

JOURNAL OF THE  
BOSTON SOCIETY OF CIVIL ENGINEERS

Volume 60

JULY 1973

Number 3

**GENERALIZED SIMULATION MODELS FOR MASSACHUSETTS  
STREAMS**

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**Introduction**

The value of computerized river simulation models in pollution assimilation capacity studies and in planning water quality management programs is widely recognized. Such models can be a valuable aid to pollution control agencies and enable them to more efficiently deal with river pollution control problems. In view of this, the Division of Water Pollution Control of the Commonwealth of Massachusetts contracted with Quirk, Lawler & Matusky Engineers (QL&M) to develop generalized stream models that could be employed to study most of the streams and rivers in the state. In addition, the State retained QL&M to train its personnel in the development and use of mathematical models. This paper presents the results of this effort.

A mathematical model is simply a mathematical representation of the major mechanisms in a natural system in such a form that a cause and effect relationship can be analytically approximated. A complicated system may require the combination of several mathematical models. These models can be transformed and incorporated into a computer program to form the computerized simulation model.

Generally, it is pollution resulting from biodegradable wastes that is of most concern in stream systems. Hence, the level of dissolved oxygen (DO) in a stream is considered to be the significant parameter in assessing its water quality and assimilation capacity. Therefore, most stream models are formulated to simulate the major mechanisms affecting the DO and BOD levels.

The first dynamic relationship between a single point pollution source (BOD) and stream dissolved oxygen (DO) was mathematically formulated by Streeter and Phelps<sup>1</sup> in 1925 in their study of the Ohio River. The differential equation they used to define the DO sag curve downstream of a waste discharge is:

$$\frac{dD}{dt} = K_1L - K_2D \quad \dots (1)$$

It is assumed in this equation that DO levels in a stream are governed by two first-order reaction processes, namely, biological oxidation and reaera-

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tion. The multiplication of the rate coefficient,  $K_1$ , and BOD concentration,  $L$ , represents the rate of DO consumed by biological oxidation. The multiplication of the rate coefficient,  $K_2$ , and DO deficit,  $D$ , represents the rate of DO replenishment from the air.

The Streeter - Phelps equation has been modified and expanded by many investigators<sup>2-7</sup> in order to take into account other sources and sinks contributing to the DO budget in a stream. The mathematical models presented in this paper are also based on a modification of the Streeter-Phelps equation.

### Physical Description of the Housatonic River Basin

In order to achieve the objectives of the study, a representative Massachusetts river, namely the Housatonic, was investigated.

#### *The Basin and the River*

The portion of the Housatonic River Basin in Massachusetts is located in the southwestern corner of the Commonwealth (see Figure 1). It covers an

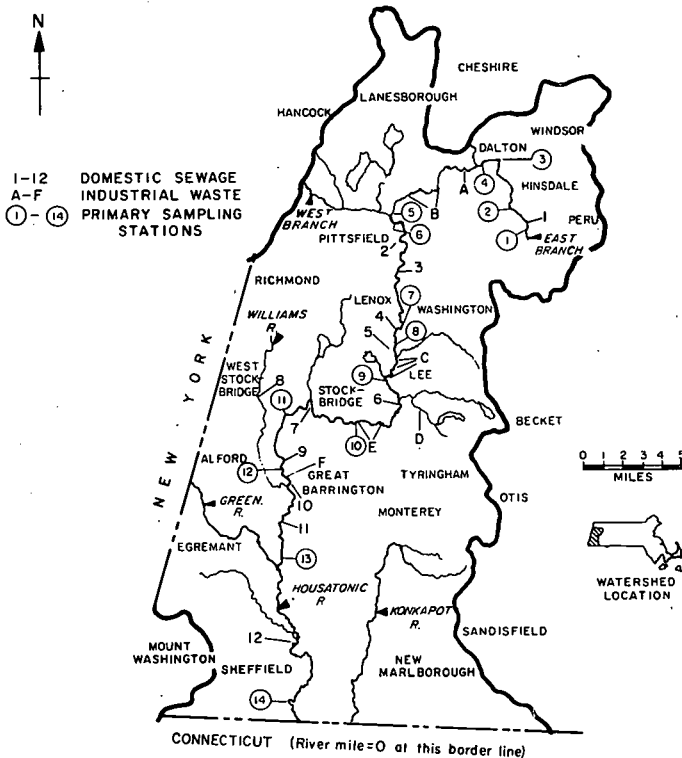


Fig. 1 The Housatonic River Basin in Massachusetts, 1969

(mile x 1.61 = km)

area of approximately 545 square miles (1,412 sq. km), representing 28 percent of the river's total watershed area. The remaining watershed area lies in New York to the west and Connecticut in the south.

The Housatonic River originates in Massachusetts and flows in a generally southerly direction crossing the Massachusetts - Connecticut state line just south of Sheffield. It continues through Connecticut, eventually draining into the Atlantic Ocean.

In this study, a total river length of 69 miles (111 km) was modeled, measured from the Massachusetts - Connecticut state line to sampling station #1 on the East Branch (see Figure 1). The West Branch was treated as a tributary to the main stem.

The river flow is relatively small. The average flows during the 1969 summer survey period were 22 cfs (37.4 cu. m/min) and 139 cfs (236.5 cu. m/min) at the gaging stations at Coltsville (river mile 60.5) and Great Barrington (river mile 23.9), respectively. For this flow condition, the total travel time determined by a dye study was approximately 200 hours for the 69 mile river section.

*Morphological Characteristics*

The Housatonic River varies in width from 45 to 140 ft (13.7 to 42.6m) and in depth from 2 to 14 ft (0.61 to 4.27m). No detailed data on the river's geometry are available.

Figure 2 shows the general profile of the river in Massachusetts. There

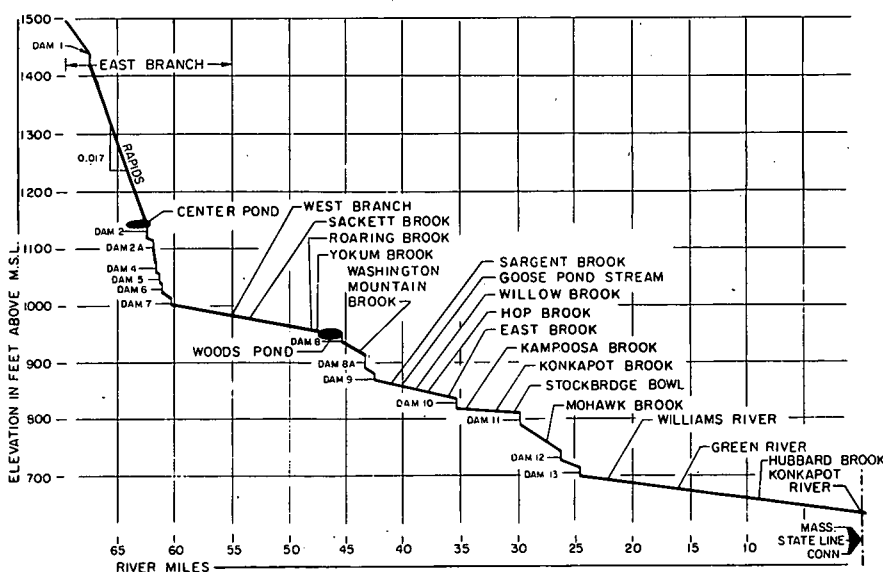


Fig. 2 The Housatonic River Profile  
(mile x 1.61 = km)

are two ponds, fourteen dams, twenty-two tributaries and a rapids section located along this reach of the river.

These morphological characteristics can affect the BOD and DO concentrations. The generally low velocities through the impoundments can cause sedimentation of wastes with a resulting benthic oxygen demand. Algal photosynthesis may also be a significant factor in the oxygen balance in these portions of the river. The high turbulence generated when water passes over a dam or passes through a rapids section can result in high oxygen transfer from the atmosphere to the river. Furthermore, most of the tributaries are unpolluted and, therefore, dilute the waste loads in the river.

