THE ANNUAL WATER BALANCE

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Preface

The tenth John R. Freeman Memorial Lecture (briefly summarized in this paper) was actually a series of five lectures presented by the writer during the months of April and May, 1977, in the lecture hall of the Center for Advanced Engineering Study at the Massachusetts Institute of Technology.

The central idea of these lectures was the introduction of the method of derived distributions in combining stochastic and deterministic hydrologies into a new approach to analyzing the annual water balance.

Intended to form an instructional introduction to new (and still-developing) ideas, the very large body of material was divided into five topics:

Lecture 1 - April 12 - Introduction to Applied Probability
Lecture 2 - April 19 - Frequency of Annual Precipitation
Lecture 3 - April 26 - Infiltration and Surface Runoff
Lecture 4 - May 3 - Evapotranspiration and Groundwater Runoff
Lecture 5 - May 10 - The Annual Water Balance

and lecture notes were provided [1]. The scope of the lectures was too great to permit even a complete summary here, thus only a discussion of the principle concepts is presented. For more detail, the interested reader is referred to references [2] through [8].

Abstract

A statistical-dynamic formulation of the vertical water budget at a land-atmosphere interface is outlined. Physically-based dynamic and conservation equations express the soil moisture movement processes during rainstorms and interstorm periods in terms of independent variables representing the precipitation, potential evapotranspiration, soil and vegetal properties and water table elevation. Uncertainty is introduced into these equations through the probability density functions of the independent climatic variables, allowing the probability distributions of the dependent water balance elements to be derived. The expected values of these quantities give a long-term average water balance which, *to the first order*, define the annual water yield and water loss in terms of the annual precipitation and potential evapotranspiration, and in terms of physical parameters of the soil, vegetation, climate and water table. This analytical framework provides physical insight into the dynamic coupling of climate-soil-vegetal systems.

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Introduction

Growing concern over the possible long-term climatic effects of man's modifications to the land-surface of our planet has prompted increased efforts to improve our understanding of the coupling, across this interface, among physical processes of the atmosphere, soil and vegetation. Past efforts in this direction have been largely of two types:

1. *Empirical studies* which provide validated interrelationships among the principle variables but which, due to their weak physical basis, lack both the generality, and the parametric incorporation of climate, soil and vegetal properties which are necessary for the generation of understanding. Prominent among these works are the early water balance studies of Thornthwaite [9], [10], [11], who used an empirical expression for evapotranspiration in a monthly moisture accounting process based upon a soil's moisture-holding capacity. More recently, Lettau and his co-workers [12], [13] have refined the water balance evapotranspiration term through use of an energy balance but have included no explicit consideration of the soil and vegetal properties which will control the evapotranspiration under most practical circumstances.

2. *Numerical studies* which utilize detailed formulations of the physics at the "micro-process" scale but which, due to their complexity, impose infeasible validation data requirements and impede the generation of overall behavioral insight. Recent examples of such studies are those of Sasamori [14] and of Deardorff [15]. The purpose of these numerical formulations is to simulate the system response to specific climatic inputs, and they usually do so in terms of a large number of climate, soil and vegetal parameters. Both of these characteristics make it difficult to draw generalizations of system behavior.

The objective of the present work is to present and discuss a *generalized* water balance model based upon simplified physics of the component processes. The model is detailed enough to capture the *essential* system dynamics yet simple enough to permit analytical (as opposed to numerical) solution. It produces valuable insights into the interactive role of soil moisture in the determination of climate, and provides a tractable basis for deriving generalized probability distributions of such important water balance components as annual basin yield.

Derivation of the water balance equation has been presented elsewhere [2] through [8], and will not be repeated here.

Dynamic Coupling of Atmosphere, Soil and Vegetation

The atmosphere is coupled to the soil-vegetal system through the exchange of momentum, energy, water mass and chemical elements across the land surface. Here we will deal only with energy and water as is indicated schematically in Figure 1. This exchange can be defined by five relationships:

1. *Energy Conservation* — Considering energy fluxes into and out of the soil surface boundary layer we can write the energy equation [16, page 218]:

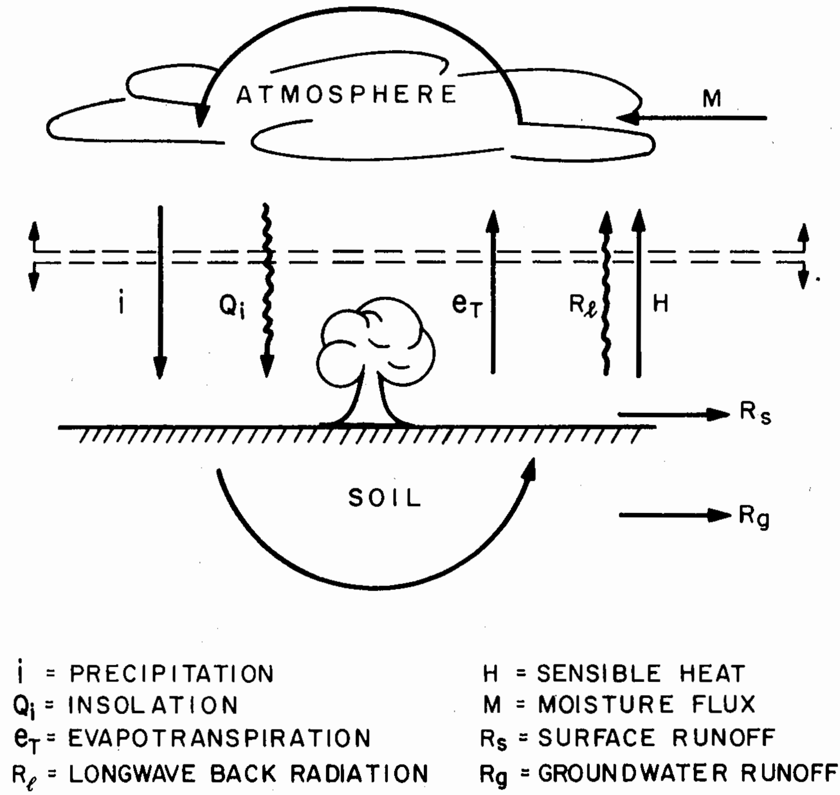


Figure 1. Coupled Climate-Soil-Vegetation System

$$\left.\begin{aligned} \rho_s c_g Z_r \frac{\partial T_g}{\partial t} &= Q_i(A_s, N) - R_\ell(T_g, N) - \rho_w L_e e_T - H(T_g, T_a) \\ &\quad + q_a - q_p \end{aligned}\right\} \quad (1)$$

in which

ρ_s = mass density of soil-water system, g cm^{-3}
c_g = specific heat of soil-water system, cal g^{-1} $°C^{-1}$
Z_r = thickness of soil surface boundary layer, cm
T_g = surface temperature, °K
t = time, min

