STRATIFIED FLOW

By Donald R. F. Harleman*

(Presented as one of the John R. Freeman Lectures on Fundamental Hydraulic Processes in Water Resources Engineering, Boston Society of Civil Engineers, Fall, 1963.)

I. Introduction

Fluid motions in a gravitational field which are originated or influenced by variations in density within the fluid are characterized by the term stratified flow. In a strict interpretation all free surface liquid motions are stratified flows in the sense that the lower fluid (e.g., water) is overlain by a lighter fluid which is the earth's atmosphere. In this case the difference in fluid densities is so large (1 to 800) that the density and therefore the inertia effects of the upper fluid can be neglected in comparison with that of the lower. Hence, such problems are reduced to the analysis of a single homogeneous fluid under the category of free surface flow. In conventional usage the term stratified flow refers to motions involving fluid masses of the same phase. A heavier liquid flowing beneath a lighter liquid, or a heavier gas moving under a lighter gas, will be subject to gravitational effects which depend upon the difference between the two specific weights. The less dense fluid may then be regarded as if it were weightless and the more dense as if it were subjected to a reduced gravitational acceleration $g(\rho_2 - \rho_1)/\rho_2 = \frac{\Delta\gamma}{\rho_2}$, wherein the ratio $\Delta\gamma/\rho_2$ represents the effective gravitational acceleration which may vary from the limit g to zero.

Nature has been generous in providing mechanisms for bringing about changes in temperature and for providing solutions of dissolved solids and solid particle suspensions which give rise to variations in density and therefore specific weight. Many of the familiar meteorological phenomena, the motion of "cold fronts" in particular, are manifestations of stratified flow. In parallel with atmospheric phenomena, the mechanism of the vast ocean currents owes much of its complexity to the existence of stratified flows. In the realm of control and utilization of surface waters, broadly characterized by the field of water

* Professor of Civil Engineering, Hydrodynamics Laboratory, Massachusetts Institute of Technology, Cambridge 39, Massachusetts.

resources engineering, the internal motions in reservoirs due to temperature and the inflow of sediment laden streams and the understanding and control of salinity intrusion in tidal estuaries are among the most challenging of present day problems dealing with stratified flow.

There is little to be gained by an attempted enumeration of all of the possible natural phenomena in which the mechanics of stratified flow are involved. Many investigations have been directed toward analyses of highly specialized problems and their discussion is precluded by the space allotted to this topic. The emphasis will therefore be placed on some of the fundamental mechanics of stratified flow and on the presentation of certain applications to technological problems. A more complete summary of stratified flow has been given by Harleman (1).

For convenience of discussion, the subject matter may be divided into three broad categories:

1. *Two-layer systems.* Such systems are the simplest to treat analytically and in many cases they represent a reasonable approximation to physical problems.

2. *Multi-layered systems and continuous density gradients.* The more complex situation in which the fluid density varies continuously.

3. *Diffusion in stratified flow.* In addition to the convective motions associated with stratified flow, a non-convective or diffusive phenomena may be present when the flow is turbulent.

It is not surprising that most of the developments have been centered around the first of the three subdivisions. However, it is apparent that the state of knowledge in all three categories is expanding rapidly.

In the analysis of stratified flow, the fluids (whether liquid or gaseous) are considered as incompressible because of the inherently small magnitude of the velocities. The density differences are therefore due to temperature gradients, variations in solute concentrations or variations in suspended solid concentrations, and are independent of the pressure intensity and the elastic properties of the fluid.

II. Two-Layer Systems

Two-layer systems represent the simplest case of a stratified flow. In a stable flow the denser fluid always tends to occupy the lowest position; for purposes of analysis the interface is assumed to be a streamline separating the two fluids and coincident with the density discontinuity. The similarity with the familiar free surface flows can be demonstrated by considering the two-layer system, shown in Fig. 1, which might represent the downflow of cold air in a mountainous region or the spilling of salt water over a submerged barrier in an estuary. If both upper and lower fluids are assumed to be non-viscous, the interface also represents a discontinuity in velocity between the moving, lower layer and the stationary upper layer. For the moving,

