DISPERSION ESTIMATES OFF NOBSKA POINT USING RECORDING CURRENT METERS

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Abstract

Dilution estimates were made from the data collected by a recording current meter operated for two weeks off Nobska Point. The estimates were compared to estimates made by other methods. The 5 and 6 hour dilution rates for the ebb and flood tides range from 4690/1 to 8100/1 with the lower value occurring during the ebb tide. Five hour dispersion coefficients of 8.6 and 8.2 x 10^5 cm^2/sec were determined from the current meter data. Generally, the hourly dispersion predictions from the current meter data were larger than estimates by other means and considered more representative. On the other hand, the steady state dispersion predictions from the current meter data require some other validation.

Background

The Town of Falmouth, Massachusetts, has proposed building a domestic sewerage system with disposal of the effluent to take place in Vineyard Sound off Nobska Point. Bumpus,[1] as the result of drogue and drift bottle studies, has proposed that the site of the outfall should be located in approximately 30 meters of water 0.504 km south of Nobska Point. Bumpus[1] and Charnews and Jaffee[2] have applied several techniques to determine the probable extent of the sewage plume and what dilution might be expected if the outfall was located as mentioned above. The intent of this paper is to provide similar estimates from data collected concurrently by a current meter. The data were supplied by EG&G, Woods Hole.

Charnews and Jaffee[2] undertook dye studies to determine dispersion patterns for a tracer released at the location of the outfall. From these results, surface dispersion due to wind driven and tidal currents may be studied. Bumpus[1] bases his steady state mixing estimates on the assumption that one-tenth of the sound is new water in a tidal cycle. This is considered a conservative estimate by Bumpus. The volume of water in the sound changing every tidal cycle was determined from the low water volume and mean tidal rise. Similar computations were made for a net flow assuming 10 percent water exchange on a tidal cycle. In addition, immediate mixing estimates were made by multiplying the sound cross-section above a diffuser by an estimated mean velocity. An eddy diffusion coefficient was also determined employing an estimated velocity. The dispersion estimates determined from current meter data

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presented subsequently do not require velocity estimates. The time history sequence of the measured currents are maintained in the analysis. However, current meter methods are based upon currents measured at a single fixed point and it is necessary to extrapolate the measurements to areas for dispersion estimates. This requires a reasonably homogeneous velocity field over the area of interest. Therefore current meter measurements should be supplemented by concurrent dye and drogue measurements.

The two techniques for determining dispersion and dilution are complementary. Dye and drogue measurements are Lagrangian techniques which provide area characteristics. The predominant variable is space. These measurements are normally limited to daylight hours during reasonably calm sea conditions. Current meter records are continuous with respect to time at one location. The predominant variable is time. The two techniques are obviously not independent. Resultant dispersion and dilution characteristics are a mixture of both, although one may predominate in a particular case. Unfortunately, it is not possible to determine which variable is the more important in this case as concurrent Lagrangian measurements are not available. This means that the current meter estimates require some validation. Comparison with the estimates of Bumpus[1] provide some validation. However, the assumptions involved in Bumpus's computations are qualitative.

The results of the current meter study are presented in the following. The techniques are summarized. Further details on the techniques can be obtained from the appropriate references.

Results

The current meter was operated at the proposed site of the outfall from November 26, 1969, to December 10, 1969, at a depth of 10m. A longer period of record would be desirable. The data set resulting from the operation consisted of readings of current speed and direction reduced to 10-minute averages. The data time sequence was smoothed using a moving average technique over ±20 minutes to make the data suitable for analysis. A measure of the smoothing can be obtained by comparing some data statistics. The resultant and arithmetic mean velocities for the complete record were 3.8 cm/sec at 154°N and 59.3 cm/sec for the smoothed data and 1.9 cm/sec at 154°N and 59.3 cm/sec for the raw data set. The smoothed data were reduced to obtain a two-dimensional frequency table (Table 1). The tidal effect is clearly evident with some 40 percent of the readings occurring in the north-east direction, and 45 percent occurring in the south-west direction.

To determine a resultant velocity magnitude, it is necessary to find a mean velocity during an ebb tide, then compare it to the mean of the flood tide.