Q_i = net short wave solar radiation, cal $cm^{-2}min^{-1}$
A_s = short wave albedo of surface
N = fractional cloud cover
R_ℓ = net long wave terrestrial radiation, cal cm^{-2} min^{-1}
ρ_w = mass density of evaporated water, g cm^{-3}
L_e = latent heat of vaporization, cal g^{-1}
e_T = actual rate of evapotranspiration, cm min^{-1}
H = net rate of transfer of sensible heat, cal cm^{-2} min^{-1}
q_a = rate of advected energy input, cal cm^{-2} min^{-1}
q_p = rate of use of energy in photosynthesis, cal cm^{-2} min^{-1}

2. *Atmospheric Vapor Transfer Capacity* — The "potential" (i.e., maximum) rate of evaporation is given by the mechanical ability of the atmosphere to transfer vapor away from a surface in the absence of any restriction upon moisture supply to that surface. This may be written [16, pg. 216]:

$$e_p = C\frac{\rho_a|V_a|}{\rho_w p_a}[e_s(T_g) - e_a] \qquad (2)$$

in which

e_p = potential rate of evaporation from the particular surface, cm min^{-1}
C = dimensionless resistance coefficient
V_a = wind velocity at reference elevation, cm min^{-1}
ρ_a = mass density of moist air, g cm^{-3}
$e_s(T_g)$ = saturated atmospheric vapor pressure at ground temperature, dynes cm^{-2}
e_a = atmospheric vapor pressure, dynes cm^{-2}

Realization of this potential depends of course upon an adequate supply of water from the soil and of energy from the sun.

3. *Moisture Transfer in Soil* — The *actual* rate of evapotranspiration from soil moisture is determined by the ability of the soil-vegetal system to deliver water to the surface under the particular atmospheric and soil moisture conditions. Functionally, this is written [5]

$$\left.\begin{aligned} e_T &= e_T(s, t; \text{climate, soil, vegetation}), \quad e_T \leqslant e_p \\ e_T &= e_p, \text{ otherwise} \end{aligned}\right\} \qquad (3)$$

and in which

s = a measure of the soil moisture concentration in surface boundary layer

4. *Water Mass Conservation* — Considering moisture fluxes into and out of the soil surface boundary layer we can write

$$nZ_r \, \partial s/\partial t = i - e_T - y - \dot{S} \quad (4)$$

in which

n = effective porosity of the soil

i = precipitation intensity, cm min^{-1}

y = yield (i.e., surface runoff + groundwater runoff per unit of surface area), cm min^{-1}

$\dot{S}$ = rate of moisture storage on surface and in saturated zone per unit of surface area, cm min^{-1}

5. *Yield* — The soil moisture movement processes controlling the infiltration of moisture during periods of precipitation, and controlling the net percolation to the zone of saturation will determine the rate of generation of yield. Functionally, this is

$$y = y(s, t; \text{climate, soil}) \quad (5)$$

These five equations define the five land surface variables, T_g, s, y, e_T and e_p in terms of the six climatic variables, e_a, T_a, ρ_a, V_a, N and i and of several land-surface parameters. To consider the true interactive nature of the atmosphere-land surface system as is illustrated in Figure 2, we would add six atmospheric equations (two momentum, two mass conservation, one energy, and one state) as is done in climate-modeling.

Aside from biological growth processes, the atmosphere-soil-vegetal system is dynamic in the sense that the interfacial flux of heat and water is modulated by the presence of resistance and of storage volume. Weather-determined time windows of random length regulate the duration of these fluxes, thus the physical properties of the soil and vegetation determining the flux rates become important. We wish to find a way to incorporate both the process dynamics and the atmospheric statistics into the computation of the long-term water balance because these are the factors which distinguish one climatic region from another.

System Reduction for Water Balance Computation

To isolate the land-surface system, we will omit the energy equation (Eq. (1)) which is the analytical interface through which the atmospheric and land-surface systems are coupled. This changes the role of the land-surface temperature, T_g, from that of a soil state variable to that of an independent "climate" variable. With this modification, the potential rate of evaporation, e_p, becomes an independent climate variable and thus Eq. (2) may be dropped from consideration.

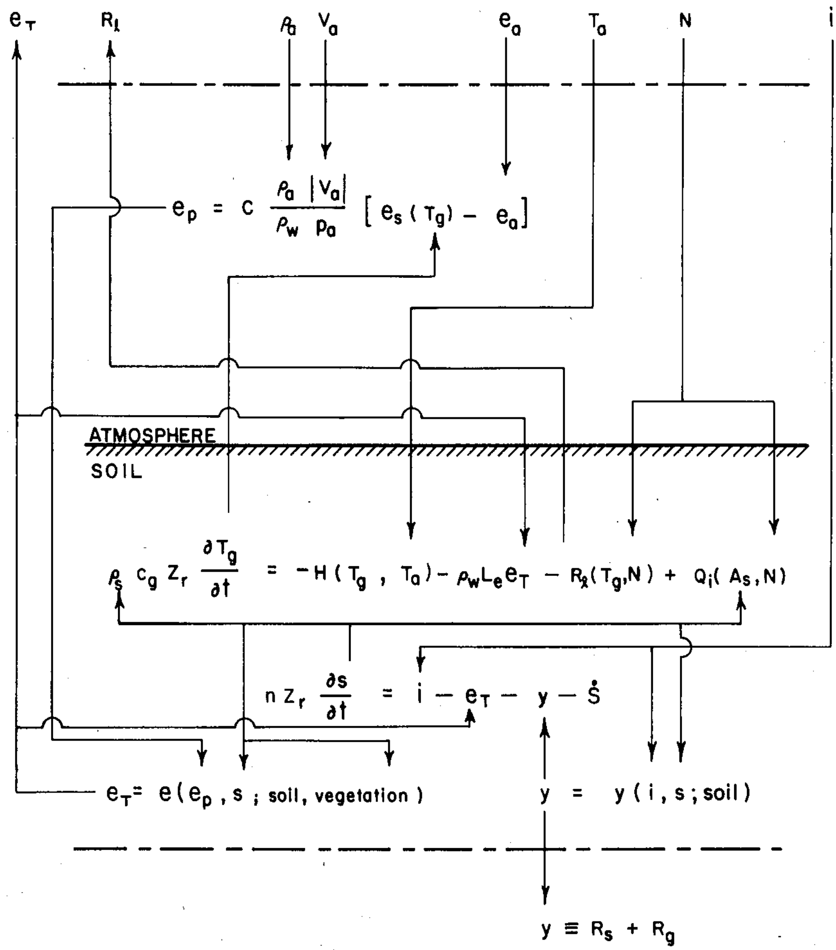


Figure 2. Atmosphere-Land Surface Couplings

The system now reduces to Eqs. (3), (4) and (5) defining s, y and e_T in terms of the climatic variables i and e_p. We can retain the independent climatic variables; T_g, e_a, ρ_a, T_a, V_a and N *implicitly* through use of the modified Penman equation [16, pg. 221] to define e_p.