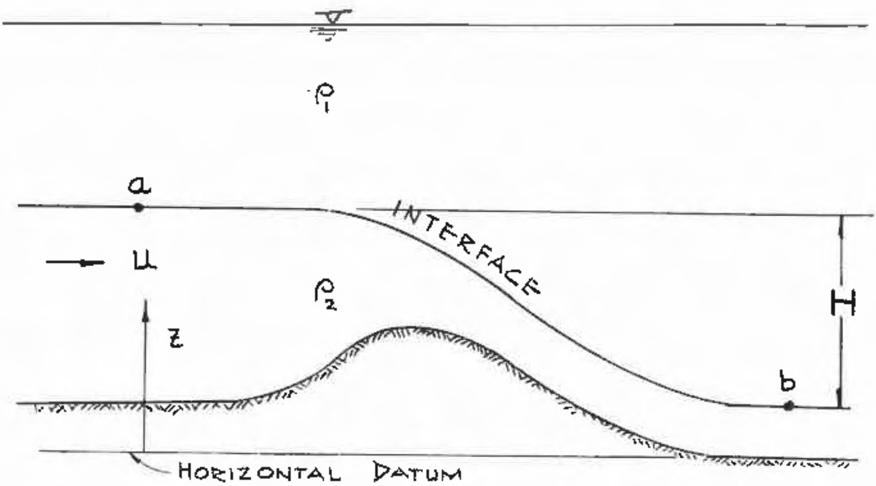


FIG. 1.—GRAVITY FLOW IN A TWO-LAYERED SYSTEM

lower layer fluid, between points a and b, on the interface the energy equation is:

$$p_a + \rho_2 \frac{u_a^2}{2} + \rho_2 g\, z_a = p_b + \rho_2 \frac{u_b^2}{2} + \rho_2 g\, z_b \qquad (1)$$

For the static upper layer, between the same interfacial points,

$$p_a + \rho_1\, g\, z_a = p_b + \rho_1\, g\, z_b \qquad (2)$$

Since the pressures are continuous across the interface, they may be eliminated from Eq. (1) by the above equation.

If, $H = z_a - z_b$ and $\Delta\rho = \rho_2 - \rho_1$, then:

$$\frac{u_b^2 - u_a^2}{2} = \frac{\Delta\rho}{\rho_2}\, g\, H \qquad (3)$$

For the case in which the lower fluid is a liquid and the upper fluid is air, $\Delta\rho \cong \rho_2$, and Eq. (3) becomes identical with the free surface equation for flow over a weir. Therefore, as stated in the introduction, the lower layer fluid may be considered as an analogous free surface flow if the gravitational acceleration is reduced by the factor $\frac{\Delta\rho}{\rho_2}$. For convenience, the reduced gravitational acceleration will be designated by a prime, $g\frac{\Delta\rho}{\rho_2} = g'$. It follows that the Froude number remains as the primary similitude parameter for the subsurface flows. In the generalized form it is called the *densimetric Froude number* and may be written as

$$F' = \frac{u}{\sqrt{g' z}} \tag{4}$$

The foregoing presentation, in emphasizing the similarity with familiar free surface flows, represents an important simplification in analysis which is very often useful in rapidly estimating orders of magnitude for velocities or depths of stratified flows. Nevertheless, as in many fluid mechanics problems, the effects of viscosity cannot always be ignored if more accurate answers are to be obtained. The differences between stratified and free surface flows therefore become important. For example, in stratified flows, resistance at the fluid interface in addition to that at the fixed boundaries must be considered since discontinuities in velocity cannot exist at an interface between real fluids. In the following subsections more detailed analyses of the mechanics of two-layer systems are presented.

2.1 *Uniform Flow in Two-Layer Systems*

A steady, uniform flow of a lower layer fluid will occur along an incline when the driving gravity force per unit area (due to the density difference between the two fluids) is in equilibrium with the shear stresses exerted by the fixed boundary and the moving interfacial boundary. These flows have been referred to by various names; density currents, gravity currents and underflows. Gravity currents, due to temperature differences have been observed in the T.V.A. system (2) where velocities of the underflow ranging from 0.15 to 0.35 ft./sec. have been measured. Numerous instances are known of the passage of underflows in reservoirs (3) where the density difference is due to suspended sediment.

If the depth of the upper fluid is assumed to be large compared to that of the lower fluid, the induced velocities in the upper layer may be neglected. The flow is shown schematically in Fig. 2. For two-dimensional flow (at a distance far enough from the origin of the current so