### *Water Quality*

A survey conducted in the summer of 1969<sup>8</sup> showed that twelve municipalities and six major industries, five of them paper manufacturers, discharged their wastes into the Massachusetts' section of the Housatonic (see Figure 1). Most of these pollution sources are subjected to primary treatment only or no treatment at all.

Data from this survey indicated that the DO concentration reached a minimum of 2.0 mg/l between river mile points 60 and 45. Color, turbidity and suspended solids were high downstream of the discharges from the paper mills.

Total phosphate concentrations were generally greater than 0.4 mg/l and ammonia nitrogen concentrations lay between 0.05 and 0.15 mg/l. Algal growth was observed in the two ponds and in those sections of the river where the velocity dropped below 0.5 fps (0.152 m/sec). Light and dark bottle studies were conducted to evaluate the photosynthetic effect. The photosynthetic oxygen production rate at the water surface ranged from 24.1 to 57.5 mg/l/day, and the respiration rate ranged from 1.04 to 2.9 mg/l/day.

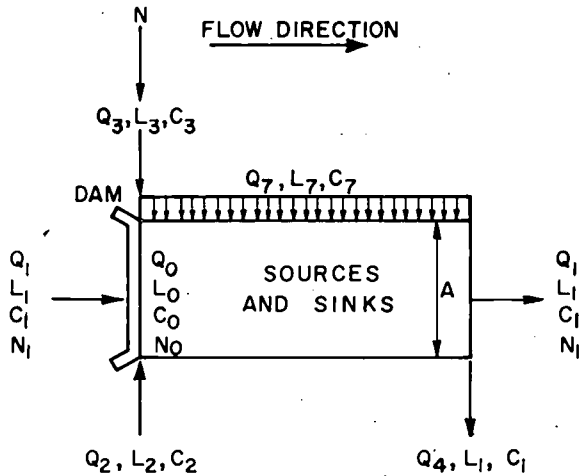
## **Development of the Generalized Simulation Models**

### *Basic Concepts*

The generalized stream model stems from the basic concept that the BOD and the DO profiles of a waterway can be generated by repeatedly solving BOD and DO models for a series of generalized reaches. This process is referred to as the "multiple-reach" technique.

A generalized reach is one in which all possible boundary conditions, sources, and sinks that can affect the level of BOD and DO in a stream are included. The generalized reach that was developed for Massachusetts waterways is shown in Figure 3. The following conditions apply to the generalized reach:

1. Dams, point waste discharges and tributaries are located at the upstream end of a reach.



## LEGEND

- $Q_1, L_1, C_1, N_1$  = FLOW, CARBONACEOUS BOD, DO, NITROGENOUS BOD, IN THE WATERWAY (COULD BE EITHER INPUT OR OUTPUT OF A REACH)
- $Q_2, L_2, C_2$  = FLOW, CARBONACEOUS BOD, DO IN PLANT WASTE DISCHARGE
- $Q_3, L_3, C_3$  = FLOW, CARBONACEOUS BOD, DO IN A TRIBUTARY
- $Q_0, L_0, C_0, N_0$  = FLOW, CARBONACEOUS BOD, DO, NITROGENOUS BOD AT THE BEGINNING OF A REACH
- $Q_7, L_7, C_7$  = FLOW, CARBONACEOUS BOD, IN THE UNIFORMLY DISTRIBUTED FLOW
- $Q_4$  = DIVERSION FLOW (THE BOD AND DO CONCENTRATIONS ARE EQUAL TO THOSE IN THE OUTPUT OF A REACH)
- A = CROSS SECTIONAL AREA
- N = NITROGENOUS BOD LOADING WITH ZERO LAG TIME

Fig. 3 Boundary conditions of the generalized reach

2. Nitrogenous BOD loadings with zero lag time are considered as point inputs at the upstream end of a reach. (The "virtual" input point is determined by the actual lag time, the stream velocity and the real input location.)
3. Groundwater, surface runoff, and distributed waste flows are input uniformly along a reach.
4. Diversion flows are located at the downstream end of a reach.
5. All geometrical, hydrological, biological, chemical and physical parameters affecting BOD and DO levels are constant within a reach.

The sources and sinks included in any given reach are two types of biological oxygen demand (i.e., carbonaceous and nitrogenous), sedimentation of BOD, deoxygenation, atmospheric reaeration, algal photosynthesis and

respiration, and benthic oxygen demand. These factors have been generally recognized and included by many investigators<sup>1-7</sup> in river model studies. The formulation and measurement of these factors is beyond the scope of this paper. This material is presented in Reference 9.

A waterway is segmented, according to the definition of the generalized reach, into a number of reaches at points where there are significant changes in boundary conditions or in the values of the parameters involved in the source and sink terms. Examples of changes that would require segmenting a river are the introduction of (1) a dam, (2) a point waste discharge, (3) a change in the stream cross-sectional area, (4) a change in flow (i.e., the entry of a tributary or the branching off of a diversion), (5) a change of reaction rate coefficients, or (6) a change of temperature.

The computational program is used to calculate the BOD and DO profiles reach by reach, starting at the upstream end of the river and proceeding downstream. The output from an upstream reach is the input to the next downstream reach. By use of the multiple-reach technique, the distribution of BOD and DO for the entire waterway is obtained.

#### *Mathematical Formulation*

The distribution of BOD or DO in the generalized reach is obtained by integrating the differential equation of mass transport of a single chemical species in a fluid medium. These differential equations can be derived from the concept of the conservation of mass by making a material balance over an elemental volume of a waterway, thus:

$$\text{Input} - \text{Output} + \text{Production} - \text{Losses} = \text{Accumulation} \quad \dots (2)$$

All terms in Equation (2) are expressed as rates, i.e., the quantity of the material per unit of time. The general form of the unit of these rates is pounds per day or milligrams per day.

The "input" and "output" terms are the sums of convective and dispersive transport across the upstream and downstream faces of the volume element, per unit time.

The "production" and the "losses" terms are rates at which material is produced and consumed by the reaction processes within the volume element. The "accumulation" term completes the inventory and accounts for any increase or decrease of material that takes place in the elemental volume. In other words, "accumulation" is equal to the time rate of change of the particular material within the segment volume.

Before using the material balance equation to formulate the mathematical models, some basic assumptions are discussed.

The first assumption is that the concentration of the substance (or pollutant) being investigated is uniform over the cross-sectional area of the river. That is, the concentration of the substances varies only in the longitudinal

direction. Therefore, only the longitudinal profiles of the cross-sectional area-averaged concentrations of the substance are generated, and hence the mathematical models are one-dimensional.

Secondly, it is assumed that longitudinal dispersion in the direction of flow can be neglected. In other words, mass transport is assumed to be due to convection only. This does not introduce any significant error since, in most streams and rivers, mean velocity is the major component of the flux.

Finally, it is assumed that a steady state condition exists in the system. This means that at any point in a waterway the quantitative change of the substance with respect to time is zero. A steady state condition may occur in a river during the late summer and early fall, when the flow is relatively low and steady. This period is certainly of most interest to a sanitary engineer because the minimal dilution and high temperature combine to produce the most critical waste assimilation conditions.

Based upon these assumptions, material balances are made (1) at the beginning plane of a reach, (2) between two consecutive planes within a reach, and (3) at the ending plane of a reach.

In Figure 3, a dam, a tributary, and a point waste discharge are shown at the beginning plane of a reach. By segmenting the stream so that waste discharges and tributaries are located downstream of a dam, the material balance at this plane is separated into two steps. In the first step, the upstream DO concentration is modified by the reaeration resulting from passage of water over the dam and the upstream BOD concentration is unchanged. The output from this balance is the upstream input into the second step where the loadings from the waste discharge and the tributary are added to give the initial BOD concentration, DO concentration and flow of the reach.

