JOURNAL OF THE
BOSTON SOCIETY OF CIVIL ENGINEERS SECTION
AMERICAN SOCIETY OF CIVIL ENGINEERS

Volume 62 **January 1976** **Number 4**

FLOW APPROACHING FILTER WASHWATER TROUGHS*

By
Jonathan A. French**

Abstract

A potential flow model is used to study the flow of backwash water, laden with filtered solids, in its progress from the top of the filter bed to the washwater troughs above the filter. For backwashing effectiveness and economy, it is of interest to minimize the region of nonuniform flow between troughs. The best practical dimensions for promoting uniform flow beneath and between the troughs are determined. For this optimal trough design and two alternative designs, the paths and travel times of fluid elements and suspended particles are studied. Factors affecting trough spacing are also discussed.

Introduction

Granular-bed filtration is a popular process for preparing water for human consumption and for treating wastes, particularly industrial wastes, before discharge. Usually, the water is allowed to flow downward through a bed of sand, granulated coal, or a combination of these or other media, until the bed is sufficiently clogged with removed solids. Filtration is then temporarily halted while the filter is cleaned by backwashing, in which a relatively brief but intense upward flow of clean water scrubs the solids from the media grains and flushes them out of the filter. Backwash of sufficiently great intensity will cause the bed to fluidize, wherein the overall bed volume expands so that the filter grains do not rest packed against each other, but are held individually suspended in the upflow. Backwash water bearing the removed solids is removed from the filter unit either by gutters in the sidewall (Fig. 1a) or by a set of troughs spanning the filter (Fig. 1b). Troughs are common in North America only (1); elsewhere the side weir design finds favor.

Such filters have operated with great success in many parts of the world

*Paper submitted June 27, 1975 for consideration for the John R. Freeman Hydraulics Prize, and awarded Honorary Mention citation on January 21, 1976.

**Senior Engineer, Camp Dresser & McKee, Inc., Boston, Mass.

for more than a century. Design has evolved slowly, guided by a few rules of thumb, except for a few early thinkers such as Hazen, and for a recent surge in experimental and theoretical research. However, design changes since the turn of the century cannot be called profound; the 1909 Cleveland, Ohio, water filters as described by Ellms and Gettrust (2) bear a jolting resemblance to today's designs, the differences being principally in a few structural materials.

Most of the recent research has been on the filtration process itself (3), with some attention given to backwashing (4). Yet there is also a need for more reasoned study of some of the concomitant problems, such as the means by which the underdrains support the bed, collect the filtrate, and distribute the backwash; and the means by which the backwash water emerging from the top of the filter bed is collected and removed.

This paper will treat the upflow of backwash water from the bed to the troughs or sidewall weirs. After a brief discussion of fluidization and introduction of the potential flow model, the importance of uniformity in the upflow will be discussed. The potential flow model will be used to show (a) which sizes and shapes of troughs provide the most uniform flow; and (b) how suspended floc particles behave in a nonuniform upflow.

Fluidization

Fluidization is employed not only in water treatment but also in widespread applications in industry as a contact process. The phenomenon is described in general terms by Kunii and Levenspiel (5):

"Pass a fluid upward through a bed of fine particles . . . . At a low flow rate, fluid merely percolates through the void spaces between stationary particles. This is a *fixed bed.*

"With an increase in flow rate, particles move apart and a few are seen to vibrate and move about in restricted regions . . . .

"At a still higher velocity, a point is reached when the particles are all just suspended in the upward flowing gas or liquid. At this point, the frictional force between a particle and fluid counterbalances the weight of the particle, the vertical component of the compressive force between adjacent particles disappears, and the pressure drop through any section about equals the weight [per unit area] of the . . . . particles in that section. The bed is considered to be just fluidized and is referred to as an *incipiently fluidized bed* or a bed at *minimum fluidization.*

"In liquid-solid systems [such as in a water filter] an increase in flow rate above minimum fluidization usually results in a smooth, progressive expansion of the bed. Gross flow instabilities are damped and remain small, and large-scale bubbling or heterogeneity is not observed under normal condition. A bed such as this . . . . [may be] called a . . . . *smoothly fluidized bed* . . . .

"Gas-solid systems generally behave in quite a different manner. With an increase in flow rate beyond minimum fluidization, large instabilities with bubbling and channeling of gas are observed. At higher flow rates agitation becomes more violent and the movement of solids becomes more vigorous. In addition, the bed does not expand much beyond its volume at minimum fluidization . . . .

"Both gas and liquid fluidized beds are considered to be *dense phase fluidized beds* as long as there is a fairly clearly defined upper limit or surface to the bed. However, at a sufficiently high fluid flow rate the terminal velocity of the solids is exceeded, entrainment becomes appreciable, and [filter media] solids are carried out of the bed with the fluid stream. In this state we have a *disperse-*, *dilute-* or *lean-phase fluidized bed* . . . ."

In this paper we shall be concerned largely with a *smoothly fluidized* bed of solids in water, with a fairly clearly defined upper surface to the bed.

The Potential Flow Model

It is assumed that the reader is familiar with the technique of modelling ideal, two-dimensional flows with potential flow theory,* describing flow boundaries by conformal mapping. In this case, the usual assumptions will be made: that viscous effects and molecular and turbulent diffusion effects are negligible, and that the flow is incompressible and irrotational. Furthermore, it is assumed that neither the bed of fluidized filter particles nor the suspended floc particles being washed from the filter affects the flow pattern appreciably.

Consider a filter structure of infinite depth and without filter media. A section of such a structure, taken in a plane normal to the weirs, is shown in Fig. 1a for sidewall weirs and in Fig. 1b for troughs. In each figure, the total flow pattern is seen to be merely the pattern within a single vertical strip of width s, reflected and/or replicated one or more times.

Let us examine first the pattern in Fig. 1a, bounded by the x and y axes (note the unconventional downward orientation of the y axis) and by the vertical line x = s. If the top water surface is assumed plane despite the slight drawdown due to flow over the weir, and if the water depth over the weir is neglected, the flow in the region ($0 \leq x \leq s$, $y \geq 0$) may be modelled by a sink of strength $v_r s$ at the coordinate origin. The potential streamline function for this is

$$\psi = \frac{2 s v_r}{\pi} \tan^{-1} \left[\frac{\tan \frac{\pi}{2} \frac{x}{s}}{\tanh \frac{\pi}{2} \frac{y}{s}} \right] \tag{1}$$

where

$$-\frac{\partial \psi}{\partial x} = \frac{dy}{dt} = -v \tag{2}$$

and

$$\frac{\partial \psi}{\partial y} = \frac{dx}{dt} = u \tag{3}$$

and v_r is the upward velocity far down at great y, where the velocity may be expected to be uniform.

*Potential flow theory is discussed in many texts on hydrodynamics, applied mechanics, and hydraulic engineering. The treatment that is perhaps most appropriate to this paper is that of Streeter (6,7), who not only provides a rather complete theoretical exposition, but includes among his examples a flow configuration essentially equal to that used herein; compare Fig. 2 of this paper with one quadrant of either Fig. 4.24 in Reference (6) or Fig. 56 in Reference (7) — after rotating the Streeter figure through 90 degrees. A derivation of this paper's Equation 1 accompanies Fig. 56 of Reference (7).