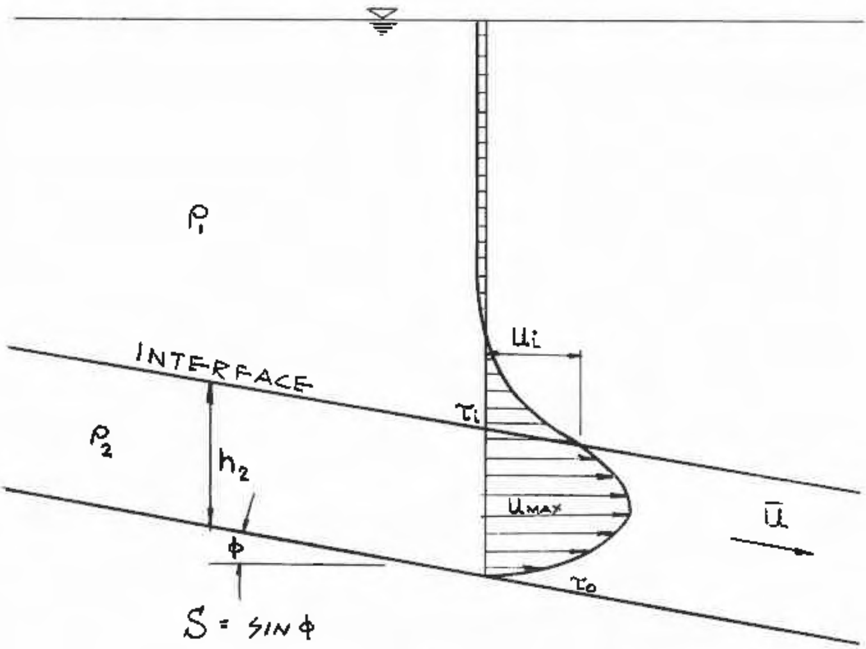


FIG. 2.—STEADY, UNIFORM FLOW IN LOWER LAYER FLUID

that the velocity distribution is fully developed) the equilibrium equation is

$$\tau_o + \tau_i = \Delta\rho \, g \, h_2 S \tag{5}$$

Under the assumption that the interface is smooth and distinct, use may be made of an analogy with flow between parallel boundaries in which the lower boundary is stationary and the upper boundary (interface) has a velocity u_i. The shear stress varies linearly from τ_o at the bottom, to zero at the point of maximum velocity, to τ_i at the interface. Letting $\tau_i = \alpha\tau_o$, the interfacial shear stress is a constant proportion of the bottom shear and depends only on the vertical location of the maximum velocity. In the notation of the shear distribution shown in Fig. 3

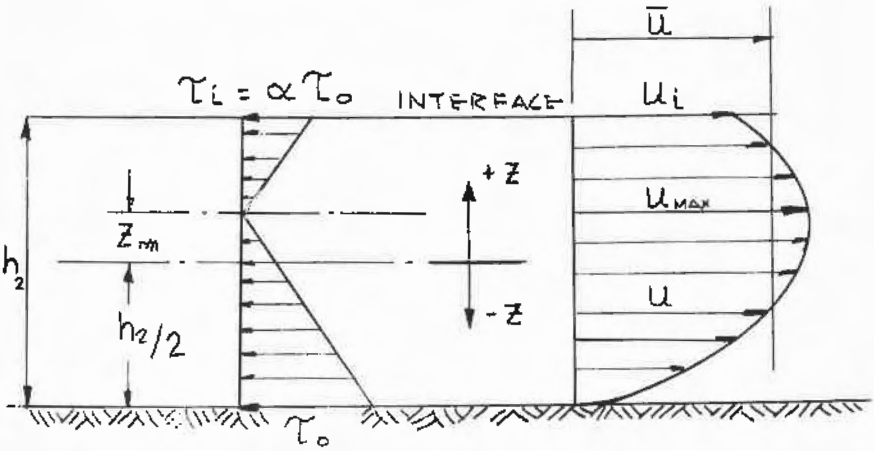


FIG. 3.—SHEAR AND VELOCITY DISTRIBUTION IN LOWER LAYER FLUID

$$\alpha = \frac{1 - 2\, z_m/h_2}{1 + 2\, z_m/h_2} \tag{6}$$

Eliminating τ_i, Eq. (5) may be written,

$$\tau_o = \Delta\rho g \left(\frac{h_2}{1+\alpha}\right) S \tag{7}$$

The shear stress τ_o may also be expressed in terms of the friction factor f,

$$\tau_o = \frac{f}{4} \rho_2 \frac{\bar{u}^2}{2} \tag{8}$$

Equating (7) and (8) and solving for $\bar{u}$, the average velocity of the lower layer becomes,

$$\bar{u} = \sqrt{8g' \frac{h_2 S}{f(1+\alpha)}} \tag{9}$$

Equation (9) is a generalized form of the uniform flow equation for open channels. For two-dimensional free surface flow, $g = g'$, $\alpha = 0$ and $h_2 = R_H$, hence, the Chezy equation

$$\bar{u} = C \sqrt{R_H S} \tag{10}$$

is obtained as a special case of Eq. (9).

a. *Laminar Flow*

For the case of laminar flow, the shear stress ratio can be found and the variation of the friction factor f with the Reynolds number can be determined analytically as shown by Ippen and Harleman (4) and (5). Eq. (9) can then be written as,

$$\bar{u} = 0.375\ R_2^{1/2}\ (g'h_2S)^{1/2} \tag{11}$$

where R_2 is the Reynolds number defined by $R_2 = \bar{u}h_2/\nu$.

Eq. (11) has been verified experimentally (4) for Reynolds numbers up to 1000 which can be regarded as the lower critical value for turbulent flow.

b. *Turbulent Flow*

In common with most turbulent resistance problems, the uniform underflow in turbulent motion is not subject to exact analysis. In addition, experimental observations are relatively meager and due to the difficulties of field measurements are largely confined to laboratory flumes. Eq. (9) provides a method of estimating orders of magnitude for velocities in turbulent flows. For two-dimensional flows the factor $(1 + \alpha)$ represents the amount by which the resistance coefficient, f, is increased by the presence of the interface, since $\alpha = 0$ for free surface flows. By analogy with the corresponding free surface flow, f may be obtained from the resistance diagram for flow in conduits (Moody diagram) using $4h_2$ as the hydraulic radius. The value of f thus obtained is to be increased by the factor $(1 + \alpha)$. Experimental velocity distributions by Bata and Bogich (6) in the lower layer turbulent flow indicate that, on the average, the maximum velocity occurs at 0.7 h_2 and therefore $\alpha = 0.43$ for turbulent flow. No systematic variation with Reynolds numbers was observed.

