and is defined from the expression given by Krey by the equation:

$$\frac{U_{\max}}{U_*} = \frac{a U_*}{\nu} \ell n \left(1 + \frac{y_0}{a}\right) \quad (10)$$

Therefore equation (9) can be expressed also in the form

$$\frac{u - U_{\max}}{U_*} = \left(\frac{a U_*}{\nu}\right) \ell n \frac{\left(1 + \frac{y}{a}\right)}{\left(1 + \frac{y_0}{a}\right)} \quad (11)$$

This relation was not developed by the writer until early in 1969 in this form [10]. It shows however that it is equivalent to the so-called Karman-Prandtl velocity defect relation established in 1934 after Krey's equations (9) and (10), which is:

$$\frac{u - U_{\max}}{U_*} = \frac{1}{k} \ell n \left(\frac{y}{y_0}\right) \quad (12)$$

Neglecting the small distance a in equation (11) versus y and y_0 under the logarithm, as was done in equation (12) before, a comparison of equations (11) and (12) then results in the equality:

$$\frac{1}{k} = \left(\frac{aU_*}{\nu} \right) \quad (13)$$

Calculating Values of $a \cdot U_*/\nu$ from experimental evidence in accordance with equation (10) gives indeed values very close to the average value of the Von Karman universal constant k stated usually as .40 for equation (12).

The velocity gradients are as follows from both equations (11) and (12)

$$\frac{du}{dy} = \left(\frac{aU_*}{\nu} \right) \cdot \frac{U_*}{(y+a)} \quad (14)$$

$$\frac{du}{dy} = \frac{1}{k} \cdot \frac{U_*}{y} \quad (15)$$

It may be added that the small distance a is proportional to the laminar sublayer thickness δ' which is defined often by $(\delta' U_*/\nu) = 11.6$, although smaller values of the constant are often quoted down to values of 4. Assuming $k = .40$, it is seen that equation (13) results in $(a \cdot U_*/\nu) = 2.5$. At any rate the distance a is of the order 10^{-3} to 10^{-4} for usual conditions of clear water flow.

Introducing equation (14) to express the velocity gradient, equation (8) may be integrated between the limits $y = y_0$ and $y = a$

$$\frac{c}{c_a} = \left[\frac{y_0 - y}{y+a} \cdot \frac{2a}{y_0} \right]^{Z_1} = \left[\frac{y_0 - y}{y_0 - a} \cdot \frac{2a}{y+a} \right]^{Z_1} \quad (16)$$

wherein:

$$Z_1 = \frac{1}{\beta} \cdot \frac{w}{U_*} \left(\frac{aU_*}{\nu} \right) \quad (17)$$

This relation was first given by the writer in 1934/35 but has been modified slightly for ready comparison with the following equations.

With equation (15) the relation (8) results in the integrated form

$$\frac{c}{c_b} = \left[\frac{y_0 - y}{y_0 - b} \cdot \frac{b}{y} \right]^{Z_2} \quad (18)$$

in which a depth b has been assumed for reference and may be any depth for which the concentration has been measured. Equation (18) was first established by H. Rouse [11] in the years 1936/37 with the value of the exponent given by

$$Z_2 = \frac{1}{\beta} \frac{w}{U_*} \cdot \frac{1}{k} \quad (19)$$

It may be noted that equations (17) and (19) are identical if the equality of $1/k = aU_*/\nu$ is assumed. If a reference concentration c_b for depth $y = b$ is determined from equation (16), and the ratio c/c_b is formed, the result agrees with equation (18). Near the bottom, good agreement of the concentration distributions with the equations is doubtful in any case. Krey's equation (9) for the velocity distribution gives zero velocity at the boundary while Von Karman specifically excluded the validity of the logarithmic velocity distribution near the boundary.

Relation Between the Coefficient k and Sediment Concentration

Experience since the publication of equation (18) has demonstrated that the concentration of particles near the wall has an important effect on the distribution of velocity as well as of sediment, a finding which is generally evaluated by determining the coefficient k . Large variations in k for velocity distributions in streams carrying relatively small concentrations of sediment have generally been established [12] [13]. Such large variations in k in the writer's opinion are due to the large concentrations of particles at the boundary which result in large variations of the effective viscosity μ' from the values μ for the clear ambient fluid.

H. A. Einstein [14] gave many years ago a well-known approximation for the variation of the dynamic viscosity μ' in terms of the clear fluid viscosity μ with particle suspensions of concentration C_0

$$\mu' = \mu (1 + 2.5 C_0) \quad (20)$$

Eilers [15] established from experimental data a better correlation which was verified in one of our own studies [16].

$$\left(\frac{\mu'}{\mu}\right)^{1/2} = 1 + 2.5 C_0 \frac{1}{2(1 - 1.35 C_0)} \quad (21)$$

For the present purposes equation (20) may suffice to redefine the shear stress at the bottom boundary in terms of the maximum concentration C_0 :

$$\tau'_o = \mu' \frac{du}{dy} = \mu \frac{du}{dy} (1 + 2.5 C_o) \quad (22)$$

The sediment load in the stream also produces an increase in the sheartstress over the clear water shear so that

$$\tau'_o = \gamma y_o S_o \left[1 + (S - 1) \int_0^{y_o} \frac{c dy}{y_o} \right] \quad (23)$$

Since the integral simply expresses the mean concentration over the depth, this equation can be rewritten as

$$\tau'_o = \gamma y_o S_o \left[1 + C_m (S - 1) \right] \quad (24)$$

wherein $S = \gamma_s/\gamma_f$, the specific gravity of the sediment.

Combining equations (22) and (24) yields:

$$\frac{du}{dy} = \gamma y_o S_o \frac{1}{\mu} \left[\frac{1 + C_m (S - 1)}{1 + 2.5 C_o} \right] \quad (25)$$

With $\gamma y_o S_o = \tau_o = \rho U_*^2$ for clear water this relation may be rewritten as

$$\frac{du}{dy} = \frac{U_*^2}{\nu} \left[\frac{1 + C_m (S - 1)}{1 + 2.5 C_o} \right] \quad (26)$$

This equation represents the modification in the velocity gradient near the boundary with suspended sediment. For clear water, equation (14) may be used near the boundary with y of the order a :

$$\frac{du}{dy} = \frac{a U_*}{\nu} \cdot \frac{U_*}{y + a} = \frac{1}{k} \frac{U_*}{y + a} \quad (14)$$

From equations (26) and (14) that distance $(y + a) = a'$ for which the velocity gradients become equal is now defined. Further defining $1/k' = a' U_*/\nu$ it is seen

that

$$\frac{a' U_*}{\nu} = \frac{1}{k'} \frac{1}{k} \cdot \left[\frac{1 + 2.5 C_0}{1 + C_m(S-1)} \right] \quad (27)$$

Since the factor by which $1/k$ is multiplied is always larger than unity for the usual sediments, it follows that k' is always smaller than k or that a' is always larger than a . This statement is amply supported by experimental and field studies.

Through the Krey equation, the universal constant k of Von Karman is shown to be governed by the sediment concentrations near the boundary and is changed to smaller values of k' when multiphase flow exists. Similarity assumptions for the region of flow at considerable distance from the bottom remain quite acceptable for the usual small concentrations C_m when the inertial interactions of particles with the turbulent flow can be neglected.

Equation (27) permits a number of important observations with regard to suspended sediment transport.