TABLE 1 Frequency Analysis for Current Meter Data

SPEED (cm/sec)	DIRECTION (DEGREES) 337.50- 22.49	22.50 67.49	67.50- 112.49	112.50- 157.49	157.50- 202.49	202.50- 247.49	247.50- 292.49	292.50- 337.49	ROW SUMS
0.00 – 12.99	0.000	0.049	0.345	0.839	0.691	0.000	0.049	0.049	2.023
13.00 – 25.99	0.099	1.085	2.713	2.368	3.355	3.453	0.296	0.099	12.468
26.00 – 38.99	0.000	2.713	2.269	0.543	0.247	5.131	0.148	0.000	11.051
39.00 – 51.99	0.000	5.969	0.888	0.000	0.000	6.364	0.049	0.000	13.271
52.00 – 64.99	0.000	7.943	0.049	0.000	0.000	7.745	0.000	0.000	15.738
65.00 – 77.99	0.000	7.795	0.000	0.000	0.247	9.423	0.000	0.000	17.464
78.00 – 90.99	0.000	8.337	0.000	0.000	0.000	5.871	0.000	0.000	14.208
91.00 – 103.99	0.000	3.207	0.000	0.000	0.000	4.489	0.000	0.000	7.696
104.00 – 116.99	0.000	2.368	0.000	0.000	0.000	1.332	0.000	0.000	3.700
117.00 – 129.99	0.000	0.296	0.000	0.000	0.000	1.085	0.000	0.000	1.381
COLUMN SUMS	0.099	39.763	6.265	3.749	4.539	44.894	0.543	0.148	

Towards this end, twelve tidal cycles were selected from the record. Eight of the tidal cycles were selected so that they were sequential in time, while the other four were selected in sequential pairs. The results are summarized in Table 2.

From Table 2 it can be seen that the direction of the current off Nobska Point remains relatively constant varying between 56°-65° on the flood tide and between 231°-237° on the ebb tide. However, the resultant velocity magnitude varies widely up to 50 percent on the flood tide and 40 percent on ebb. In the first group of eight half tidal cycles, if taken in pairs starting with the first two, it would appear that there is a large net flow alternately on the ebb and flood cycles. However, starting with the second half cycle, and taking pairs, there is little net flow. This phenomenon also appears in the other pairs of tidal cycles. These records indicate that misleading results can occur by assessing the net flow on the basis of measurements over one tidal cycle. They may also indicate the need for several current meters operated at different locations. Considering the twelve tidal cycles presented in Table 2, the resultant speeds for all twelve on the flood and ebb tides are 54.10 cm/sec and 54.29 cm/sec respectively. Thus, for the twelve cycles chosen here, representing 45 percent of the data, the net transport velocity is 0.16 cm/sec to the south-west (ebb tide direction). The overall arithmetic mean velocity magnitude is 54.20 cm/sec.

These results may be compared with those found by Bumpus[1]. The arithmetic mean velocity magnitude is 1.06 knots (54.20 cm/sec) compared to 1.15-2 knots found by Bumpus.[1] The net speed found here is 0.004 knots (0.16 cm/sec) whereas Bumpus found a value of 0.1 knots in his net flow analysis. The effect of this difference is to lower the net transport from 200 million m^3 to 10 million m^3 and to decrease the dilution from 2850/1 to 134/1. Thus the dilution is 20 times lower than found by Bumpus. The meaning of this small dilution or net transport integrated over a period of 14 days is not clear. The mean velocities for each ebb and flood cycle are so large that material will be transported long distances during a single cycle into regions which may have different dispersive qualities.

The summary of the twelve tidal cycles indicates that the directions of the current at the proposed outfall site remain constant at 233° and 60° for the ebb and flood tides respectively. During the 14 days when the current meter operated, it is likely that meteorological conditions varied. It would thus appear that meteorological effects have little effect on subsurface currents at the depth at which the current meter was located. It is difficult to estimate the current characteristics at other depths and locations from one current meter site without the aid of other current meter installations. However, initially dispersion should be in a direction slightly more offshore than found by Charnews and Jaffee[2] in their surface dye experiments. The difference in direction is not surprising since subsurface and surface currents are generally different.