Any attempt to refine the analysis for turbulent flows is hindered by the fact that, as the degree of turbulence increases, the interface becomes increasingly difficult to define due to mixing and resulting density gradients.

c. *Sediment Transportation by Gravity Currents*

Reservoir underflows or density currents are caused by sediment-laden river water entering a lake as shown in Fig. 4. The conditions at the so-called plunge point, where the river inflow disappears beneath the surface, are difficult to define due to the intense local mixing. The inflow becomes diluted and the rapid reduction of velocities near the entrance is responsible for the deposition of the larger sediment particles and the formation of delta areas. Studies of gravity currents in reservoirs have shown that they consist primarily of particles in suspension less than 20 microns in diameter. The settling rate for particles of this diameter is approximately 0.001 ft./sec. Thus, turbulent fluctuations of the order of 1% in a current having a mean velocity of only 0.1 ft./sec. would be sufficient to keep such particles in suspension.

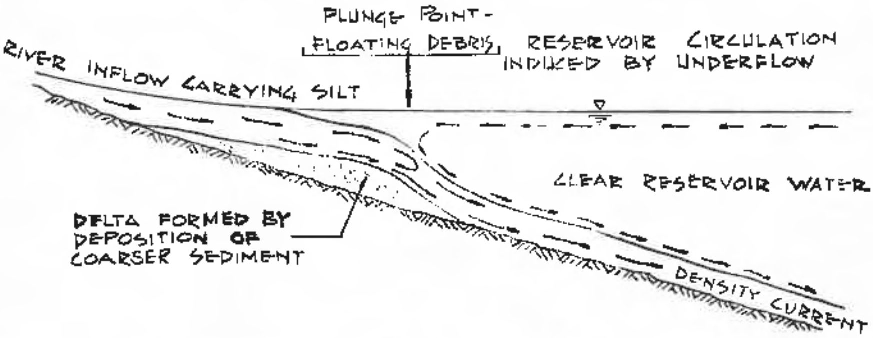


FIG. 4.—UPSTREAM END OF RESERVOIR SHOWING FORMATION OF DENSITY CURRENT

Lake Mead surveys (3) have indicated the existence of underflows with a density difference as small as $\Delta\rho/\rho = 0.0005$. If the average bottom slope is taken as 5 feet per mile ($S = 0.0009$) and measurements indicate a depth of 15 feet for the moving current, the order of magnitude of the underflow velocity can be obtained from Eq. (9).

Example:

Assume $f = 0.010$ and $\alpha = 0.43$ for turbulent flow. Eq. (9) $\bar{u} = \sqrt{\dfrac{8(0.0005)(32.2)(15)(0.0009)}{0.010\,(1.43)}} = 0.35$ ft./sec. The Reynolds number, for two-dimensional flow, is therefore

$$\frac{\bar{u}h_2}{\nu} = 5 \times 10^5$$

The friction factor f for this Reynolds number is within 10% of the assumed value and a further refinement of the calculation is not warranted. The computed velocity of 0.35 ft./sec. (6 miles per day) is consistent with actual measurements in Lake Mead.

The percent concentration of the sediment (by weight), P, is related to the density difference as follows,

$$\frac{P}{100} = \frac{\Delta\rho}{\rho} \cdot \frac{1}{1 - s_f/s_s}$$

where s_f and s_s are the specific gravities of the water and sediment respectively. For the above conditions $P = 0.08\%$ and the sediment discharge per unit width is,

$$q_s = \frac{0.08}{100} (0.35)\ 15 = 0.004 \text{ cfs/ft.}$$

which is equivalent to a sediment transport rate of 30 tons per day per foot of width of the underflow.

2.2 *Non-Uniform Flow*

A number of examples of the analysis of non-uniform motion in a two-layer system are discussed in reference (1) together with bibliographical sources. Space does not permit derivation of specific equations, however, Figs. 5 and 6 illustrate two interesting problems in this category.