1. The maximum concentrations C_0 moving near the bottom affect primarily the value of k .
2. The mean concentrations C_m are usually much less than the maximum concentrations C_0 and therefore should show little correlation with changes in k .
3. Large changes in k observed with suspensions of near neutrally buoyant particles are seen to depend on C_0 , while the term $C_m(S-1)$ tends towards zero. Values of C_0 are approximately equal to C_m .
4. The value of k is affected only by the maximum volume concentration of particles near the boundary, not by other properties of the particles such as diameter and size distribution. The effective viscosity depends only on concentration in first approximation. However, particle sizes are assumed to be of the order of a or δ' in diameter.
5. By plotting velocity distributions in the upper portion of the depth, the constant k' may be determined for streams carrying suspended load, and hence the absolute concentration C_0 may be obtained.

The developments up to this point may be illustrated by a series of graphs. Figure 1 shows for comparison two velocity distributions plotted to linear scale, one for clear water and the other for a suspension of neutrally buoyant particles of $C_0 = .27$. The characteristic decrease of the gradient near the boundary as well as its increase in the upper part of the depth is quite evident and stands confirmed by many experiments. When plotted to a semi-logarithmic scale the same runs show a straight line for the clear water with a value of $k = .376$, while for $C_0 = .27$ the $k' = .248$ and a curve results over almost the entire depth (see

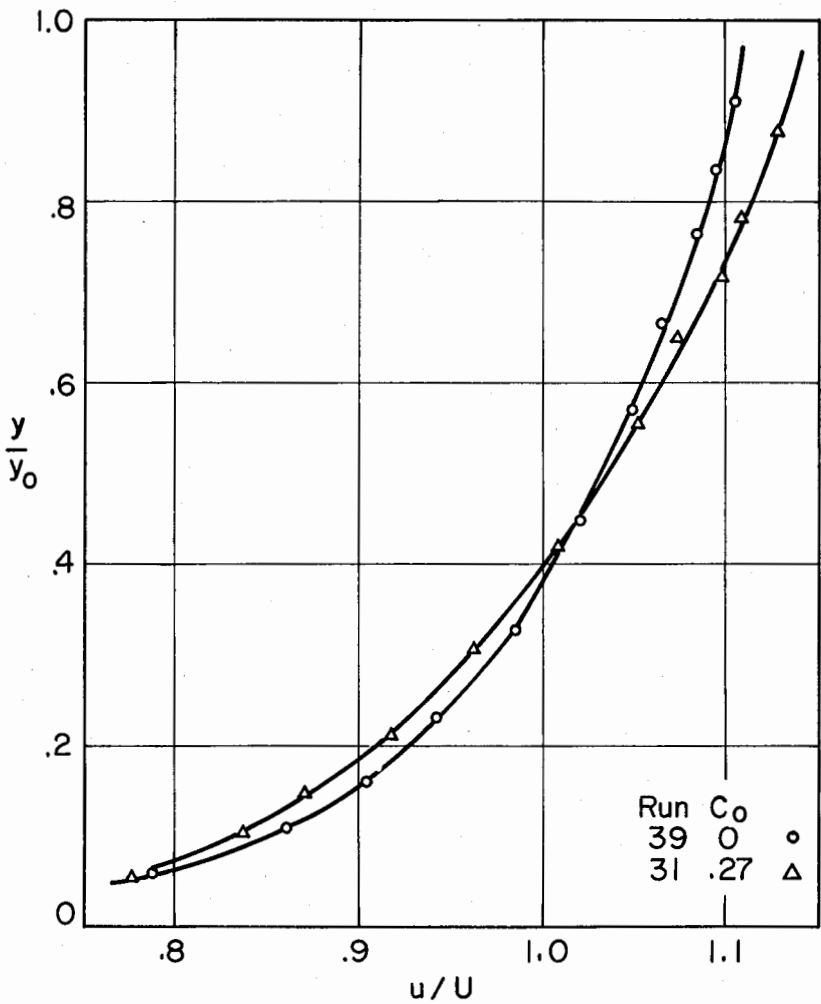


Fig. 1 Comparison of Velocity Profiles for Flows with and without Suspensions (Linear Plot). (Ref. 16)

equation (12)). In Figure 2 the variation of the effective viscosity with increasing concentration is given in accordance with equations (20) and (21) as confirmed by experiments in our laboratory [16].

Figure 3 shows the distribution of sediment over the depth in terms of absolute concentrations C in grams per liter for various values of Z , as presented in reference [16]. The experimental evidence clearly shows general agreement with the theory as long as the absolute concentrations C remain very small. When concentration distributions are plotted to logarithmic scale, Figure 3 shows that, in accordance with equations (16) and (18), the slopes of the concentration profiles with depth represent the exponents Z in the equations.

As has been pointed out, the correlation of the mean concentration C_m with the values of k obtained from various experiments cannot be satisfactory. But Vanoni and Nomicos [17] and Einstein and Chien [13] recognized that the higher concentrations near the boundary were largely responsible for the effect on k . A graph by Einstein and Chien showing the variation of k with a ratio of the power P_s to suspend the sediment in a vertical column of unit area to the power P_f to overcome the bottom resistance to flow in the unit area (i.e., $U \cdot \tau_0$), is given in Figure 4. The plot is noteworthy in view of the field and laboratory data included but the scatter, even for the laboratory data alone, is quite large. When Vanoni and Nomicos [17] analyzed their data for fine sand of .10 mm diameter in terms of the same power ratio, large scatter was again observed as seen in the upper part of Figure 5. However, when they concentrated the power to suspend particles to a layer only from 0.001 to .01 y_0 near the bed given as P'_s , the correlation of k with the power ratio P'_s/P_f is seen to improve substantially. It is felt that the latter approach succeeded somewhat better because the mean concentration in the thin bottom layer tended towards the maximum concentration C_0 used in the analysis developed in equation (27). This approach fails, however, to account for the material reduction of k for near neutrally buoyant particles since for these the power P'_s would approach zero. It was successful in Figure 5 only because for all runs the specific gravity was that for sand.

When all experimental data of these investigators were extrapolated towards the bottom boundary, a maximum concentration C_0 could be defined. The results of our laboratory tests with neutrally buoyant particles [16] as well as later tests by Ordóñez [18] with sand were also analyzed to establish values of C_0 for equation (27). Figure 6 shows the comparison of equation (27) with these experimental results relating the maximum concentrations C_0 to the values of k as obtained from semi-logarithmic plots of u/U_* versus y/y_0 . The agreement between the simple theory proposed in equation (27) and experimental results is quite satisfactory.