The dam reaeration model used in the first step was developed by Quirk and Eder<sup>6</sup> and is formulated as follows:

$$C_e = \frac{f_d(Q) \cdot \theta^{(T-20)} \cdot C_s + C_i}{f_d(Q) \cdot \theta^{(T-20)} + 1} \quad \dots (3)$$

where:

$C_e$  = effluent (or downstream) DO concentration

$f_d(Q)$  = a function of the river flow,  $Q$ , over the dam

$\theta$  = temperature coefficient; = 1.02

$T$  = temperature, °C

$C_s$  = saturation DO concentration

$C_i$  = influent (or upstream) DO concentration

The BOD and the DO models for the second step are, respectively:

$$L_o = \frac{Q_1 L_1 + Q_2 L_2 + Q_3 L_3}{Q_o} \quad \dots (4)$$

and

$$C_o = \frac{Q_1 C_1 + Q_2 C_2 + Q_3 C_3}{Q_o} \quad \dots (5)$$

Assuming that the densities of these flows (i.e., river, tributary and waste discharge) are approximately equal, the following relationship is obtained:

$$Q_o = Q_1 + Q_2 + Q_3 \quad \dots (6)$$

Material balances of carbonaceous BOD and of DO over an elemental volume within a reach are shown in Figure 4. Using Equation (2), four linear, ordinary differential equations of the first order are obtained. The derivation process includes: (1) dividing through by  $[-A\Delta\bar{X}]$ , (2) rearranging terms, (3) taking limits as  $\Delta X \rightarrow 0$  and (4) transforming terms such as flow velocity  $U = Q/A = dX/dt$ , oxygen deficit  $D = C_s - C$  and hydraulic radius  $R_H = A/P_w$ . The differential equations and solutions are presented in Table I.

As noted in Figure 4, when the incremental flow,  $Q_7$  is equal to zero, the uniformly distributed waste load,  $L_7$ , does not have to be equal to zero. With the model structured in this manner it is possible to simulate the situation where organic matter is introduced into a stream without a concomitant increase in flow. Examples of this are: (1) the death of algae, (2) scour of bottom deposits, and (3) resuspension of sludge particles by gas bubbles resulting from anaerobic decomposition. Therefore, when  $Q_7 \neq 0$ ,  $L_7$  is expressed as mass per unit volume, and when  $Q_7 = 0$ ,  $L_7$  is expressed as mass per unit time per unit length.

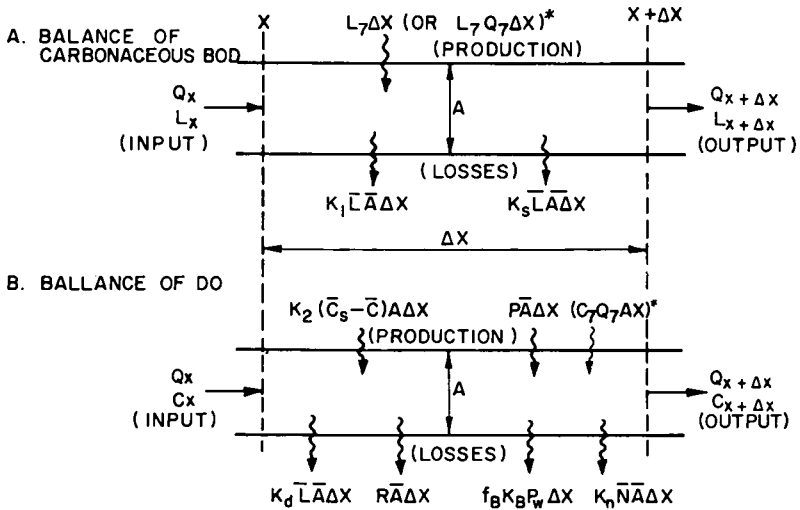
Furthermore, if the  $Q_7$ 's are set to zero in Equations (9) and (10), it can be shown that these equations cannot be converted to their corresponding models for constant flow, i.e., Equations (7) and (8), respectively, and the resulting equations, which are independent of distance, are not correct.

The reason this occurs is that the spatially varied flow is originally defined as:

$$Q_x = Q_o + Q_7 X \quad \dots (11)$$

If  $Q_7$  is set to zero in these equations, the distance variable,  $X$ , is also canceled along with  $Q_7$ . For this reason, both constant flow and spatially





\* WHEN INCREMENTAL FLOW  $Q_7 \neq 0$ , THE TERM,  $L_7 Q_7 \Delta X$ , SHOULD REPLACE THE TERM,  $L_7 \Delta X$ , AND ADD THE TERM  $C_7 Q_7 \Delta X$  INTO DO BALANCE.

LEGEND

- Q = FLOW
- L = BOD CONCENTRATION
- C = DO CONCENTRATION
- $K_1$  = BIOLOGICAL OXIDATION RATE COEFFICIENT
- $K_s$  = SEDIMENTATION RATE COEFFICIENT
- $K_2$  = ATMOSPHERIC REAERATION RATE COEFFICIENT
- $C_s$  = SATURATED DO CONCENTRATION
- P = PHOTOSYNTHETIC OXYGEN PRODUCTION RATE COEFFICIENT
- R = ALGAL RESPIRATION RATE COEFFICIENT
- $K_d$  = DEOXYGENATION RATE COEFFICIENT
- $K_n$  = NITROGENOUS OXIDATION RATE COEFFICIENT
- N = NITROGENOUS BOD CONCENTRATION
- $f_B$  = FRACTION OF BOTTOM COVERED BY SLUDGE
- $K_B$  = BENTHAL OXYGEN UPTAKE RATE COEFFICIENT
- $P_w$  = WETTED PERIMETER

SUBSCRIPTS:

- X = AT PLANE X
- $X + \Delta X$  = AT PLANE  $(X + \Delta X)$
- 7 = RELATED TO DISTRIBUTED WASTE

SYMBOLS:

- "—" = AVERAGE VALUE

Fig. 4 Mass balance over volume element,  $A \Delta X$ , within a reach. (One dimensional, steady-state, no longitudinal dispersion)

varied flow models of BOD and DO are included in the final computerized simulation models.

The formulation for the nitrogenous oxygen demand is not given in Fig-ure 4 since it is analogous to that for the carbonaceous matter. Since nitro- genous BOD is generally removed by biological oxidation, the mathemati-

TABLE I

ONE-DIMENSIONAL, STEADY-STATE BOD AND DO MODELS

Incremental Flow, $Q_7$	Ordinary Differential Equation	Boundary Condition	Solution
$Q_7 = 0$ (Constant flow condition)	BOD Model: $\frac{dL}{dt} + (k_1 + k_d)L = \frac{L_0}{A}$	$L = L_0 \text{ at } t = 0$	$L = \frac{L_0}{A(k_1 + k_d)} \left[ 1 - e^{-(k_1 + k_d)t} \right] + L_0 e^{-(k_1 + k_d)t}$ <p style="text-align: right;">(7)</p>
	DO Model: $\frac{dD}{dt} + k_2 D = k_d L + k_n N - \frac{f_B K}{R_H} - (R - P)$	$D = D_0 \text{ at } t = 0$	$D = \frac{K_d L_0}{A(k_1 + k_d)k_2} (1 - e^{-k_2 t}) - \frac{K_d L_0}{A(k_1 + k_d)(k_2 - (k_1 + k_d))} [e^{-(k_1 + k_d)t} - e^{-k_2 t}] + \frac{K_d L_0}{[k_2 - (k_1 + k_d)]} [e^{-(k_1 + k_d)t} - e^{-k_2 t}] + \frac{K_n N_0}{k_2 - k_n} (e^{-k_n t} - e^{-k_2 t}) + \frac{f_B K}{k_2 R_H} (1 - e^{-k_2 t}) + \frac{R - P}{k_2} (1 - e^{-k_2 t}) + D_0 e^{-k_2 t}$ <p style="text-align: right;">(8)</p>
$Q_7 \neq 0$ (Spatially varied flow condition)	BOD Model: $\frac{dL}{dx} + \frac{1}{(Q_0 + Q_7 x)} [A(k_1 + k_d) + Q_7] L = \frac{L_0 Q_7}{Q_0 + Q_7 x}$	$L = L_0 \text{ at } x = 0$	$L = \frac{L_0}{1 + A(k_1 + k_d)/Q_7} \left[ 1 - \left( \frac{Q_0}{Q_0 + Q_7 x} \right)^{1 + A(k_1 + k_d)/Q_7} \right] + L_0 \left( \frac{Q_0}{Q_0 + Q_7 x} \right)^{1 + A(k_1 + k_d)/Q_7}$ <p style="text-align: right;">(9)</p>
	DO Model: $\frac{dC}{dx} + \frac{(Q_7 + AK_2)}{(Q_0 + Q_7 x)} C = \frac{A}{(Q_0 + Q_7 x)} \left[ K_d L - k_n N - \frac{f_B K}{R_H} - (R - P) + \frac{C_7 Q_7}{A} + K_2 C \right]$	$C = C_0 \text{ at } x = 0$	$C = \frac{AK_d L_0}{(PK1)(AK_2 + Q_7)} [(QF)(PK2) - 1] - \frac{K_d L_0}{(PK1)(k_2 - (k_1 + k_d))} [(QF)(PK2) - (QF)(PK1)] + \frac{K_d L_0}{k_2 - (k_1 + k_d)} [(QF)(PK2) - (QF)(PK1)] + \frac{K_n N_0}{k_2 - k_n} [(QF)(PK2) - (QF)(PK1)] + \frac{A(R - P)}{AK_2 + Q_7} [(QF)(PK2) - 1] + \frac{A f_B K}{(AK_2 + Q_7) R_H} [(QF)(PK2) - 1] - \frac{C_7 Q_7}{AK_2 + Q_7} [(QF)(PK2) - 1] + C_0 (QF)(PK2)$ <p style="text-align: right;">(10)</p>