The Water Balance Equation

Physically-based dynamic equations [17] are used to express the storm infiltration volume, V_i, and the interstorm evapotranspiration volume, V_e, in terms of the respective climatic potential rates, i and e_p; the respective durations of the storm (t_r) and interstorm (t_b) periods; the initial soil moisture, s_o, averaged over the surface boundary layer; and in terms of physical parameters describing the soil and vegetation.

These equations are used along with assumed probability density functions (pdf) for the independent climatic variables to derive the pdf of the dependent flux volumes. Taking the expectation of these last random variables and multiplying by the average number of (independent) events per year gives the long-term average annual infiltration, $E[I_A]$ and the long-term average annual evapotranspiration, $E[E_{T_A}]$.

Gravitational percolation to the water table and capillary rise from the water table to the surface are assumed to be steady and their difference over the year is averaged to give the long-term average ground-water component of yield, $E[R_{g_A}]$.

Assuming as a first approximation that all evapotranspiration comes from soil moisture and considering only systems which are steady-state in the long-term average, the mean annual precipitation, $E[P_A] \equiv m_{P_A}$, is partitioned above the interface according to

$$E[P_A] = E[I_A] + E[R_{s_A}] \tag{6}$$

in which $E[R_{s_A}]$ is the long-term average annual surface water component of yield.

With this result, Eq. (4) can be appropriately time-averaged to give the long-term average soil moisture water balance.

$$E[I_A] = E[E_{T_A}] + E[R_{g_A}] \tag{7}$$

By combination of Eqs. (6) and (7), we have the full water balance equation.

$$E[P_A] = E[E_{T_A}] + E[R_{s_A}] + E[R_{g_A}] \tag{8}$$

By definition, the long-term average annual yield, $E[Y_A]$, is

$$E[Y_A] = E[R_{s_A}] + E[R_{g_A}] \tag{9}$$

Dividing Eq. (7) by the mean annual precipitation and inserting the expectations derived from the physical equations of soil moisture movement, we have the dimensionless average annual water balance equation:

$$\underbrace{[\overbrace{1}^{\text{Precip.}} - \overbrace{e^{-G-2\sigma}\Gamma(\sigma+1)\sigma^{-\sigma}}^{\text{Surface Runoff}}]}_{\text{Infiltration}} = \underbrace{\frac{E[E_{P_A}]}{m_{P_A}} J(E, M_o, k_v)}_{\text{Evapotranspiration from Soil Moisture}} + \underbrace{\overbrace{\frac{m_\tau K(1)}{m_{P_A}} s_o^c}^{\text{Groundwater Recharge}} - \overbrace{\frac{Tw}{m_{P_A}}}^{\text{Loss}}}_{\text{Groundwater Runoff}} \tag{10}$$

in which

G = gravitational infiltration parameter

$$= \frac{\alpha K(1)}{2}[1 + s_o^{c}] - \alpha w \tag{11}$$

σ = capillary inflation parameter

$$= \left[\frac{5n\eta^2 K(1)\psi(1)(1 - s_o^2)\phi_i}{6\pi\delta m}\right]^{1/3} \tag{12}$$

$m_{P_A}/E[E_{P_A}]$ = potential humidity

$E[E_{P_A}]$ = long-term average annual potential evapotranspiration

$J(\ )$ = evapotranspiration function

E = evaporation parameter

$$= \frac{2\beta n K(1)\psi(1)\phi_e}{\pi m \bar{e}_p^{2}} s_o^{d+2} \tag{13}$$

M_o = vegetal canopy density at natural equilibrium
k_v = ratio of potential rates of transpiration and soil surface evaporation
w = apparent velocity of capillary rise from water table [18]

$$= K(1)\left[1 + \frac{3/2}{mc - 1}\right]\left[\frac{\psi(1)}{Z}\right]^{mc} \tag{14}$$

m_τ = long-term average length of rainy season, sec
$K(1)$ = saturated effective hydraulic conductivity, cm sec^{-1}
s_o = long-term average effective soil moisture concentration in the surface boundary layer
c = pore disconnectedness index $= \ell n\,(K(s_o)/K(1))/\ell n\, s_o$
T = one year, sec
α = reciprocal of mean storm intensity $\equiv m_i^{-1}$, sec cm^{-1}
n = effective soil porosity = volume of active voids/total volume
η = reciprocal of mean storm depth $\equiv m_H^{-1}$, cm^{-1}
$\Psi(1)$ = saturated soil moisture potential, cm (suction)

$$= \frac{\sigma_w}{\gamma_w}\left[\frac{n}{k(1)}\right]^{1/2} 10^{-0.33-0.28/m-0.07/m^2}, \text{ [4]}$$

σ_w	= surface tension of pore fluid, dynes cm^{-1}
γ_w	= specific weight of pore fluid, dynes cm^{-3}
k(1)	= saturated effective instrinsic permeability of soil, cm^2 $\equiv K(1)\, \mu_w/\gamma_w$
μ_w	= dynamic viscosity of pore fluid, poises
ϕ_i	= dimensionless infiltration diffusivity (see Fig. 3)
δ	= reciprocal of mean storm duration $\equiv m_{t_r}^{-1}$, $days^{-1}$
m	= pore size distribution index = 2/(c - 3), [19]
β	= reciprocal of mean interstorm period $\equiv m_{t_b}^{-1}$, $days^{-1}$
ϕ_e	= dimensionless exfiltration diffusivity (see Fig. 4)
e_p	= potential rate of evaporation from a bare soil surface, cm/sec
d	= diffusivity index = (c + 1)/2, [4]