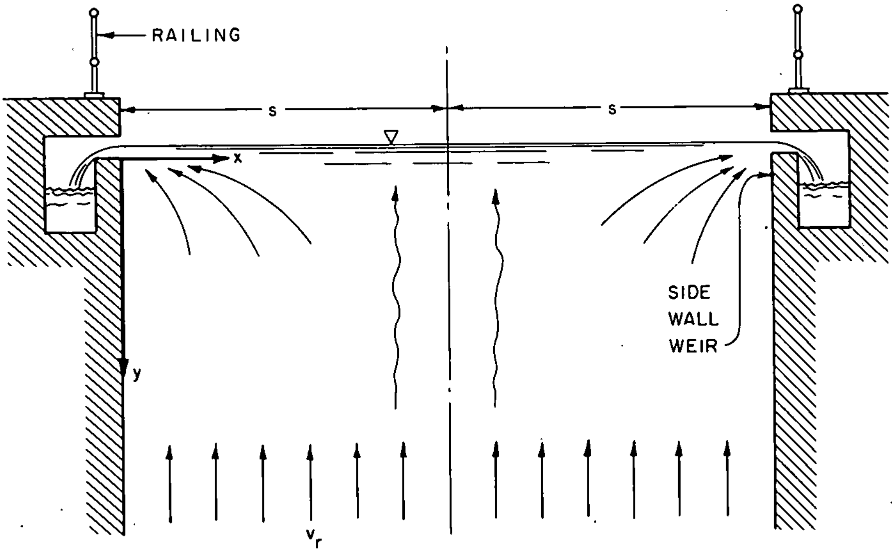

Figure 1a. Upflow to side wall weirs.

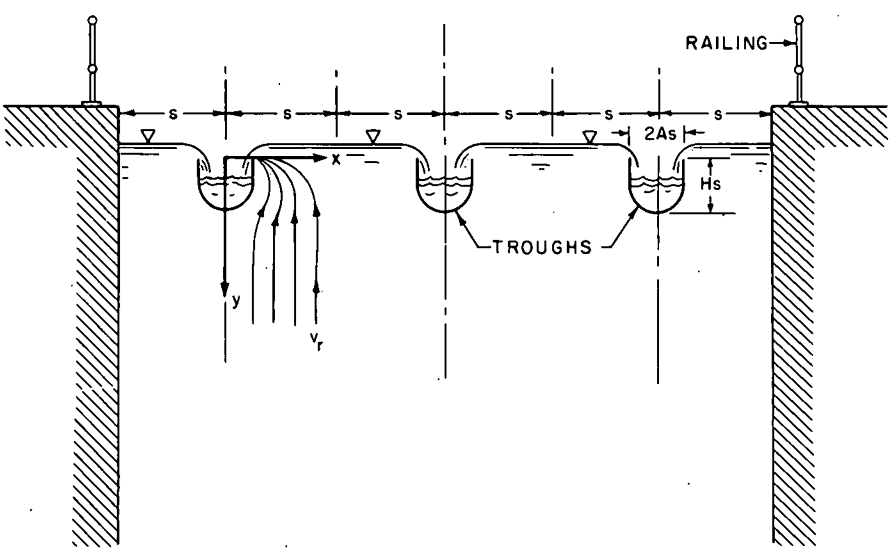

Figure 1b. Upflow to troughs.

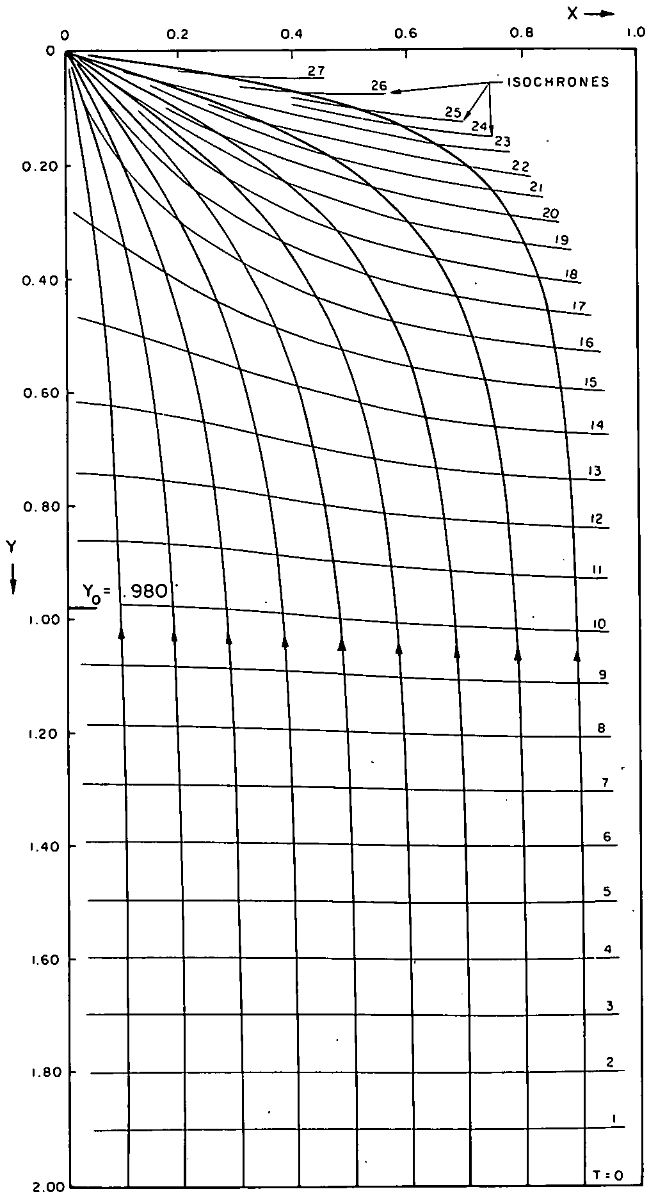

Figure 2. Pattern of flow approaching side wall weir (A=O, B=O).

It is convenient to introduce now the normalized variables $X = x/s$, $Y = y/s$, $T = v_r t/s$, $\Psi = \psi/v_r s$, and $V = v/v_r$, so that

$$\Psi = \frac{2}{\pi} \tan^{-1} \left[\frac{\tan \frac{\pi}{2} X}{\tanh \frac{\pi}{2} Y} \right] \tag{4}$$

$$-\frac{\partial \Psi}{\partial X} = \frac{dY}{dT} = -\frac{(\tanh \frac{\pi}{2} Y)\ (\sec \frac{\pi}{2} X)^2}{(\tanh \frac{\pi}{2} Y)^2 + (\tan \frac{\pi}{2} X)^2} \tag{5}$$

$$\frac{\partial \Psi}{\partial Y} = \frac{dX}{dT} = -\frac{(\tan \frac{\pi}{2} X)\ (\mathrm{sech}\ \frac{\pi}{2} Y)^2}{(\tanh \frac{\pi}{2} Y)^2 + (\tan \frac{\pi}{2} X)^2} \tag{6}$$

The normalized source strength is now unity, as is the upflow velocity at great Y. The flow pattern in terms of the normalized variables is shown in Fig. 2. Note the stagnation region near ($X = 1$, $Y = 0$).