Note:  $t =$  travel time  
 $= X/U$

Where:

$$QF = \frac{Q_0}{Q_0 + Q_7 x} \cdot PK2 + 1 + AK_2/Q_7$$

$$PK1 = 1 + A(k_1 + k_d)/Q_7, PKN = 1 + AK_n/Q_7$$

cal model for predicting its concentrations in a reach is written:

1) for the constant flow condition

$$N = N_0 e^{-k_n t} \dots (12)$$

and

2) for the spatially varied flow condition

$$N = N_0 \left( \frac{Q_0}{Q_0 + Q_7 X} \right)^{(1 + AK_n / Q_7)} \dots (13)$$

Since carbonaceous and nitrogenous oxidation are both assumed to be first order reactions, the total BOD is equal to the sum (or a linear combination) of these two components.

The material balance at the downstream end of a reach is exactly the same as that at the upstream end. In Figure 3, one diversion flow,  $Q_4$ , which can be treated as a negative waste discharge, is located at the downstream plane. Furthermore, the BOD and the DO concentrations, in the diversion flow are assumed to be the same as those in the river flow. Therefore, a volume balance relationship is obtained as follows:

$$Q_{\text{downstream}} = Q_1 - Q_4 \dots (14)$$

*Diurnal Variation of Photosynthesis and the One-Dimensional Unsteady State DO Models*

Photosynthetic activity and the oxygen production rate are assumed to vary with sunlight intensity during the day and to be zero at night. If the sunlight period is assumed to be 12 hours, the periodic variation of the oxygen production rate may be approximately described by a Fourier series, i.e.,

$$P_t = P_m \left( \frac{1}{\pi} + \frac{1}{2} \sin \frac{\pi t'}{p} - \frac{2}{3\pi} \cos \frac{2\pi t'}{p} \right) \dots (15)$$

where:

$P_t$  = photosynthetic oxygen production rate at time  $t'$

$P_m$  = maximum photosynthetic oxygen production rate during the period

$t'$  = time

$p$  = period of sunlight = 0.5 day

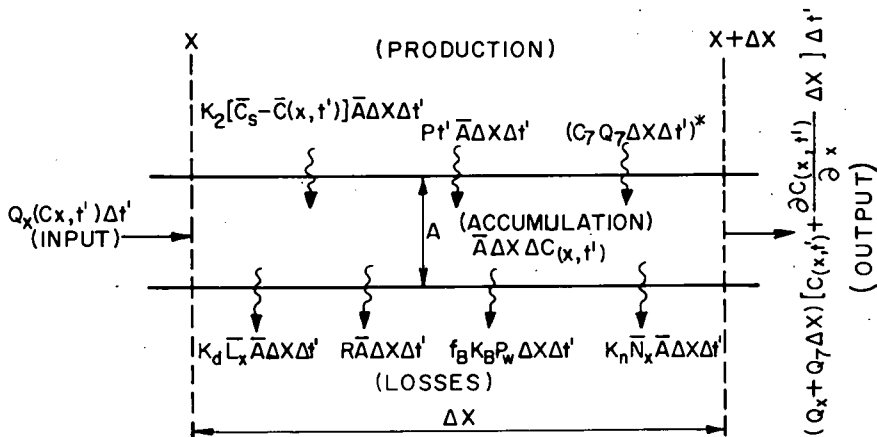
If the term  $P$  is replaced by the term  $P_t$  in the steady-state DO models, the unsteady-state DO models can be constructed. The one-dimensional, unsteady-state DO models are developed by making a material balance over an elemental volume,  $A\Delta X$ , in a waterway and over an elemental time interval,  $\Delta t'$ . Figure 5 illustrates these balances. Following a derivation process similar to the one used in developing the steady-state models, two quasi-linear partial differential equations of the first order are obtained. These equations are solved by using Laplace transforms and their solutions are presented in Table II.

The only difference between the unsteady-state and the steady-state DO models is that the photosynthetic oxygen production rates in the former are expressed as time functions. Therefore, the one-dimensional unsteady-state models can only be used to determine the diurnal temporal and spatial DO distribution in a river which is otherwise at steady-state except for the daily variation in sunlight.

*Atmospheric Reaeration and BOD in Rapids Sections*

Quirk and Eder<sup>6</sup> proposed the following model for a rapids section:

$$C_e = \frac{f_r(Q) \cdot \theta^{(\tau-20)} \cdot C_s + C_i + \Delta L}{f_r(Q) \cdot \theta^{(\tau-20)} + 1} \dots (18)$$



\* THIS TERM IS INCLUDED WHEN THE INCREMENTAL FLOW  $Q_7 \neq 0$

Fig. 5 Mass balance of DO over an elemental volume,  $A\Delta X$ , and over an elemental time interval,  $\Delta t'$  (One dimensional, unsteady-state, no longitudinal dispersion)