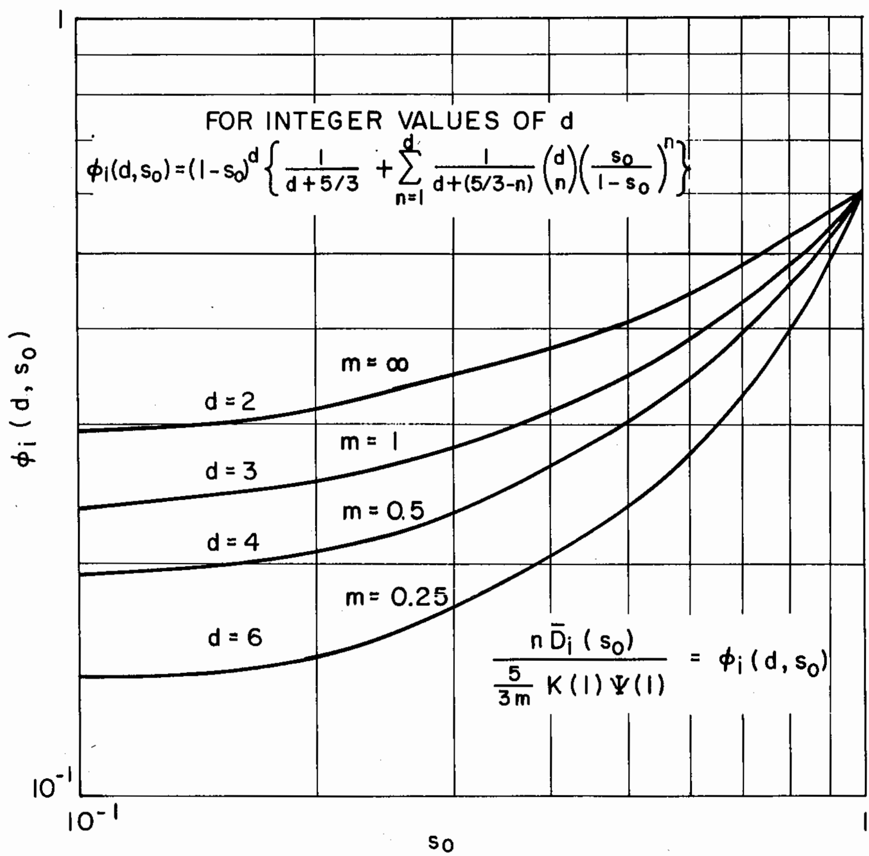


Figure 3. Weighted Mean Diffusion Coefficient-Sorption

For the special case of bare soil, $M_O = k_V \equiv 0$ and $w/\bar{e}_p << 1$, the evaporation function becomes

$$J(E) = \frac{E[E_{T_A}]}{E[E_{P_A}]} = 1 - [1 + 2^{1/2}E]e^{-E} + (2E)^{1/2}\Gamma[\tfrac{3}{2}, E] \qquad (15)$$

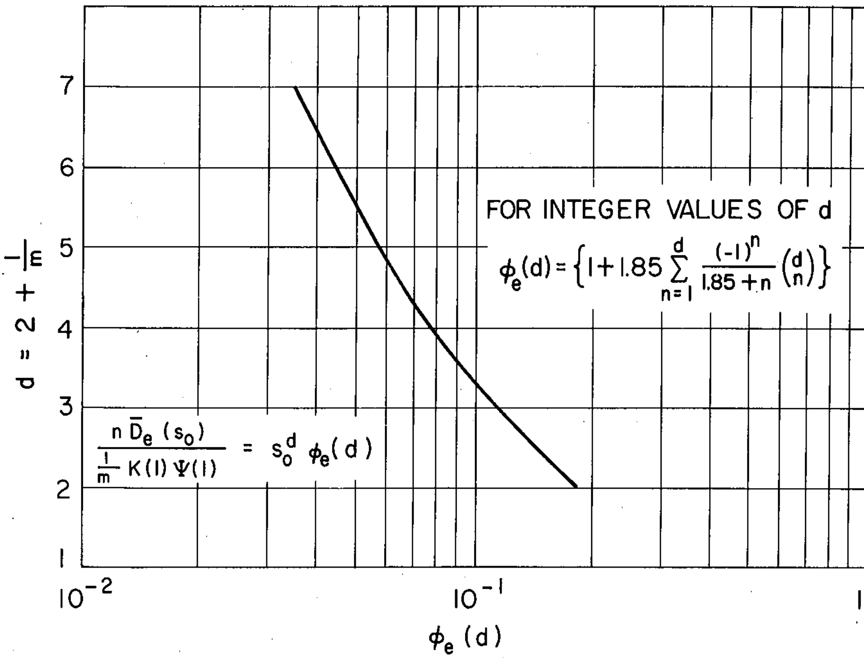


Figure 4. Weighted Mean Diffusion Coefficient-Desorption

This function is plotted in Fig. 5 along with its asymptotes and is the key to understanding water balance behavior in different climates. Looking at Fig. 5 and the definition of E, we see that as the precipitation events occur more frequently (increasing β); as the soil sorptivity increases (increasing ϕ_e); and/or as the potential rate of evaporation decreases (such as in a cold, moist climate), the parameter E increases. These conditions all indicate an increase in the relative evaporation and, as expected, we see the actual average annual evaporation approach the potential in the limit. The actual evaporation is thus controlled primarily by the *climate* for large E, through the potential rate of evaporation, and we describe it as being under *demand control.*

At the other extreme, where there are few rainstorms and the times between them are large; where the soil has low sorptivity and/or the rate of potential evaporation is large, the parameter E decreases. These conditions all indicate a decrease in relative evaporation. Here the actual evaporation is controlled by the availability of water through insufficient precipitation and/or through inability of the soil to bring moisture to the surface. The evaporation is thus *supply controlled.*

The intersection of the asymptotes of J(E) define

$$E_{critical} = 2/\pi \tag{16}$$

which may be used as a criterion for the classification of climate-bare soil systems as either supply controlled ($E < 2/\pi$) or demand controlled ($E > 2/\pi$) as far as relative evaporation is concerned.

The surface runoff function

$$\frac{E[R_{s_A}]}{E[P_A]} = e^{-G-2\sigma}\Gamma(\sigma+1)\sigma^{-\sigma} \tag{17}$$

is plotted in Figure 6 continuing the assumption that the surface retention is negligible. For small σ, the soil behaves as though wet and the average annual surface runoff approaches e^{-G} $E[P_A]$ in the limit. As σ increases, the soil becomes effectively drier and the average annual surface runoff becomes a decreasing fraction of the mean annual precipitation.