The presence of round-bottom troughs is modelled by not one but two sinks of unit strength, located at ($X \pm A$, $Y = 0$), and two sources of half strength, located at ($X = 0$, $Y = \pm B$). The equations corresponding to Equations 4, 5, and 6 for the stream function and the two velocity components are:

$$\Psi = \frac{2}{\pi} \tan^{-1} \left[\frac{\tan \frac{\pi}{2} (X + A)}{\tanh \frac{\pi}{2} Y} \right] + \frac{2}{\pi} \tan^{-1} \left[\frac{\tan \frac{\pi}{2} (X - A)}{\tanh \frac{\pi}{2} Y} \right]$$

$$- \frac{1}{\pi} \tan^{-1} \left[\frac{\tan \frac{\pi}{2} X}{\tanh \frac{\pi}{2} (Y + B)} \right] - \frac{1}{\pi} \tan^{-1} \left[\frac{\tan \frac{\pi}{2} X}{\tanh \frac{\pi}{2} (Y - B)} \right] \tag{7}$$

$$\frac{dY}{dT} = -\frac{\tanh\frac{\pi}{2}Y \quad \sec^2\frac{\pi}{2}(X+A)}{\tanh^2\frac{\pi}{2}Y + \tan^2\frac{\pi}{2}(X+A)} - \frac{\tanh\frac{\pi}{2}Y \quad \sec^2\frac{\pi}{2}(X-A)}{\tanh^2\frac{\pi}{2}Y + \tan^2\frac{\pi}{2}(X-A)}$$

$$+\frac{1}{2}\cdot\frac{\tanh\frac{\pi}{2}(Y+B)\ \sec^2\frac{\pi}{2}X}{\tanh^2\frac{\pi}{2}(Y+B)+\tan^2\frac{\pi}{2}X} + \frac{1}{2}\cdot\frac{\tanh\frac{\pi}{2}(Y-B)\quad \sec^2\frac{\pi}{2}X}{\tanh^2\frac{\pi}{2}(Y-B)+\tan^2\frac{\pi}{2}X} \quad (8)$$

$$\frac{dX}{dT} = -\frac{\tan\frac{\pi}{2}(X+A)\ \mathrm{sech}^2\frac{\pi}{2}Y}{\tanh^2\frac{\pi}{2}Y + \tan^2\frac{\pi}{2}(X+A)} - \frac{\tan\frac{\pi}{2}(X-A)\ \mathrm{sech}^2\frac{\pi}{2}Y}{\tanh^2\frac{\pi}{2}Y + \tan^2\frac{\pi}{2}(X-A)}$$

$$+\frac{1}{2}\ \frac{\tan\frac{\pi}{2}X\quad \mathrm{sech}^2\frac{\pi}{2}(Y+B)}{\tanh^2\frac{\pi}{2}(Y+B)+\tan^2\frac{\pi}{2}X} + \frac{1}{2}\ \frac{\tan\frac{\pi}{2}X\ \mathrm{sech}^2\frac{\pi}{2}(Y-B)}{\tanh^2\frac{\pi}{2}(Y-B)+\tan^2\frac{\pi}{2}X} \quad (9)$$

The flow pattern among these sources and sinks, plotted for one quadrant in Fig. 3, consists of one zone inside a roughly elliptical shape (half of which is shown) centered on the coordinate origin, and another zone outside. The inside flow consists of flow from the two sources. The streamline separating the flows when $Y>0$ represents the exterior cross-sectional shape of a round-bottom trough of normalized width 2A and normalized depth H; where H is the positive value of Y, for $X = 0$, at which the right-hand side of Equation 8 equals zero. Note the stagnation region near ($X = 0$, $Y = H$), as well as at ($X = 1$, $Y = 0$). This profile may be considered a reasonable approximation of a typical trough shape; further refinement of the round-bottom shape, or the modelling of other shapes, could be attained with a more complex array of sources and sinks, and with other conformal mappings.

The flow nets shown in Figs. 2, 3, and 6 were plotted largely by computer. The program consisted of Equations 8 and 9 for the two velocity components, values of A and B, initial positions for each streamline, instructions to increment X and Y according to the local velocity components, and instructions to plot X and Y at all times and to tick the resulting plotted streamlines at values of $T = .1, .2, .3 \ldots.$ The initial positions are (X = small positive value, $Y = H$) for the trough boundary streamline, and ($X = 0.1, 0.2, \ldots 0.9$, $Y = 2.0$) for the other streamlines; $Y = 2.0$ is chosen as a starting point because it is found to be in the zone of essentially uniform flow.

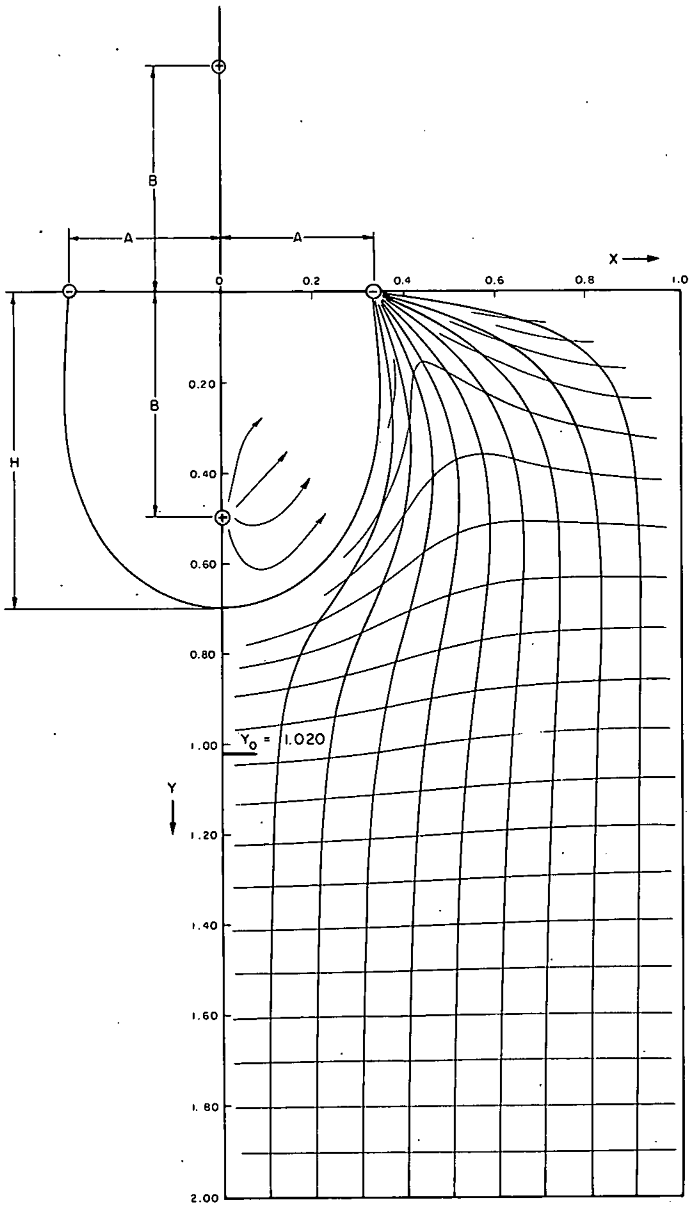

Figure 3. Sources and sinks for trough model (A=.33, B=.50).

Isochrones

In Figs. 2, 3, 6, and 7, curves connecting ticks representing equal elapsed time were faired in by hand to obtain what may be called isochrones or isochronal curves. In Fig. 2, consider the fluid elements along the lowest, horizontal isochronal curve ($T = 0$), perhaps tagged with a streak of dye. At a time $0.1n$ later, the dye streak will be congruent with the nth isochronal curve above the first.