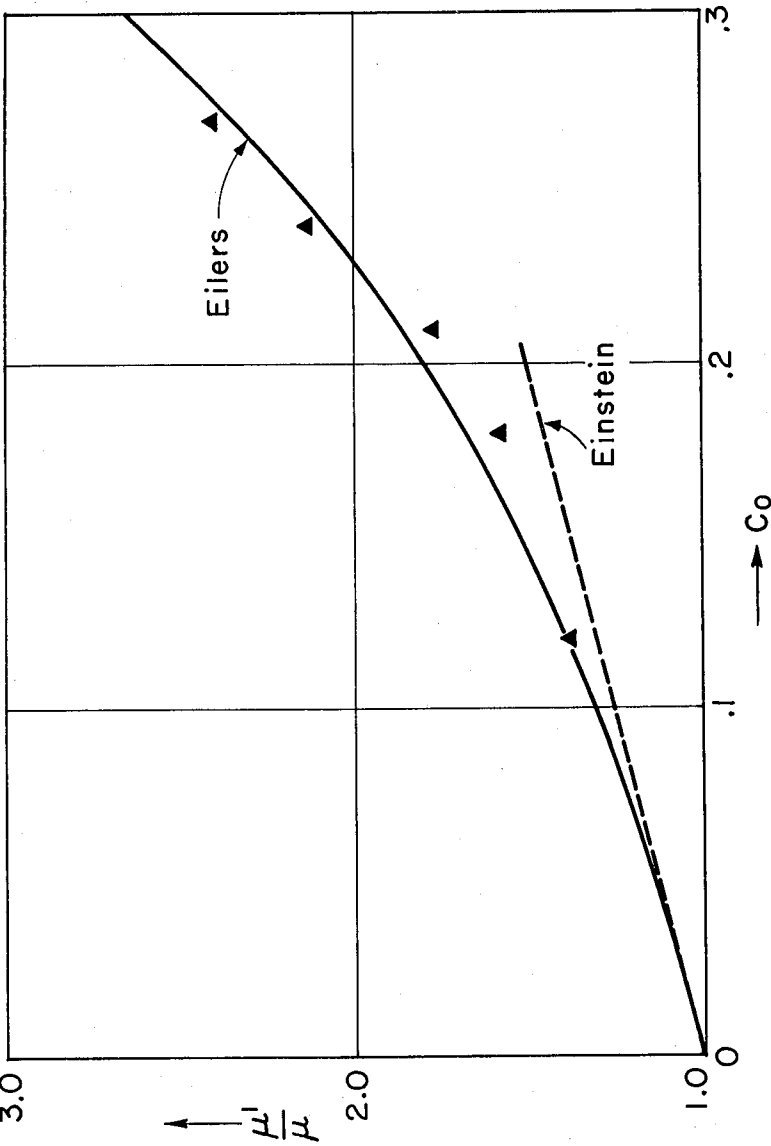


Fig. 2 Variation of Effective Viscosity μ' in Terms of Clear Water Viscosity μ with Volume Concentration C_0 . (Ref. 16)

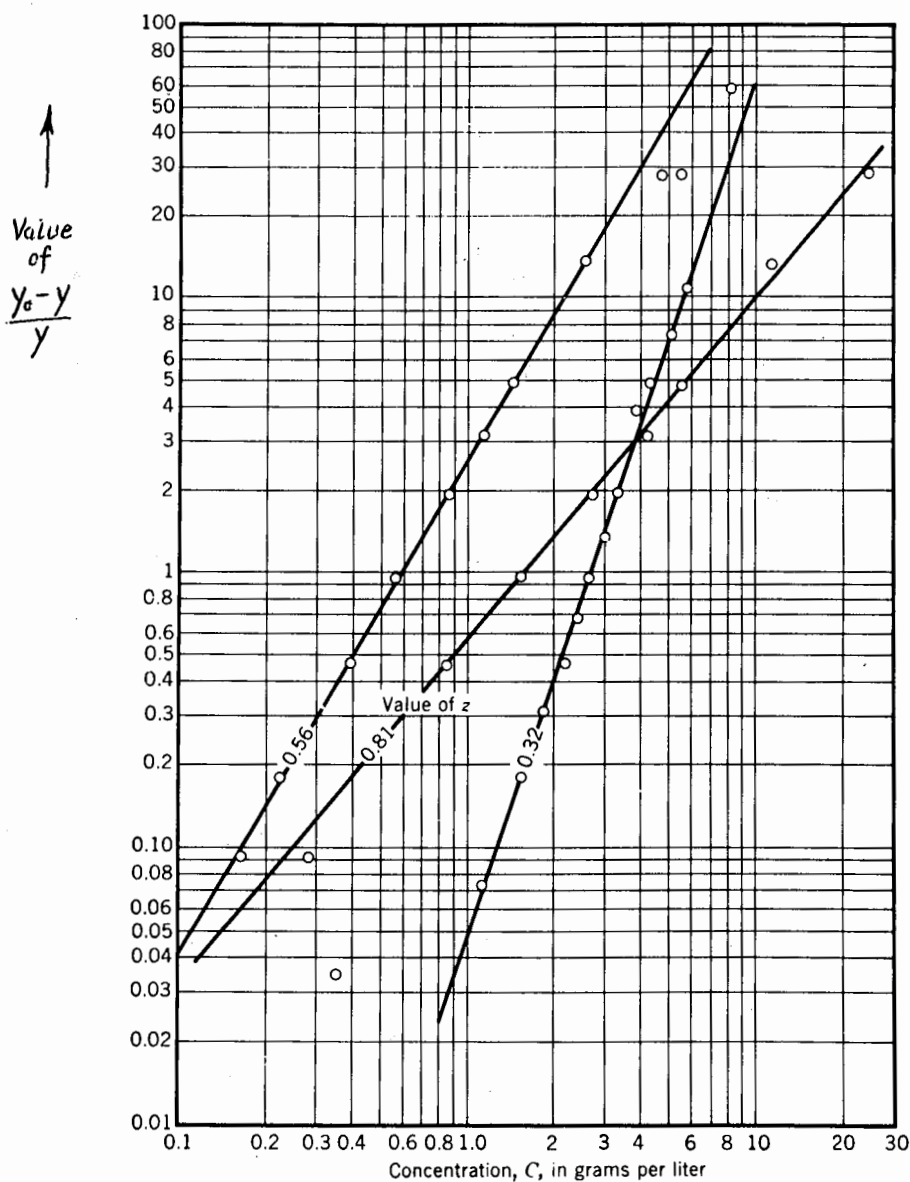


Fig. 3 Distribution of Sediment Concentration C over the Depth for Different Values of Exponent z . (Ref. 1b)

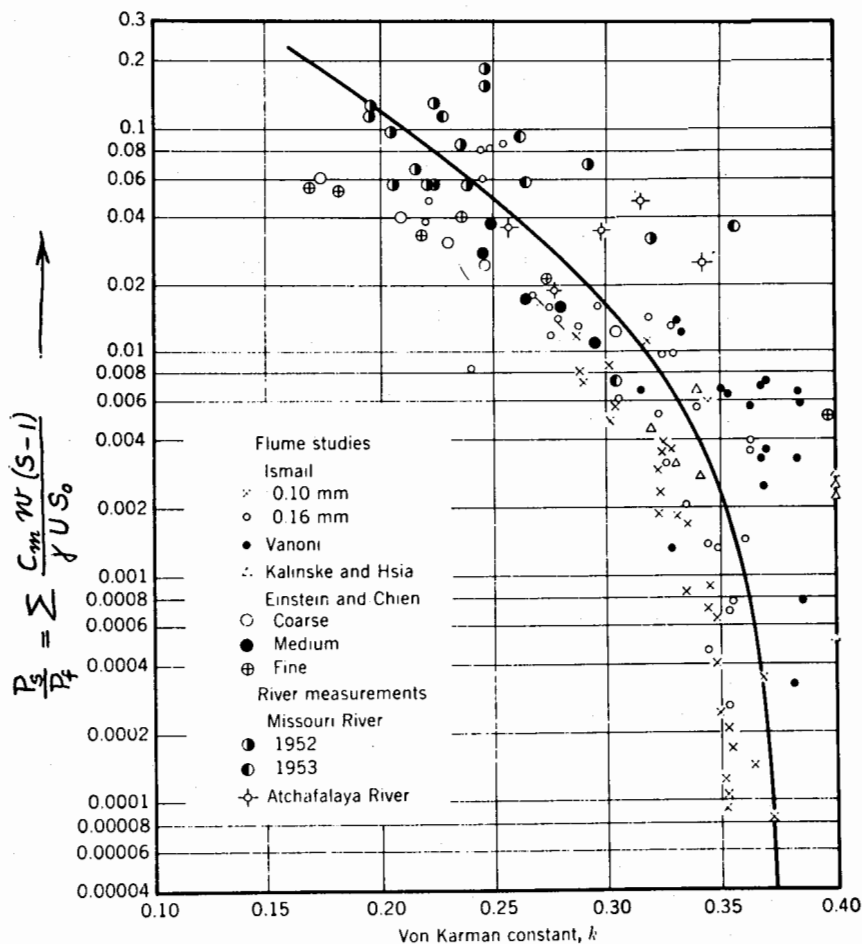


Fig. 4 Variation of k with Ratio of Power P_s to Suspend Particles to Power P_f to Overcome Bottom Resistance. (ref. 1b)