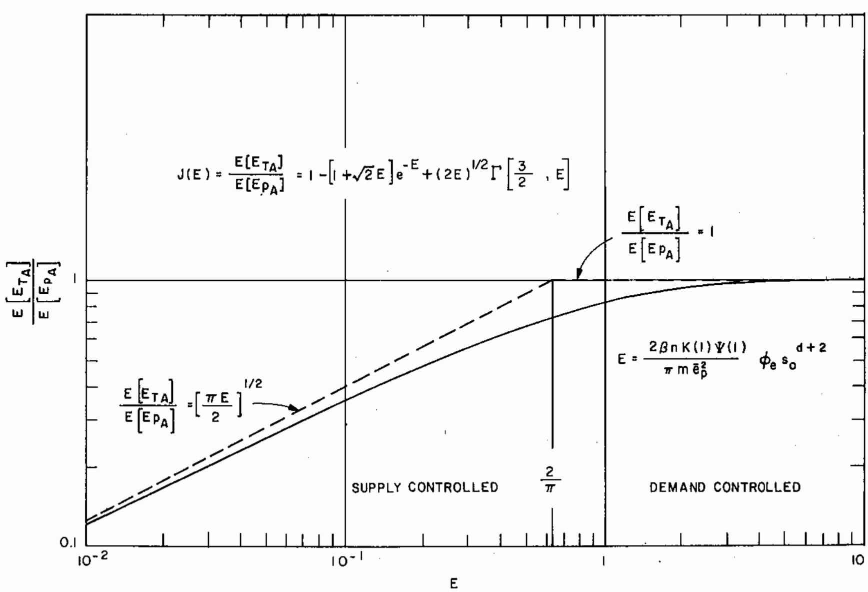


Figure 5. Bare Soil Evaporation Function ($w/\bar{e}_p << 1$)

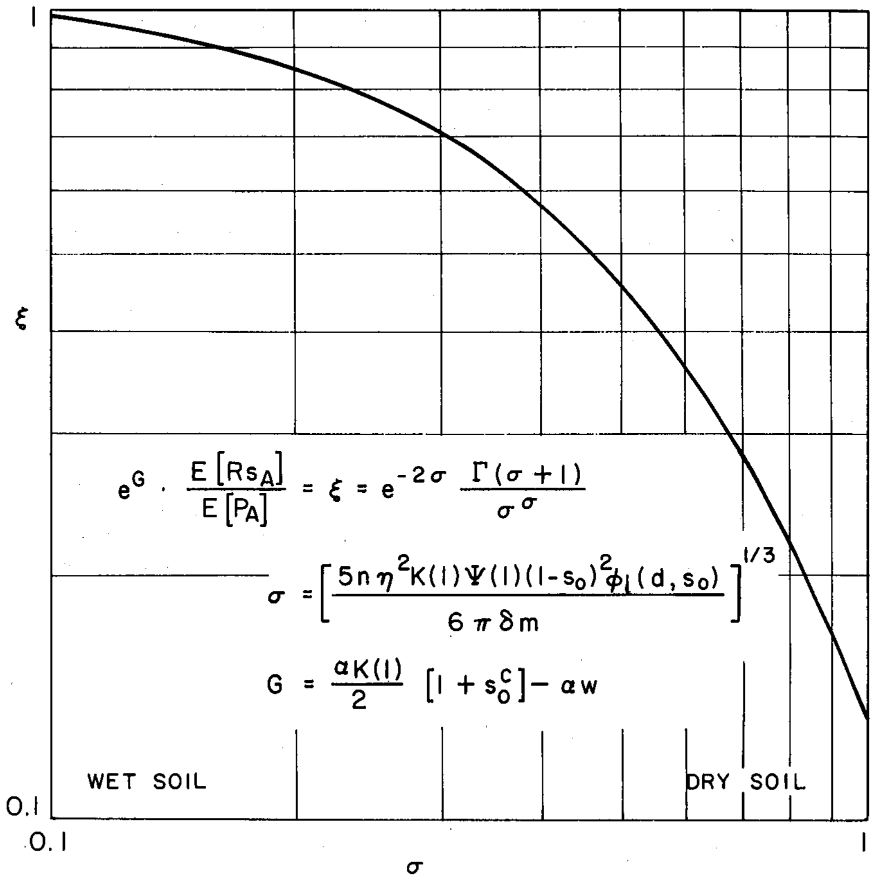


Figure 6. Surface Runoff Function ($h_o = 0$)

From Eqs. (6) and (17), the infiltration function is

$$\frac{E[I_A]}{E[P_A]} = 1 - e^{-G-2\sigma}\Gamma(\sigma + 1)\sigma^{-\sigma} \tag{18}$$

By definition, the groundwater runoff function is

$$\frac{E[R_{g_A}]}{E[P_A]} = \frac{m_\tau K(1)}{m_{P_A}} s_o{}^c - \frac{Tw}{m_{P_A}} \tag{19}$$

With Eqs. (15) - (19), the average annual water balance can be displayed graphically in a variety of ways. One of these is illustrated in Figure 7. Here the expected value notation has been omitted for convenience, the surface retention capacity, h_o, has been neglected, and the individual water balance components are sketched as a function of average annual precipitation *with everything else held constant.* For very large P_A, of course, E_{p_A} must decline (to zero in the limit) and E_{T_A} with it. The continuously rising groundwater component requires an unlimited lateral transmissivity if we are not to contend with a water table rising to the surface. Both of these very practical limitations are discussed elsewhere [7].

In the lower half of Figure 7, the sketched yield curve displays the commonly-observed linearity in the humid regions where the climatic moisture capacity (i.e., E_{p_A}) controls the losses, while for small P_A, the evaporation is limited by moisture supply.