Obviously, isochrones are not to be confused with the equipotential curves conventionally found in a flow net. In a conventional net, the spacing of the streamlines equals the spacing of the equipotential curves at any point, and both are inversely proportional to the local speed. In this flow net, however, the spacing between isochronal curves is directly proportional to local speed, a fact which aids in the visualization of velocity profiles.

Degree of Uniformity in the Upflow

A uniform flow has everywhere the same speed and direction. It has been noted that for sufficiently large Y, the upflow towards a trough is uniform, but that near the trough the flow is unavoidably not uniform.

Hirsch (8) has stressed the importance of flow uniformity in the fluidized bed during backwash:

> "Operating difficulties which in many cases stem from insufficient or poorly distributed backwash include: mounding of gravel, jetting of sand, formation of mud balls, early turbidity breakthrough, high washwater consumption, and short filter runs. So important is an evenly spread backwash that equipment manufacturers offer this feature as their most appealing sales argument . . . .
>
> ". . . Filter plant operators are aware of the unevenness of a backwash, their usual observations being the early appearance of clear areas and the persistence of muddy spots in corners . . . .
>
> ". . . Similarly, toward the end of a wash a band of clear water extends the length of the trough lip before clearing is discernible in the intertrough areas . . . .
>
> ". . . Even when uniformity of washwater introduction from the underdrains is assured, thc troughs themselves, by their discrete linear positions, are the cause of the nonuniformity."

Clearly, the troughs themselves by their discrete linear positions can indeed be the cause of the nonuniformity if the bed is allowed to expand into the nonuniform region near the trough. Furthermore, if the bed expands into the nonuniform region near the trough, the nonuniformity may be exacerbated, rather than damped. The student of flow in porous media can see that the presence of a fixed bed in Figs. 2 or 3, with a plane, horizontal top surface at, say, $Y = .8$, would tend to make more uniform the flow within the bed at $Y > .8$, because flow emerging from a porous medium tends to do so in a direction normal to the exit surface. However, if the bed is not fixed but is fluidized, the bed will expand more in high velocity zones than in low velocity zones, thus causing the top surface to mound up where the isochronal curves are seen to mound up.

The filter grains on the high-velocity mound will tend to sidle downslope, i.e., to restore the horizontal surface of the top of the bed. Such removal of grains from the high-velocity area will reduce the resistance to upflow there, inviting a greater upflow there than elsewhere, thus amplifying the nonuniformity caused by the rigid boundaries of the flow region. This argument, while qualitative, is presented here merely to underscore the importance of establishing a sufficiently uniform flow within the bed.

In this paper, nonuniformity is conveniently defined as V_{max}/V_{min}, a function of Y and the trough dimensions, where V_{max} is the greatest normalized upward velocity at a given value of Y, and V_{min} is the smallest velocity at that Y. It is, of course, desirable to have V_{max}/V_{min} as close to unity as possible; in this paper an arbitrary value of $V_{max}/V_{min} = 1.2$ is chosen as an acceptable tolerance. (The choice of tolerance deserves more study.)

In Fig. 2, for the sidewall weir (or for a very small trough), notice that for Y less than about 1.0, V_{max} is found at $X = 0$ and V_{min} is found at $X = 1$, with nonuniformity increasing as Y decreases, reaching the value 1.2 at $Y_o = .980$. The isochrones tilt upward to the left.

In Fig. 3, for a trough relatively large with respect to its spacing, the isochrones tilt upward not to the left but to the right; and V_{min} is now found at $X = 0$. The V_{max} is found at $X < 1$, at the "hump in the middle" of the isochrones. The nonuniformity of 1.2 is attained at $Y_o = 1.020$; the pattern in Fig. 3 may, therefore, be considered slightly "worse" than that in Fig. 2, because a greater depth, Y_o, below the weirs is required to establish acceptable uniformity.

Contemplation of Figs. 2 and 3, with their rather opposite patterns of nonuniformity for very small and rather large troughs, respectively, suggests, a la Goldilocks, that a trough of intermediate size would engender a flow pattern with isochrones tilting neither to the left nor to the right, and with V_{max}/V_{min} 1.2 occurring at a Y_o that is less than in either of the first two patterns. A systematic search for an optimal trough size and shape (i.e., with a minimal Y_0) was thus undertaken.

In this search, B/A ranged from 0.6 to 1.4. For each B/A, the value of A was varied between .18 and .33. For each value of A and B studied, the vertical velocity, using Equation 8, was computed for $X = 0, .1, .2, \ldots, 1.0$, and for $Y = .6, .7, \ldots, 1.2$. (The range of Y was chosen and amended as necessary to include Y_o, when found.) The value of H was also computed. For each Y, the values of V were scanned, V_{max} and V_{min} noted, and their ratio computed.

For each B/A, the nonuniformity V_{max}/V_{min} was plotted as a function of H for the several values of Y, as shown by an example in Fig. 4. For each Y, there is clearly a value of H for which nonuniformity is least, for a given B/A. The curve for each Y has two limbs; conditions on the left limb represent conditions shown in Fig. 2, or for small troughs, where V_{min} is found at $X = 1$. The right limb represents conditions shown in Fig. 3, where V_{min} is found at $X = 0$. Where the limbs cross, V_{min} is both at $X = 0$ and $X = 1$, and nonuniformity is minimized. Note that on Fig. 4 the value of H for least V_{max}/V_{min} changes little with Y; see also that $V_{max}/V_{min} = 1.2$ can be attained for Y less than 0.7, compared with Y_0 values of .980 and 1.020 for Figs. 2 and 3.

For B/A other than 1.4, plots were made that were qualitatively similar to Fig. 4. Interest centered on conditions at point P, where for a given B/A the optimum H provides the lowest V_{max}/V_{min}, at $Y = 0.7$.

To specify B/A and H also is to specify A; conditions at P are plotted in Fig. 5 as a curve relating A to H, with values of V_{max}/V_{min} noted at points along the curve (the value $V_{max}/V_{min} = 1.168$ in Fig. 5 is the one for $B/A = 1.4$, shown in Fig. 4). The A-curve in Fig. 5 indicates, for any given H, the value of A that will give the least nonuniformity in the upflow at $Y = 0.7$.

While for a given H we now can find the A for best uniformity, we still have not specified an optimum H. There is a problem here; note that V_{max}/V_{min}min varies along the curve, becoming least at $H = 0$, $A = 0.5$. While one would normally search for the point on the curve with the least V_{max}/V_{min}, it is obviously not practical to have a trough of zero depth H;* what then shall we call a practical optimum?

The trough's internal capacity is roughly proportional to $Ah^{1.5}$, a parameter which has also been plotted on Fig. 5. There is a maximum in this parameter at $H = .41$, $A = .25$; the corresponding value of B is .25. These values, therefore, define what may be called a practical optimum.

The flow pattern for $A = B = .25$, $H = .41$ is shown on Fig. 6. In this case, the depth to $V_{max}/V_{min} = 1.2$ is $Y_0 = .675$, considerably less than for the cases shown in Figs. 2 and 3. Depths to other values of V_{max}/V_{min} are shown on the right-hand margin.