Fig. 5 shows the arrested saline wedge near the ocean entrance of

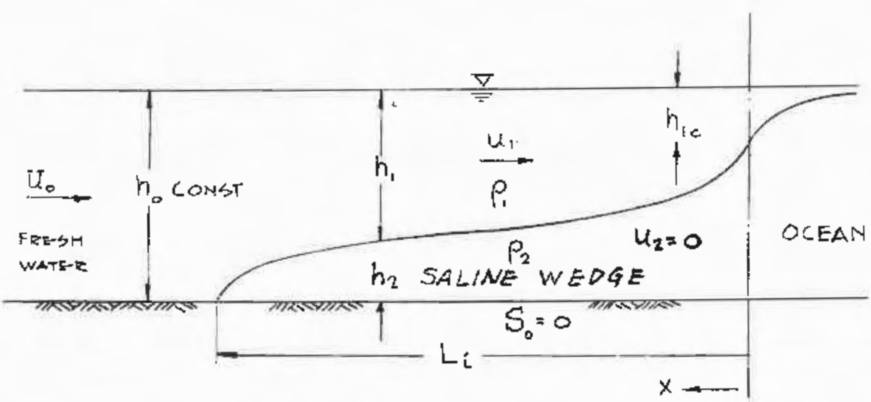
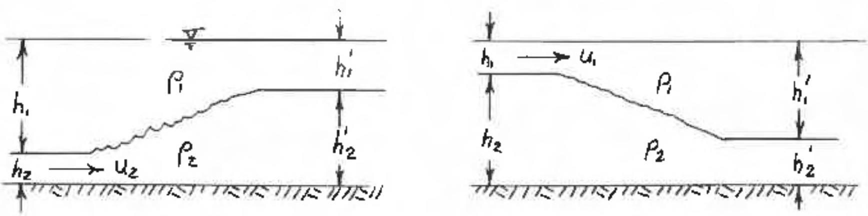


FIG. 5.—ARRESTED SALINE WEDGE NEAR THE OCEAN ENTRANCE OF A FRESH WATER CHANNEL.

a fresh water channel. Keulegan (7) has investigated the equilibrium position of the salt water wedge in laboratory flumes. The wedge type of salinity intrusion is relatively uncommon in actual estuaries. In most cases there is a longitudinal oscillation of the wedge due to tidal motion in the ocean which is sufficient to cause vertical mixing. This invalidates the assumption of a two-layer system and both horizontal and vertical diffusion must be considered. The saline wedge near the mouth of the Mississippi River probably comes closest to satisfying the conditions shown in Fig. 5.

Yih and Guha (8) have analyzed the hydraulic jump in a two-layer system. Both normal and inverted forms of the jump are possible, as shown in Fig. 6. The form of the jump depends on the relative initial



$$\frac{h_2'}{h_2} = \frac{1}{2}\left(\sqrt{1 + 8\,(F_2')} - 1\right) \qquad \frac{h_1'}{h_1} = \frac{1}{2}\left(\sqrt{1 + 8\,(F_1')^2} - 1\right)$$

Fig. 6.—Hydraulic Jumps in a Two-Layered System

depth and the layer which is in motion. The equations are identical with the free surface counterpart except for the use of the densimetric Froude number.

2.3 *Internal Wave Motion*

In a two-layer system of fluid of different density, waves at the interface are known as internal waves. Much of the basic theory was developed before 1900 (beginning with the work of Stokes in 1847) and represents the first analytical approach to problems of stratified flow. A number of examples of oscillatory and solitary internal wave motions are discussed in reference (1). One of the interesting problems in the category of unsteady, non-uniform flow in a two-layer system is the motion following the removal of a vertical dividing wall separating liquids of different densities, as shown in Fig. 7. In the case of a channel

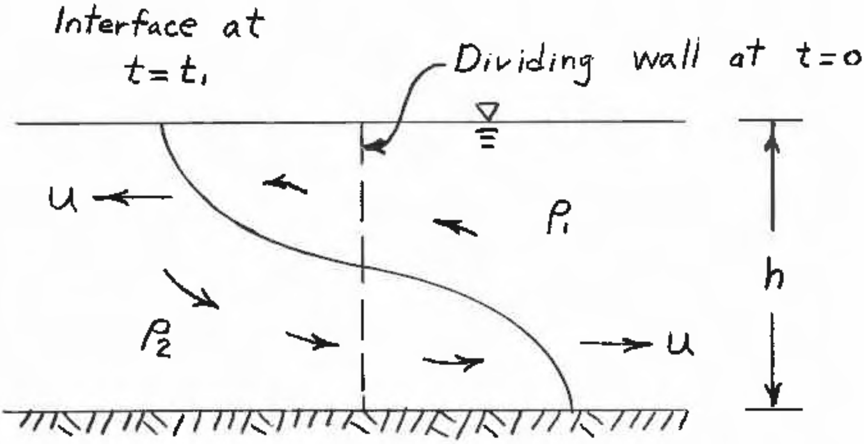


FIG. 7.—INTERNAL SURGE FOLLOWING REMOVAL OF A DIVIDING WALL

of uniform width (such as a lock between fresh and salt water) the initial velocity of the lower interface is

$$u = 0.45 \sqrt{g' h} \tag{12}$$

For surges arising from a tideless sea and entering a rectangular channel with the water initially at rest, the numerical constant in Eq. (12) was found by Keulegan (9) to be 0.57.