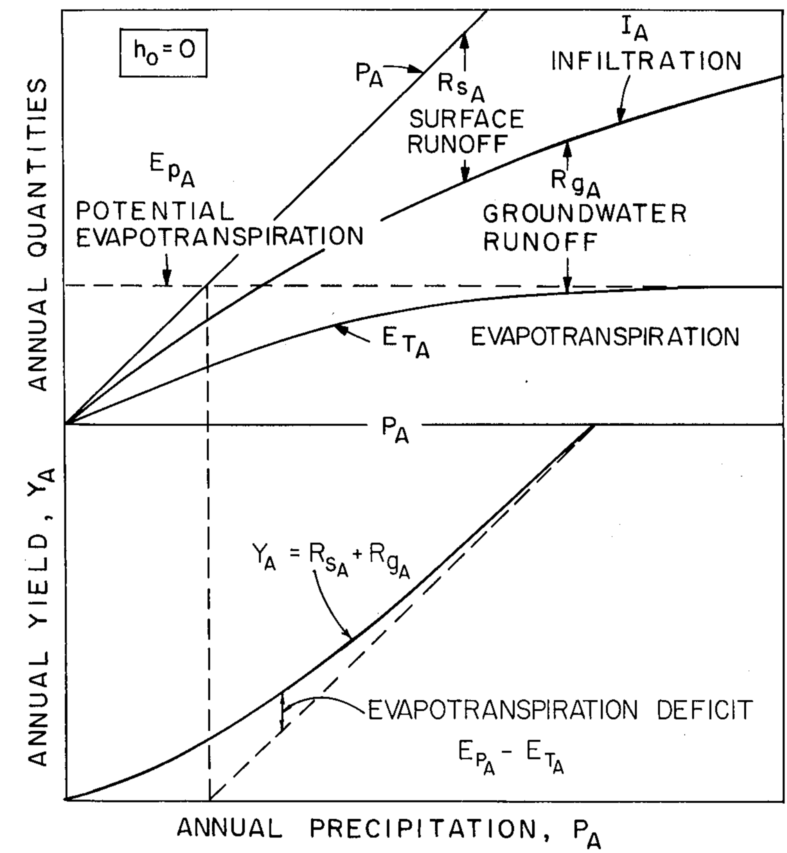
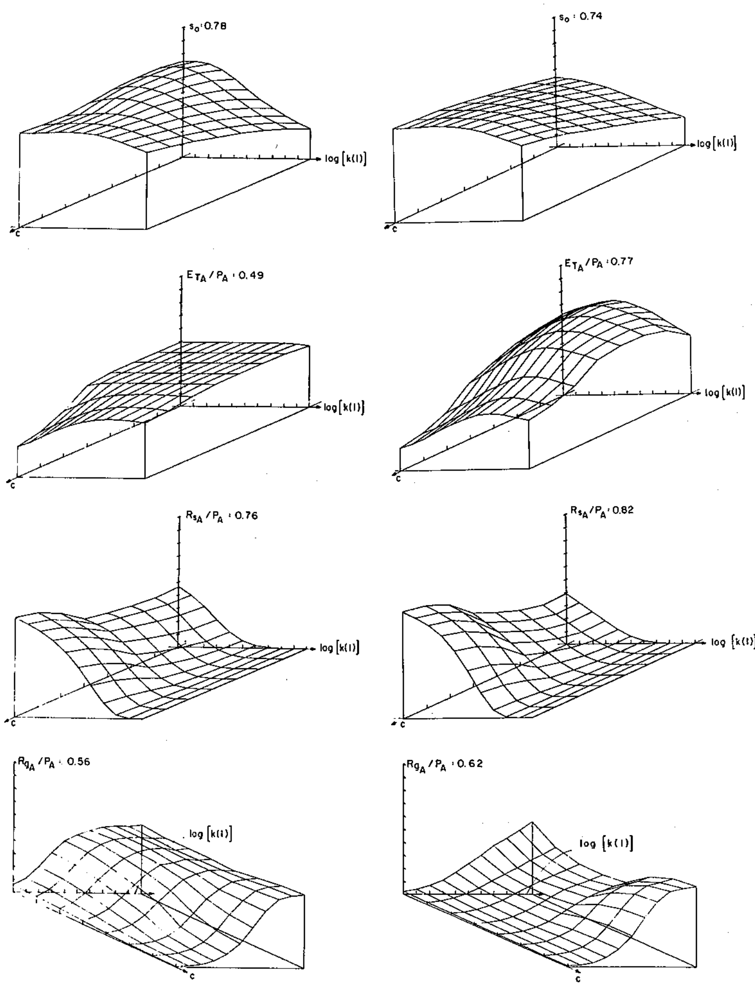


Figure 7. Climate Influence on the Annual Water Balance

Water Balance Sensitivity

Equation (10) and its components as given by Eqs. (15), (17) and (19) may be used to study the sensitivity of the water balance to variations in any of the climate or soil parameters. To illustrate this, Figure 8 is presented in which the average annual soil moisture, evapotranspiration and runoff components are presented as a function of the two primary soil properties, permeability and pore disconnectedness index, under each of two contrasting climates, one sub-humid and the other arid. The climatic properties are summarized in



a. SUB-HUMID CLIMATE (CLINTON, MA.)   b. ARID CLIMATE (SANTA PAULA, CA.)

Figure 8. Sensitivity of Annual Water Budget to Changes in Soil Parameters ($M = 0$, $h_o = 0$, $w/\bar{e}_p << 1$)

Table 1. In each case, the soil permeability was taken as $n = 0.35$. Once again in this figure, the expectation notation has been omitted and we continue the simplifications, $M = 0$, $h_o = 0$ and $w/\bar{e}_p << 1$). In interpreting the figures, we should remember that as log [k(1)] increases, the soil becomes intrinsically more permeable, while an increase in c indicates a greater pore disconnectedness and a less permeable soil. The numerical value of each ordinate is the maximum plotted value of that variable.

TABLE 1

CLIMATE PARAMETERS FOR SENSITIVITY ANALYSIS

PARAMETER	UNITS	LOCATION	
		Clinton, Ma.	Santa Paula, Ca
m_{P_A}	cm	111.3	54.4
$\bar{e}_p$	cm/day	0.15	0.27
m_{t_b}	days	3.0	10.4
m_{t_r}	days	0.32	1.4
m_τ	days	365	212
$\bar{T}_a$	°C	8.4	13.8
Z	m	∞	∞