TABLE 2 Mean Resultant Velocities for Selected Half Tidal Cycles

START DATE	END DATE	DIRECTION	RESULTANT SPEED cm/sec.
Nov. 27 11:56	Nov. 27 18:00	61.6°	55.0
Nov. 27 18:00	Nov. 28 00:06	233.2°	68.6
Nov. 28 00:44	Nov. 28 06:04	56.1°	64.2
Nov. 28 06:44	Nov. 28 12:44	233.1°	45.4
Nov. 28 12:25	Nov. 28 18:31	62.5°	48.0
Nov. 28 18:31	Nov. 29 00:25	232.5°	66.0
Nov. 29 1:25	Nov. 29 7:25	59.8°	64.6
Nov. 29 7:31	Nov. 29 13:31	233.6°	45.0
Dec. 1 2:25	Dec. 1 8:31	59.3°	64.2
Dec. 1 8:31	Dec. 1 14:25	237.6°	45.6
Dec. 1 14:50	Dec. 1 20:50	64.8°	34.8
Dec. 1 20:50	Dec. 1 02:50	234.5°	64.7
Dec. 1 20:25	Dec. 2 3:13	233.5°	58.5
Dec. 2 3:25	Dec. 2 9:25	58.2°	41.0
Dec. 2 9:31	Dec. 2 16:00	233.5°	50.5
Dec. 2 16:31	Dec. 2 22:25	59.4°	45.4
Dec. 2 22:31	Dec. 3 3:13	233.0°	36.3
Dec. 3 3.25	Dec. 3 10:44	60.4°	70.0
Dec. 3 10:50	Dec. 3 15:50	235.3°	49.1
Dec. 3 17:00	Dec. 3 22:31	60.1°	45.4
Dec. 5 18:06	Dec. 6 00:13	61.3°	51.7
Dec. 6 0:13	Dec. 6 6:06	231.2°	46.0
Dec. 6 6:13	Dec. 6 12:13	60.2°	65.1
Dec. 6 12:38	Dec. 6 18:00	233.2°	75.7
	Resultant	233°	0.16 cm/sec

The current meter data may also be used as input to stochastic models for the prediction of dilution rates. The currents were classified into states by speed and direction. For example, 10 speed states (0-13 cm/sec; 13-26 cm/sec, etc.) and 8 direction states (337.5°-22.5°; 22.5°-67.5° etc.) were defined. Thus, it is possible to categorize each current reading by its speed and direction. By comparing each current state with the one following, it is possible to find the probability of a current transferring from one state to another in one time period. Using the complete record of readings, a matrix called the first order Markov Chain transition probability matrix, is developed. For this study two transition matrices were developed, one for ten-minute interval data and one for hourly interval data. A more complete outline of the method is contained in publications by Palmer and Izatt.[6,5]

The frequency table shown in Table 1 gives the probability of being in any one of the 80 possible states based on the period of record. The probability of going from one state to another in one hour is given by the probability transition matrix. Thus, the probability of starting in state A and proceeding in 5 hours to state B through 4 intermediate states may be found. The distance travelled may be calculated by assuming that the speed for each state remains constant for one hour, then multiplying the speed and time together. The distance travelled in five hours may be weighted by the probability of that sequence of states occurring. The weighted distance travelled can be resolved into components in the 4 major directions. If all the possible sequences for all initial and final states are considered, then a weighted mean distance and standard deviation of distance in each direction can be found for the time period. The number of possible sequences increases by the power of the number of hours considered and computer costs limit the practical sequence to 5 states or 5 hours for hourly transition matrices. Diffusion coefficients can be found from the standard deviations using the following equation (Hinze[4]).

$$\langle y^2 \rangle = 2\epsilon t \qquad (1)$$

in which $\langle y^2 \rangle$ is the variance of particle separation in cm^2; ϵ is the diffusion coefficient in cm^2/sec; and t is the time in seconds. The mean velocity for five hours may be found by dividing the weighted distance by the time. In the case of strong tidal action, as occurs off Nobska Point, the two directions perpendicular to the tidal current do not have enough readings in the current meter record to give meaningful results. A summary of results for 5 hours is given in Table 3. The mean velocities shown in Table 3 are a bit low since the Markov Chain analysis ignores some of the smaller probability sequences in order to economize on computer time. The diffusion coefficients for both ebb and flood tide direction are about the same.