2.4 *Interfacial Mixing*

The development of internal wave motion contributes to internal mixing in stratified fluids through the mechanism of breaking of the internal waves. From dimensional reasoning a stability parameter has been defined,

$$\theta = \frac{\nu_2 g'}{u_2^3} \tag{13}$$

where u_2 is the velocity of the lower layer relative to the upper layer. For a two-dimensional flow, with h_2 taken as the characteristic length in the Reynolds number, the stability parameter is readily transformed into

$$\theta = \frac{1}{(F_2')^2 R_2} \tag{14}$$

On the basis of experiments at M.I.T. (4) and the Bureau of Standards (10), the critical values of θ are as follows:

$$\text{laminar flow (lower layer)} \quad \theta_c = \frac{1}{R_2}$$

$$\text{turbulent flow (lower layer)} \quad \theta_c = 0.18.$$

No mixing should occur for flows with θ values greater than θ_c. The transition to turbulent flow begins at a Reynolds number of approximately 1000 in the tests cited above.

An observer of the process of interfacial mixing is impressed by the extreme stability exhibited by adjacent layers with density differences as small as a few tenths of a percent. Even after the interfacial waves have begun to break, the amount of lower layer fluid entrained in the upper layer is relatively small. Ultimately this type of mixing process tends to form a new layer with a density intermediate to that of the original layers. In this way a three-layered system is produced. In all of the foregoing it is assumed that the heavier fluid is beneath the lighter inasmuch as the reverse case is unstable under all conditions.

2.5 *Selective Withdrawal of Fluid*

The ability to selectively withdraw fluid from a region in which the fluid density varies in the vertical direction has been brought about by an understanding of the mechanics of stratified flow. Density differences may occur due to temperature differentials, suspended sediment, dissolved salts or other chemicals. Consequently, a large number of technological fields are finding applications for selective withdrawal or some degree of control of stratified fluids.

Control structures are in use to provide cooler water at the condenser intakes of thermal power plants located near sources of water which are stratified due to temperature differences. Lakes or artificial reservoirs are used as large heat exchangers for nuclear power plants making it necessary to design intakes to withdraw the low level cool water and to distribute the inflow of warm water near the surface. The reduction of reservoir sedimentation by removal, at the dam, of water containing large amounts of suspended sediment has been suggested as a means of prolonging the useful life of major structures. In the area of density variations due to dissolved salts, the control of salinity intrusion by barriers and locks separating fresh water channels from an ocean or estuary may be mentioned.

A common feature of the applications described is the fact that in all cases the fluids are miscible, essentially incompressible, similar in viscosity and the density differences are small.

Craya (11) has treated the withdrawal of fluid from a two-layer system with a horizontal intake located on a vertical boundary (as in the case of the upstream face of a dam). The intake is located above the initially horizontal interface, Fig. 8(a). It is desired to determine

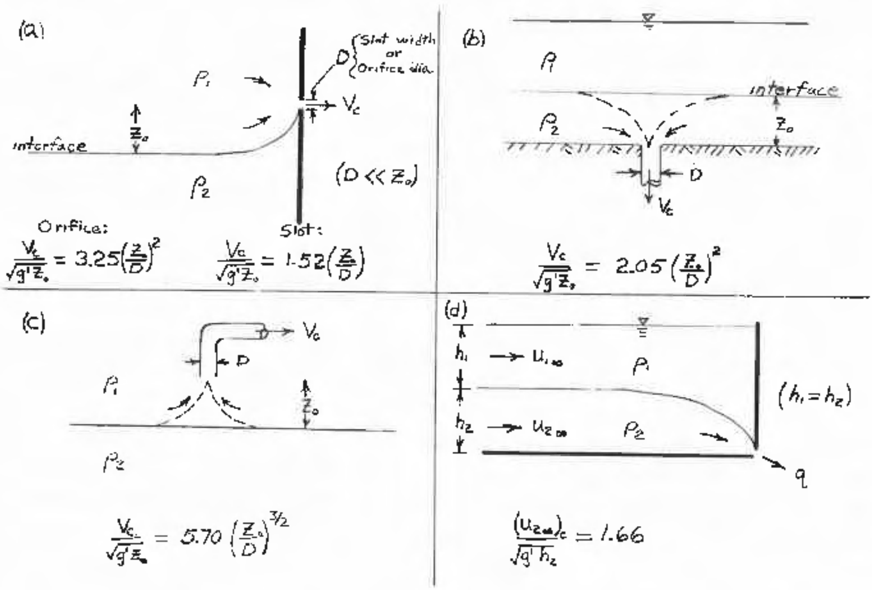


FIG. 8.—SELECTIVE WITHDRAWAL OF FLUID FOR VARIOUS BOUNDARY CONDITIONS IN TWO-LAYERED SYSTEMS.

the flow of the upper fluid necessary to raise the interface locally to the level of the intake at which time discharge from the lower layer begins. The efflux velocity at this critical condition is designated V_c. Intakes in the form of horizontal line sinks and three-dimensional point sinks are considered for the case in which the vertical extent of both the upper and lower layers is unlimited.