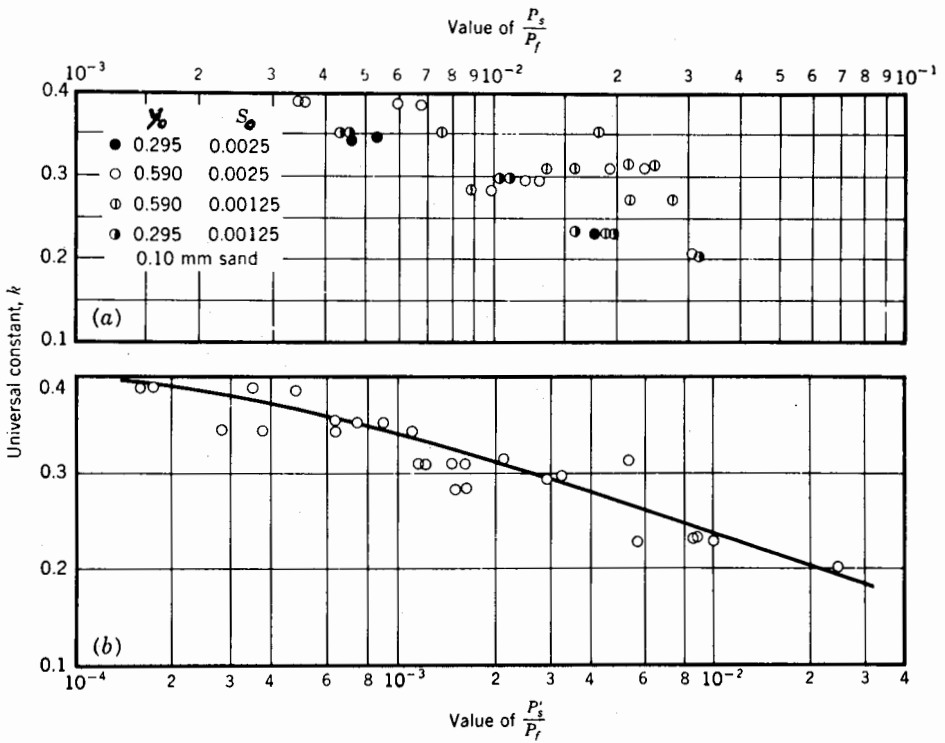


Fig. 5a Variation of k with Ratio of Power P_s to Suspend Particles to Power P_f to Overcome Bottom Resistance for Sand of .10 mm dia. and Various Flow Conditions. (Ref. 1b)

Fig. 5b The Same Data as in Figure 5a with P_s' Confined to Bottom Layer of .001 to .01 y_0 .

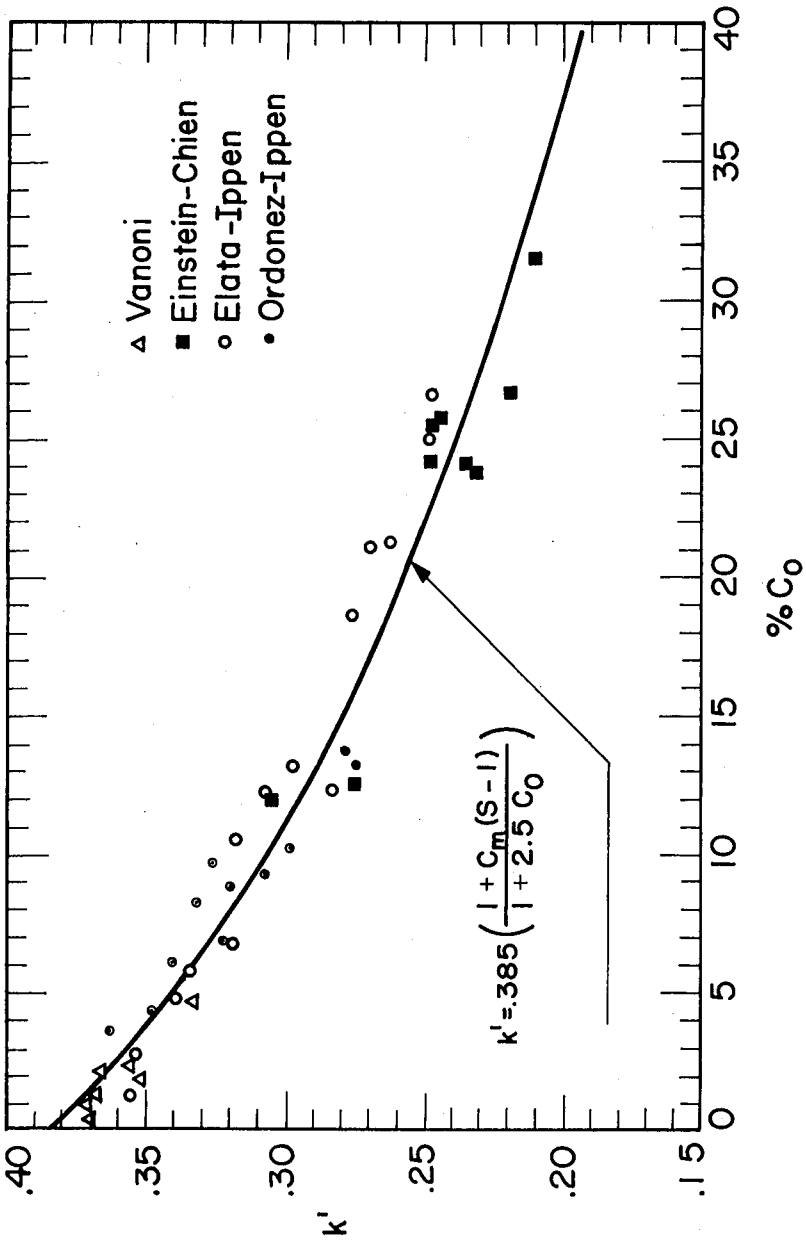


Fig. 6 Comparison of the Constant k' as Defined by Equation (27) with Various Experimental Results. (ref. 10)

Velocity Distributions with Suspended Sediment

In the preceding section a relation was developed between the value of the coefficient aU_*/ν in the velocity distribution functions stated by equations (10) and (12) and the maximum sediment concentration C_0 near the bed. With values of k' as determined for C_0 , the velocity profiles over the upper portion of the stream can be described. In Figure 7 typical velocity distributions obtained by Einstein-Chien [13] for various concentrations of sand are seen, however, to exhibit a markedly different behavior from this upper portion particularly for the 10% of depth near the bottom. This departure from the linear condition of the semi-logarithmic law was analyzed for many similar data sets, and an essentially empirical correction in the velocity distribution equation was introduced by Ordonez [18] as follows:

$$\frac{u - U_{\max}}{U_*} = \frac{1}{k'} \ln \left(\frac{y}{y_0} - \psi \ln \frac{y}{y_0} \right) \quad (28)$$

The term involving ψ representing a small depth ratio modifies the velocity distribution particularly in the critical region near the bed, as illustrated by Figure 8, for various values of ψ , which in most cases are of the order .001 to .016. For all runs in our laboratory as well as for those of Einstein-Chien and of Vanoni, experimental values of ψ were determined and good agreement was achieved between the function (28) and the measurements. The solid lines in Figure 8 represent the function (28) with the appropriate constant ψ for a number of laboratory experiments with different concentration distributions.

The next step was to establish a correlation between the sedimentation parameter Z in equations (17) and (19) which was evaluated from the experimental runs, the effective value of k' which depends primarily on the maximum concentration C_0 , and the small factor ψ in equation (28). Figure 9 illustrates that such a correlation does exist as far as the four sets of experimental data obtained by different investigators for a wide variety of conditions can serve as evidence. In view of the difficult experimental evaluations, the correlation appears to be quite convincing, but additional work is in progress to establish further clarification.