TABLE II - ONE-DIMENSIONAL, UNSTEADY-STATE DO MODELS

Incremental Flow, $Q_7$	Partial Differential Equation	Boundary and Initial Conditions	Solution
$Q_7 = 0$ (Constant flow condition)	$\frac{\partial D(X,t')}{\partial t'} = -U \cdot \frac{\partial D(X,t')}{\partial X} - K_2 D(X,t') - P_D \left( \frac{1}{H} \right)$ $+ h \sin \frac{\pi X}{P} - \frac{2}{3L} \cos \frac{2\pi X}{P} \cdot K_d L_x$ $+ R \cdot \frac{f_D Q_0}{K_H} \cdot K_N N_x$	$D(0,t') = D_0$ at a given $t'$  $D(X,0) = D$ computed by Equation (8)	$D(X,t') =$ Right-side terms of Equation (8) $\cdot P_m f(X,t')$  Where: $f(X,t') = \frac{1}{2(K_2^2 - u^2/P^2)} \left\{ \left[ K_2 \sin \frac{\pi X}{P} - \frac{u}{P} \cos \frac{\pi X}{P} \right] \right.$ $\left. - \left[ K_2 \sin \frac{u(t' - \frac{X}{U})}{P} - \frac{u}{P} \cos \frac{u(t' - \frac{X}{U})}{P} \right] e^{-\frac{K_2 X}{U}} \right\}$ $+ \frac{2}{3n(K_2^2 - 4n^2/P^2)} \left[ K_2 \cos \frac{2\pi X}{P} + \frac{2n}{P} \sin \frac{2\pi X}{P} \right]$ $- \left[ K_2 \cos \frac{2u(t' - \frac{X}{U})}{P} + \frac{2n}{P} \sin \frac{2u(t' - \frac{X}{U})}{P} \right] e^{-\frac{K_2 X}{U}}$
$Q_7 \neq 0$ (spatially varied flow condition)	$\frac{\partial C(X,t')}{\partial t'} = -\frac{Q_0}{A} \cdot \frac{\partial C(X,t')}{\partial X} - Q_2 C(X,t') - K_2 (C_0 - C(X,t'))$ $+ P_m \left( \frac{1}{H} \cdot h \sin \frac{\pi X}{P} - \frac{2}{3L} \cos \frac{2\pi X}{P} \right)$ $+ \frac{C_0 Q_2}{A} - K_d L_x - R \cdot \frac{f_D Q_0}{K_H} \cdot K_N N_x$	$C(0,t') = C_0$ at a given point  $C(X,0) = C$ computed by Equation (10)	$C(X,t') =$ Right-side terms of Equation (10) $\cdot P_m f(X,t')$  Where: $f(X,t') = \frac{1}{2[(Q_2/A + K_2)^2 - u^2/P^2]} \left\{ \left[ (Q_2/A + K_2) \sin \frac{\pi X}{P} - \frac{u}{P} \cos \frac{\pi X}{P} \right] \right.$ $\left. - \left[ (Q_2/A + K_2) \sin \frac{u}{P} \left[ t' + \frac{A}{Q_2} \ln \left( \frac{Q_0}{Q_0 + Q_2 V} \right) \right] \right] \right\}$ $\cdot \cos \frac{u}{P} \left[ t' + \frac{A}{Q_2} \ln \left( \frac{Q_0}{Q_0 + Q_2 V} \right) \right] \cdot \left( \frac{Q_0}{Q_0 + Q_2 V} \right)^{(1 + AK_2/Q_2)}$ $- \frac{2}{3n[(Q_2/A + K_2)^2 - 4n^2/P^2]} \left[ (Q_2/A + K_2) \cos \frac{2\pi X}{P} \right.$ $\left. + \frac{2n}{P} \sin \frac{2\pi X}{P} \right] - \left[ (Q_2/A + K_2) \cos \frac{2u}{P} \left[ t' + \frac{A}{Q_2} \ln \left( \frac{Q_0}{Q_0 + Q_2 V} \right) \right] \right]$ $+ \frac{2n}{P} \sin \frac{2u}{P} \left[ t' + \frac{A}{Q_2} \ln \left( \frac{Q_0}{Q_0 + Q_2 V} \right) \right] \cdot \left( \frac{Q_0}{Q_0 + Q_2 V} \right)^{(1 + AK_2/Q_2)}$

where:

$C_0, \theta, T, C_s, C_i =$  as defined in Equation (3)

$f_r(Q) =$  similar to  $f_r(Q)$  but for a rapids section

$\Delta L =$  DO consumed due to deoxygenation

This equation is analogous to the equation for dam reaeration except that it includes a term for biological deoxygenation.

If the length of a rapids is short in comparison to the river section of interest, biological deoxygenation can be considered negligible due to the short travel time in the rapids as compared to the travel time in the reach. In that case, the term  $\Delta L$  can be eliminated from Equation (18), and the

equation takes the same form as Equation (3), i.e., the rapids is considered as a point oxygen source analogous to a dam.

On the other hand, if this assumption cannot be made or the rapids has a rock bed with biological growths, biological deoxygenation can be significant since the rapids will act like a trickling filter. In that case, the rapids section should be considered as a reach rather than a point.

#### *Temperature*

All the reaction rate coefficients used in the model are adjusted to account for temperature effect according to the following general expression:

$$K_T = K_{20} \cdot \theta^{(T-20)} \quad \dots (19)$$

where:

$K_T$  = coefficient at temperature  $T$  °C

$K_{20}$  = coefficient at 20° C

$\theta$  = temperature coefficient

$T$  = temperature, °C

#### *Distribution of Stream Flow*

The stream flow measured at a given gaging station on a river is equal to that measured at the station immediately upstream plus discharges between the two points, minus all diversions, and plus runoff. The total amount of runoff is arrived at by taking a mass balance between the two stations.

When modeling a large river, in which the stream flow is much greater than the total waste discharge, the runoff term may not be important. However, this is usually not the case in small rivers. If it should be required, the runoff flow can be distributed into each reach by application of the variable,  $Q_7$ , defined in Figure 3.

To accomplish this, the uniformly distributed flow,  $Q_7$ , is computed by the following equation:

$$Q_7 = \frac{F1(A2-A1)}{|X2-X1|} \quad \dots (20)$$

where:

$F1$  = areal distribution flow.

$A1$  and  $A2$  = total drainage area at the upstream and downstream ends of a reach, respectively.

$X1$  and  $X2$  = the river mile at the upstream and downstream ends of a reach, respectively.

The areal distribution flow,  $F1$ , in Equation (20) is defined as follows:

$$F1 = \frac{QG2 - \Sigma QT - \Sigma QP + \Sigma QD - QG1}{DAG2 - \Sigma DAT - DAG1} \dots (21)$$

where:

$QG1$  and  $QG2$  = known flow at the upstream and the downstream gaging station, respectively.

$\Sigma QT$ ,  $\Sigma QP$  and  $\Sigma QD$  = summation of all tributary flows, waste discharges and diversions, respectively, between the two gaging stations.

$DAG1$  and  $DAG2$  = drainage area at the upstream and downstream gaging stations, respectively.

$\Sigma DAT$  = summation of the drainage areas for all tributaries between the two gaging stations.

#### *Manipulation of the Lag Time of Nitrification*

Courchaine<sup>3</sup> reports that second stage (nitrogenous) oxidation generally occurs from 1.0 to 10 days after the first stage (carbonaceous) oxidation starts, depending on the degree of treatment. This lag time influences when or where a nitrogenous BOD loading starts nitrification in a waterway.

In the stream simulation model, when the travel time of a nitrogenous BOD loading is equal to its lag time, a new segment is started and the loading is input at that point as an oxygen sink.

Uniformly distributed nitrogenous BOD loadings with lag time are handled in the model in the same way as point loads. This is achieved by converting the distributed loadings to an equivalent series of point discharges. Naturally, the more the distributed loading is segmented, the better the approximation.

#### **Model Verification**

The 1969 summer survey data of the Housatonic River were used to set up and verify the computerized simulation model. Second state (nitrogenous) oxidation was not considered because the results from the long-term BOD studies did not indicate that it was present.

This river was segmented into 58 reaches in accordance with the generalized reach concept presented previously. The input data for each of these reaches are presented in Table III. The model results and field survey data are shown in Figure 6.

As shown in Figure 6, the predicted and measured BOD and DO profiles differed somewhat. However, the results were considered to be satisfactory as far as the limited data allowed.