The critical efflux velocity equations for both the two- and three-dimensional conditions are shown in Fig. 8(a). It is assumed that the size of the slit or orifice is small in comparison with the height of the opening above the original interface (z). The equations have been substantiated by the experiments of Gariel (12) for the same boundary conditions.

For the case in which the lower layer is limited in vertical extent, Harleman et al. (13) have investigated, analytically and experimentally, the efflux from a vertical circular intake in the bottom boundary, as shown in Fig. 8(b). The maximum discharge (from the lower layer) without simultaneous withdrawal from the upper layer is determined. In addition to the efflux equation given in Fig. 8(b), the effect of reentrant intake geometries is discussed in the reference cited.

In a brief exploratory study, Rouse (14) has given some experimental results for a vertical axis, circular intake pipe withdrawing the lighter of two stratified fluids from a point above the interface. Both fluids are assumed to be of unlimited extent, as shown in Fig. 8(c). The efflux equation has been reduced to a form comparable to cases (a) and (b) of the same figure.

Huber (15) has obtained an analytical solution for the steady, two-dimensional flow induced by a line sink located at the bottom corner of a rectangular channel in which two fluid strata (of equal depth) extend to infinity in the $-x$ direction, Fig. 8(d). If the fluids are at rest the interface will be horizontal. As the strength of the sink is increased, the lower layer will start to flow until the interface is drawn down to the sink and the upper fluid begins to take part in the flow. The shape of the interface is determined by relaxation techniques. The critical densimetric Froude number for which the upper layer is stationary is $F_2' = 1.66$. As the flow from the slit is increased, the Froude number of the upper layer increases and that of the lower layer decreases. A point is reached, however, when this trend is changed and both discharges then increase together as the densimetric Froude numbers approach equality.

The efflux problem described above may be considered as a limiting case of a more general boundary configuration in which the vertical height of the slot is comparable to the depth of the lower layer. The boundary conditions shown in Fig. 9 have been investigated by Harleman and Goda (16) in connection with the design of condenser water intakes for thermal power plants. It is desired to determine, for a given gate opening b and interface elevation h_r, the limiting discharge for the colder stratum of water. Engineering applications of this type of control structure have been described by Elder (17). The prevention of recirculation of heated condenser water, which has been discharged into a river, has also been investigated by Harleman and Garrison (18).

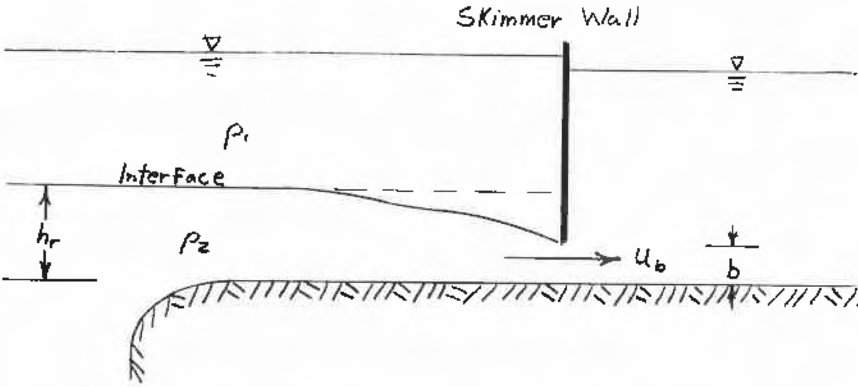


FIG. 9.—TYPE I SKIMMER WALL WITH VERTICAL INTAKE

III. CONTINUOUS DENSITY GRADIENTS

The state of knowledge in the case of multi-layered, stratified fluids and fluids with continuous density gradients is much less well developed than in the two-layer systems. One interesting aspect is the stiffening of the fluid against vertical displacements when a vertical density gradient is present. This can be shown to have applications to internal currents and circulation patterns in large bodies of water having a vertical temperature gradient. The stiffening phenomena can be demonstrated [Yih (19)] by considering very weak motions in a non-viscous fluid. For a steady flow, neglecting convective accelerations and viscosity, the Navier-Stokes equations [see Eagleson, Eq. (2.22)]* reduce to the following:

$$\frac{\partial p}{\partial x} = 0$$

$$\frac{\partial p}{\partial y} = 0 \qquad (15)$$

$$\frac{\partial p}{\partial z} + \gamma = 0$$

The continuity equation [see Eagleson, Eq. (1.31)]* is:

$$u\frac{\partial \rho}{\partial x} + v\frac{\partial \rho}{\partial y} + w\frac{\partial \rho}{\partial z} + \rho\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) = 0 \tag{16}$$

since the fluid is incompressible,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

and if the density gradient is confined to the vertical (z) direction,

$$\frac{\partial \rho}{\partial x} = \frac{\partial \rho}{\partial y} = 0$$

hence, from Eq. (16),

$$w\frac{\partial \rho}{\partial z} = 0$$

Since $\frac{\partial \rho}{\partial z}$ is not equal to zero, it follows that the vertical velocity component,

$$w = 0.$$

Therefore, motion must be confined to the horizontal (x,y) plane, since,

$$u \neq 0$$
$$v \neq 0.$$

Water withdrawn at the face of a dam in a reservoir having a vertical temperature distribution will therefore tend to come from a horizontal layer of restricted vertical extent. In a reservoir of homogeneous density, the water is withdrawn in essentially equal amounts from all depths. This has an important bearing on the quality of reservoir water releases. The above phenomenon has been demonstrated on a laboratory scale by Debler (20) and in the field by studies at Fontana Dam (21).