In more real-world terms, the basic dimensions of this practical - optimum trough are: total external width (2As) equal to .25 times the center-to-center trough spacing (2s) and external drafts (Hs) equal to $.41/2 = .20$ times the center-to-center spacing (2s). The filter may be designed so that when the bed is fluidized during backwash, the top of the bed comes as high as $Y_0 = .675$ (or $y_0 = .34$ times the center-to-center spacing, 2s), or another value if a tolerance of nonuniformity other than 1.2 is adopted.

*It may be of interest to note that the pattern for $A = .5$, $H = O$ is the same as for $A = O$, $H = O$, but to one-half the scale.

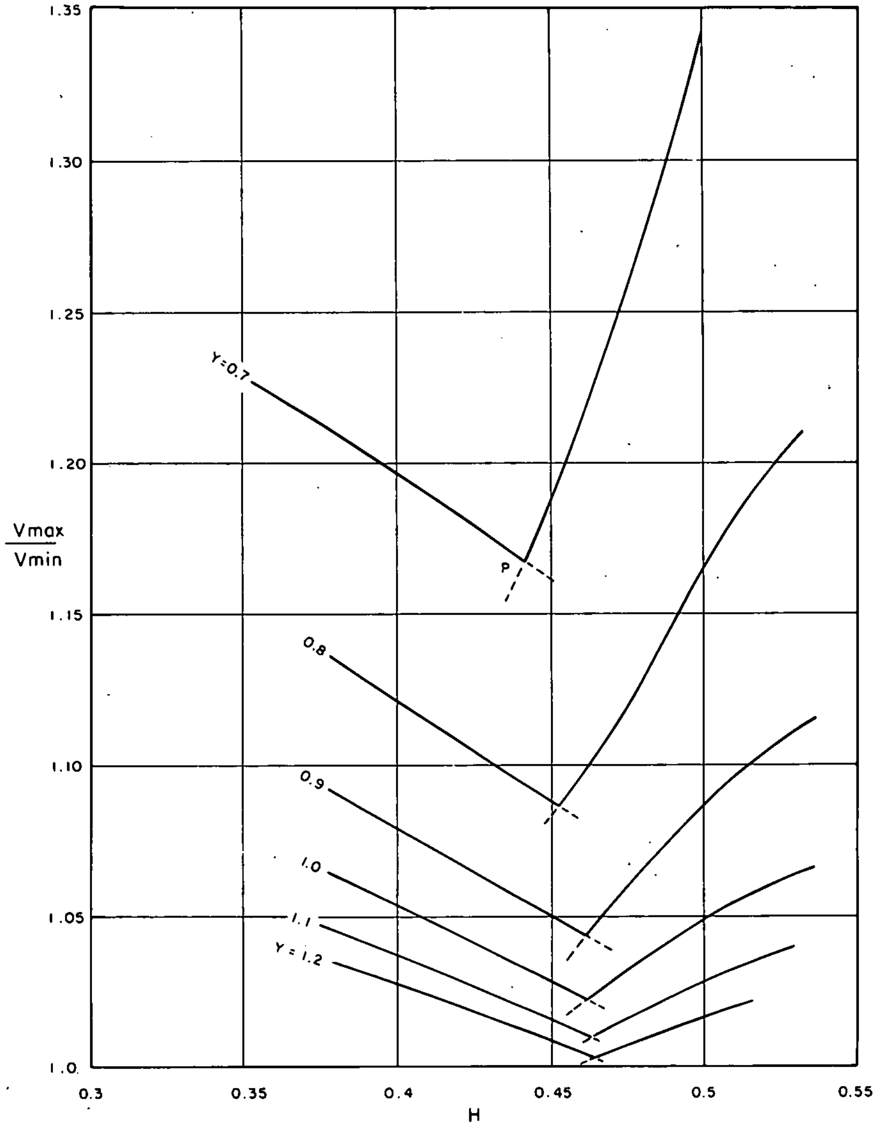

Figure 4. Uniformity of upflow as a function of trough size, H, and water depth, Y, for B/A = 1.4.

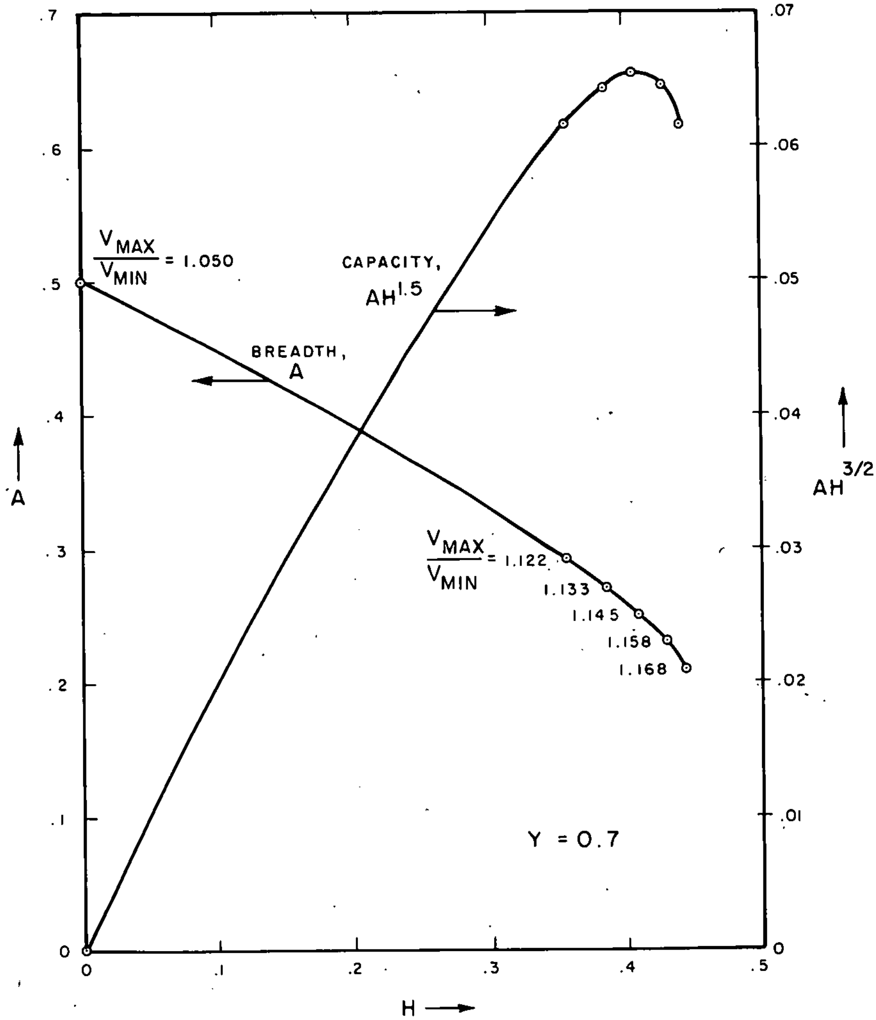

Figure 5. Breadth and capacity functions of H for optimum uniformity.