TABLE III - INPUT PARAMETERS

Reach No.	River Mile*	Drainage Area (sq miles/day)*	Mean Depth (ft)	Travel Time (hr)	River Flow (cfs)**	Temp. (°C)	Oxidation Rate Coeff. K (day <sup>-1</sup> )	Waste Discharge		Photo-synthetic oxygen (mg/l/day)	Algal Respiration (mg/l/day)	Dam f <sub>3</sub> (Q)	Rapids f <sub>2</sub> (Q)	Bottom Deposits	
								Flow (cfs)	5-Day BOD (mg/l)					% covered	Oxygen Uptake Rate p/l-sq.m/day
0	69	18.0	-	0.00	-	-	-	-	-	-	-	-	-	-	-
1	67.4	23.2	2.0	5.04	4.6	15.6	0.25	-	-	-	-	-	-	50	2.5
2	67.3	23.9	3.8	0.31	6.1	16.7	0.25	0.23	20	-	-	-	-	100	2.5
3	66.5	25.3	1.9	0.85	6.3	16.1	0.28	-	-	-	-	0.30	0.87	-	-
4	63.7	30.2	1.9	2.98	6.6	16.1	0.28	-	-	-	-	-	3.46	-	-
5	62.9	52.3	2.5	12.25	7.8	16.1	0.33	-	-	4.05	1.20	-	-	-	-
6	62.6	52.5	4.5	0.56	13.3	17.2	0.33	-	-	-	-	0.07	-	-	-
7	62.4	54.5	4.0	0.37	15.3	18.3	0.33	-	-	-	-	0.07	-	-	-
8	61.4	54.9	4.0	1.89	13.8	18.3	0.32	-	-	-	-	0.04	-	-	-
9	61.2	55.3	3.7	0.31	13.9	18.3	0.31	-	-	-	-	-0.02	-	-	-
10	60.9	56.1	3.2	0.63	14.0	18.3	0.31	-	-	-	-	0.04	-	-	-
11	60.7	56.4	3.0	0.31	21.0	18.3	0.30	7.70	41	-	-	-	-	100	2.5
12	59.3	60.7	2.5	2.5	22.0	18.3	0.28	-	-	-	-	0.06	-	-	-
13	57.1	66.5	2.8	3.9	25.5	17.8	0.25	0.39	162	-	-	-	-	-	-
14	56.9	67.3	3.2	0.4	28.3	17.8	0.24	2.72	31	-	-	-	-	100	2.5
15	55.4	68.3	3.5	2.6	35.0	17.8	0.23	6.41	89	-	-	-	-	100	2.5
16	55.0	129.0	4.3	0.6	57.9	17.8	0.28	-	-	-	-	-	-	-	-
17	53.8	131.4	4.5	3.3	58.6	17.8	0.35	-	-	3.22	1.04	-	-	-	-
18	52.8	141.0	5.0	3.14	62.1	17.8	0.37	-	-	2.96	1.04	-	-	-	-
19	50.9	142.4	5.5	6.06	62.8	17.8	0.36	-	-	2.73	1.04	-	-	-	-



20	49.1	143.4	5.7	6.10	74.4	18.3	0.35	11.4	20	2.64	1.04	-	50	2.5
21	48.1	144.0	5.7	5.00	74.8	18.3	0.32	0.1	160	2.64	1.04	-	10	2.5
22	47.5	153.0	5.7	3.00	78.2	18.3	0.27	-	-	2.64	1.04	-	-	-
23	46.3	161.4	6.0	6.00	81.2	18.6	0.14	-	-	2.53	1.04	-	-	-
24	45.7	164.2	6.0	9.50	82.4	18.9	0.08	-	-	2.53	1.04	-	100	2.5
25	45.1	166.2	6.0	9.50	83.1	19.2	0.08	-	-	2.53	1.04	-	100	2.5
26	45.0	166.2	7.0	0.20	83.4	19.4	0.15	-	-	2.20	1.04	0.08	-	-
27	43.7	168.6	2.7	2.52	83.6	19.4	0.20	0.16	32	4.58	1.04	-	-	-
28	43.6	168.6	2.7	0.20	84.8	19.4	0.25	0.34	142	4.58	1.04	-	-	-
29	43.2	168.7	2.7	0.78	85.0	19.4	0.26	0.16	150	4.58	1.04	-	-	-
30	43.0	177.2	3.0	0.40	88.1	19.4	0.26	-	-	4.30	1.04	-	-	-
31	42.0	180.8	3.0	1.94	84.9	19.4	0.26	-	-	4.30	1.04	-	-	-
32	41.3	181.5	2.7	1.62	96.1	19.4	0.26	10.77	106	4.58	1.04	0.15	100	2.5
33	41.1	184.6	2.6	0.65	97.6	19.4	0.26	-	-	4.68	1.04	-	100	2.5
34	40.0	187.1	3.1	3.60	97.6	19.4	0.26	-	-	4.21	1.04	0.05	100	2.5
35	39.2	201.3	4.0	2.60	103.6	19.4	0.26	-	-	3.53	1.04	-	-	-
36	38.4	201.6	4.7	2.60	104.8	19.4	0.26	1.0	19.0	3.11	1.04	-	-	-
37	37.5	206.9	5.5	2.92	106.0	19.4	0.25	-	-	2.73	1.04	-	-	-
38	36.3	229.6	6.0	3.9	114.9	19.4	0.24	-	-	2.53	1.04	-	-	-
39	35.5	239.2	7.0	2.60	118.7	19.4	0.23	2.92	126	2.20	1.04	-	-	-
40	35.3	239.6	6.0	0.64	119.2	18.9	0.23	-	-	2.53	1.04	0.10	-	-
41	34.4	239.8	6.0	2.86	120.5	18.9	0.23	2.86	80	2.53	1.04	-	100	2.5
42	32.7	243.4	3.7	5.54	121.5	18.9	0.21	-	-	3.73	1.04	-	-	-
43	30.5	257.8	8.2	7.00	126.9	18.9	0.20	-	-	1.89	1.04	-	-	-
44	28.6	274.9	7.0	5.74	133.4	18.9	0.19	0.3	75	2.20	1.04	-	-	-

TABLE III - INPUT PARAMETERS CONT'D

Reach No.	River Mile*	Drainage Area (sq miles/day)*	Mean Depth (ft)	Travel Time (Hr)	River Flow (cfs)**	Temp. (°C)	Oxidation Rate Coeff. K (day <sup>-1</sup> )	Waste Discharge		Photo-synthetic oxygen (mg/l/day)	Algal Respiration (mg/l/day)	Dam f <sub>d</sub> (Q)	Rapids f <sub>r</sub> (Q)	Bottom Deposits	
								Flow (cfs)	5-Day BOD (mg/l)					% covered	Oxygen Uptake Rate g/sq.m/day
45	27.6	275.5	8.2	2.18	133.8	18.9	0.19	-	-	1.89	1.04	0.31	-	20	2.5
46	26.0	277.7	7.0	3.5	134.5	18.9	0.20	-	-	2.20	1.04	-	-	20	2.5
47	25.7	278.5	4.8	0.65	134.9	18.9	0.20	-	-	3.06	1.04	0.39	-	20	2.5
48	24.7	279.4	7.0	6.27	135.4	18.9	0.21	0.3	152	2.2	1.04	-	-	100	2.5
49	24.4	279.5	3.9	1.88	135.8	18.9	0.21	-	-	3.59	1.04	0.03	-	15	2.5
50	23.9	280.3	4.0	1.94	138.0	18.9	0.22	3.19	235	3.53	1.04	-	-	100	2.5
51	23.3	281.0	4.0	2.32	138.3	18.9	0.22	-	-	3.53	1.04	-	-	15	2.5
52	19.5	329.6	4.0	7.10	153.5	18.9	0.22	-	-	3.53	1.04	-	-	15	2.5
53	15.9	339.2	4.0	7.12	157.8	18.6	0.21	1.2	152	3.53	1.04	-	-	15	2.5
54	9.0	398.1	5.4	15.30	180.6	18.6	0.20	-	-	2.77	1.04	-	-	15	2.5
55	8.5	461.6	6.0	1.11	206.0	18.6	0.20	-	-	5.10	2.90	-	-	15	2.5
56	6.2	462.4	5.8	5.1	206.3	18.6	0.19	0.15	204	5.25	2.90	-	-	15	2.5
57	0.2	483.8	5.2	13.32	210.7	18.6	0.19	-	-	5.77	2.90	-	-	15	2.5
58	0.0	545.2	5.0	0.40	237.2	18.6	0.19	-	-	5.96	2.9	-	-	15	2.5

\*At the downstream end of a reach  
 \*\*At the upstream end of a reach

sq mile/day x 2.59 = sq km/day; ft x 0.3048 = m; cfs x 1.7 = cu.m/min.