Distribution of Sediment Concentrations

The major consequence of the modified velocity distribution equation (28) with regard to the distribution of suspended sediment is that equation (8) must be integrated anew with a local velocity gradient du/dy obtained as the derivative of equation (28). The effect of the sediment on the shearstress is usually very minor and is therefore neglected in the integration. The derivative of equation (28) is

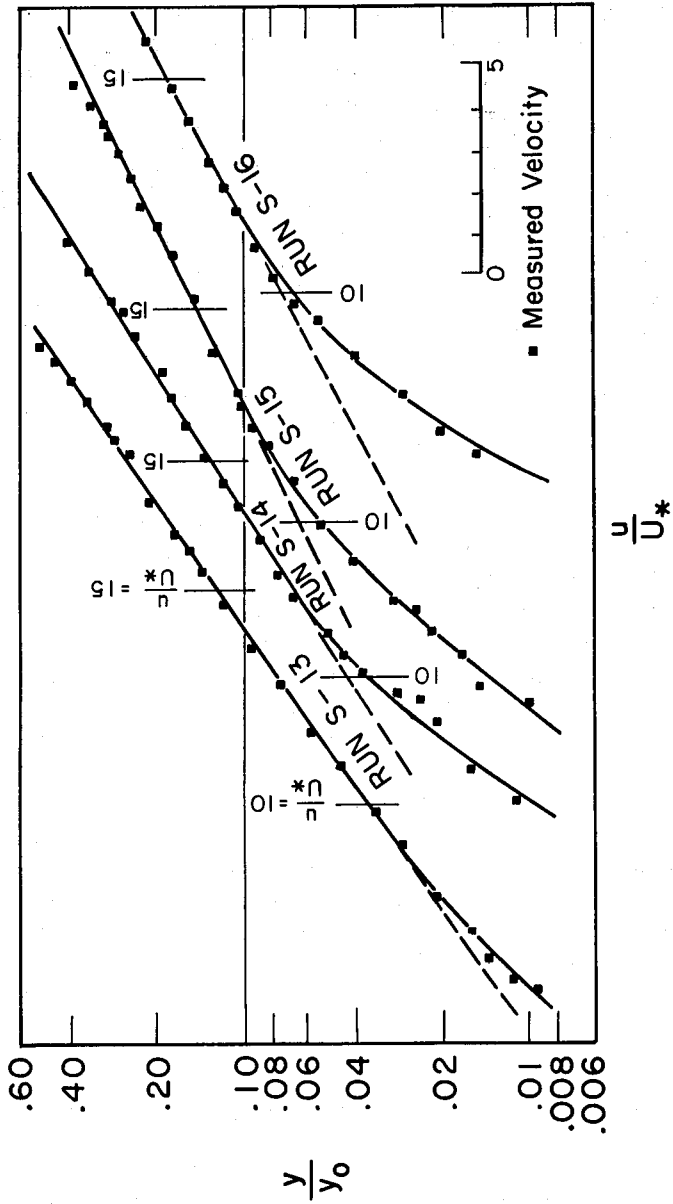


Fig. 7 Typical Velocity Distributions for Different Concentrations of Sand (Ref. 13)

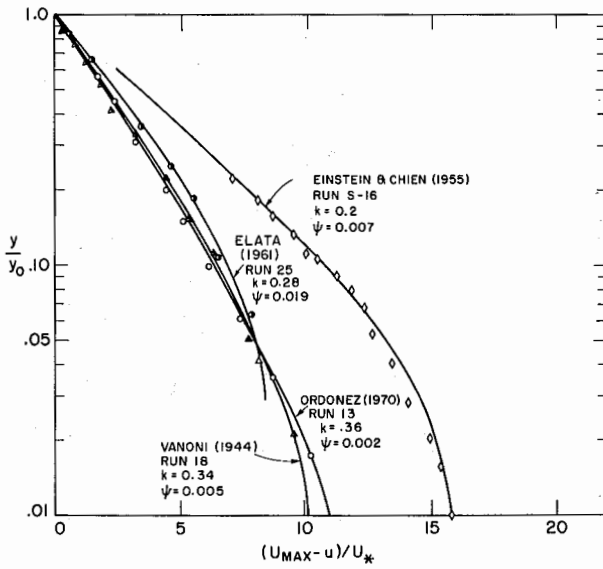


Fig. 8a Comparison of Experimental Velocity Distributions from Various Investigators with the Modified Velocity Distribution Equation (28).

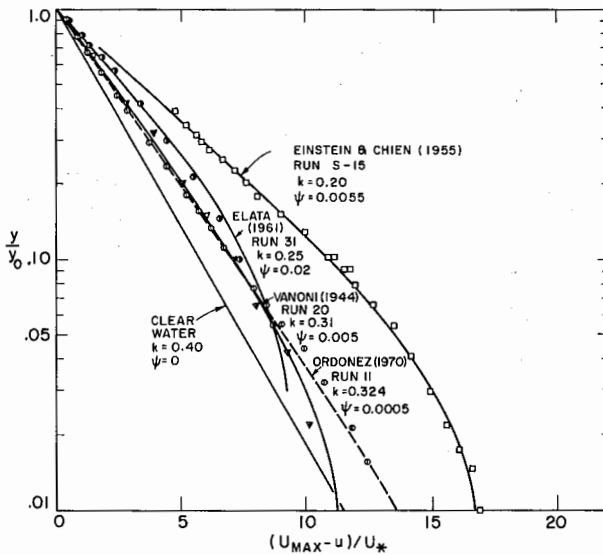


Fig. 8b Comparison of Experimental Velocity Distributions from Various Investigators with the Modified Velocity Distribution Equation (28)