Suspended Solids Flushing Histograms

Not only is it important to keep the expanded bed below the region of nonuniform flow; it is also important to have a trough design that engenders a flow pattern that efficiently removes the filtered floc particles from the filter. Baylis et al (9) state that except for problems caused by uneven distribution of washwater and improper grading of filter materials, nearly all

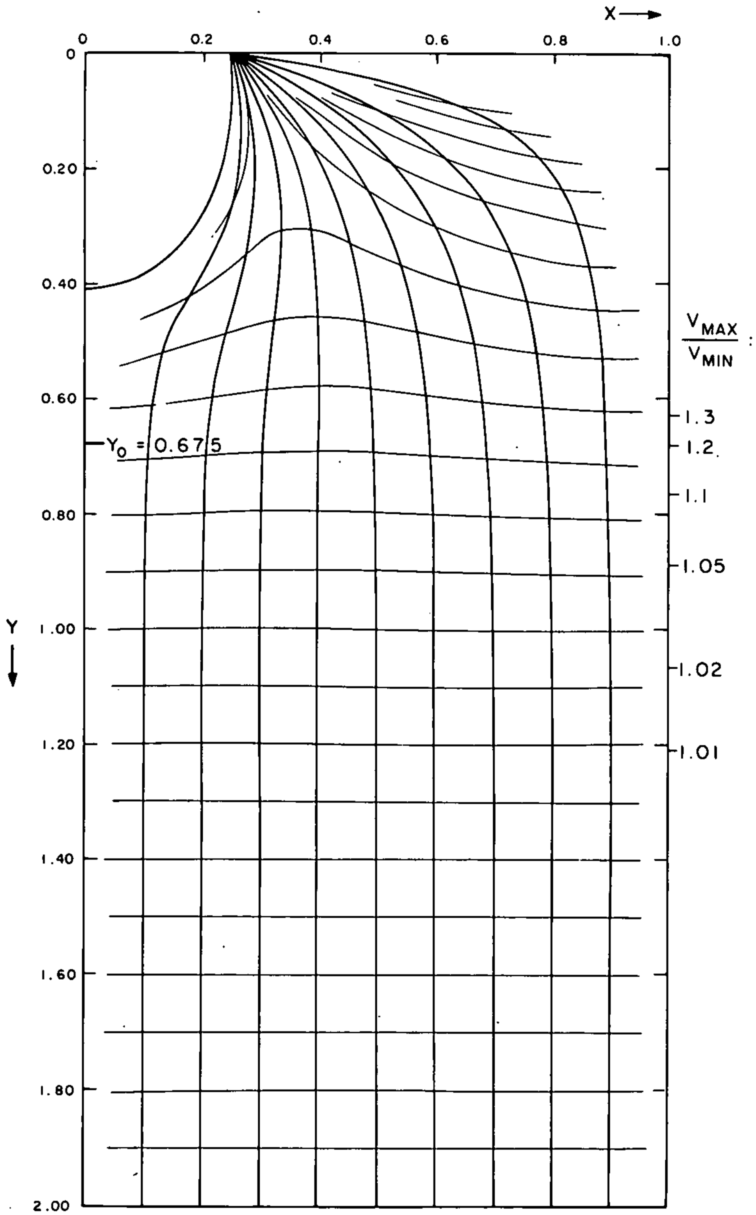

Figure 6. "Optimal" flow pattern (A=B=.25, H=.41).

filter bed trouble is attributable to failure of the washing process to remove from the filter the material which has been filtered from the water. Part of the washing process is to dislodge floc from the filter grains and bring it to the surface of the bed. A second and important part is to lift the floc clear of the bed and up to the trough weirs, not difficult for floc that has a small settling velocity, but increasingly costly and time consuming as the settling velocity of the floc increases. Since it has been estimated that 92 percent of the cost of backwashing is merely the cost of the pumped water used (10), it would appear worthwhile to seek a design that would minimize the time, hence the volume, of water used per backwash.

At several filter plants, Hirsch (8) performed backwash tests in which suspended solids concentration in the washwater was measured at frequent time intervals during the wash. Composite water samples were taken at the point of trough discharge to the sewer; when possible, samples were also taken at troughside bottom (in terms of our coordinates, at about $X = A$, $Y = H$) and midway between the troughs ($X = 1$).

Curves of suspended solids concentration plotted against time typically showed an early peak followed by a gradually decreasing tail. Generally, the troughside samples showed higher peaks and shorter tails than did the composite samples, or the samples from midway between troughs. In such plots, one may define a "good wash" as that for which the composite sample shows an early, high peak and a short tail.

In the present study, the paths of floc particles are computed as follows: Consider a particle of unhindered settling velocity, Wv_r, or in normalized terms, W. Assume that the particle settles at this velocity with respect to the surrounding fluid, so that with respect to fixed coordinates its velocity is the vectoral sum of W and the local fluid velocity:

$$\frac{dY_p}{dT} = \frac{dY}{dT} + W \tag{10}$$

$$\frac{dX_p}{dT} = \frac{dX}{dT} \tag{11}$$

where the subscript (p) denotes the particle, and dY/dT and dX/dT are as given by Equations 8 and 9. The path of the particle may be computed and traced, with time recorded, in the same manner as for streamlines. Since the depth, Y_o, to a sufficiently small nonuniformity v_{max}/v_{min} can be determined, computations are begun at $Y = Y_o$ instead of the more arbitrary $Y = 2$, it being assumed that the top of the expanded bed is at $Y = Y_o$.

Consider first the "practical optimum" trough configuration, $A = B = .25$, $H = .41$. For $W = 0$, the particle paths are identical to the streamlines in Fig. 6, for $Y \leq Y_o$. For $W = 0$, the paths may be quite different, as shown for $W = 0.9$ in Fig. 7. The values T_f are the normalized times required to flush the particles from $Y = Y_o$ to the weir at $X = A$, $Y = 0$. Comparison of the

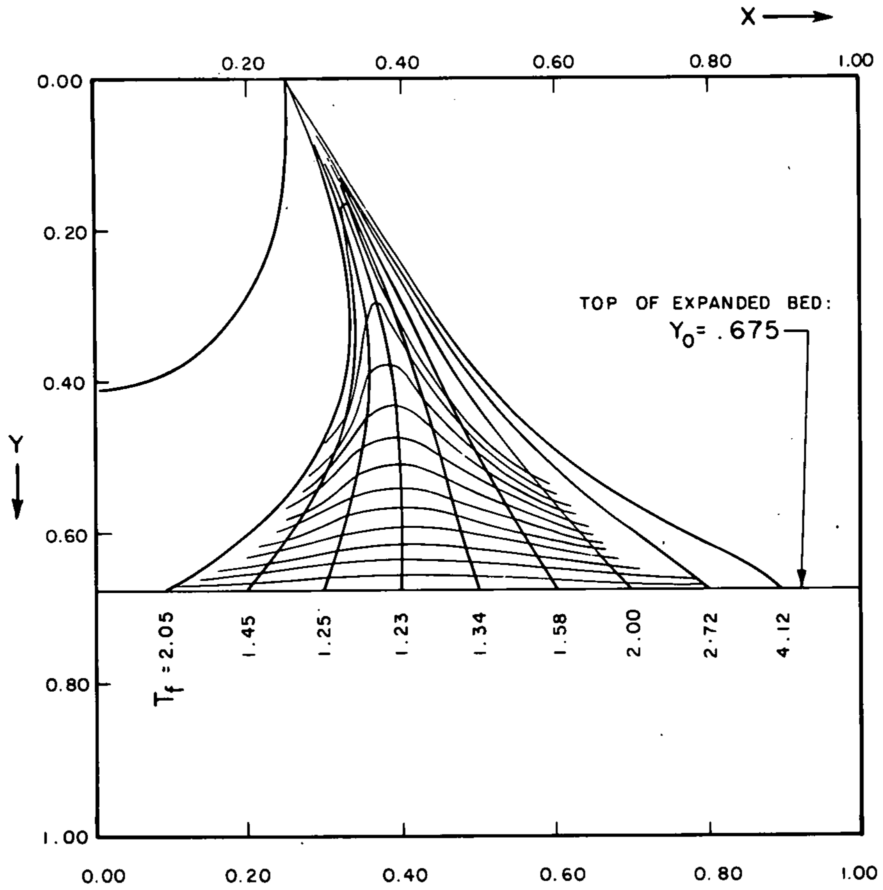

Figure 7. Particle paths for W = 0.9, with A=B=.25, H =.41.

particle paths in Fig. 7 with the streamlines in Fig. 6 shows that a heavy particle lifted towards a stagnant region makes little progress vertically until it can drift laterally, crossing streamlines, sufficiently far into a high velocity zone to be carried up and out.