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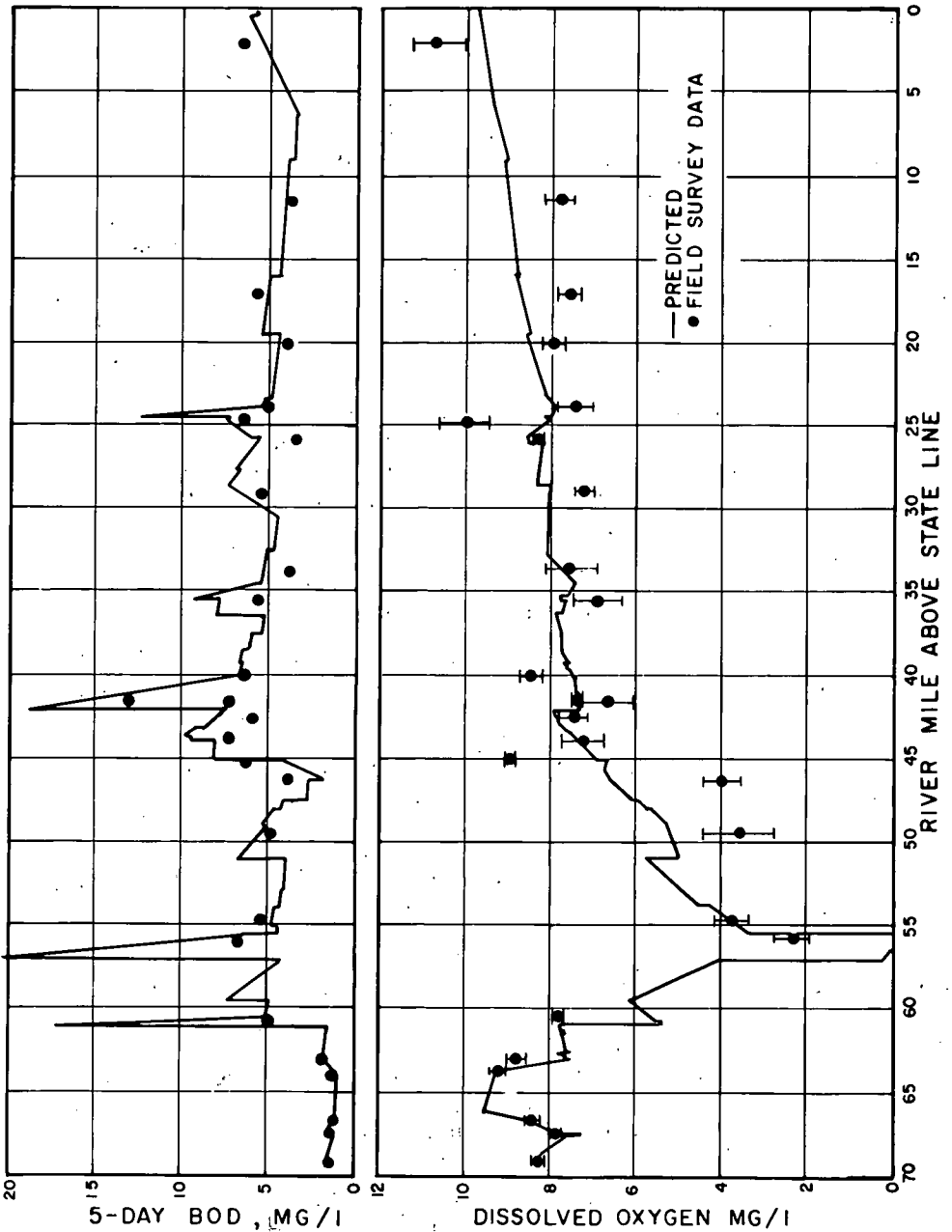


Fig. 6 Comparison of model predicted BOD and DO profiles with field survey data

As mentioned before, detailed geometrical data were not available for the Housatonic River. Therefore, most of the mean depths for the reaches used in the simulation model were estimated. Errors in estimating these depths will affect the evaluation of (1) atmospheric reaeration rate coefficients, (2) photosynthetic oxygen production rates, and (3) deoxygenation by bottom sludge deposits. The use of travel time and flow data to compute the river cross-sectional area and depth was tried, but the results were not successful.

Travel time for each reach was determined by assuming that it is linearly proportional to the distance between two survey stations. This assumption may hold true if the river section between the two survey stations has approximately uniform flow, velocity and geometry. The 14 primary sampling stations (see Figure 1) which were selected for the purpose of collecting water quality data probably did not meet this criterion, and hence, errors were introduced.

The measured DO and BOD values were based on samples taken from mid-channel and mid-depth of the river. How well these one-point measurements represent cross-sectional average values is questionable.

In short, the quality of the input data determines the accuracy and reliability of a simulation model.

### **Application of the Model for Future Water Quality Management**

A verified simulation model can be used to predict the spatial responses of BOD and DO to changes in the amount or strength of pollutants entering a stream, flow augmentation, discharge location or any of the other parameters that define the physical system under study. To demonstrate this, the model was used to evaluate the effect of various treatment schemes on the future water quality of the Housatonic River.

Massachusetts water quality standards specify that the dissolved oxygen concentration in the Housatonic River should be at least 5.0 mg/l. The 1969 river survey data indicate, however, that DO levels were below the standard between river mile points 57.1 and 46.3.

In order to meet the DO standard in the river for projected 1990 conditions, the Massachusetts Division of Water Pollution Control proposed two treatment plans (see Table IV). The projected 1990 waste loads are listed in this table and the design river flow conditions are listed in Table V.

Besides the waste loads, other parameters had to be projected for river conditions expected in 1990. The travel time and mean depth of a given reach for the critical flow conditions were estimated from the 1969 summer survey data and the following relationships:

$$t_1 = \frac{t_2}{\left(\frac{Q_1}{Q_2}\right)^{2/5}} \dots (22)$$

TABLE IV. - PROPOSED WASTE TREATMENT PLANS

Sources and Mile Point	Season	Characteristics of the Expected Waste Discharge			Percentage of BOD re- moval %
		Q cfs	BOD <sub>5</sub> mg/l	DO mg/l	
Upstream Background (62.9)	July & August	25.11	2.05	8.4	-
	Fall	6.54	2.05	11.3	-
Crane Mill (60.9)	July & August	7.9	19.24	2.0	75
	Fall				
G.E. (57.1)	July & August	12.7	12.0	2.0	83
	Fall				
Pittsfield S.T.P. (50.9)	July & August	10.0	23.2	3.0	93
	Fall	16.65	23.2	3.0	93
Lenox S.T.P. (45.0)	July & August	1.32	43.0	2.0	85
	Fall				
Lenoxdale S.T.P. (43.6)	July & August	0.43	43.0	2.0	85
	Fall				
Schweitzer Mill (42.0)	July & August	7.0	21.0	2.0	90
	Fall	7.0	52.5	2.0	75
LEE S.T.P. (39.2)	July & August	1.35	22.2	2.0	85
	Fall				

Q = Waste Discharge

BOD<sub>5</sub> = 5-day biochemical oxygen demand

DO = dissolved oxygen

S.T.P. = Sewage Treatment Plant

cfs x 1.7 = cu.m/min

TABLE V. - CRITICAL CONDITIONS FOR FLOW AND TEMPERATURE

Critical Condition	Date	Design Flow* cfs		Aerial Distributed Flow cfs/sq.miles	Temp. °C	Waste Loads
		at Coltsville	at Great Barrington			
No. 1	1990 July 6					Projected loads in 1990
	August	31.9	82	0.15	25	
No. 2	1990 Fall	15	65	0.12	10	Projected loads in 1990

\*The minimum average seven consecutive day low flow with a frequency of occurrence of once in ten years.

cfs x 1.7 = cu. m/min; cfs/sq miles x 0.657 = cu.m/min/sq km

and

$$H_1 = \left( \frac{Q_1}{Q_2} \right)^{3/5} \cdot H_2 \quad \dots (23)$$

where

$t_1, H_1, Q_1$  = travel time, mean depth and average flow of a given reach for future conditions, respectively.

$t_2, H_2, Q_2$  = travel time, mean depth, and average flow of a given reach for present conditions, respectively.

These relationships were developed from the Manning equation and are based on the following assumptions:

1. the hydraulic radius can be approximated by the depth;
2. the width of the river does not change significantly with changes in flow; and
3. the roughness and bottom slope of a given reach does not change significantly over the design period.

Travel times and depths estimated from these relationships were used to calculate river velocities and atmospheric reaeration coefficients for each reach under future conditions.