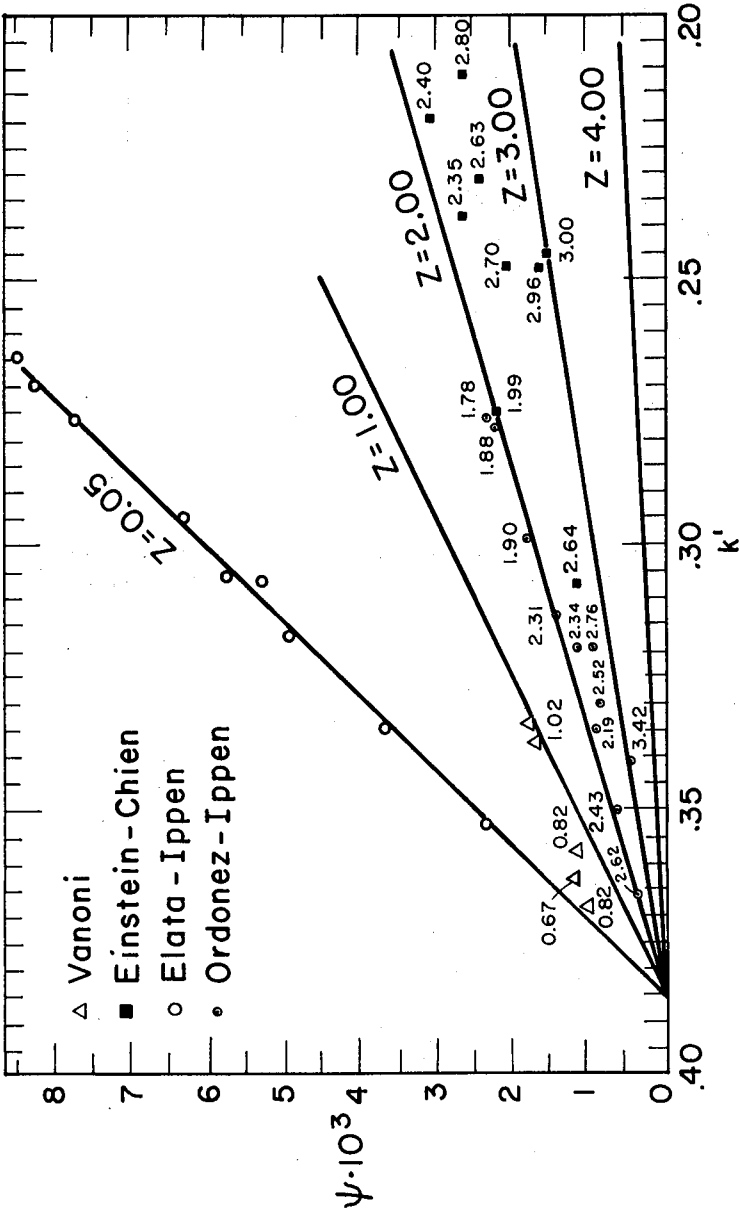


Fig. 9 Correlation between ψ and k' for Different Values of Z_{exp}

$$\frac{du}{dy} = \frac{U_*}{k'y} \frac{\frac{y}{y_0} - \psi}{\frac{y}{y_0} - \psi \ln \frac{y}{y_0}} \quad (29)$$

Introducing also the maximum concentration C_0 near the bed as the reference concentration where $y/y_0 = \psi$, the integration of equation (8) with equation (29) for the velocity gradient results in

$$\ln \frac{C}{C_0} = \frac{w}{k'U_*} \int_{\frac{y}{y_0}}^{\psi} \frac{\frac{y}{y_0} - \psi}{\frac{y}{y_0} (1 - \frac{y}{y_0}) (\frac{y}{y_0} - \psi \ln \frac{y}{y_0})} d(\frac{y}{y_0}) \quad (30)$$

This equation was first given by Ordonez and Ippen [18] in 1970. The integral was evaluated by computer for a wide range of numerical values of ψ and $Z = w/k'U$. The function differs from the previous equations (16) and (18) by the introduction of k' from equation (27) and of the velocity gradient in accordance with equation (29). The former effect provides for a change in slope in a log-log plot of the sediment concentrations and the latter effect produces the increasing deviation from this slope particularly for the concentration distribution near the bed. This deviation from the exponential law is usually confined to the region below $y_0/10$.

Figure 10 reproduces several experimental data sets by Einstein and Chien [13]. These were obtained for rather large concentrations and are shown here for emphasis. When compared to the curves obtained by means of equation (30) they show generally good agreement. It is to be noted that the curves terminate at specific points near the bed where $y/y_0 = \psi$ and $C = C_0$. It is clear that the maximum concentration C_0 had to be determined by extrapolation of the experimental data to ψ and from equation (27) since measurements to this level are not available due to experimental difficulties. It may be surmised that ψ is more of a reference parameter than a physical quantity, as it was determined indirectly by recourse to measured velocity distributions.

Many other experimental comparisons were made with comparable success [18]. Vanoni's experiments result in good agreement with both equations (18) and (30). This is due to his relatively low concentrations of fine sand (mean diameter $d = .16$ mm.), which extend only to within 5% of the depth near the bed. Einstein and Chien's measurements were carried out with mixtures of particles with D_{50} from .27 mm. to 1.30 mm. With the coarser particles the concentrations showed particularly the trends near the bed expressed by equation (30). The measurements also extended closer to the bottom boundary as shown in Figure 10.

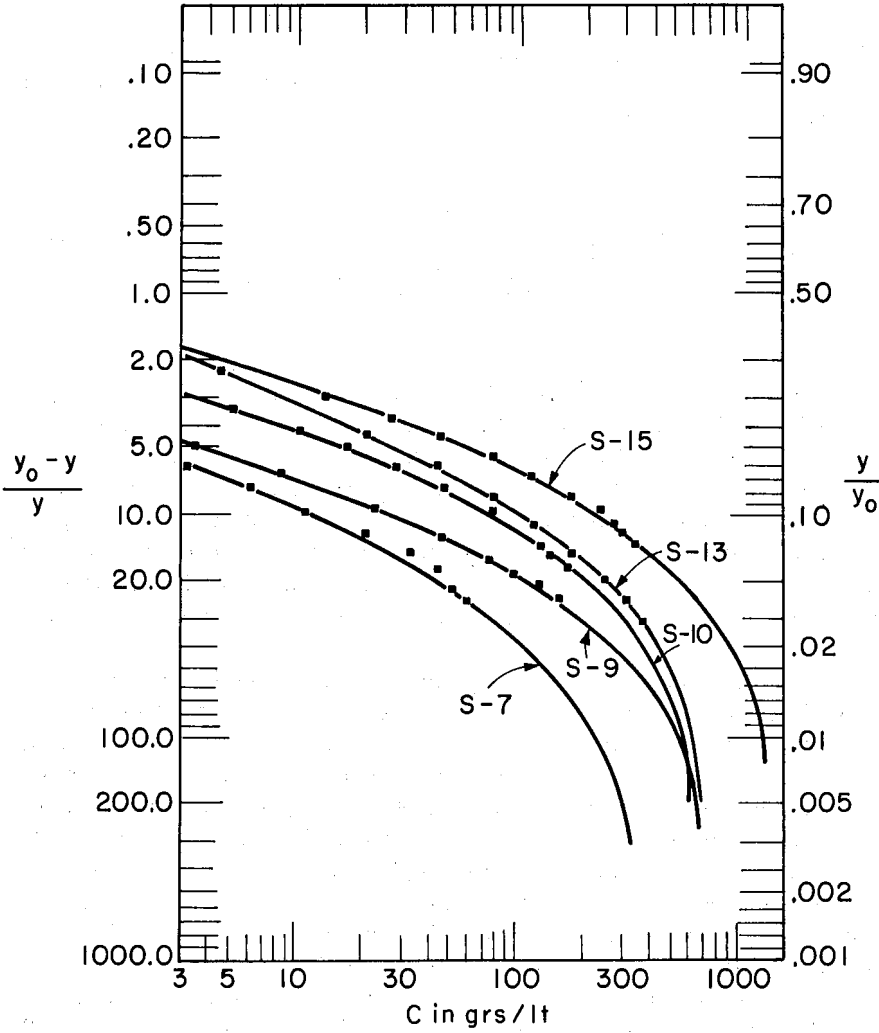


Fig. 10 Concentration Distributions C in Grams per Liter vs. Relative Depth, y/y_0 (Einstein - Chien (13))