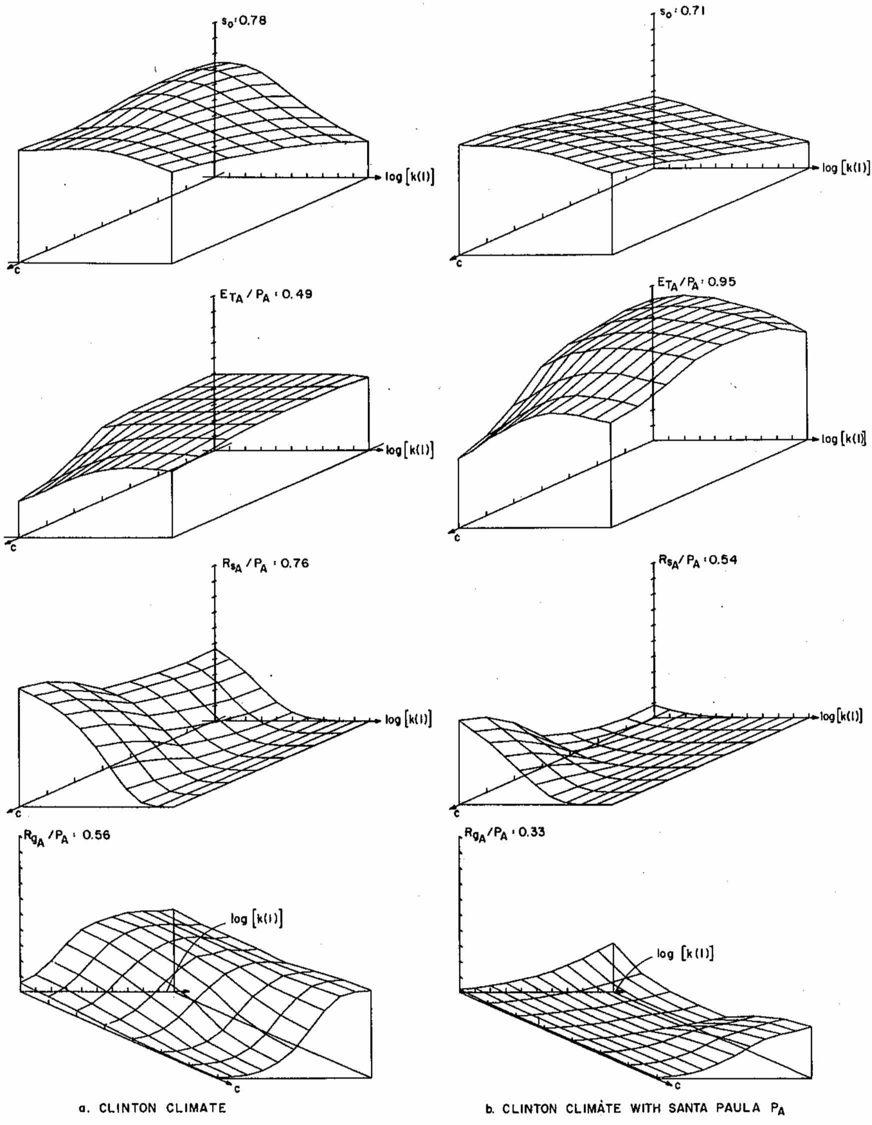
Comparing the two columns of Figure 8, we see contrasting behavior only in evapotranspiration and soil moisture. Beginning with the former, we see insensitivity of E_{T_A} to soil properties in the sub-humid climate except when the soil gets very impermeable. For the arid climate, however, E_{T_A} is sensitive to the soil poperties over their full range. This basic difference in behavior was pointed out earlier in discussion of Figure 5 and it allows us to understand why the soil moisture is highest, in the humid case, where c is small, and in the arid case, where c is large.

In the humid case, the supply of water is adequate and the soil moisture will be largest where the permeability readily admits water (*and* holds it against gravity). This requires a small σ which occurs for small k and large m (i.e., small c).

In the arid case where the evapotranspiration is controlled by the moisture supply to the surface, s_o will be largest where the moisture movement to the surface, as given by E, is smallest. This will occur for small k(1) and large d (i.e., large c).

The runoff behavior is qualitatively the same in both climates. For small k(1), the total yield is predominately surface runoff because the water cannot enter the soil. This component increases with c due to decreasing permeability and it decreases with increasing k(1) due to increasing permeability. The groundwater component also increases with k(1). The "saddle" in the Santa Paula groundwater component with increasing c results from the behavior of the factor s_o^c where s_o is less than one and is increasing with c.

A graphic demonstration of the importance of the potential rate of evaporation in determining climate-soil behavior is given in Figure 9. Here we have substituted the Santa Paula, $\bar{e}_p$, for the Clinton value in the Clinton climate parameter set. The result is to change the Clinton climate from sub-humid to arid wiping out the qualitative difference (just described) between the soil moisture and evapotranspiration sensitivities.



a. CLINTON CLIMATE

b. CLINTON CLIMATE WITH SANTA PAULA P_A

Figure 9. Effect on Annual Budget Due to Decreasing Mean Annual Precipitation

Similarity Parameters

Equation (10) defines the dependent dimensionless variable, s_o, in terms of a set of independent dimensionless variables having physical significance and being the similarity parameters for the average annual water balance. We identify these parameters as follows

Ξ	$= m_{P_A}/E[E_{P_A}]$ = Potential Humidity	(20)
c	= Pore Disconnectedness Index	(21)
$G(1)$	$= \alpha K(1)$ = Gravitational Infiltration Potential	(22)
Υ	$= w/K(1)$ = Index of Water Table Influence	(23)
$2\sigma(0)$	= Capillary Infiltration Effectiveness	(24)
$2E(1)$	= Exfiltration Effectiveness	(25)
Ω	$= m_\tau K(1)/m_{P_A}$ = Groundwater Recharge Potential	(26)
Λ	$= Tw/m_\tau K(1)$ = Groundwater Loss Index	(27)

The average annual water balance for a climate-bare soil system is thus defined in terms of 8 dimensionless parameters. One is a climate parameter (Ξ), one is a soil parameter (c), and the remaining 6 are climate soil parameters.

For the special case of negligible water table influence, two of the climate-soil parameters vanish (Υ and Λ), leaving a total parameter set of 6.

Interestingly, with the incorporation of vegetation (in situations which are in a natural equilibrium state at least) it appears necessary [7] to add only one additional similarity parameter:

$$k_V = \text{Potential Transpiration Efficiency} = \frac{\text{potential rate of transpiration}}{\text{potential rate of evaporation from bare soil surface}} \tag{28}$$

The intersections of the asymptotes of Eq. (15) and of Eq. (18) can be expressed in terms of $G(1)$, $\sigma(0)$, Ω, and $E(1)$. Along with the potential humidity, Ξ, these dimensionless intersections provide a rational means of climate classification [7].