In the analysis of the proposed treatment schemes, algal photosynthesis and respiration were not included in the model runs. Present bottom sludge conditions were assumed to be indicative of 1990 conditions and present biological oxidation rate coefficients were used. This latter assumption should make the predicted DO profiles conservative because the future rate coefficients should be lower since the waste will receive at least secondary treatment.

The predicted DO profiles for the proposed waste treatment plans for 1990 summer and fall conditions are shown in Figure 7. These profiles indicate the following:

1. Temperature is an important factor in the determination of the critical condition with respect to oxygen depletion in the Housatonic River. Even though the river flow in the fall is less than that in the summer, the DO levels in the river for the latter period are lower because of higher temperatures.
2. The proposed plans will not insure that the DO standard is complied with throughout the river. Modification of these plans will be necessary.
3. The relatively large volumes of wastes with low DO concentrations introduced at various points in the river result in significant reductions in the DO level in the river just below these points. This means that aeration of certain effluents may be required in order to maintain the river DO level above the standard.

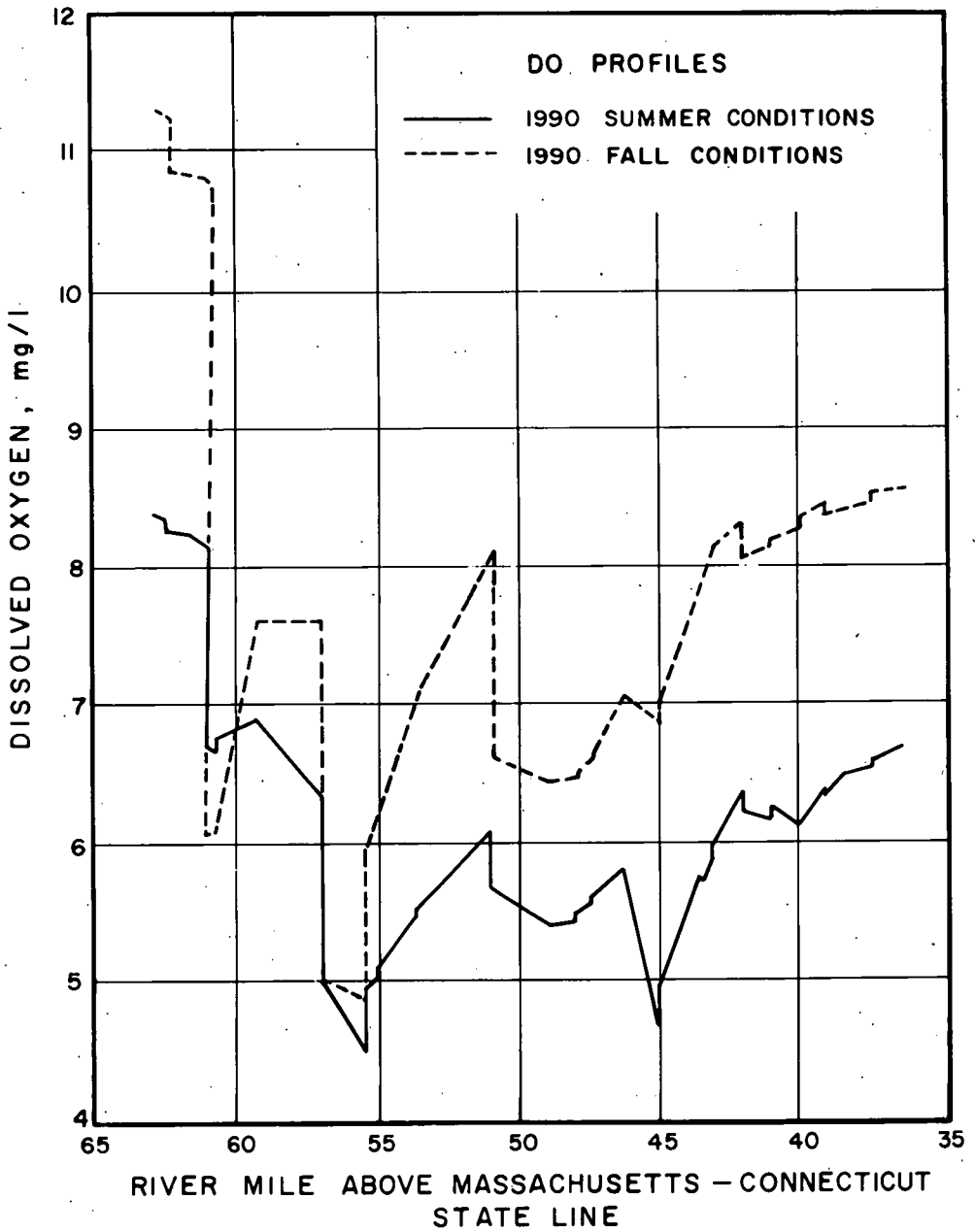


Fig. 7 The Housatonic River water quality control study



## Conclusion

A generalized stream simulation model was developed to provide the Division of Water Pollution Control of the Commonwealth of Massachusetts with the in-house capability to evaluate the assimilative capacities of Massachusetts' streams. The model can be used to predict BOD and DO profiles in a stream, and also to determine the fate of other pollutants. Furthermore, the model is useful in determining the location of sampling stations and in planning waste treatment facilities.

The model was verified with Housatonic River data and its generality is currently being tested in applications on other streams and rivers in the Commonwealth, e.g., Upper Taunton, Blackstone, Tenmile, Assabet and Millers.<sup>10</sup> We are informed that Upper Taunton has been verified, and that Blackstone and Tenmile are reaching the finishing stage of verification. Nitrogenous oxidation and diurnal variation of photosynthesis are included in these studies. The results are reported as being very satisfactory.

## Acknowledgments

### Credits

The study on which this paper is based was performed under a contract with the Division of Water Pollution Control of the Commonwealth of Massachusetts, by Quirk, Lawler & Matusky Engineers. The authors wish to thank Robert A. Norris, Associate and Director of Computer Applications of the firm, for his advice in constructing the computational program, and the staff in the Division of Water Pollution Control for their assistance in verifying the simulation model.

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## CARROLL A. FARWELL

1883-1973

Carroll A. Farwell, retired Treasurer and Director of Fay, Spofford & Thorndike, Inc., died at his home in Sharon, Massachusetts, on May 20, 1973.

Born on September 13, 1883, in Bolton, Massachusetts, Mr. Farwell was graduated from Massachusetts Institute of Technology with a degree of Bachelor of Science in Civil Engineering in 1906. After graduation, he was with the U. S. Reclamation Service in several western states, followed by an assignment in Spain as resident engineer on the construction of the then largest concrete dam in Europe. He joined Fay, Spofford & Thorndike in 1915 becoming a partner in 1922. He became Treasurer of the Partnership in 1951.

When the firm incorporated in 1956 he became Director and Treasurer. In 1958 he terminated as Treasurer. He remained as Consultant and Director of the firm until the year 1970.

Mr. Farwell was involved in the design and construction of some of the largest public works projects in New England, including the Boston Army Base, the Memorial Bridge in Springfield, Massachusetts, the Bourne and Sagamore Bridges over the Cape Cod Canal, and the Lake Champlain Bridge at Crown Point, New York; also, the New Jersey Turnpike and the New Jersey Garden State Parkway, as well as other turnpikes and toll roads. During and following World War II, he represented his firm in the construction of military complexes in Alaska, Newfoundland, and Labrador.

Mr. Farwell's interest in his profession is evidenced by his active participation in the engineering societies. He was president of each of the following: American Institute of Consulting Engineers (1952), the Engineering Societies of New England (1949), the Boston Society of Civil Engineers (1945), and the Northeastern Section of the American Society of Civil Engineers (1941). He was a leader in establishing the Northeastern Chapter of American Institute of Consulting Engineers and served as its first president in 1956.

Mr. Farwell was married to Alice Sargent on August 19, 1912. He is survived by three daughters, 14 grandchildren, and 7 great grandchildren.

Mr. Farwell will be remembered as a friendly gentleman, with great sensibility for the feelings of others, and as a true leader. His contribution to the engineering profession will be long remembered.