TABLE 3 Summary of Dispersion Characteristics for 5 Hours

Direction (Degrees)	Weighted Standard Deviation $<y^2>^{1/2}$ (cm)	Weighted Diffusion Coefficient ϵ (cm^2/sec.)	Mean Velocity $<U>$ (cm/sec.)
60°	1.76×10^5	8.60×10^5	51.6
150°	$.11 \times 10^5$	$.03 \times 10^5$	3.8
240°	1.72×10^5	8.22×10^5	55.0
330°	$.001 \times 10^5$	$.00 \times 10^5$	1.3

A measure of the dilution taking place can be found using the following equation (Foxworthy et al[3])

$$<Cmax\ (\chi)> = \frac{Q}{(2\pi)^{1/2} <y^2>^{1/2} <\bar{U}>} \tag{2}$$

in which $<Cmax\ (x)>$ is the maximum concentration at a distance x in mg/cm^2; Q is the mass discharge in mg/sec.; $<y^2>$ is the weighted mean spread in cm^2; and $<\bar{U}>$ is the weighted mean velocity in cm/sec. This is essentially a two-dimensional equation as current meter readings are two-dimensional. From the consultant's report, the source concentration of PO_4 as P is 30 mg/1 and the effluent is being discharged at the rate of 0.156 m^3/sec.

In order to make the source two-dimensional it is necessary to assume that the outfall consists of a circular pipe discharging at a speed of 1 m/sec. The diameter of the pipe is then 0.492 meters. Taking a 1 cm wide slice at the maximum diameter gives a source of 150 mg/sec. Substituting in Equation (2) yields $<Cmax\ (x)> = 4.7 \times 10^{-6}$ mg/cm^3 on the flood cycle and $<Cmax\ (x)> = 4.5 \times 10^{-6}$ mg/cm^3 on the ebb cycle for a 1 cm thick layer. The percentage dilution is then 0.016 percent and 0.015 percent on the flood and ebb tides respectively.

An alternative way of finding dilution characteristics involves defining an initial state vector and multiplying it by the transition probability matrix (Palmer and Izatt[5]). This defines a final state probability vector after one time period. Weighted mean distances travelled can be found by multiplying the probability of a state by the speed and the time interval and summing those in the same direction. This produces a mean distance travelled in one time period for each of the eight directions. The final state vector can be used as an initial state vector for the next time step. The procedure may be carried on for as many steps as desired. Figure 1 illustrates the contours predicted for 1, 3 and 6 hours for an initial state with 100 percent probability of a current going north-east at 33.5 cm/sec. and for an initial state with 100 percent probability of a current going south-east at 33.5 cm/sec.

The areas represented by the contours do not represent the shape of an effluent plume released at the source. On the average, an effluent released at the source will form a plume which will be contained somewhere within the contours after 1, 3 and 6 hours. The plume would not necessarily occupy the total area within the contour. If a constant depth for an effluent plume is assumed then a pessimistic estimate of its volume may be obtained by measuring the area of the contours in Figure 1 and multiplying by the constant depth. The dilution rate may then be found if the amount of effluent released in 1, 3 and 6 hours is known.

A depth of 10 meters has been assumed and the areas, volumes, and dilutions for 1-6 hours for a flood tide and an ebb tide have been calculated and summarized in Tables 4 and 5. The dilution is 4690/1 parts seawater to effluent for an ebb tide after 6 hours, and 8100/1 for a flood tide. There is a 42 percent difference in initial dilution in the two directions after 6 hours. This method is restricted to times less than a half tidal cycle because the probabilistic nature of the solution forces flow in both directions even after the currents have reversed direction. An approximation for the steady state dilution may be found by averaging the dilution for the two directions. This yields a steady state dilution of 6345/1 parts of seawater to sewage.

Discussion

The current meter data indicates that the average tidal current varies up to 50 percent in magnitude but remains constant in direction. Analysis of mean tidal resultant currents shows that several tidal cycles must be considered when a net current is sought. Definition of a net current based on one tidal cycle may be in error if the current is to be used as a mean value to represent exchange processes for many tidal cycles.