First Order Analysis

When we have a function of two random variables

$$Y_A = g_1(P_A) \tag{29}$$

in which $g(P_A)$ is non-linear, we may expand $g(P_A)$ about the mean of P_A (i.e., m_{P_A}) in a Taylor series. Taking the expected value of this expansion term-by-term gives

$$E[Y_A] = g_1(m_{P_A}) + \frac{1}{2}\left[\frac{d^2g}{\partial P_A{}^2}\bigg|_{m_{P_A}}\right]\sigma_{P_A}{}^2 + \ldots \tag{30}$$

As long as both $\frac{d^2g}{\partial P_A{}^2}\bigg|_{m_{P_A}}$ and σ_{P_A} are small, we may represent Eq. (30) by the "first order approximation"

$$E[Y_A] = g_1(m_{P_A}) \tag{31}$$

The average annual water balance relation, Eq. (10) can be combined with Eqs. (17) and (19) to eliminate the soil moisture, s_o, from the latter equations. Equation (9) can then be written

$$E[Y_A] = g_2(m_{P_A}, E[E_{P_A}], m_\tau) \tag{32}$$

Assuming that all Y_A variability comes from P_A and none from E_{P_A} and/or τ, we can consider $g_2(m_{P_A})$ to be a first order approximation to $g_1(P_A)$.

We thus drop the expectation symbols in the *average annual* water balance equation to get a first order approximation to the *annual* water balance equation. Using the above notation, this gives, for annual yield, the monotonic function

$$Y_A = g_2(P_A) \tag{33}$$

Given the cumulative distribution function (cdf) of annual precipitation either from observed annual totals or as derived phenomenologically using storm observations [3], we can use Eq. (33) to derive the cdf of annual yield [8]

$$\text{Prob}[Y_A \leqslant z] = \text{Prob}[P_A \leqslant g_2{}^{-1}(z)] \tag{34}$$

This derived yield distribution will be a function of the physical properties of the climate and the soil as well as of the parameters of the precipitation probability distributions. It, therefore, provides a means for quantitatively assessing the effect of land surface change upon the stochastic structure of basin water yield.

The same derived distribution approach is applicable to other elements of the annual water balance.

Summary and Conclusions

The average annual one-dimensional water balance is expressed for natural surfaces in terms of physically-significant dimensionless parameters thereby providing the basis for dynamic similarity of the process and for an improved understanding of climate-soil-vegetation coupling.

A sensitivity analysis points out the critical importance of the potential rate of evaporation in defining water balance variations with other climate and soil parameters.

A first-order analysis of the average annual water balance gives an equation for the annual water balance which can be used to estimate the cumulative distribution functions (cdf) of the components of the annual water balance in terms of the cdf of the annual precipitation and of observable parameters of the physical system. This provides a rational basis for assessing the risk of physical changes to the land surface and for estimating the recurrence interval of such water balance components as basin yield.

Notation

SYMBOL	DEFINITION
A_s	short wave albedo of surface
C	dimensionless resistance coefficient
c	pore disconnectedness index
c_g	specific heat of soil-water system, cal g^{-1} °C^{-1}
d	diffusivity index
E	exfiltration parameter
E_{P_A}	annual potential evapotranspiration, cm
E_{T_A}	annual total evapotranspiration, cm
e_a	atmospheric vapor pressure, dynes cm^{-2}
e_p	potential (soil surface) evaporation rate, cm sec^{-1}
$\bar{e}_p$	time average potential evaporation rate, cm sec^{-1}
e_s	saturated atmospheric vapor pressure at ground temperature, dynes cm^{-2}
e_T	actual rate of evapotranspiration, cm min^{-1}
G	gravitational infiltration parameter
H	net rate of transfer of sensible heat, cal cm^{-2} min^{-1}
h_o	surface retention capacity, cm
I_A	annual infiltration, cm
i	precipitation rate, cm sec^{-1}
K(1)	saturated effective hydraulic conductivity, cm sec^{-1}
k(1)	saturated effective intrinsic permeability, cm^2
k_v	potential transpiration efficiency
L_e	latent heat of vaporization, cal g^{-1}
M_o	growth-equilibrium vegetated surface fraction
m	pore size-disconnectedness index
m_{P_A}	average annual precipitation, cm
m_τ	mean length of rainy season, days
n	effective medium porosity = volume of active voids/total volume
P_A	annual precipitation, cm

Notation (continued)

SYMBOL	DEFINITION
Q_i	net short wave solar radiation, cal cm^{-2} min^{-1}
q_a	rate of advected energy input, cal cm^{-2} min^{-1}
q_p	rate of use of energy in photosynthesis, cal cm^{-2} min^{-1}
R_{g_A}	annual groundwater runoff, cm
R_ℓ	net long wave terrestrial radiation, cal cm^{-2} min^{-1}
R_{s_A}	annual surface runoff, cm
$\dot{S}$	rate of moisture storage on surface and in saturated zone per unit of surface area, cm min^{-1}
s	effective saturation of medium (i.e., effective soil moisture concentration) = volume of water/volume of active voids
s_o	time and spatial average effective soil moisture concentration in surface "boundary layer"
T	one year, sec
T_a	atmospheric temperature, °C
T_g	surface temperature, °K
t	time, sec
t_a	storm interarrival time, days
t_b	time between storms, days
t_r	storm duration, days
V_a	wind velocity, cm min^{-1}
$\mathcal{V}_e$	interstorm evapotranspiration volume, cm
$\mathcal{V}_i$	storm infiltration volume, cm
w	upward apparent pore fluid velocity representing capillary rise from the water table, cm sec^{-1}
Y_A	annual water yield, cm
y	yield rate, cm sec^{-1}
Z	depth to water table, cm
Z_r	thickness of soil surface boundary layer, cm
z	value of water balance term
α	reciprocal of mean storm intensity, sec cm^{-1}
β	reciprocal of average time between storms, $days^{-1}$
δ	reciprocal of mean storm duration, $days^{-1}$
γ_w	specific weight of pore fluid, dynes cm^{-3}
μ_w	dynamic viscosity of pore fluid, poises
ρ_a	mass density of moist air, g cm^{-3}
ρ_s	mass density of soil-water system, g cm^{-3}
ρ_w	mass density of water, g cm^{-3}
σ	capillary infiltration parameter
σ_w	surface tension of pore fluid, dynes cm^{-1}
η	reciprocal of mean storm depth, cm^{-1}
ϕ_e	dimensionless exfiltration diffusivity
ϕ_i	dimensionless infiltration diffusivity
$\Psi(1)$	saturated soil moisture potential, cm (suction)
E[ ]	expected value of [ ]
g( )	functional notation
J( )	evapotranspiration function
Γ( )	Gamma function

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