This condensed outline of some of the engineering implications of stratified flow has necessarily omitted many important references. In particular, the area of diffusion in stratified flow includes a wide range of technical problems including air pollution, ocean outfalls, pollution and salinity intrusion under conditions of mixing in tidal estuaries.

* Eagleson, P. E., "Some Fundamental Concepts of Incompressible Fluid Mechanics," this Journal, January, 1965.

References

1. Harleman, D. R. F. Stratified Flow, Ch. 26, *Handbook of Fluid Dynamics*, (V. Streeter Ed.) McGraw-Hill, 1961.
2. Fry, A. S., Churchill, M. A. and Elder, R. A. "Significant Effects of Density Currents in TVA's Integrated Reservoir and River System," *Proc. Minnesota Int. Hyd. Convention, I.A.H.R.*, pp. 335-354, Sept. 1953.
3. Howard, C. S. "Density Currents in Lake Mead," *Proc. Minnesota Int. Hyd. Convention, I.A.H.R.*, pp. 355-368, Sept. 1953.
4. Ippen, A. T. and Harleman, D. R. F. "Steady-State Characteristics of Subsurface Flow," *Nat'l Bur. Standards Circ.* 521, pp. 79-93, 1952.
5. Harleman, D. R. F. "Sediment Transportation Mechanics: Density Currents," *Proc. ASCE*, Vol. 89, No. HY5, pp. 77-87, Sept. 1963.
6. Bata, G. L. and Bogich, K. "Some Observations on Density Currents in the Laboratory and in the Field," *Proc. Minnesota Int. Hyd. Convention, I.A.H.R.*, pp. 387-400, Sept. 1953.
7. Keulegan, G. H. "Form Characteristics of Arrested Saline Wedge," *Nat'l Bur. Standards Report* 5482, Oct. 1957.
8. Yih, C. S. and Guha, C. R. "Hydraulic Jump in a Fluid System of Two Layers," *Tellus*, Vol. 7, No. 3, pp. 358-365, 1955.
9. Keulegan, G. H. "The Motion of Saline Fronts in Still Water," *Nat'l Bur. Standards Report* 5831, April 1958.
10. Keulegan, G. H. "Interfacial Instability and Mixing in Stratified Flows," *Nat'l Bur. Standards* Research Paper 2040, Vol. 43, Nov. 1949.
11. Craya, A. "Theoretical Research on the Flow of Non-Homogeneous Fluids," La Houille Blanche, pp. 44-55, Jan./Feb. 1949.
12. Gariel, P. "Experimental Research on the Flow of Non-Homogeneous Fluids," La Houille Blanche, pp. 56-64, Jan./Feb. 1949.
13. Harleman, D. R. F., Morgan, R. L. and Purple, R. A. "Selective Withdrawal from a Vertically Stratified Fluid," *Int. Assoc. for Hydr. Res.*, 8th Congress, Montreal, Aug. 1959.
14. Rouse, H. "Seven Exploratory Studies in Hydraulics," *Proc. ASCE*, Vol. 82, No. HY4, Aug. 1956.
15. Huber, D. G. "Irrotational Motion of Two Fluid Strata Towards a Line Sink," *Journal Eng. Mech. Division, ASCE*, 1960.
16. Harleman, D. R. F. and Goda, Y. "Control Structures in Stratified Flows." MIT Hydrodynamics Lab. Tech. Report No. 54, May 1962.
17. Elder, R. A. and Dougherty, G. B. "Thermal Density Underflow Diversion, Kingston Steam Plant," *Proc. ASCE*, Vol. 84, No. HY2, Part 1, April 1958.
18. Harleman, D. R. F. and Garrison, J. M. "The Effect of Intake Design on Condenser Water Recirculation." MIT Hydrodynamics Lab. Tech. Report No. 56, August 1962.
19. Yih, C. S. "On the Flow of a Stratified Fluid," *3rd U.S. Nat'l Congress of Appl. Mech.*, Brown Univ., 1958.
20. Debler, W. R. "Stratified Flow Into a Line Sink," *Proc. ASCE*, Vol. 85, No. EM3, Part 1, July 1959.
21. Harleman, D. R. F., Elder, R. E. and Duncan, W. "Internal Density Currents Created by Withdrawal from a Stratified Reservoir." TVA Eng. Laboratory, Norris, Tenn., Feb. 1962.