The flushing times T_f were computed not only for W= 0.9, as shown in Fig. 7, but also for W = 0, .1, .2, ..., .8, for floc particles initially located at Y = Y_0, X = .1, .3, .5, .7, .9. Values of T_f are presented in Table 1. As one would expect, the lowest T_f at any W occurs at about X = .3, where the stream velocity is greatest. As W increases, T_f also increases for any given X, but T_f for X = .3 for a given W is usually less than T_f at X = .1 for the next lesser W. Thus, a heavy particle initially in a high velocity zone may be flushed out more quickly than a lighter particle initially in a low velocity zone.

The data in Table 1 are plotted as a frequency histogram in Fig. 8a. This histogram may be considered comparable to Hirsch's plots of suspended solids concentration vs. time reported earlier, provided (a) we can assume that all floc particles begin the flush-out process at the same time from $Y = Y_o$ (whereas, in fact, some may still be deep within the filter when the first particles begin to rise from $Y = Y_o$); (b) the distribution of floc settling velocities is uniformly graded from W = .0 to W = .9. The computation process may be quite easily amended to represent more realistic conditions than these two provisions, should one wish; but the procedure as given is quite adequate for the expository purpose of the paper.

TABLE 1

FLUSHING TIMES T_f FOR $A = B = .25$, $H = .41$, $Y_o = .675$

W\X	.1	.3	.5	.7	.9
0	.479	.374	.420	.584	1.024
.1	.479	.439	.459	.639	1.119
.2	.579	.479	.509	.689	1.189
.3	.579	.519	.539	.759	1.299
.4	.669	.539	.589	.849	1.429
.5	.749	.599	.649	.909	1.569
.6	.949	.689	.739	1.029	1.799
.7	1.069	.779	.859	1.199	2.169
.8	1.399	.949	1.049	1.479	2.749
.9	2.049	1.249	1.339	1.999	4.119

In Fig. 8a, the histogram shows an early peak and a slowly decreasing tail, quite like Hirsch's graphs as described. (The subsidiary peaks, valleys, and gaps are due to computational coarseness and have no physical meaning; inclusion of more paths would smooth out the histogram.) The peak occurs at $T_f = .5$ to .6; 50 percent of the T_f values are less than .8, and 80 percent are less than 1.3.

Histograms for A = .33, H = .70 (the trough of Fig. 3) and for A = B = H = 0 (the sidewall weir of Fig. 2) are shown in Figs. 8b and 8c, respectively, for comparison. For these the peaks and 50 and 80 percentile marks occur much later than in Fig. 8a. In large part, but not entirely, this is due to the differing values of Y_o; it is also due in part to the less uniform flow patterns for the large trough and the sidewall weir. Certainly the fact that the peak is less well

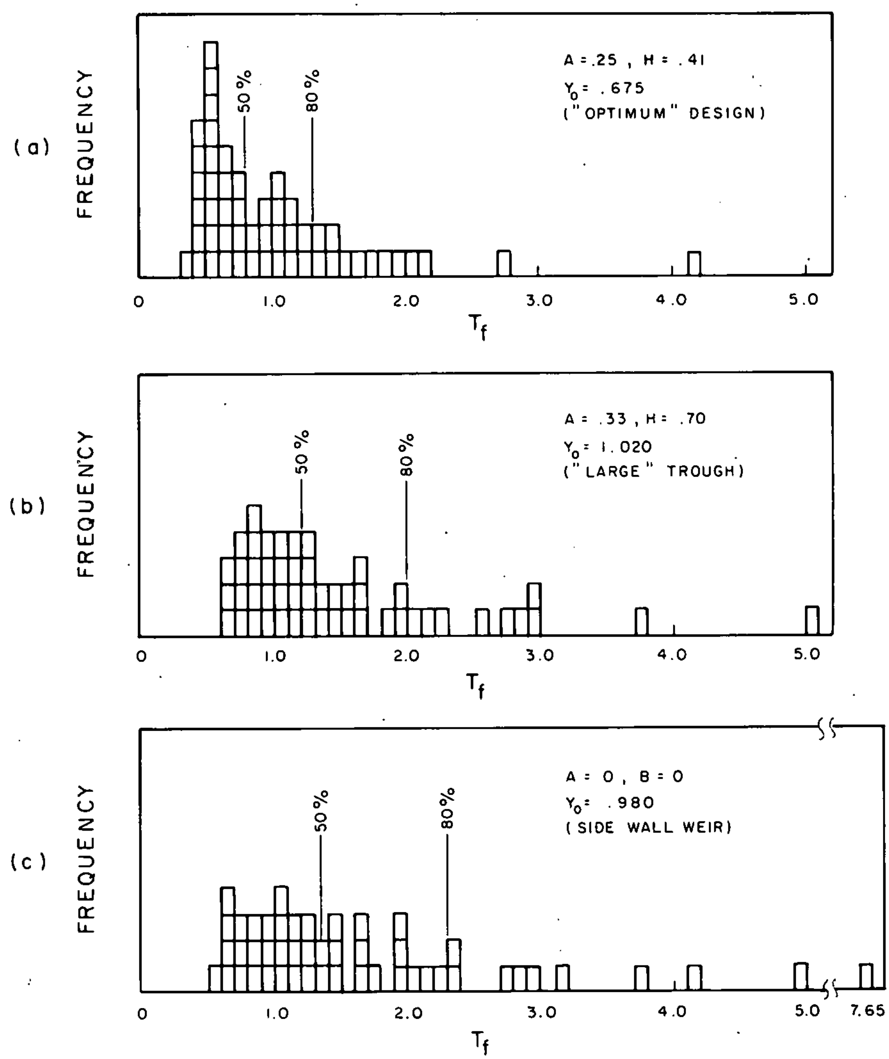

Figure 8. Flushing histograms for three trough designs.

defined and the tail longer in Figs. 8b and 8c is due to less uniformity in the respective flow patterns, and points out an advantage of the "optimum" design beyond the fact that its Y_0 is least: it also removes the floc more quickly. A wash of duration $T_f = 2.2$ would theoretically be sufficient for the conditions of Fig. 8a; the volume per unit area of washwater required (perhaps a more important factor than the time) is theoretically $T_f s = 2.2s$.