The dispersion estimates determined from the current data are summarized in Table 6. It is observed that the current meter estimates are much larger than those predicted by Bumpus[1] confirming his qualifications that the estimates are conservative. For hourly predictions the current meter method is probably more representative as the temporal variations are large. The mean resultant current directions for the ebb and flood tides are probably more representative of the initial dilution for a sub-surface outfall than are the results obtained by Charnews and Jaffee[2] in their surface dye tracing experiments. The current meter data indicate that initially dilution will take place in a direction more off

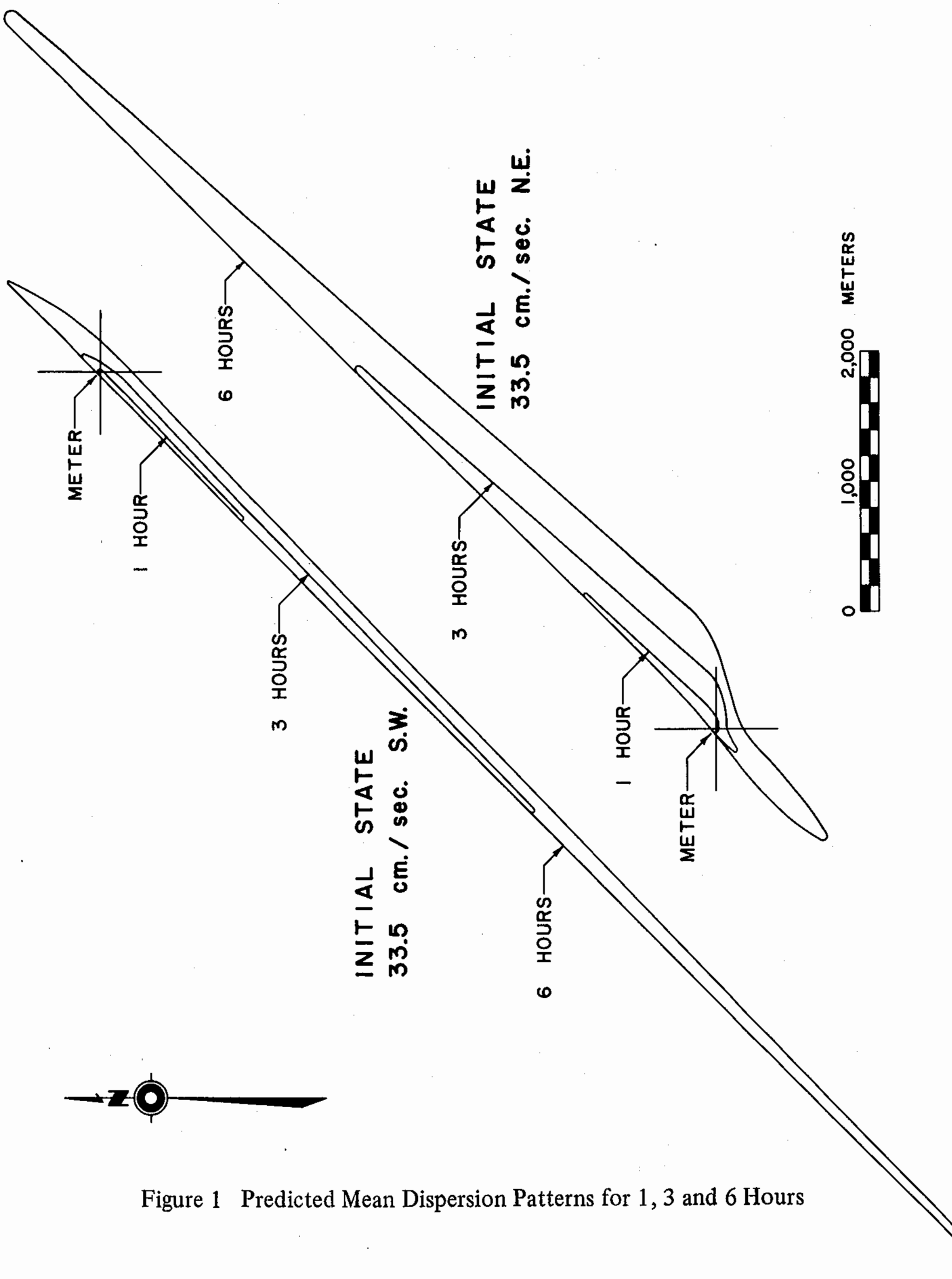

Figure 1 Predicted Mean Dispersion Patterns for 1, 3 and 6 Hours

TABLE 4 6 Hour Dilution Characteristics for Flood Tide

HOURS	AREA (m^2)	VOLUME (m^3)	SOURCE (m^3)	DILUTION (Parts Seawater/Effluent)	
1	$.86 \times 10^5$	8.6×10^5	584	1472/1	(.07%)
2	3.56×10^5	35.6×10^5	1168	3050/1	(.03%)
3	9.25×10^5	92.5×10^5	1752	5270/1	(.02%)
4	13.98×10^5	139.8×10^5	2336	5980/1	(.02%)
5	21.58×10^5	215.8×10^5	2920	7390/1	(.01%)
6	28.40×10^5	284.0×10^5	3504	8100/1	(.01%)

TABLE 5 6 Hour Dilution Characteristics for Ebb Tide

HOURS	AREA (m^2)	VOLUME (m^3)	SOURCE (m^3)	DILUTION (Parts Seawater/Effluent)	
1	$.2 \times 10^5$	2×10^5	584	343/1	(.3%)
2	1.3×10^5	13×10^5	1168	1111/1	(.09%)
3	3.7×10^5	37×10^5	1752	2105/1	(.05%)
4	6.7×10^5	67×10^5	2336	2870/1	(.03%)
5	11.7×10^5	117×10^5	2920	4000/1	(.02%)
6	16.4×10^5	164×10^5	3504	4690/1	(.02%)

shore than found by Charnews and Jaffee.[2] The steady state current meter dilution predictions from single fixed point measurements require extrapolation to large areas. Consequently these predictions require validation by other methods. It is the authors' opinion that the steady state prediction from the meter data is too large.