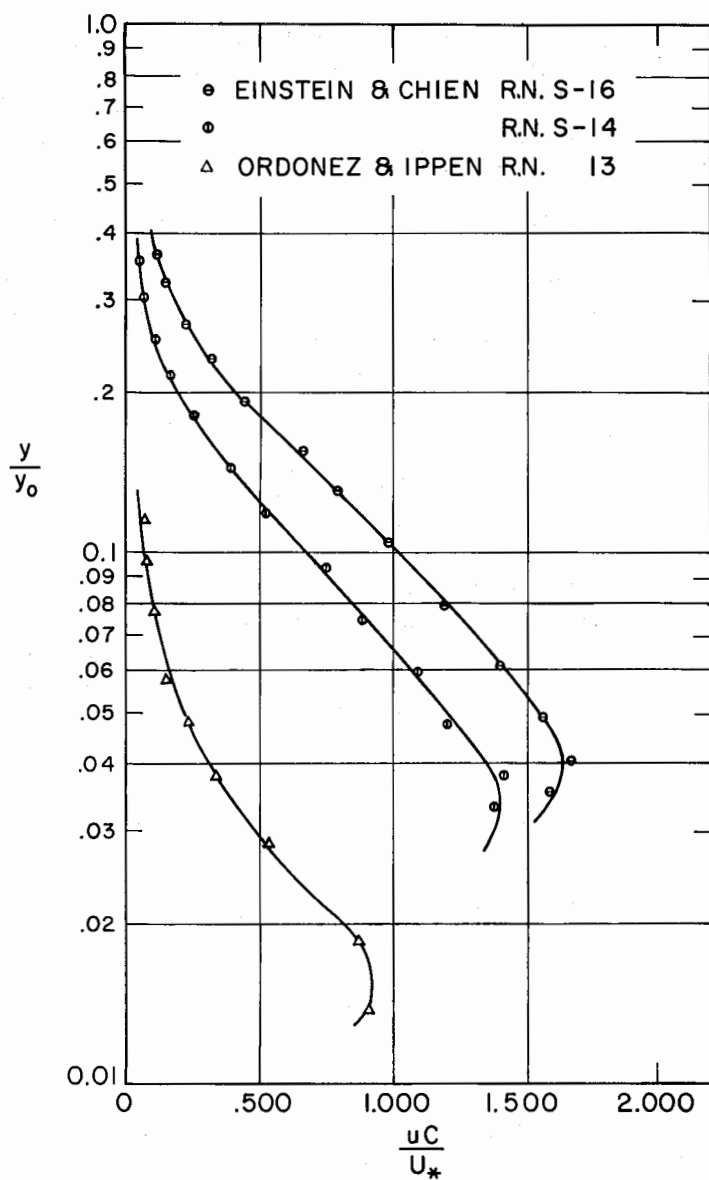


Fig. 11 Variation of Transport rates of Sediment uC/U_* over the Depth.

In the M.I.T. study by Ordonez and Ippen, fairly uniform Ottawa sand was used with a diameter $D_{50} = .25$ mm. However, the variation of the grain sizes from this mean contributed to some scatter. Measurements of concentrations were carried out to within $.02 y/y_0$ near the bed.

Modifications of Hydraulic Parameters

Several interesting hydraulic aspects of flow with suspensions in contrast to clear water flow may be evaluated if the preceding relations are accepted as representing a satisfactory approach to the analysis of flow of suspensions. In accordance with the Darcy-Weisbach equation, the resistance coefficient is generally defined by:

$$\frac{U_*}{U} = \sqrt{\frac{f}{8}}. \quad (31)$$

Since the experimental evidence points to changing values of the Darcy-Weisbach resistance coefficient f for flow with suspensions, the mean velocity U must also change for a given shear velocity $U_* = \sqrt{g y_0 S_0}$. By integrating equation (28) over the entire depth and dividing by y_0 the relation between maximum velocity U_{\max} and the mean velocity U is obtained as:

$$\frac{U_{\max}}{U} = 1 + \frac{1}{k} \sqrt{\frac{f}{8}} \left[\frac{k}{k'} \int_1^{\psi} \ln \left(\frac{y}{y_0} - \psi \ln \frac{y}{y_0} \right) d \left(\frac{y}{y_0} \right) \right]. \quad (32)$$

For clear water flow with $\psi = 0$ and $k/k' = 1$ this reduces to:

$$\frac{U_{\max}}{U} = 1 + \frac{1}{k} \sqrt{\frac{f}{8}} \quad (33)$$

The integral expression in equation (32) has been evaluated for the range of ψ from 0 to .016 to give a factor only 14% smaller than unity for the maximum ψ . The ratio k/k' , however, may reach values of up to 2. The expression in brackets in equation (32) is therefore always larger than unity. Since all velocity profiles with suspended sediment show larger values of U_{\max}/U than for clear water, it follows that the value in brackets in (32) must more than compensate for any possible decrease in f for a given shear velocity as explained for equation (31).

Table 1 gives a summary of basic data for a number of representative experimental runs selected from the four sets of references [12], [13], [16],

	1	2	3	4	5	6	7	8	9	10	11	12
Source of Data	S_o	y_o (ft)	U (fps)	U_{max} (fps)	f	U_* (fps)	k'	ψ	$\frac{U_{max}}{U}$ expt.	$\frac{U_{max}}{U}$ computed by Eq. 32	C_m vol. mean $\times 10^4$	$\frac{U_{max} - U}{U_*}$
Vanoni (12) Run No. 1 18 20	0.0025 0.00125 0.0025	0.481 0.462 0.461	3.62 2.50 3.24	4.175 2.95 3.86	0.0236 0.0235 0.0283	0.197 0.136 0.193	0.375 0.31 0.34	0 0.005 0.005	1.151 1.181 1.190	1.146 1.163 1.161	0 4.42 7.05	2.67 3.013 2.747
Einstein-Chien [13] Run No. S-15 S-16	0.0262 0.0267	0.408 0.391	10.00 9.30	12.80 12.05	0.0271 0.0310	0.586 0.580	0.20 0.20	0.0055 0.007	1.28 1.295	1.269 1.281	250 293	4.648 4.568
Ordenez-Ippen [18] Run No. 11 13	0.005 0.005	0.291 0.335	5.89 5.20	6.55 5.86	0.0109 0.0160	0.218 0.232	0.324 0.360	0.0005 0.002	1.110 1.129	1.114 1.120	21.0 27.0	3.044 2.690
Elaia-Ippen [16] Run No. 25 31	0 0	0.123 0.134	5.31 4.59	6.021 5.265	0.0157 0.0154	0.237 0.202	0.28 0.25	0.019 0.020	1.131 1.148	1.132 1.147	1320 2660	3.005 3.345

TABLE 1: Summary of Basic Data

[18]. They represent a large variety of hydraulic and concentration conditions. It is seen that the values of U_{\max}/U computed from equation (32) compare very favorably with the experimental values. They could not have been obtained from the equation (33) for clear water.

It has been observed that the resistance coefficient f may decrease or increase with increasing sediment concentrations. This problem is as yet not resolved clearly, but equations (31) and (32) provide at least further insight into this aspect.

Many of the conclusions with regard to the dependence of the hydraulic parameters on sediment concentrations are independently confirmed by the contributions to this field of M. Hino [19], who developed the hydrodynamic theory of flow of suspensions on the basis of energy dissipation in the flow including suspended particles and total energy production.

Evaluation of Total Transport

With the velocity distribution defined by equation (28) and the distribution of volume concentration given by equation (30), the distribution of the sediment transport rate over the depth can be determined. Such distributions are given for illustration in Figure 11 for three cases as indicated. The volume rate of transport is given in dimensionless form as uC/U_* and plotted against y/y_0 . The logarithmic scale for y/y_0 emphasizes the predominance of sediment flux in the lower 10 to 20% of the depth.

From these plots the total sediment flux can be obtained by integration which was done numerically using the relation

$$\alpha U y_0 C_m = \int_0^{y_0} u C dy \quad (34)$$

wherein the mean concentration

$$C_m = \frac{1}{y_0} \int_0^{y_0} C dy \quad (35)$$

The factor α by which the mean concentration C_m as defined by the expression (35) is multiplied in equation (34) represents a correction factor to obtain the true mean concentration C_m' from the integration of the local transport rates over the depth. In Table 2 a few such evaluations of α have been made from available data. The more uniform the distribution of sediment, the closer is the value of α to unity, as is obvious for the nearly neutrally buoyant particles of the Elata-Ippen runs.

Source of Data	Run No.	$C_m \times 10^2$	$C_m' \times 10^2$	α
Elata-Ippen [16]	31	26.6	26.6	1.00
Vanoni [12]	20	.071	.067	.945
Ordenez-Ippen [18]	13	.27	.185	.686
Einstein-Chien [13]	14	1.38	.77	.557
	16	2.93	1.48	.505

TABLE 2

The value of α for various conditions of sediment transport can be predicted only through the integration of the product of velocity (equation (28)) and concentration (equation (30)) over the depths. Such evaluations are quite complex and only possible by computer. No simple correlations are possible since the variations of α depend on the interdependent functions for velocity and sediment distribution. The values of α given in Table 2, however, show that actual concentrations C_m' differ materially from the arithmetic average concentration C_m obtained from suspended sediment concentration surveys.