Trough Spacing

To minimize the volume of washwater required, $T_f s$, it appears that one should select the "optimal" trough dimensions as determined herein to minimize T_f, and then minimize s. What are the practical lower limits on the center-to-center trough spacing, 2s?

There are at least two important hydraulic considerations. The first is that the troughs must be large enough to carry the flow entering them, yet they should not be larger than necessary. When discharge out of the trough is free (unhindered by any conditions in the collector channel) and flow into the trough is uniformly distributed along its length, the discharge Q is related to the *internal* breadth 2a as

$$Q^2 = g\,(2a)^5 \text{ times a constant} \tag{12}$$

where the constant is a function of the trough's cross-sectional shape, and the ratio of flow depth to trough breadth.

For U-shaped troughs with flow depth about equal to breadth 2a, the constant is about equal to 0.1 (11). The discharge Q is also equal to 2 sLv_r, where L is the trough length and 2 sL is the filter area served by the trough, so that

$$(2\,sLv_r)^2 = 0.1\,g(2a)^5 \tag{13}$$

If the trough wall is relatively thin, 2a is nearly equal to 2As, the trough external breadth.

If A is the optimal value of .25,

$$(2sLv_r)^2 \cong 0.1\,g(2 \times .25\,s)^5 \tag{14}$$

or

$$L \cong .03\,g^{0.5}\,v_r^{-1}\,s^{1.5} \tag{15}$$

The trough length L is thus quite sensitive to the half-spacing, s; to reduce s, while maintaining constant A and H, would entail a concomitant reduction in trough length, hence filter width, L.

The second hydraulic consideration is that reasonable uniformity of discharge *per unit length of weir* should be assured, in addition to flow uniformity in a plane normal to the weir as considered previously.

Although the troughs should be installed with the greatest care given to the elevation of the weirs (1/32 of an inch is sometimes specified as a tolerated maximum variation in weir elevation in one filter), a certain variation is unavoidable, and design should, therefore, render the flow acceptably insensitive to such variation. Insensitivity is increased by increasing the depth of flow over the weir: If the water surface elevation midway between troughs is h_o and the weir elevation is h_1, the head on the weir is $h_0 - h_1 = h$, varying slightly from point to point. The discharge per unit length of weir is

$$q = Ch^{3/2} \tag{16}$$

where C is a coefficient and $q = v_r$ s.

A variation in head, dh, will result in a variation in discharge

$$dq = \frac{3}{2} C h^{1/2} \, dh \tag{17}$$

The fractional variation in discharge is

$$\frac{dq}{q} = \frac{3}{2} \frac{dh}{h} = \frac{3}{2} \left(\frac{C}{v_r s}\right)^{2/3} dh \tag{18}$$

For a given tolerance dh, and given v_r and C, an acceptably small dq/q depends, therefore, on a sufficiently large value of s.

Besides these two hydraulic considerations, one may note simply that a low value of s implies a greater number of troughs, and a greater construction cost. Furthermore, troughs too close together would prevent access to the bed for maintenance or inspection.

If the filter is to be provided with facilities for simultaneous air and water backwash, there is yet another design factor: the rising air bubbles lift considerable amounts of filter media to the water surface. These grains would be lost from the filter if water were passing over weirs as conventionally designed. One solution, practiced in some European designs, is to design the weirs with a quiescent zone sheltered from air bubbles, to permit filter grains to settle out before the water reaches the weir crest; the settled grains fall back to the bed (12). The other solution is to set the troughs high enough so that the water surface, initially drawn down to the top of the bed, never reaches the weirs during the relatively brief air-water wash. Weir elevations set by this consideration will thus be some given distance y_o above the top of the expanded bed at the greatest backwash rate. If $Y_o = y_o/s$ is to be the optimum value of 0.675, s is then established at $y_o/0.675$; there is no point in making it less.

Under what conditions are sidewall weirs feasible, according to this discussion, and how are they able to operate with relatively little vertical clearance between the top of the bed and the weirs? Filters with sidewall weirs and relatively small Y_o, common in many European designs, can only be, and obviously are, backwashed with only very slight fluidization; any intention to expand the bed in backwash must be accompanied by a design to permit removal of water at numerous points across the top of the filter, as by troughs, and not just by sidewall weirs, unless one is prepared to keep the top of the bed a depth y_0 below the weirs equal to about half the filter width, 2s.

Conclusions

For a given trough spacing, the round-bottom trough that minimizes the region of nonuniform flow above a filter bed, consistent with an efficient shape for internal hydraulic capacity, is one with external total width equal to .25 times the center-to-center spacing, and with a total external draft equal to .20 times the center-to-center spacing. This design also promotes more efficient flushing of floc particles from the filter than do either the much larger or much smaller trough sizes studied.

Diminution of the nonuniform flow region, and more efficient flushing, can also be promoted by decreasing the trough spacing; but there are practical lower limits to trough spacing.

The computational techniques described have not been employed exhaustively herein. Rather, it is hoped that they may be of use to those who may care to refine and extend them, to model other shapes of trough, or perhaps more realistic frequency distributions of settling velocity of the floc particles.

Acknowledgments

The author wishes to thank his employer, Camp Dresser & McKee Inc., for permission to prepare this paper for the Freeman Hydraulics competition and for publication; for making available the CDM computer; and for the programming skills of Mrs. Nancy LeSaffre, the drafting skills of Mrs. Erika Kalam, and the typing skills of Mrs. Esther MacLaughlin.

REFERENCES

1. Fair, G. M., J. C. Geyer, and D. A. Okun, *Water and Wastewater Engineering,* Vol. 2, Wiley, New York, 1968, Chapter 27.
2. Ellms, J. W., and J. S. Gettrust, "A Study of the Behavior of Rapid Sand Filters Subjected to the High-Velocity Method of Washing," *Trans. ASCE,* Vol. 80 (1916), pp 1342-1428.
3. "State of the Art of Water Filtration," Report of the Committee on Filtration Problems, R. L. Woodward, Chairman, *J. Amer. Water Works Assoc.,* October 1972, pp 662-665.
4. Cleasby, J. L., Annual Progress Reports for 1972, 1973, 1974, "Backwash of Granular Filters Used in Waste Water Filtration," EPA Project R 802140, Engineering Research Institute, Iowa State University, Ames, Iowa.
5. Kunii, D., and O. Levenspiel, *Fluidization Engineering,* Wiley, New York, 1969, pp 1-3.
6. Streeter, V. L., ed., *Handbook of Fluid Dynamics,* McGraw-Hill, New York, 1961, Chapter 4.
7. Streeter, V. L., *Fluid Dynamics,* McGraw-Hill, New York, 1948.
8. Hirsch, A. A., "Filter Backwashing Tests and Upflow Equalization," *J. San. Eng. Div., ASCE,* Vol. 94, No. SA1, February 1968, p 129 (Proc. Paper 5813).
9. Baylis, J. R., O. Gullans, and H. E. Hudson, Jr., "Filtration," Chapter 7a in *Water Quality and Treatment: A Handbook of Public Water Supplies* by Amer. Water Works Assoc., McGraw-Hill, New York, 1971.
10. Koenig, L., "The Cost of Water Treatment," *J. Amer. Water Works Assoc.,* Vol. 59, pp 290-336, March 1967.
11. French, J. A., "Internal Hydraulics of Washwater Troughs," Unpublished Memorandum, June 1973.
12. Verbal Communication by Representatives of Degremont, S. A., 1974.