The application of the dispersion estimates to an actual effluent would require some consideration on both positive and negative buoyancy effects. None of the estimates considers buoyancy directly. The dye studies of Charnews and Jaffee[2] do provide a qualitative indication of the possible dispersion pattern of a positively buoyant effluent. Normally surface effluent releases dilute more slowly than subsurface releases. Buoyancy effects should be incorporated into the estimates if the range of expected specific gravities for the effluent and receiving water were known.

TABLE 6 Comparative Dilution Rate Estimates

PERIOD	BUMPUS			CURRENT METER		
	(DILUTION ESTIMATES) (Parts Seawater/Effluent)		DISPERSION COEFFICIENT	(DILUTION ESTIMATES) (Parts Seawater/Effluent)		5 HOUR DISPERSION COEFFICIENT
	Tidal Exchange	Net Flow	(cm^2/sec.)	5 hr. Mean	6 hr. Mean	(cm^2/sec.)
Ebb Tide	—	—	0.1×10^5	6660/1	4690/1	8.6×10^5
Flood Tide	—	—	0.1×10^5	6250/1	8100/1	8.2×10^5
Steady State	1700/1	2850/1	—	6455/1	6345/1	—

References

1. Bumpus, D.F., Wright, W.R. and Vaccaro, R.F., "Considerations on a Sewer Outfall off Nobska Point", *Reference No. 69-87,* Woods Hole Oceanographic Institution, Woods Hole, Mass., BSCE Journal, Oct. 1971.

2. Charnews, D.P. and Jaffee, R.J., "Water Transport off Nobska Point by Dye Tracing", Ocean Engineering, Woods Hole Oceanographic Institution, Woods Hole, Mass., Aug. 1970.

3. Foxworthy, J.E., Tibby, R.B. and Barsom, G.M., "Dispersion of a Surface Waste Field on the Sea", Journal Water Pollution Control Federation, Vol. 38, No. 7, 1966, p. 1170.

4. Hinze, J.O., *Turbulence,* 1st Ed., McGraw-Hill, 1959, p. 332.

5. Palmer, M.D. and Izatt, J.B., "Lakeshore Two-Dimensional Dispersion", *Proceedings of 13th Conference on Great Lakes Research,* International Association for Great Lakes Research, pp. 495-507.

6. Palmer, M.D. and Izatt, J.B., "Lake Hourly Dispersion Estimates from a Recording Current Meter", Journal of Geophysical Research, Vol. 76, No. 3, Jan. 1971, pp. 688-693.