Conclusions and Summary

The study presented in its essential phases is part of a comprehensive investigation into the characteristics of flow with solid suspensions at MIT. The preceding sections dealt with analytical and experimental phases of this study pertaining to two-dimensional steady and uniform flow over a smooth boundary with particle suspensions in various concentrations.

It was shown that the variation of the Von Karman constant k' is primarily a function of the maximum concentrations of sediment C_0 .

A velocity distribution function is given which depends on the shear velocity, the value of k' , and a parameter ψ . The value of ψ is related to k' and C_0 .

The concentration distribution has been redeveloped with the modified velocity distribution function and evaluated. The total suspended load for individual particle sizes may be determined from the mean concentration and, therefore, also the transport rate of suspended sediment for the two-dimensional case.

The effects of the suspensions on the mean velocity, maximum velocity, and resistance coefficient have been shown to be consistent with experimental observations and the analytical approaches employed.

The general problem of transport of sediments in turbulent streams is still unsolved, but important insights have been gained in recent years, and the results

of the MIT research in this field hopefully have contributed to this advance in our knowledge. It is an important area of research in the present era of concern with our water environment, and therefore merits further attention and support in analytical, experimental and field explorations.

Acknowledgements

Much of the introductory material presented was taken from the professional literature and is acknowledged in the list of references. A part of the analytical approach originated from unpublished notes of the writer. Special recognition, however, is recorded here to Dr. Nelson A. Ordonez-C., of Lima, Peru, who carried out his doctoral work in this area of suspended sediment transport at the Ralph M. Parsons Laboratory for Water Resources and Hydrodynamics in the Civil Engineering Department of MIT in the years 1968-1970 under the supervision of the author. Many of the figures, experiments, and analytical evaluations cited in the text originated through his efforts. Grateful acknowledgment is due also to Mr. Sergio Montes, Research Assistant in the same laboratory, for his extensive assistance on many details of the material prepared for the final draft of this paper.

REFERENCES

1. "Sediment Transportation Mechanics," Progress Reports of Task Committee on Sedimentation of the Hydr. Div. of the ASCE.
 - 1a. "Initiation of Motion," *Proc. A.S.C.E.*, Vol. 92, No. HY2, 1966; also "Sediment Discharge Formulas," *Proc. A.S.C.E.*, Vol. 97, No. HY4, 1971.
 - 1b. "Suspension of Sediment," *Proc. A.S.C.E.*, Vol. 89, No. HY5, 1963.
2. Graf, Walter H., *Hydraulics of Sediment Transport*, McGraw-Hill Book Co., New York, 1970.
3. Shields, A., "Anwendung der Aehnlichkeitsmechanik und Turbulenzforschung auf die Geschiebebewegung," *Mitteil. Preuss. Versuchsanstalt fuer Wasser, Erd, und Schiffsbau*, Berlin, No. 26, 1936.
4. Kennedy, John F., "The Mechanics of Dunes and Antidunes in Erodible Bed Channels," *Journal of Fluid Mechanics*, Vol. 16/4, 1963.
5. O'Brien, M. P., "Review of the Theory of Turbulent Flow and Its Relation to Sediment Transport," *Trans. American Geophysical Union*, 1933.
6. Schmidt, W., "Der Massenaustausch in Freier Luft Und Verwandte Erscheinungen," *Probleme der Kosmischen Physik*, Vol. 7, Hamburg, 1925.
7. Hurst, H. E., "The Suspension of Sand in Water," *Proc.*, Royal Society, London, Series A. Vol. 157, 1929.
8. Rouse, H., "Experiments on the Mechanics of Sediment Suspension," *Proc. Fifth International Congress of Applied Mechanics*, Cambridge, Mass., 1938.
9. Krey, H., "Die Quergeschwindigkeitskurve bei Turbulenter Stroemung," *Zeitschrift fuer Ang. Mathematik und Mechanik*, Vol. 7, No. 2, 1927.
10. Ippen, A. T., "Transport of Solids by Turbulent Flow in Channels and Conduits," IUTAM Symposium on Flow of Fluid-Solid Mixtures, Cambridge, England, 1969 (unpublished invited lecture).
11. Rouse, H., "Modern Conceptions of the Mechanics of Turbulence," *Trans. A.S.C.E.*, Vol. 102, 1937.
12. Vanoni, Vito A., "Transportation of Suspended Sediment by Water," *Trans. A.S.C.E.*, Vol. 111, 1946.
13. Einstein, H. A., and Chien, N., "Effects of Heavy Sediment Concentration Near the Bed on Velocity and Sediment Distribution," University of California, Inst. of Eng. Research, No. 8, 1955.
14. Einstein, H. A., *Annalen der Physik*, Vol. 19, p. 286, 1906 and Vol. 34, p. 591, 1911.
15. Eilers, H., "Die Viskositat von Emulsionen Hochviskoser Stoffe als Funktion der Konzentrationen," *Kolloid-Zeitschrift*, Vol. 97, p. 313, 1941.
16. Elata, C., and Ippen, A. T., "The Dynamics of Open Channel Flow with Suspensions of Neutrally Buoyant Particles," M.I.T., Hydrodynamics Laboratory, Techn. Report No. 45, 1961.
17. Vanoni, V. A., and Nomicos, G. N., "Resistance Properties of Sediment-Laden Streams," *Trans. A.S.C.E.*, Vol. 125-I, 1960.
18. Ordenez-C., Nelson A., "The Absolute Concentration Distribution of Suspended Sediment in Turbulent Streams," Ph.D. Thesis, M.I.T., R. M. Parsons Lab. for Water Resources and Hydrodynamics, July 1970.
19. Hino, Mikio, "Turbulent Flow with Suspended Particles," *Proc. A.S.C.E.*, Vol. 89, No. HY4, 1963.

TABLE OF NOTATIONS

Roman Letters

a	small distance from bottom defined by equation (10)
b	arbitrary reference distance from bottom
c or C	local concentration of sediment
C _o	maximum sediment concentration
C _m	average sediment concentration
d or D	mean particle diameter
f	Darcy-Weisbach resistance coefficient
f ()	function of ()
F	Froude number of flow
g	acceleration of gravity
k	Von Karman universal constant
k'	effective constant for velocity distributions with sediment
P _s	power to suspend sediment particles per unit area
P _f	power to overcome bottom resistance per unit area
q	discharge per unit width
q _s	sediment transport rate per unit width
S	specific gravity of sediment
S _o	slope of bottom or energy gradient
u	local velocity
U	average velocity for cross-section
U*	shear velocity $\sqrt{\tau_o/\rho}$
U _{max}	maximum velocity at surface
w	settling velocity of particles in still water
x	distance in direction of flow
y	local depth measured from bottom
y _o	total depth of stream
Z	sedimentation parameter as defined by equations (17) and (19)

Greek Letters

α	=	proportionality factor used as defined locally
β	=	ratio of mass exchange coefficient to momentum transfer coefficient
γ	=	specific weight
δ'	=	thickness of laminar sublayer
ϵ_m	=	turbulent momentum transfer coefficient
ϵ_s	=	kinematic mass exchange coefficient
μ	=	dynamic viscosity
μ'	=	dynamic viscosity for fluid-particle mixtures
ν	=	kinematic viscosity = μ/ρ
ρ	=	density = γ/g
τ	=	local shear stress
τ_0	=	maximum shear stress at bottom
ψ	=	small experimental factor in equation (28)